Computers & Industrial Engineering 197 (2024) 110622

Contents lists available at ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

Enhancing group decision-making: Maximum consensus aggregation for fuzzy cross-efficiency under hesitant fuzzy linguistic information ☆

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ARTICLE INFO

Keywords:
Group decision-making
Hesitant fuzzy linguistic term set
Fuzzy envelope
Fuzzy DEA cross-efficiency
Maximum consensus

ABSTRACT

Group decision-making (GDM) is essential as it recognizes the inherent complexity of many decision scenarios, which frequently require the collective wisdom and knowledge of multiple decision-makers (DMs) to be effectively resolved. The proposed method aims to develop fuzzy data envelopment analysis (DEA) cross-efficiency models tailored to address GDM challenges, wherein attribute values are provided by DMs using hesitant fuzzy linguistic term sets (HFLTSs). For this purpose, we initially transform HFLTSs into their corresponding fuzzy envelopes, defined as trapezoidal fuzzy numbers (TrFNs). This conversion strategy effectively minimizes the loss in assessments based on HFLTSs while retaining the inherent ambiguity of the original information. Building upon this foundation, we develop fuzzy cross-efficiency models by leveraging the α-level sets of fuzzy envelopes. These models are designed to handle fuzzy input and output variables under various α-level sets, which are capable of considering all possible attribute values for each alternative. Following this, we implement a maximum consensus model using fuzzy cross-efficiency to assign weights to DMs. These weights facilitate the aggregation of individual fuzzy cross-efficiency intervals obtained from DMs' assessments into collective ones, which serve to rank alternatives. Finally, we showcase the effectiveness and superiority of our proposal through numerical validation and comparative analysis.

1. Introduction

Group decision-making (GDM) (Hwang & Lin, 2012) is a process where multiple decision-makers (DMs), often with different expertise, work together to solve problems or make decisions. This approach is used in various fields like advertising (Akram et al., 2021), supplier selection (Xing et al., 2022), and supply chain management (Carrera et al., 2020). By gathering the knowledge of several DMs, GDM leads to more effective and comprehensive decisions than those made individually. In many cases, DMs must evaluate options based on attributes that cannot always be measured numerically (García-Zamora et al., 2024), such as product quality or after-sales service. These qualitative factors add complexity and subjectivity to the decision-making process, where linguistic information often reflects DMs' thoughts more accurately. When faced with uncertainty, DMs may hesitate between several linguistic terms. To capture this hesitation, the hesitant fuzzy linguistic term set (HFLTS) (Rodríguez et al., 2011) was proposed, allowing DMs to express varying degrees of uncertainty in their choices. A GDM model was later developed to use comparative linguistic expressions (CLEs) (Rodríguez et al., 2013), which let DMs incorporate linguistic preferences more flexibly. Many GDM methods convert linguistic information into precise numerical values, reducing ambiguity. To better preserve the inherent fuzziness of linguistic terms, Liu and Rodríguez (2014) developed the transformed method which can obtain the fuzzy envelopes of HFLTSs, further the fuzzy envelopes represented as corresponding trapezoidal fuzzy numbers (TrFNs). This method ensures minimal information loss while maintaining computational efficiency, providing a solid foundation for future GDM methods.

The literature on GDM offers valuable methods and insights to address the complexities of decision-making processes. The framework of existing GDM methods encompasses, but is not limited to, the following key research aspects: (1) Expression of individual preferences. Preferences are typically expressed through pairwise comparisons of

☆ The work was partly supported by the National Natural Science Foundation of China (Project No. 72371077), Young Project of the Social Science Planning of Fujian Province (Project No. FJ2024C075, FJ2023BF036) and Young Project of the Natural Science Foundation of Fujian Province (No. 2024J08286).

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https://doi.org/10.1016/j.cie.2024.110622
Received 3 May 2024; Received in revised form 25 September 2024; Accepted 30 September 2024
Available online 4 October 2024

alternatives or evaluation matrices under multiple attributes. For example, Wan et al. (2024) proposed a fuzzy best–worst GDM method based on intuitionistic fuzzy preference relations, using a linear goal programming model to calculate priority weights and maintain additive comparison consistency. Liu et al. (2016) introduced a multi-attribute GDM method to help firms select the best cloud computing vendor by considering both objective and subjective factors. (2) Group consensus reaching process. Consensus models aim to harmonize differing preferences among DMs to ensure the final decision is widely accepted. Research has shown that optimization-based consensus models are more efficient than identification and direction rules (Zhang, Dong et al., 2019). In this context, Zhang, Kou et al. (2019) developed soft cost consensus models that assess costs and opinions at various consensus levels, even analyzing economic impacts. To handle trust issues in consensus, Ji et al. (2024) proposed a framework that manages decayed trust while balancing the validity and acceptability of recommendations. (3) Aggregation of evaluation information. This process typically involves assigning weights to individual opinions and then combining them into a collective decision. Methods for determining weights include similarity-based approaches, consensus and consistency-based techniques, and clustering methods (Koksalmis & Kabak, 2019). Additionally, aggregation operators (Ashraf & Abdullah, 2019) and classical decision methods (Grošelj et al., 2015) have been used in this process. (4) Selection of the optimal alternative. After group opinions are evaluated and ranked using specific decision-making techniques (Triantaphyllou et al., 2020), the best alternative is selected based on these rankings.

Some traditional decision-making methods have been applied to GDM, enhancing its scientific rigor to some extent (Grošelj et al., 2015; Pang et al., 2016). However, these methods often rely on prior information, such as attribute weights or pairwise comparison matrices, which can compromise objectivity. In contrast, data envelopment analysis (DEA) (Charnes et al., 1978), a non-parametric method, addresses GDM challenges without requiring such prior information. In DEA-based GDM, alternatives are treated as decision-making units (DMUs), with cost attributes as inputs and benefit attributes as outputs. The efficiency of each alternative is quantified by calculating the ratio of the weighted sum of evaluation values under benefit attributes to the weighted sum of evaluation values under cost attributes (Song et al., 2023). In related research, Wang and Chin (2009) proposed a DEA method to avoid illogical priorities in GDM, while Liu et al. (2019) introduced an integrated approach using interval fuzzy preference relations, DEA, and stochastic simulation to handle uncertainty. Later, Liu et al. (2021) developed a GDM method based on DEA cross-efficiency and hesitant fuzzy preference relations to improve decision accuracy. Recently, Song et al. (2023) created a DEA-based GDM framework that addresses multi-granular hesitant linguistic information, accounting for non-rational behavior in GDM. However, many DEA-based methods simplify fuzzy information into precise numbers, leading to information loss. To address this, Huang and Wang (2024) developed a fuzzy DEA cross-efficiency model that preserves ambiguity by transforming information into intervals. Despite this advancement, the model's reliance on the CCR approach can result in multiple sets of optimal attribute weights, highlighting the need for new fuzzy DEA models to better handle GDM in fuzzy environments.

The GDM framework involves two crucial phases form above analysis: aggregation and exploration. In the aggregation phase, the evaluations from all DMs are collected, and assigning appropriate weights to each DM is essential, as these weights reflect their expertise and significantly influence the final decision. The exploration phase then involves calculating the overall evaluations of alternatives and determining their rankings. In fuzzy cross-efficiency models for GDM, objectively determining DM weights is crucial, as these weights are used to aggregate cross-efficiency scores from various evaluations, which directly influence the final choice of alternatives, making the determination of convincing weights essential for informed and effective decision-making. However, methods specifically designed to determine expert weights in cross-efficiency aggregation are limited. Most studies just rely on decision matrices provided by DMs. For instance, Geng et al. (2017) used a least squares optimization model to assign weights by comparing each DM's evaluation to an ideal decision, while Liu et al. (2021) introduced a linear programming model to minimize differences between individual and group preferences. Additionally, Song et al. (2023) used a stochastic neutral cross-efficiency model to rank alternatives by considering expert evaluations. Despite these efforts, many DEA-based GDM methods fail to consider the importance of group consensus when aggregating cross-efficiency scores. This oversight can lead to comprehensive evaluations that are not fully accepted by all DMs, resulting in decisions that lack collective agreement and may not be entirely supported by the group.

Building upon prior research findings, although many DEA models have been proposed in the domain of GDM, there remain several issues that still need either resolution or significant improvements.

(1) The processing of evaluation information is a crucial step in developing GDM methods based on DEA. Some studies have derived preference relations by comparing alternatives using numerical values (Lin & Wang, 2019) or provided crisp values for alternatives across multiple attributes (Liu et al., 2022), and then applied DEA models to determine the optimal solution based on these preference relations. However, in practice, many decision-making situations cannot be accurately measured using precise numerical values, and DMs often rely on linguistic terms to express their judgments in such cases (Dutta et al., 2024). Among DEA models utilizing linguistic information, most studies (Jin et al., 2022) focus on converting linguistic information into precise numerical values before developing corresponding DEA models to evaluate alternative performance. However, these transformation methods often result in significant loss of original linguistic information and fail to capture the inherent fuzziness of GDM.

(2) Some existing linguistic-based DEA models often solely emphasize self-evaluation, allowing each DMU to optimize multipliers to enhance its performance (Geng et al., 2017). However, such models may not sufficiently distinguish between all DMUs, resulting in irrational rankings due to the variation in multipliers, which could lead to biased weightings. Cross-efficiency, therefore, offers an effective solution by combining self-evaluation with peer-evaluation. Nonetheless, in current fuzzy cross-efficiency DEA frameworks in GDM, the handling of fuzzy variables typically involves only the endpoints, meaning that the identification of optimal and worst cross-efficiency generally relies on the upper and lower bounds of the input and output data (Liu & Chen, 2022).

(3) For the aggregation in GDM, two strategies exist for aggregating individuals' opinions. One aggregates data first, then calculates collective results; Another calculates individual results first and then aggregates them. Among the current DEA-based GDM models, some research (Liu et al., 2022; Song et al., 2023) focus on the first strategy, which aggregates evaluation data from diverse experts first to obtain a comprehensive assessment, this is followed by computing the efficiency of alternatives and ultimately selecting the optimal one. However, such aggregation for DEA-based GDM typically results in only collective decision outcomes, leaving the preferences of each DM regarding the alternatives unknown. Conversely, aggregating at the efficiency level rectifies this drawback (Kao & Liu, 2022). Furthermore, in aggregating the perspectives of each DM, it is crucial to determine the weight of each individual to account for their differing degrees of significance, rather than indiscriminately treating all DMs as equally influential.

Given previous limitations, our proposal aims to introduce the fuzzy cross-efficiency models based on the transformation of hesitant fuzzy linguistic information and the determination of the weights of individuals for aggregation. The main contributions can be outlined as follows:

(1) We employ the fuzzy envelopes for HFLTSs to derive TrFNs-based representations of hesitant fuzzy linguistic assessments, effectively mitigating the loss of original linguistic information. These transformation rules capitalize on the computational simplicity of numerical values while retaining the inherent fuzziness of the evaluation data within the framework of fuzzy numbers.

(2) We apply the α level sets derive the intervals of TrFNs-based assessments under different attributes and develop fuzzy DEA cross-efficiency models based on these intervals. These models are instrumental in determining the optimal and worst cross-efficiency for alternatives, thus establishing crucial cross-efficiency intervals for assessing the performance of available alternatives. By treating inputs and outputs as fuzzy variables, the model thoroughly considers all possible attribute values within the intervals of alternatives across various α-levels, thereby offering a more nuanced evaluation framework.

(3) We calculate the weights of DMs by constructing a maximum consensus model based on individual cross-efficiency intervals. These optimal weights play a crucial role in aggregating the individual cross-efficiency intervals obtained from each DM's assessments, leading to collective cross-efficiency intervals. These aggregated intervals are more broadly acceptable to all DMs, facilitating the ranking of alternatives.

The remainder of this research is structured as follows: Section 2 presents key concepts and models relevant to the proposed method. In Section 3, we delve into the proposed framework, encompassing the formulation of the GDM problem, the transformation of HFLTSs into TrFNs through the utilization of fuzzy envelopes method, the development of fuzzy cross-efficiency based on α-levels of fuzzy envelopes, the determination of DMs' weights by a maximum consensus model on cross-efficiency levels, and the ranking of alternatives based on collective fuzzy cross-efficiency. Section 4 provides a numerical example and comparative analysis to confirm the effectiveness and superiority of our proposed method.

2. Preliminaries

This section focuses on reviewing a range of related concepts and models used in our proposal, including the concept of HFLTS, fuzzy envelopes for HFLTSs, conventional cross-efficiency and fuzzy DEA models.

2.1. Hesitant fuzzy linguistic term sets

In decision-making environments characterized by complexity and uncertainty, a singular linguistic term falls short of accurately reflecting DMs' cognition of the subject evaluated, leading to hesitation among several linguistic expressions. To tackle this challenge and increase the applicability of linguistics in contexts of uncertainty, Rodríguez et al. (2011) introduced the definition of HFLTS, which is outlined as follows:

Definition 1. Consider a linguistic term set represented as $S = \{s_0, \dots, s_g\}$. An HFLTS, denoted as H, on S is defined as an ordered finite subset containing consecutive linguistic terms form S:

$$H = \{s_l, s_{l+1}, \dots, s_{l+z}\}, \text{ where } s_h \in S, h \in \{l, l+1, \dots, l+z\}$$

Comparative linguistic expressions (CLEs) are formulated using context-free grammar G_H and serve as a universal language employed by DMs in addressing real-world decision-making scenarios. However, these expressions cannot be directly utilized in executing computing with words (CWW) processes. Therefore, Rodríguez et al. (2013) established a conversion function to transform CLEs into HFLTSs.

Definition 2 (*Rodríguez et al., 2013*). Consider G_H as the context-free grammar and $S = \{s_0, \dots, s_g\}$ as a set of linguistic terms. The CLEs generated by G_H are transformed into HFLTSs through the transformation function E_{G_H}. E_{G_H} involves mapping CLEs produced by G_H to the equivalent representation in the form of HFLTSs, which can be defined as:

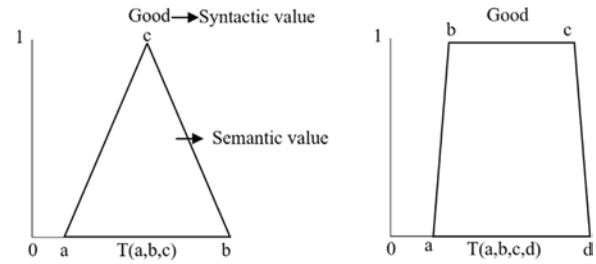


Fig. 1. Linguistic label.

$$E_{G_H}(s_h) = \{s_h | s_h \in S\}$$

$$E_{G_H}(\text{at most } s_h) = \{s_l | s_l \le s_h \text{ and } s_h \in S\}$$

$$E_{G_H}(\text{at least } s_h) = \{s_l | s_l \ge s_h \text{ and } s_h \in S\}$$

$$E_{G_H}(\text{between } s_h \text{ and } s_l) = \{s_o | s_h \le s_o \le s_l \text{ and } s_o \in S\}$$

2.2. The fuzzy envelope for HFLTSs

Zadeh (1975) pioneered the concept of linguistic variable, revolutionizing the way linguistic information is quantified (Zadeh et al., 1996). These variables are represented through parametric membership functions, often visualized as triangular or trapezoidal shapes (refer to Fig. 1). To facilitate computations involving HFLTSs, Liu and Rodríguez (2014) introduced the concept of a fuzzy envelope. This approach captures the semantics of CLEs through fuzzy membership functions, defined in the following manner:

Definition 3 (*Liu & Rodríguez, 2014*). The fuzzy envelope, denoted as $env_F(H)$, is characterized by a trapezoidal fuzzy membership function (refer to Definition 4), expressed as follows:

$$env_F(H) = T(a, b, c, d)$$

where H represents a HFLTS and $T(a, b, c, d)$ denotes a TrFN. The general procedure for obtaining the fuzzy envelope for HFLTSs is presented in Fig. 2.

Definition 4 (*Dubois & Prade, 1978*). If $T = (a, b, c, d)$ and $0 < a \le b \le c \le d$, T is defined as a TrFN, characterized by a membership function expressed as:

$$\mu_T(x) = \begin{cases} (x-a)/(b-a), & a \le x \le b, \\ 1, & b \le x \le c, \\ (d-x)/(d-c), & c \le x \le d, \\ 0, & \text{otherwise}. \end{cases} \quad (1)$$

The concept of α-levels is crucial in fuzzy decision-making and optimization, as it allows for the comparison and manipulation of fuzzy numbers by converting them into interval numbers at various levels of α. This simplifies the problem and makes it more tractable while preserving the essence of the uncertainty represented by the original fuzzy numbers.

Definition 5 (*Klir & Yuan, 1995*). The α-level set of a TrFN $T = (a, b, c, d)$ is a crisp set containing all elements x of the universe of discourse, which can be mathematically defined as:

$$T_\alpha = \{x \in R | \mu_T(x) > \alpha\}, \text{ where } 0 < \alpha < 1 \quad (2)$$

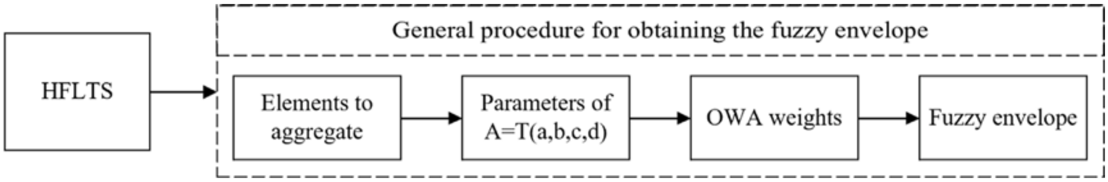


Fig. 2. The general procedure for obtaining the fuzzy envelope.

For a given α, T_α can be expressed as:

$$T_\alpha = [a + (b - a) \cdot \alpha, d - (d - c) \cdot \alpha] \quad (3)$$

This set includes all x values between two points, which are calculated based on linear interpolations on the left and right slopes of the trapezoid. It is worth noting that the choice of the α level typically depends on the DMs' preference for balancing optimism and pessimism in the evaluation process. By considering factors such as risk attitude, sensitivity analysis, historical data, expert judgment, and other relevant considerations, DMs can select an α level that best aligns with their preferences and the specific requirements of the application.

2.3. The conventional cross-efficiency

The traditional DEA model named CCR was proposed by Charnes et al. (1978) for evaluating the efficiency of a set of DMUs. The efficiency of each DMU is determined by the ratio of the weighted sum of outputs to the weighted sum of inputs. A DMU is considered efficient only when this ratio equals 1; Otherwise, it is classified as inefficient. Assuming a set of DMUs, denoted as $DMU_1, \ldots, DMU_m$, requiring evaluation with respect to P inputs and Q outputs, let $x_{pm}(p = 1, \ldots, P)$ and $y_{qm}(q = 1, \ldots, Q)$ be the inputs and outputs values of $DMU_i (i = 1, \ldots, m)$. The linear CCR model can be utilized to compute the efficiency of the evaluated DMU_d:

$$\max E_{dd} = \sum_{q=1}^{Q} u_{qd} y_{qd}$$
$$s.t. \begin{cases} \sum_{p=1}^{P} v_{pd} x_{pd} = 1, \\ \sum_{q=1}^{Q} u_{qd} y_{qi} - \sum_{p=1}^{P} v_{pd} x_{pi} \le 0, i = 1, \ldots, m, \\ u_{qd} \ge 0, v_{pd} \ge 0, q = 1, \ldots, Q, p = 1, \ldots, P. \end{cases} \quad (4)$$

where $u^*_{qd}(q = 1, \ldots, Q)$ and $v^*_{pd}(p = 1, \ldots, P)$ are the optimal weights of outputs and inputs in terms of DMU_d, the efficiency E^*_{dd} is relative efficiency or self-efficiency. This model seeks the optimal weights that are most favorable to DMU_d, potentially leading to an over-estimation of DMU_d's efficiency. Then, the cross-efficiency $E_{di} = \sum_{q=1}^{Q} u^*_{qd} y_{qi} / \sum_{p=1}^{P} v^*_{pd} x_{pi}$ stands for the peer-evaluation of DMU_i from DMU_d, the final efficiency of DMU_i is determined by the average operator: $\bar{E}_i = \frac{1}{m} \sum_{d=1}^{m} E_{di}$. Therefore, cross-efficiency techniques can mitigate the tendency for overestimation of efficiency to a certain extent.

2.4. The fuzzy DEA model

Traditional DEA models rely on precisely determined input and output values, which constrains the applicability of the DEA method when dealing with uncertain or fuzzy problems. To address this limitation, numerous fuzzy DEA models have been developed to handle imprecise or fuzzy input and output data caused by unquantifiable, incomplete, and unavailable information (Kao & Liu, 2000; Wang & Chin, 2011). Sengupta (1992) pioneered the incorporation of fuzziness into the DEA model by introducing a fuzzy mathematical programming approach, the fuzzy CCR model has been formulated as:

$$\max \tilde{E}_{dd} = \sum_{q=1}^{Q} u_{qd} \tilde{y}_{qd}$$
$$s.t. \begin{cases} \sum_{p=1}^{P} v_{pd} \tilde{x}_{pd} = 1, \\ \sum_{q=1}^{Q} u_{qd} \tilde{y}_{qi} - \sum_{p=1}^{P} v_{pd} \tilde{x}_{pi} \le 0, i = 1, \ldots, m, \\ u_{qd} \ge 0, v_{pd} \ge 0, q = 1, \ldots, Q, p = 1, \ldots, P. \end{cases} \quad (5)$$

where $\tilde{x}_{pd}$ and $\tilde{y}_{qd}$ represent the p_{th} fuzzy input and q_{th} fuzzy output for the DMU_d, respectively.

Zhu (2004) developed the fuzzy DEA method based on interval data to determine the maximum and minimum self-efficiency for DMU_d. Suppose both inputs and outputs for DMUs are represented as intervals, denoted by $[x^L_{pi}, x^U_{pi}]$ and $[y^L_{qi}, y^U_{qi}]$ respectively, the lower and upper bounds of self-efficiency can be determined by the following models:

$$\max E^U_{dd} = \sum_{q=1}^{Q} u_{qd} y^U_{pd}$$
$$s.t. \begin{cases} \sum_{p=1}^{P} v_{pd} x^L_{pd} = 1, \\ \sum_{q=1}^{Q} u_{qd} y^U_{qd} - \sum_{p=1}^{P} v_{pd} x^L_{pd} \ge 0, \\ \sum_{q=1}^{Q} u_{qd} y^L_{qi} - \sum_{p=1}^{P} v_{pd} x^U_{pi} \ge 0, i = 1, \ldots, m, i \ne d, \\ u_{qd} \ge 0, v_{pd} \ge 0, q = 1, \ldots, Q, p = 1, \ldots, P. \end{cases} \quad (6)$$

In this model, the maximum self-efficiency E^U_{dd} for the evaluated DMU_d is derived by using the lowest inputs and highest outputs. However, for other DMU_i $(i \ne d)$, they generate the lowest outputs while utilizing the highest inputs within their respective intervals. Similarly, the minimum self-efficiency E^L_{dd} of the evaluated DMU_d is obtained using the following model:

$$\max E^L_{dd} = \sum_{q=1}^{Q} u_{qd} y^L_{qd}$$
$$s.t. \begin{cases} \sum_{p=1}^{P} v_{pd} x^U_{pd} = 1, \\ \sum_{q=1}^{Q} u_{qd} y^L_{qd} - \sum_{p=1}^{P} v_{pd} x^U_{pd} \ge 0, \\ \sum_{q=1}^{Q} u_{qd} y^U_{qi} - \sum_{p=1}^{P} v_{pd} x^L_{pi} \ge 0, i = 1, \ldots, m, i \ne d, \\ u_{qd} \ge 0, v_{pd} \ge 0, q = 1, \ldots, Q, p = 1, \ldots, P. \end{cases} \quad (7)$$

3. The GDM framework based on fuzzy cross-efficiency models with HFLTSs

This section outlines the GDM method based on the fuzzy DEA cross-efficiency models with hesitant fuzzy linguistic information. First, the

GDM problem is defined by using HFLTSs to transform the original assessment information. Next, fuzzy envelopes are derived for HFLTSs, producing TrFN-based representations of the assessments. Using these fuzzy envelopes, we develop cross-efficiency models to determine the lower and upper bounds of cross-efficiency intervals for alternatives at various α-levels. A maximum group consensus model is then constructed to assign weights to the DMs, enabling the aggregation of individual cross-efficiency intervals into collective ones. Finally, the degree of preference based on collective intervals is used to rank the alternatives under different α-levels. A summary of the main steps in the proposed framework is provided at the end of this section.

3.1. Formulate the GDM problem

Suppose there are K DMs, denoted as $DM = \{dm_1, \ldots, dm_K\}$, who are invited to evaluate m available alternatives $X = \{X_1, \ldots, X_m\}$ with regard to n attributes $A = \{A_1, \ldots, A_n\}$, where there are P cost-based attributes and Q benefit-based attributes, satisfying $P + Q = n$. DMs often use linguistic terms aligned with human cognition in decision-making scenarios filled with qualitative attributes and uncertainty. HFLTSs are particularly effective in these situations, capturing DMs' hesitations more accurately than single linguistic term. The representation of decision matrices incorporating DMs' opinions modeled using HFLTSs can be expressed as:

$$HL^{(k)} = [hl_{ij}^{(k)}]_{m\times n} = \begin{bmatrix} hl_{11}^{(k)} & \cdots & hl_{1j}^{(k)} & \cdots & hl_{1n}^{(k)} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ hl_{i1}^{(k)} & \cdots & hl_{ij}^{(k)} & \cdots & hl_{in}^{(k)} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ hl_{m1}^{(k)} & \cdots & hl_{mj}^{(k)} & \cdots & hl_{mn}^{(k)} \end{bmatrix}. \tag{8}$$

where $hl_{ij}^{(k)}$ represents an HFLTS of dm_k for alternative X_i under the attribute A_j. Further, there is $hl_{ij}^{(k)} = \{s_{ij,c}^{(k)} | c = 1, \ldots, C\}$, where C is the number of linguistic terms in $hl_{ij}^{(k)}$. For any HFLTS-based assessments under cost attributes, denoted by $hl_{ij}^{c(k)} = \{s_h, \ldots, s_l\}(h < l$ for $h, l \in \{0, \ldots, g\})$, it should be normalized as $\bar{hl}_{ij}^{c(k)} = \{s_{g-l}, \ldots, s_{g-h}\}$. Therefore, the normalized HFLTS-based evaluation matrix of DM_k is represented as:

$$\bar{HL}^{(k)} = [\bar{hl}_{ij}^{(k)}]_{m\times n} = \begin{bmatrix} \bar{hl}_{11}^{c(k)} & \cdots & \bar{hl}_{1P}^{c(k)} & hl_{11}^{b(k)} & \cdots & hl_{1Q}^{b(k)} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \bar{hl}_{i1}^{c(k)} & \cdots & \bar{hl}_{ip}^{c(k)} & hl_{11}^{b(k)} & \cdots & hl_{iq}^{b(k)} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \bar{hl}_{m1}^{c(k)} & \cdots & \bar{hl}_{mP}^{c(k)} & hl_{11}^{b(k)} & \cdots & hl_{mQ}^{b(k)} \end{bmatrix}. \tag{9}$$

3.2. Obtain the fuzzy envelopes of HFLTSs

In this section, we will demonstrate the transformation rules from HFLTSs to their respective fuzzy envelopes, laying the foundation for further representing hesitant fuzzy linguistic information as the TrFNs-based assessments. Such transformations will adhere to specific considerations to ensure a comprehensive representation (Liu & Rodríguez, 2014):

a. The hesitation among various linguistic terms suggests varying levels of significance associated with each term.

b. Employing a trapezoidal fuzzy membership function adequately captures the ambiguity inherent in CLEs.

c. The parameters of the trapezoidal fuzzy membership function are determined by an aggregation operator, reflecting the varying importance of linguistic terms in the HFLTSs.

Based on this, we will further discuss three different kinds of fuzzy envelops for HFLTSs as follows:

- At least s_h: DMs employ this expression when they hesitate between linguistic terms but are certain about the worst assessment, such expression can be transformed into HFLTS as E_{G_H}(at least s_h) $= \{s_h, s_{h+1}, \ldots, s_g\}$ according to Definition 2. To compute the fuzzy envelope for this expression, follow these steps:
  - *Gather the elements for aggregation.* The aggregation set comprises:

$$T = \{e_L^h, e_M^h, e_L^{h+1}, e_R^h, e_M^{h+1}, e_L^{h+2}, e_R^{h+1}, \ldots, e_L^g, e_R^{g-1}, e_M^g, e_R^g\} \tag{10}$$

  Consider $e_R^{o-1} = e_M^o = e_L^{o+1}, o = 1, 2, \ldots, g-1$, the elements for aggregation are acquired as:

$$T = \{e_L^h, e_M^h, e_M^{h+1}, \ldots, e_M^g, e_R^g\} \tag{11}$$

  - *Calculate the parameters of the trapezoidal fuzzy membership function $A = T(a, b, c, d)$.*

$$\begin{aligned} a &= \min\{e_L^h, e_M^h, e_M^{h+1}, \ldots, e_M^g, e_R^g\} = e_L^h, \\ d &= \max\{e_L^h, e_M^h, e_M^{h+1}, \ldots, e_M^g, e_R^g\} = e_R^g, \\ b &= OWA_{W^{(2)}}(e_L^h, e_M^h, e_M^{h+1}, \ldots, e_M^g, e_R^g), \\ c &= OWA_{W^{(2)}}(e_L^h, e_M^h, e_M^{h+1}, \ldots, e_M^g, e_R^g), \end{aligned} \tag{12}$$

  The trapezoidal fuzzy membership function for *at least* s_h is shown in Fig. 3.
  - *OWA weights.* The importance of linguistic terms in the HFLTS is reflected by computing OWA weights for expressions with at least s_h. The weights utilized for computing b are represented as $W^{(2)}$ with $c = g - h + 1$, denoted as $W^{(2)} = (w_1^{(2)}, w_2^{(2)}, \ldots, w_{g-h+1}^{(2)})$, where:

$$w_1^{(2)} = \gamma^{g-h}, w_2^{(2)} = (1-\gamma)\gamma^{g-h-1}, w_3^{(2)} = (1-\gamma)\gamma^{g-h-2}, \ldots, \\ w_{g-h}^{(2)} = (1-\gamma)\gamma, w_{g-h+1}^{(2)} = 1-\gamma \tag{13}$$

  Further, the orness measure related to $W^{(2)}$ weights can be obtained as follows:

$$orness(W^{(2)}) = \frac{\gamma - \gamma^c}{(c-1)(1-\gamma)} \tag{14}$$

Remark 1. An orness measure $orness(W^{(2)}) > 0.5$ suggests b is closer to the maximum, emphasizing the significance of the highest linguistic term s_g in the HFLTS. Conversely, $orness(W^{(2)}) < 0.5$, it indicates b is near to the minimum, highlighting the importance of the lowest linguistic term s_h.

  - *Fuzzy envelop.* For the HFLTS derived from the CLEs with *at least* s_h, its fuzzy envelope is defined as a TrFN (e_L^h, b, e_M^g, e_R^g), where b is computed using Definition 2 with associated weights $W^{(2)}$ as per Eq. (13).
- At most s_h: DMs employ this expression when they hesitate between linguistic terms but are certain about the best assessment, such expression can be transformed into HFLTS as E_{G_H}(at most s_h) $= \{s_0, s_1, \ldots, s_h\}$ according to Definition 2. The fuzzy envelope for such expressions can be obtained as the following steps:
  - *Gather the elements for aggregation.*

$$T = \{e_L^0, e_M^0, e_L^1, e_R^0, e_M^1, e_L^2, e_R^1, \ldots, e_L^h, e_R^{h-1}, e_M^h, e_R^h\} \tag{15}$$

  Consider $e_R^{o-1} = e_M^o = e_L^{o+1}, o = 1, 2, \ldots, g-1$, the elements for aggregation are acquired as:

$$T = \{e_L^0, e_M^0, e_M^1, \ldots, e_M^h, e_R^h\} \tag{16}$$

– *Calculate the parameters of the trapezoidal fuzzy membership function* $A = T(a, b, c, d)$.

$$
\begin{aligned}
a &= \min\{e_L^0, e_M^0, e_M^1, \dots, e_M^h, e_R^h\} = e_L^0,\\
d &= \max\{e_L^0, e_M^0, e_M^1, \dots, e_M^h, e_R^h\} = e_R^h,\\
b &= OWA_{W^{(1)}}(e_L^0, e_M^0, e_M^1, \dots, e_M^h, e_R^h),\\
c &= OWA_{W^{(1)}}(e_L^0, e_M^0, e_M^1, \dots, e_M^h, e_R^h).
\end{aligned} \tag{17}
$$

The trapezoidal fuzzy membership function for *at most* s_h is shown in Fig. 4.

– *OWA weights.* The importance of linguistic terms in the HFLTS is reflected by computing OWA weights for expressions with *at most* s_h. The weights utilized for computing c are represented as $W^{(1)}$ with $c = h+1$, denoted as $W^{(1)} = (w_1^{(1)}, w_2^{(1)}, \dots, w_{h+1}^{(1)})$, where:

$$
\begin{aligned}
w_1^{(1)} = \gamma, w_2^{(1)} = \gamma(1-\gamma), w_3^{(1)} = \gamma(1-\gamma)^2, \dots,\\
w_h^{(1)} = \gamma(1-\gamma)^{h-1}, w_{h+1}^{(1)} = (1-\gamma)^h
\end{aligned} \tag{18}
$$

where $\gamma \in [0, 1]$ is a parameter. Further, the orness measure related to $W^{(1)}$ weights can be obtained as follows:

$$
orness(W^{(1)}) = \frac{c}{c-1} - \frac{1-(1-\gamma)^c}{(c-1)\gamma} \tag{19}
$$

Remark 2. An orness measure $orness(W^{(1)}) > 0.5$ suggests c is closer to the maximum, emphasizing the significance of the highest linguistic term s_h in the HFLTS. Conversely, $orness(W^{(1)}) < 0.5$, it indicates c is near to the minimum, highlighting the importance of the lowest linguistic term s_0 in the HFLTS.

– *Fuzzy envelop.* For the HFLTS derived from the CLEs with *at most* s_h, its fuzzy envelope is defined as a TrFN (e_L^0, e_M^0, c, e_R^h), where c is computed using Definition 2 with associated weights $W^{(1)}$ as per Eq. (18).

- Between s_h and s_l: Such expression can be transformed into HFLTS as E_{G_H}(between s_h and s_l) $= \{s_h, s_{h+1}, \dots, s_l\}$ according to Definition 2. The fuzzy envelope for such expressions can be obtained as the following steps:

– *Gather the elements for aggregation.*

$$
T = \{e_L^h, e_M^h, e_L^{h+1}, e_R^h, e_M^{h+1}, e_L^{h+2}, e_R^{h+1}, \dots, e_L^l, e_R^{l-1}, e_M^l, e_R^l\} \tag{20}
$$

Consider $e_R^{o-1} = e_M^o = e_L^{o+1}, o = 1, 2, \dots, g-1$, the elements for aggregation are acquired as:

$$
T = \{e_L^h, e_M^h, e_M^{h+1}, \dots, e_M^l, e_R^l\} \tag{21}
$$

– *Calculate the parameters of the trapezoidal fuzzy membership function* $A = T(a, b, c, d)$.

$$
\begin{aligned}
a &= \min\{e_L^h, e_M^h, e_M^{h+1}, \dots, e_M^l, e_R^l\} = e_L^h,\\
d &= \max\{e_L^h, e_M^h, e_M^{h+1}, \dots, e_M^l, e_R^l\} = e_R^l,
\end{aligned} \tag{22}
$$

Parameters b and c (refer to Fig. 5) are calculated using the OWA operator, considering the number of linguistic terms in the HFLTS derived from the CLE.

(i) If $h + l$ is odd, then

∗ If $h + 1 = l$, then $b = e_M^h$ and $c = e_M^{h+1}$. This indicates that the linguistic terms s_h and s_l hold equal importance in HFLTS.

∗ If $h + 1 < l$, then

$$
\begin{aligned}
b &= OWA_{W^{(2)}}(e_L^h, e_M^{h+1}, \dots, e^{\frac{h+l-1}{2}}),\\
c &= OWA_{W^{(1)}}(e_L^l, e_M^{l-1}, \dots, e^{\frac{h+l-1}{2}}),
\end{aligned} \tag{23}
$$

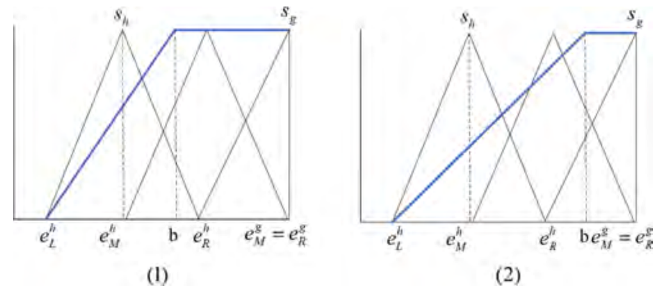


Fig. 3. The membership function of $E_{G_H} = \{s_h, s_{h+1}, \dots, s_g\}$.

(ii) If $h + l$ is even, then

$$
\begin{aligned}
b &= OWA_{W^{(2)}}(e_L^h, e_M^{h+1}, \dots, e^{\frac{h+l}{2}}),\\
c &= OWA_{W^{(1)}}(e_L^l, e_M^{l-1}, \dots, e^{\frac{h+l}{2}}),
\end{aligned} \tag{24}
$$

– *OWA weights.* Within this CLE, the importance of linguistic terms in the HFLTS is reflected through the computation of OWA weights utilizing $W^{(1)}$ and $W^{(2)}$. These weights are determined through two distinct scenarios:

(i) If $h + l$ is odd, then the $W^{(2)}$ and $W^{(1)}$ in Eq. (23) can be determined as follows:

∗ $W^{(2)} = (w_1^{(2)}, w_2^{(2)}, \dots, w_{\frac{l-h+1}{2}}^{(2)})$, with

$$
\begin{aligned}
w_1^{(2)} = \gamma_1^{\frac{l-h-1}{2}}, w_2^{(2)} = (1-\gamma_1)\gamma_1^{\frac{l-h-3}{2}}, \dots,\\
w_{\frac{l-h-1}{2}}^{(2)} = (1-\gamma_1)\gamma_1, w_{\frac{l-h+1}{2}}^{(2)} = 1-\gamma_1
\end{aligned} \tag{25}
$$

∗ $W^{(1)} = (w_1^{(1)}, w_2^{(1)}, \dots, w_{\frac{l-h+1}{2}}^{(1)})$, with

$$
\begin{aligned}
w_1^{(1)} = \gamma_2, w_2^{(1)} = \gamma_2(1-\gamma_2) \dots,\\
w_{\frac{l-h-1}{2}}^{(1)} = \gamma_2(1-\gamma_2)^{\frac{l-h-3}{2}}, w_{\frac{l-h+1}{2}}^{(1)} = (1-\gamma_2)^{\frac{l-h-1}{2}}
\end{aligned} \tag{26}
$$

(ii) If $h + l$ is even, then the $W^{(2)}$ and $W^{(1)}$ in Eq. (24) can be determined as follows:

∗ $W^{(2)} = (w_1^{(2)}, w_2^{(2)}, \dots, w_{\frac{l-h+2}{2}}^{(2)})$, with

$$
\begin{aligned}
w_1^{(2)} = \gamma_1^{\frac{l-h}{2}}, w_2^{(2)} = (1-\gamma_1)\gamma_1^{\frac{l-h-2}{2}}, \dots,\\
w_{\frac{l-h}{2}}^{(2)} = (1-\gamma_1)\gamma_1, w_{\frac{l-h+2}{2}}^{(2)} = 1-\gamma_1
\end{aligned} \tag{27}
$$

∗ $W^{(1)} = (w_1^{(1)}, w_2^{(1)}, \dots, w_{\frac{l-h+2}{2}}^{(1)})$, with

$$
\begin{aligned}
w_1^{(1)} = \gamma_2, w_2^{(1)} = \gamma_2(1-\gamma_2) \dots,\\
w_{\frac{l-h}{2}}^{(1)} = \gamma_2(1-\gamma_2)^{\frac{l-h-2}{2}}, w_{\frac{l-h+2}{2}}^{(1)} = (1-\gamma_2)^{\frac{l-h}{2}}
\end{aligned} \tag{28}
$$

Let $\gamma_1 = \frac{(L-1)-(l-h)}{L-2}$, where L denotes the granularity of the linguistic term set $S = s_0, \dots, s_{L-1}$, and $\gamma_2 = 1 - \gamma_1$.

– *Fuzzy envelop.* For the HFLTS derived from the CLEs with *between* s_h *and* s_l, its fuzzy envelope is defined as a TrFN (e_L^h, b, c, e_R^l), where b and c is computed using Eq. (23) or Eq. (24).

3.3. Develop the fuzzy cross-efficiency models

When dealing with hesitant fuzzy linguistic assessments provided by DMs, the original attribute values are represented as HFLTSs. By employing the transformation rules detailed in Section 3.2, we can

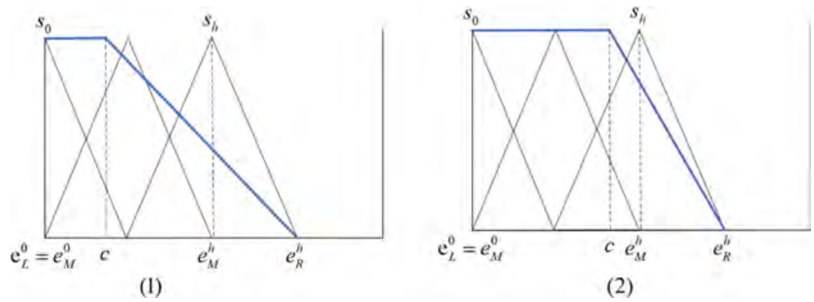


Fig. 4. The membership function of $E_{G_H} = \{s_0, s_1, \ldots, s_h\}$.

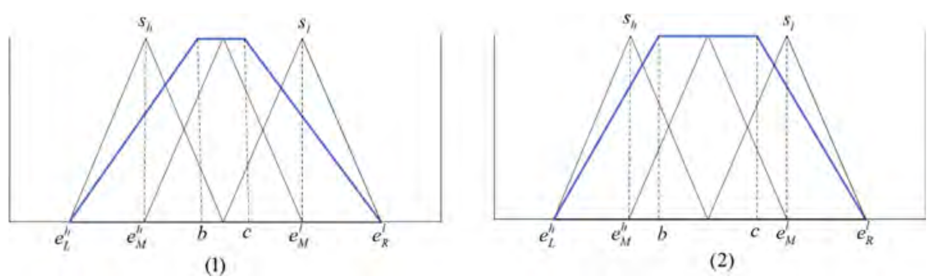


Fig. 5. The membership function of E_{G_H}(between s_h and s_l) $= \{s_h, s_{h+1}, \ldots, s_l\}$.

derive the corresponding TrFNs-based assessments of DMs, where attribute values are expressed as TrFNs without information loss. Let $\tilde{T}^k = (\tilde{T}_{ij}^{(k)})m \times n$ represent the TrFN-based assessments of DM_k, where $\tilde{T}_{ij}^{(k)}$ is a TrFN denoting the assessed value for alternative X_i under attribute A_j, expressed as $[Tij^{(k)}(a), T_{ij}^{(k)}(b), T_{ij}^{(k)}(c), T_{ij}^{(k)}(d)]$. Subsequently, we categorize the normalized cost-based attributes as inputs and the benefit-based attributes as outputs. Fuzzy cross-efficiency models are then constructed to assess the performance of the available alternatives in GDM. Given its widespread adoption in fuzzy DEA model development (Hatami-Marbini et al., 2011), the α-level based approach is employed in formulating the fuzzy cross-efficiency mathematical models. For a certain α-level, the TrFNs-based attribute values are determined as intervals, denoted as $(\tilde{T}_{ij}^{(k)})_\alpha = [(\tilde{T}_{ij}^{(k)})_\alpha^L, (\tilde{T}_{ij}^{(k)})_\alpha^U]$. Let there be n attributes, consisting of P cost-type attributes and Q benefit-type attributes. We denote $(\tilde{T}_{ip}^{c(k)})_\alpha = [(\tilde{T}_{ip}^{c(k)})_\alpha^L, (\tilde{T}_{ip}^{c(k)})_\alpha^U]$ as the α-level fuzzy inputs for alternative X_i of DM_k. Similarly, $(\tilde{T}_{iq}^{b(k)})_\alpha = [(\tilde{T}_{iq}^{b(k)})_\alpha^L, (\tilde{T}_{iq}^{b(k)})_\alpha^U]$ is denoted as the α-level fuzzy outputs.

Next, we will develop the fuzzy cross-efficiency models using α-level fuzzy inputs and outputs. The core idea involves identifying optimal and worst cross-efficiency for alternatives within input and output intervals at a specific α-level, while ensuring the average self-efficiency of the evaluated alternative remains constant. Consider the α-level self-efficiency of the evaluated alternative X_d derived from the assessments by DM_k, expressed as $(\bar{E}_{dd}^{(k)})_\alpha = \frac{1}{2}[(\tilde{E}_{dd}^{(k)})_\alpha^L + (\tilde{E}_{dd}^{(k)})_\alpha^U]$, where $(\tilde{E}_{dd}^{(k)})_\alpha^U$ is the optimal self-efficiency and $(\tilde{E}_{dd}^{(k)})_\alpha^L$ is the worst self-efficiency, which can be obtained from Model (6) and Model (7) respectively, wherein $[(\tilde{T}_{ip}^{c(k)})_\alpha^L, (\tilde{T}_{ip}^{c(k)})_\alpha^U]$ and $[(\tilde{T}_{iq}^{b(k)})_\alpha^L, (\tilde{T}_{iq}^{b(k)})_\alpha^U]$ in the previous two models are the lower and upper bounds for α-level fuzzy inputs and outputs. Based on this, the optimal cross-efficiency value of alternative X_l based on assessments provided by DM_k can be determined as the following model:

$$\max(\tilde{E}_l^{(k)})_\alpha^U = \frac{1}{m}\sum_{d=1}^{m}\sum_{q=1}^{Q} u_{dq}^{(k)} \cdot (\tilde{T}_{lq}^{b(k)})_\alpha$$
$$s.t. \begin{cases} \sum_{p=1}^{P} v_{dp}^{(k)} \cdot (\tilde{T}_{lp}^{c(k)})_\alpha = 1, d = 1, \ldots, m; l = 1, \ldots, m, \\ (\bar{E}_{dd}^{(k)})_\alpha \cdot \sum_{p=1}^{P} v_{dp}^{(k)} \cdot (\tilde{T}_{dp}^{c(k)})_\alpha - \sum_{q=1}^{Q} u_{dq}^{(k)} \cdot (\tilde{T}_{dq}^{b(k)})_\alpha = 0, d = 1, \ldots, m, \\ \sum_{q=1}^{Q} u_{dq}^{(k)} \cdot (\tilde{T}_{iq}^{b(k)})_\alpha \le \sum_{p=1}^{P} v_{dp}^{(k)} (\tilde{T}_{ip}^{c(k)})_\alpha, i, d, l = 1, \ldots, m, i \ne d, l, \\ v_{dp}^{(k)} \ge 0, u_{dq}^{(k)} \ge 0, d = 1, \ldots, l; p = 1, \ldots, P; q = 1, \ldots, Q. \end{cases} \tag{29}$$

In Model (29), the decision variables $v_{dp}^{(k)}(d = 1, \ldots, m; p = 1, \ldots, P)$ represent the weights of inputs for the alternative X_d based on the assessment of DM_k, the number of such variables is $m \times P$. Similarly, $u_{dq}^{(k)}(d = 1, \ldots, m; q = 1, \ldots, Q)$ represents the weights of outputs for the alternative X_d, with the number of such variables being $m \times Q$. Additionally, the variables like $(\tilde{T}_{iq}^{b(k)})_\alpha(q = 1, \ldots, Q)$ and $(\tilde{T}_{ip}^{c(k)})_\alpha(p = 1, \ldots, P)$ are interval variables, representing the α-level fuzzy outputs and inputs of alternative X_i. The inclusion of interval variables often requires transforming the problem into a form that can handle uncertainty. These problems are generally more complex than standard linear programming and can be NP-hard depending on the nature of the intervals. The objective function employs a benevolent strategy in Model (29), seeking to maximize the cross-efficiency value of X_l while ensuring that the average self-efficiency of the evaluated alternative X_d remains unchanged. Similarly, it switches to an aggressive strategy that aims to minimize the objective when determining the worst cross-efficiency value of X_l. The following model can determine the corresponding worst cross-efficiency of X_l:

$$\min(\tilde{E}_l^{(k)})_\alpha^L = \frac{1}{m}\sum_{d=1}^{m}\sum_{q=1}^{Q} u_{dq}^{(k)} \cdot (\tilde{T}_{lq}^{b(k)})_\alpha$$

$$s.t.\begin{cases} \sum_{p=1}^{P} v_{dq}^{(k)} \cdot (\tilde{T}_{lp}^{c(k)})_\alpha = 1, d = 1,\dots,m; l = 1,\dots,m, \\ (\bar{E}_{dd}^{(k)})_\alpha \cdot \sum_{p=1}^{P} v_{dp}^{(k)} \cdot (\tilde{T}_{dp}^{c(k)})_\alpha - \sum_{q=1}^{Q} u_{dq}^{(k)} \cdot (\tilde{T}_{dq}^{b(k)})_\alpha = 0, d = 1,\dots,m, \\ \sum_{q=1}^{Q} u_{dq}^{(k)} \cdot (\tilde{T}_{iq}^{b(k)})_\alpha \le \sum_{p=1}^{P} v_{dp}^{(k)} (\tilde{T}_{ip}^{c(k)})_\alpha, i, d, l = 1,\dots,m, i \ne d, l, \\ v_{dp}^{(k)} \ge 0, u_{dq}^{(k)} \ge 0, d = 1,\dots,l; p = 1,\dots,P; q = 1,\dots,Q. \end{cases} \tag{30}$$

The variables in Model (30) are consistent with those in Model (29). Due to the inclusion of interval variables, both models are classified as nonlinear programming problems. To simplify calculations and ensure optimality, we employ the transformation rules proposed by Despotis and Smirlis (2002) to convert both models into linear representations. There are many robust and efficient software tools and solvers available for LP problems, such as CPLEX, Gurobi, and the open-source solver GLPK. These tools are widely used in industry and academia for solving linear programming problems. For enhanced clarity, we denote intervals derived from α-level fuzzy inputs as $\tilde{T}_{ip}^{c(k)} = [(\tilde{T}_{ip}^{c(k)})_\alpha^L, (\tilde{T}_{ip}^{c(k)})_\alpha^U]$, and intervals obtained from α-level fuzzy outputs as $\tilde{T}_{iq}^{b(k)} = [(\tilde{T}_{iq}^{b(k)})_\alpha^L, (\tilde{T}_{iq}^{b(k)})_\alpha^U]$. The transformation rules are as follows:

$$\begin{aligned} &\tilde{T}_{ip}^{c(k)} = (\tilde{T}_{ip}^{c(k)})_\alpha^L + s_{ip}^{(k)} \cdot ((\tilde{T}_{ip}^{c(k)})_\alpha^U - (\tilde{T}_{ip}^{c(k)})_\alpha^L), p = 1,\dots,P; i = 1,\dots,m; \\ &\quad 0 \le s_{ip}^{(k)} \le 1, \\ &\tilde{T}_{iq}^{b(k)} = (\tilde{T}_{iq}^{b(k)})_\alpha^L + t_{iq}^{(k)} \cdot ((\tilde{T}_{iq}^{b(k)})_\alpha^U - (\tilde{T}_{iq}^{b(k)})_\alpha^L), q = 1,\dots,Q; i = 1,\dots,m; \\ &\quad 0 \le s_{iq}^{(k)} \le 1. \end{aligned} \tag{31}$$

Then, the nonlinear expressions can be transformed into linear ones as follows:

$$\begin{aligned} \sum_{p=1}^{P} v_{dp}^{(k)} \cdot \tilde{T}_{ip}^{c(k)} &= \sum_{p=1}^{P} v_{dp}^{(k)}[(\tilde{T}_{ip}^{c(k)})_\alpha^L + s_{ip}^{(k)} \cdot ((\tilde{T}_{ip}^{c(k)})_\alpha^U - (\tilde{T}_{ip}^{c(k)})_\alpha^L)] \\ &= \sum_{p=1}^{P} v_{dp}^{(k)} \cdot (\tilde{T}_{ip}^{c(k)})_\alpha^L + v_{dp}^{(k)} \cdot s_{ip}^{(k)}((\tilde{T}_{ip}^{c(k)})_\alpha^U - (\tilde{T}_{ip}^{c(k)})_\alpha^L) \\ &= \sum_{p=1}^{P} v_{dp}^{(k)} \cdot (\tilde{T}_{ip}^{c(k)})_\alpha^L + a_{ip}^{(k)}((\tilde{T}_{ip}^{c(k)})_\alpha^U - (\tilde{T}_{ip}^{c(k)})_\alpha^L), a_{ip}^{(k)} \le v_{dp}^{(k)}. \end{aligned}$$

Similarly,

$$\begin{aligned} \sum_{q=1}^{Q} u_{dq}^{(k)} \cdot \tilde{T}_{iq}^{b(k)} &= \sum_{q=1}^{Q} u_{dq}^{(k)}[(\tilde{T}_{iq}^{b(k)})_\alpha^L + t_{iq}^{(k)} \cdot ((\tilde{T}_{iq}^{b(k)})_\alpha^U - (\tilde{T}_{iq}^{b(k)})_\alpha^L)] \\ &= \sum_{q=1}^{Q} u_{dq}^{(k)} \cdot (\tilde{T}_{iq}^{b(k)})_\alpha^L + u_{dq}^{(k)} \cdot t_{iq}^{(k)}((\tilde{T}_{iq}^{b(k)})_\alpha^U - (\tilde{T}_{iq}^{b(k)})_\alpha^L) \\ &= \sum_{q=1}^{Q} u_{dq}^{(k)} \cdot (\tilde{T}_{iq}^{b(k)})_\alpha^L + b_{iq}^{(k)}((\tilde{T}_{iq}^{b(k)})_\alpha^U - (\tilde{T}_{iq}^{b(k)})_\alpha^L), b_{iq}^{(k)} \le u_{dq}^{(k)}. \end{aligned}$$

Ultimately, we present the linear representation for Model (29) and Model (30) below, which serves for calculating the optimal and worst cross-efficiency of alternatives, respectively:

$$\max(\tilde{E}_l^{(k)})_\alpha^U = \frac{1}{m}\sum_{d=1}^{m}\sum_{q=1}^{Q} u_{dq}^{(k)} \cdot (\tilde{T}_{lq}^{b(k)})_\alpha^L + b_{lq}^{(k)} \cdot ((\tilde{T}_{lq}^{b(k)})_\alpha^U - (\tilde{T}_{lq}^{b(k)})_\alpha^L),$$

$$s.t.\begin{cases} \sum_{p=1}^{P} v_{dp}^{(k)} \cdot (\tilde{T}_{lp}^{c(k)})_\alpha^L + a_{lp}^{(k)} \cdot ((\tilde{T}_{lp}^{c(k)})_\alpha^U - (\tilde{T}_{lp}^{c(k)})_\alpha^L) = 1, d = 1,\dots,m; l = 1,\dots,m, \\ (\bar{E}_{dd}^{(k)})_\alpha \cdot \sum_{p=1}^{P} v_{dp}^{(k)} \cdot (\tilde{T}_{dp}^{c(k)})_\alpha^L + a_{dp}^{(k)} \cdot ((\tilde{T}_{dp}^{c(k)})_\alpha^U - (\tilde{T}_{dp}^{c(k)})_\alpha^L) \\ -\sum_{q=1}^{Q} u_{dq}^{(k)} \cdot (\tilde{T}_{dq}^{b(k)})_\alpha^L + b_{dq}^{(k)} \cdot ((\tilde{T}_{dq}^{b(k)})_\alpha^U - (\tilde{T}_{dq}^{b(k)})_\alpha^L) = 0, d = 1,\dots,m, \\ \sum_{q=1}^{Q} u_{dq}^{(k)} \cdot (\tilde{T}_{iq}^{b(k)})_\alpha^L + b_{iq}^{(k)} \cdot ((\tilde{T}_{iq}^{b(k)})_\alpha^U - (\tilde{T}_{iq}^{b(k)})_\alpha^L) \le \\ \sum_{p=1}^{P} v_{dp}^{(k)} \cdot (\tilde{T}_{ip}^{c(k)})_\alpha^L + a_{ip}^{(k)} \cdot ((\tilde{T}_{ip}^{c(k)})_\alpha^U - (\tilde{T}_{ip}^{c(k)})_\alpha^L), i, d, l = 1,\dots,m, \\ a_{ip}^{(k)} \le v_{dp}^{(k)}, i = 1,\dots,m; p = 1,\dots,P, \\ b_{iq}^{(k)} \le u_{dq}^{(k)}, i = 1,\dots,m; q = 1,\dots,Q, \\ v_{dp}^{(k)} \ge 0, u_{dq}^{(k)} \ge 0, d = 1,\dots,m; p = 1,\dots,P; q = 1,\dots,Q. \end{cases} \tag{32}$$

and,

$$\min(\tilde{E}_l^{(k)})_\alpha^U = \frac{1}{m}\sum_{d=1}^{m}\sum_{q=1}^{Q} u_{dq}^{(k)} \cdot (\tilde{T}_{lq}^{b(k)})_\alpha^L + b_{lq}^{(k)} \cdot ((\tilde{T}_{lq}^{b(k)})_\alpha^U - (\tilde{T}_{lq}^{b(k)})_\alpha^L),$$

$$s.t.\begin{cases} \sum_{p=1}^{P} v_{dp}^{(k)} \cdot (\tilde{T}_{lp}^{c(k)})_\alpha^L + a_{lp}^{(k)} \cdot ((\tilde{T}_{lp}^{c(k)})_\alpha^U - (\tilde{T}_{lp}^{c(k)})_\alpha^L) = 1, d = 1,\dots,m; l = 1,\dots,m, \\ (\bar{E}_{dd}^{(k)})_\alpha \cdot \sum_{p=1}^{P} v_{dp}^{(k)} \cdot (\tilde{T}_{dp}^{c(k)})_\alpha^L + a_{dp}^{(k)} \cdot ((\tilde{T}_{dp}^{c(k)})_\alpha^U - (\tilde{T}_{dp}^{c(k)})_\alpha^L) \\ -\sum_{q=1}^{Q} u_{dq}^{(k)} \cdot (\tilde{T}_{dq}^{b(k)})_\alpha^L + b_{dq}^{(k)} \cdot ((\tilde{T}_{dq}^{b(k)})_\alpha^U - (\tilde{T}_{dq}^{b(k)})_\alpha^L) = 0, d = 1,\dots,m, \\ \sum_{q=1}^{Q} u_{dq}^{(k)} \cdot (\tilde{T}_{iq}^{b(k)})_\alpha^L + b_{iq}^{(k)} \cdot ((\tilde{T}_{iq}^{b(k)})_\alpha^U - (\tilde{T}_{iq}^{b(k)})_\alpha^L) \le \\ \sum_{p=1}^{P} v_{dp}^{(k)} \cdot (\tilde{T}_{ip}^{c(k)})_\alpha^L + a_{ip}^{(k)} \cdot ((\tilde{T}_{ip}^{c(k)})_\alpha^U - (\tilde{T}_{ip}^{c(k)})_\alpha^L), i, d, l = 1,\dots,m, \\ a_{ip}^{(k)} \le v_{dp}^{(k)}, i = 1,\dots,m; p = 1,\dots,P, \\ b_{iq}^{(k)} \le u_{dq}^{(k)}, i = 1,\dots,m; q = 1,\dots,Q, \\ v_{dp}^{(k)} \ge 0, u_{dq}^{(k)} \ge 0, d = 1,\dots,m; p = 1,\dots,P; q = 1,\dots,Q. \end{cases} \tag{33}$$

where $v_{dp}, u_{dq}^{(k)}, a_{ip}^{(k)}, b_{iq}^{(k)}$ are unknown variables. These two models consider all possible values within the intervals of α level fuzzy inputs and outputs. Solving both linear models to ascertain the fuzzy cross-efficiency intervals $[(\tilde{E}_l^{(k)})_\alpha^L, (\tilde{E}_l^{(k)})_\alpha^U]$ of $X_l (l = 1,\dots,i; l \ne d)$ under the α-level fuzzy inputs and outputs provided by DM_k, maintaining the average self-efficiency value of X_d unchanged. Ultimately, the individual fuzzy cross-efficiency intervals based on the assessments of $DM_k (k = 1,\dots,K)$ will be obtained as shown in Table 1. Let $\Lambda = \{\lambda_k^\alpha | k = 1,\dots,K\}$ be the weights of DMs reflecting their importance in GDM process, consequently, the collective fuzzy cross-efficiency intervals for each alternative which aggregated from the all individual fuzzy cross-efficiency intervals, is presented in the final row of Table 1.

Table 1
The α-level fuzzy cross-efficiency for alternatives obtained from the assessments of DMs.

DMs	$Weights$	$Alternatives$			
		X_1	X_2	$\cdots$	X_m
DM_1	λ_1	$[(\tilde{E}_1^{(1)})_\alpha^L, (\tilde{E}_1^{(1)})_\alpha^U]$	$[(\tilde{E}_2^{(1)})_\alpha^L, (\tilde{E}_2^{(1)})_\alpha^U]$	$\cdots$	$[(\tilde{E}_m^{(1)})_\alpha^L, (\tilde{E}_m^{(1)})_\alpha^U]$
DM_2	λ_2	$[(\tilde{E}_1^{(2)})_\alpha^L, (\tilde{E}_1^{(2)})_\alpha^U]$	$[(\tilde{E}_2^{(2)})_\alpha^L, (\tilde{E}_2^{(2)})_\alpha^U]$	$\cdots$	$[(\tilde{E}_m^{(2)})_\alpha^L, (\tilde{E}_m^{(2)})_\alpha^U]$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
DM_K	λ_K	$[(\tilde{E}_1^{(K)})_\alpha^L, (\tilde{E}_1^{(K)})_\alpha^U]$	$[(\tilde{E}_2^{(K)})_\alpha^L, (\tilde{E}_2^{(K)})_\alpha^U]$	$\cdots$	$[(\tilde{E}_m^{(K)})_\alpha^L, (\tilde{E}_m^{(K)})_\alpha^U]$
Collective		$[(\tilde{E}_1)_\alpha^L, (\tilde{E}_1)_\alpha^U]$	$[(\tilde{E}_2)_\alpha^L, (\tilde{E}_2)_\alpha^U]$	$\cdots$	$[(\tilde{E}_m)_\alpha^L, (\tilde{E}_m)_\alpha^U]$

3.4. Determine the weights of DMs

Determining the weights of DMs in GDM is essential, as it acknowledges the varying significance of each individual's contributions. By assigning appropriate weights to DMs, we can more effectively aggregate and assess their opinions, mitigating bias and enhancing fairness, which leads to more balanced and objective decision outcomes. From this perspective, we aim to determine the optimal weights for aggregating individual fuzzy cross-efficiency intervals into collective fuzzy cross-efficiency intervals for alternatives. We construct a maximum consensus model to assign these optimal weights to each individual. This strategic allocation promotes a higher degree of unanimity among all DMs concerning the cross-efficiency evaluations of alternatives, thus improving consensus.

3.4.1. Group consensus based on fuzzy cross-efficiency

Measuring consensus through cross-efficiency is a vital step, as it assesses the level of agreement among DMs regarding alternative evaluations. This process significantly contributes to the sustainability and acceptability of decision-making outcomes. Next, we will design the consensus measurement of how to determine group consensus based on the fuzzy cross-efficiency of alternatives.

Definition 6. Let $[(\tilde{E}_i^{(k)})_\alpha^L, (\tilde{E}_i^{(k)})_\alpha^U]$ denote the intervals representing the α-level fuzzy cross-efficiency of alternative X_i based on the assessments provided by DM_k, and let λ_k represent the weight assigned to DM_k. Consequently, the collective α-level fuzzy cross-efficiency of X_i, denoted as $[(\tilde{E}_i)_\alpha^L, (\tilde{E}_i)_\alpha^U]$, can be defined as follows:

$$[(\tilde{E}_i)_\alpha^L, (\tilde{E}_i)_\alpha^U] = [\sum_{k=1}^{K} \lambda_k \cdot (\tilde{E}_i^{(k)})_\alpha^L, \sum_{k=1}^{K} \lambda_k \cdot (\tilde{E}_i^{(k)})_\alpha^U] \tag{34}$$

Definition 7. Let $[(\tilde{E}_i^{(k)})_\alpha^L, (\tilde{E}_i^{(k)})_\alpha^U]$ and $[(\tilde{E}_i)_\alpha^L, (\tilde{E}_i)_\alpha^U]$ be equivalent expressions as defined in Definition 6, then the group consensus based on fuzzy cross-efficiency of alternatives can be defined as follows:

$$\begin{aligned} GC &= 1 - \frac{1}{2*m*K} \sum_{k=1}^{K} \sum_{i=1}^{m} |(\tilde{E}_i^{(k)})_\alpha^L - (\tilde{E}_i)_\alpha^L| + |(\tilde{E}_i^{(k)})_\alpha^U - (\tilde{E}_i)_\alpha^U| \\ &= 1 - \frac{1}{2*m*K} \sum_{k=1}^{K} \sum_{i=1}^{m} |(\tilde{E}_i^{(k)})_\alpha^L - \sum_{k=1}^{K} \lambda_k \cdot (\tilde{E}_i^{(k)})_\alpha^L| + |(\tilde{E}_i^{(k)})_\alpha^U - \sum_{k=1}^{K} \lambda_k \cdot (\tilde{E}_i^{(k)})_\alpha^U| \end{aligned} \tag{35}$$

3.4.2. Weight determination of decision-makers

Following this, we will develop an optimization model to ascertain DMs' weights. This model aims to identify the optimal set of weights for DMs based on fuzzy cross-efficiency to maximize group consensus. Specifically, the aggregated fuzzy cross-efficiency of alternatives by utilizing these optimal weights will be maximally accepted by the group.

$$\max GC = 1 - \frac{1}{2*M*K} \sum_{k=1}^{K} \sum_{i=1}^{m} |(\tilde{E}_i^{(k)})_\alpha^L - (\tilde{E}_i)_\alpha^L| + |(\tilde{E}_i^{(k)})_\alpha^U - (\tilde{E}_i)_\alpha^U|$$

$$s.t. \begin{cases} (\tilde{E}_i)_\alpha^L = \sum_{k=1}^{K} \lambda_k \cdot (\tilde{E}_i^{(k)})_\alpha^L, i = 1, \dots, m, \\ (\tilde{E}_i)_\alpha^U = \sum_{k=1}^{K} \lambda_k \cdot (\tilde{E}_i^{(k)})_\alpha^U, i = 1, \dots, m, \\ \sum_{k=1}^{K} \lambda_k = 1, \\ \lambda_k \geq 0, k \in \{1, 2, \dots, K\}. \end{cases} \tag{36}$$

Model (36) is initially a non-linear programming model, and we transform it into a linear programming model through specific mathematical conversions, as outlined in Theorem 1.

Theorem 1. *Model (36) can be transformed into an equivalent linear programming model by introducing a set of new variables, let* $r_i^{(k)} = (\tilde{E}_i^{(k)})_\alpha^L - (\tilde{E}_i)_\alpha^L$, $s_i^{(k)} = |r_i^{(k)}|$, $o_i^{(k)} = (\tilde{E}_i^{(k)})_\alpha^U - (\tilde{E}_i)_\alpha^U$, $z_i^{(k)} = |o_i^{(k)}|$, *then the linear model is shown as below:*

$$\max GC = 1 - \frac{1}{2*m*K} \sum_{k=1}^{K} \sum_{i=1}^{m} s_i^{(k)} + z_i^{(k)}$$

$$s.t. \begin{cases} r_i^{(k)} = (\tilde{E}_i^{(k)})_\alpha^L - (\tilde{E}_i)_\alpha^L, i = 1, \dots, m; k = 1, \dots, K, \\ s_i^{(k)} \geq r_i^{(k)}, i = 1, \dots, m; k = 1, \dots, K, \\ s_i^{(k)} \geq -r_i^{(k)}, i = 1, \dots, m; k = 1, \dots, K, \\ 0 \leq s_i^{(k)} \leq 1, i = 1, \dots, m; k = 1, \dots, K, \\ -1 \leq r_i^{(k)} \leq 1, i = 1, \dots, m; k = 1, \dots, K, \\ o_i^{(k)} = (\tilde{E}_i^{(k)})_\alpha^U - (\tilde{E}_i)_\alpha^U, i = 1, \dots, m; k = 1, \dots, K, \\ z_i^{(k)} \geq o_i^{(k)}, i = 1, \dots, m; k = 1, \dots, K, \\ z_i^{(k)} \geq -o_i^{(k)}, i = 1, \dots, m; k = 1, \dots, K, \\ 0 \leq z_i^{(k)} \leq 1, i = 1, \dots, m; k = 1, \dots, K, \\ -1 \leq o_i^{(k)} \leq 1, i = 1, \dots, m; k = 1, \dots, K, \\ (\tilde{E}_i)_\alpha^L = \sum_{k=1}^{K} \lambda_k \cdot (\tilde{E}_i^{(k)})_\alpha^L, i = 1, \dots, m, \\ (\tilde{E}_i)_\alpha^U = \sum_{k=1}^{K} \lambda_k \cdot (\tilde{E}_i^{(k)})_\alpha^U, i = 1, \dots, m, \\ \sum_{k=1}^{K} \lambda_k = 1, \\ \lambda_k \geq 0, k = 1, \dots, K. \end{cases} \tag{37}$$

3.5. The ranking of fuzzy cross-efficiency

An optimal set of weights $\Lambda^* = \{\lambda_k^* | k = 1, \dots, K\}$ of DMs can be obtained by solving model (37), and then the collective fuzzy cross-efficiency intervals of alternative $X_i (i = 1, \dots, m)$ under α-level set can be determined as $[(\tilde{E}_i)_\alpha^L, (\tilde{E}_i)_\alpha^U] = [\sum_{k=1}^{K} \lambda_k^* \cdot (\tilde{E}_i^{(k)})_\alpha^L, \sum_{k=1}^{K} \lambda_k^* \cdot (\tilde{E}_i^{(k)})_\alpha^U]$, which is the foundation for initiating the alternative ranking process, ultimately leading to the selection of the optimal alternative. Motivated by Wang et al. (2005), we employ the degree of preference for ranking among the collective fuzzy cross-efficiency intervals of alternatives under α-level. Subsequently, the detailed ranking process can be outlined as follows:

- Step 1 Calculate the degree of preference between alternatives based on collective α-level fuzzy cross-efficiency

$$PD_\alpha = \begin{bmatrix} - & (pd_{12})_\alpha & \cdots & (pd_{1m})_\alpha \\ (pd_{12})_\alpha & - & \cdots & (pd_{2m})_\alpha \\ \cdots & \cdots & \cdots & \cdots \\ (pd_{m1})_\alpha & (pd_{m2})_\alpha & \cdots & - \end{bmatrix}. \tag{38}$$

where

$$(pd_{di})_\alpha = P_\alpha(X_d > X_i) = \frac{max(0,(\tilde{E}_d)_\alpha^U - (\tilde{E}_i)_\alpha^L) - max(0,(\tilde{E}_d)_\alpha^L - (\tilde{E}_i)_\alpha^U)}{((\tilde{E}_d)_\alpha^U - (\tilde{E}_d)_\alpha^L) + ((\tilde{E}_i)_\alpha^U - (\tilde{E}_i)_\alpha^L)}, \quad d,i = 1,\dots,m. \tag{39}$$

If $P_\alpha(X_d > X_i) > 0.5$, it means that X_d is superior to X_i to the degree of $P_\alpha(X_d > X_i)$, denoted by $X_d \overset{P_\alpha(X_d>X_i)}{\succ} X_i$; If $P_\alpha(X_d > X_i) = 0.5$, it means that X_d is indifferent to X_i, denoted by $X_d \sim X_i$; Otherwise, X_d is inferior to X_i to the degree of $P_\alpha(X_d > X_i)$, denoted by $X_d \overset{P_\alpha(X_d>X_i)}{\prec} X_i$.

- Step 2 Compute the matrix of the preference relations

$$PR_\alpha = \begin{bmatrix} - & (pr_{12})_\alpha & \cdots & (pr_{1m})_\alpha \\ (pr_{12})_\alpha & - & \cdots & (pr_{2m})_\alpha \\ \cdots & \cdots & \cdots & \cdots \\ (pr_{m1})_\alpha & (pr_{m2})_\alpha & \cdots & - \end{bmatrix}. \tag{40}$$

where

$$(pr_{di})_\alpha = \begin{cases} 1, & \text{if } (pd_{di})_\alpha > 0.5, \\ 0, & \text{if } (pd_{di})_\alpha \le 0.5, \end{cases} \quad d,i = 1,\dots,m; d \neq i. \tag{41}$$

- Step 3 Compute the sum of elements within each row in the previously mentioned preference relation matrix
Alternative X_d is deemed superior to X_i when the sum of elements in the dth column exceeds that of the ith column. Based on this rule, we will obtain the α-level final ranking denoted as $R_\alpha = (R_1, R_2, \dots, R_m)$.

3.6. The process of the proposed approach

The proposed approach emphasizes addressing the GDM problems based on fuzzy cross-efficiency under hesitant fuzzy linguistic information. The detailed process of the proposal can be described as follows:

- **Step 1: Formulate the GDM problem.** The GDM problem involves K DMs selecting the optimal solution from m available alternatives under n attributes, consisting of P cost attributes and Q benefit attributes. The CLEs are utilized in the initial evaluation provided by the DMs and are then transformed into HFLTSs via Definition 2. The HFLTS-based assessments of DM_k are shown in Eq. (8), while their corresponding normalized format is displayed in Eq. (9).
- **Step 2: Obtain fuzzy envelopes of HFLTSs.** Convert the HFLTSs into their corresponding fuzzy envelopes, represented as TrFNs. For E_{G_H}(at least s_h) $= \{s_h, s_{h+1}, \dots, s_g\}$, use Eqs. (10)–(14) to obtain the TrFN as (e_L^h, b, e_M^g, e_R^g). For E_{G_H}(at most s_h) $= \{s_0, s_1, \dots, s_h\}$, apply Eqs. (15)–(19) to derive the TrFN as (e_L^0, e_M^0, c, e_R^g). In the case of E_{G_H}(between s_h and s_l) $= \{s_h, s_{h+1}, \dots, s_l\}$, employ Eqs. (20)–(28) to obtain the TrFN as (e_L^h, b, c, e_R^l).
- **Step 3: Develop fuzzy DEA cross-efficiency models.** Determine the optimal and worst cross-efficiency based on the α-level fuzzy inputs and outputs by using Model (32) and Model (33), then the corresponding individual fuzzy cross-efficiency intervals based on each DM' assessments can be determined, shown as Table 1.
- **Step 4: Determine the weights of DMs.** Acquire the optimal set of weights for aggregating individual fuzzy cross-efficiency intervals by constructing the maximum consensus model outlined in Model (37). Subsequently, derive the α-level collective fuzzy cross-efficiency intervals of alternatives, as illustrated in Table 1. Consequently, the final ranking of alternatives can be determined based on these collective intervals.
- **Step 5: Derive the final ranking of alternatives.** Rank the alternatives based on their collective fuzzy cross-efficiency intervals at different α-levels. Use Eqs. (38)–(41) to calculate the degree of preference among alternatives, ultimately identifying the optimal alternative.

To visually represent the workflow of the GDM framework, we have depicted it in Fig. 6.

4. Case study

In this section, we provide a concrete illustration of our proposed method by showcasing its application in selecting optimal sites for electric vehicle charging stations (EVCS). This demonstration underscores the effectiveness of our approach within GDM scenarios. Additionally, we conduct a comparative analysis to highlight the superior performance of our method.

4.1. Background description

The rise in energy consumption and carbon emissions in the transportation sector has worsened environmental pollution (Wang et al., 2019). However, in recent years, there has been a growing focus on clean energy transportation, particularly electric vehicles (EVs), which are seen as key to reducing environmental impact. As EV technology advances, EVs are becoming central to the future of transportation. EVCS, which provides crucial services to EVs, plays a vital role in this transition (Huang & Ge, 2019). In China, charging infrastructure has expanded rapidly, creating the largest network in the world. However, issues like uneven distribution, imbalanced services, and operational challenges still hinder the creation of a high-quality system. To address these, China has set a goal to establish a charging infrastructure by 2030 with wide coverage, appropriate scale, balanced structure, and comprehensive functions, supporting the growth of the new energy vehicle industry and meeting the public's charging needs.

In response to market demands and government policies, an Electric Power Company aims to establish the new EVCS sites. After scrutinizing the project-feasibility research report and government development plans, the company has identified four potential EVCS sites, the company now faces the task of engaging three experts to select the optimal one. Experts evaluate four potential EVCS sites, labeled as X_1, X_2, X_3, X_4, across six attributes (Feng et al., 2021). Economically, they analyze the payback period (A_1). In terms of environmental impact, they consider the effect on the surrounding area (A_2). Technologically, they assess system reliability (A_3) and security (A_4). From a social perspective, they evaluate traffic convenience (A_5). In terms of resources, they consider resource recycling (A_6). It is worth noting that attributes A_1 and A_2 are cost-type, while the remaining attributes are benefit-type. DMs rely on HFLTSs for providing evaluation information, as they can effectively manage the inherent uncertainty and hesitation that traditional fuzzy sets or exact values fail to capture. In the context of EVCS site selection, evaluation attributes are highly subjective, and the use of HFLTs enables DMs to express their preferences more flexibly and accurately. The linguistic term set utilized for this purpose is defined as $S = \{s_0 = \text{Extremely Bad(EB)}, s_1 = \text{Very Bad (VB)}, s_2 = \text{Bad (B)}, s_3 = \text{Fair (F)}, s_4 = \text{Good (G)}, s_5 = \text{Very Good (VG)}, s_6 = \text{Extremely Good (EG)}\}$ shown in Fig. 7. The assessments of four alternatives under six attributes, provided by three experts using HFLTSs, are shown in Tables 2–4.

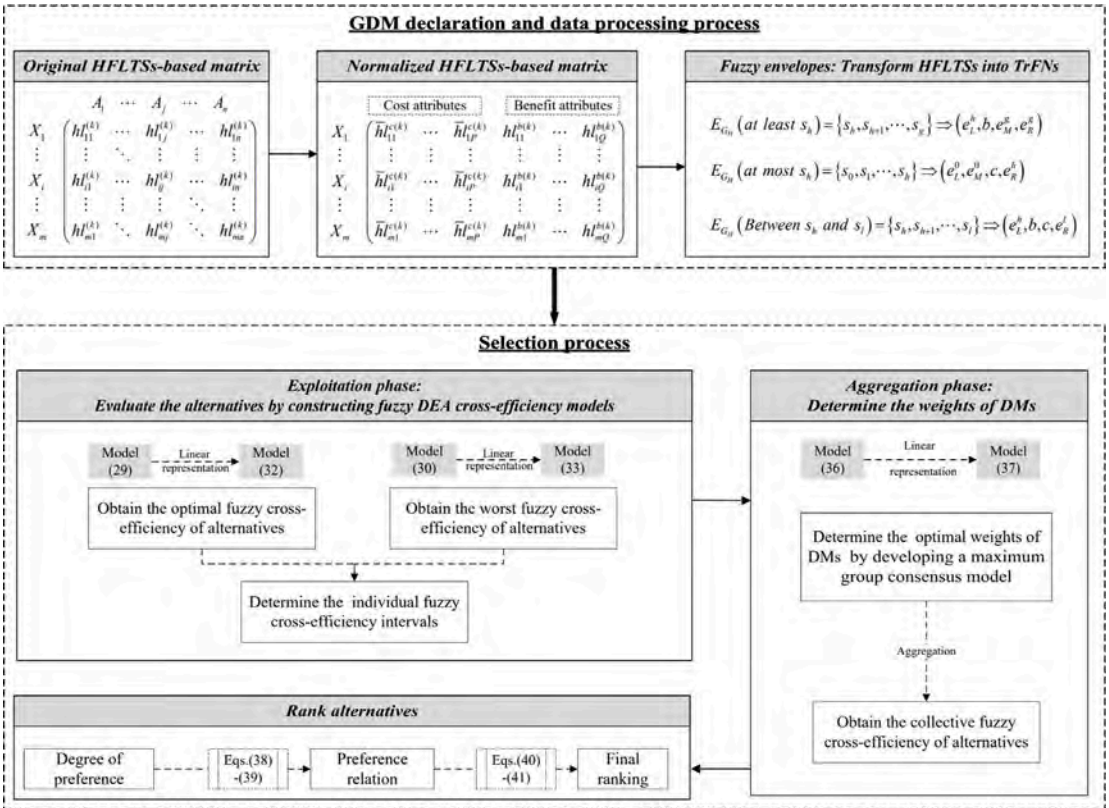


Fig. 6. The proposed GDM framework.

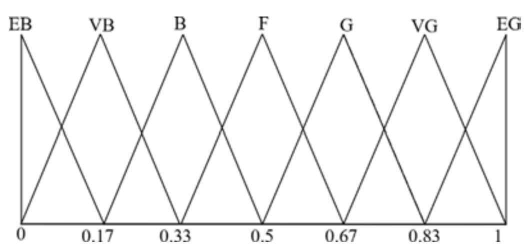


Fig. 7. The linguistic term set of $S = \{s_0, s_1, s_2, s_3, s_4, s_5, s_6\}$.

Table 2
The HFLTSs-based assessments provided by expert 1.

	A_1	A_2	A_3	A_4	A_5	A_6
X_1	VG	VG, EG	F, G	G, VG	VB, B	B, F, G
X_2	EG	G, VG	G, VG	VG, EG	VG, EG	G, VG
X_3	F, G	VG, EG	F, G	G, VG	VG, EG	G, VG
X_4	G, VG, EG	G	G, VG	VB, B	VG	F, G, VG

Table 3
The HFLTSs-based assessments provided by expert 2.

	A_1	A_2	A_3	A_4	A_5	A_6
X_1	VG, EG	VG, EG	G, VG	VG	F, G	G, VG
X_2	G	VG	G, VG, EG	VG	VG, EG	F, G
X_3	G, VG	VG	VB, B	EG	G, VG	VG, EG
X_4	G, VG	EG	G, VG	VG, EG	G	G, VG

4.2. Decision-making process

We begin by normalizing the HFLTSs-based assessments provided by three experts under the cost-type attributes A_1 and A_2. Subsequently, we convert these assessments into fuzzy envelopes, represented as

Table 4
The HFLTSs-based assessments provided by expert 3.

	A_1	A_2	A_3	A_4	A_5	A_6
X_1	G, VG	VG, EG	VB, B, F	B, F	F, G	G, VG
X_2	G	G, VG	VG, EG	G, VG	VG, EG	VG, EG
X_3	F	B	G	VG, EG	VB, B	B, F
X_4	B, F, G	VG, EG	F, G	G, VG	F	VG, EG

TrFNs, following the transformation rules outlined in Section 3.2. The results of this transformation are presented in Tables 5–7.

Next, we apply the α-level set approach to transform TrFNs into intervals, which exhibit different widths based on the α values. When α is set to 0, each interval exhibits its maximum range, whereas at α equal to 1, the range of each interval is minimum. Utilizing these varying α-level fuzzy inputs and outputs, we apply Model (6) and Model (7) to determine the upper and lower bounds of self-efficiency, and then the optimal and worst cross-efficiency can be obtained by adopting Model (32) and Model (33), respectively. The corresponding cross-efficiency intervals of alternatives based on the assessments provided by experts are presented in Table 8.

By employing Model (37), we ascertain the optimal weights of experts based on the individual cross-efficiency intervals outlined in Table 8. These optimal weights are instrumental in maximizing group consensus, thereby substantially improving the overall acceptability of collective cross-efficiency intervals. The weights allocated to experts at various α-levels are delineated in Table 9. Furthermore, Table 10 exhibits the collective cross-efficiency of alternatives, aggregated based on these optimal expert weights. For enhanced clarity, Fig. 8 depicts the lower and upper bounds of collective cross-efficiency for all available alternatives. Referring to Fig. 9, it is clear that group consensus varies alongside changes in α-levels. Overall, the collective cross-efficiency

Table 5
The fuzzy envelope for HFLTSs-based assessments provided by expert 1.

	A_1	A_2	A_3	A_4	A_5	A_6
X_1	[0, 0.17, 0.17, 0.33]	[0, 0, 0.03, 0.33]	[0.33, 0.5, 0.67, 0.83]	[0.5, 0.67, 0.83, 1]	[0, 0.17, 0.33, 0.5]	[0.17, 0.47, 0.53, 0.83]
X_2	[0, 0, 0, 0.17]	[0, 0.17, 0.33, 0.5]	[0.5, 0.67, 0.83, 1]	[0.67, 0.97, 1, 1]	[0.67, 0.97, 1, 1]	[0.5, 0.67, 0.83, 1]
X_3	[0.17, 0.33, 0.5, 0.67]	[0, 0, 0.03, 0.33]	[0.33, 0.5, 0.67, 0.83]	[0.5, 0.67, 0.83, 1]	[0.67, 0.97, 1, 1]	[0.5, 0.67, 0.83, 1]
X_4	[0, 0, 0.15, 0.5]	[0.17, 0.33, 0.33, 0.5]	[0.5, 0.67, 0.83, 1]	[0, 0.17, 0.33, 0.5]	[0.67, 0.83, 0.83, 1]	[0.33, 0.64, 0.7, 1]

Table 6
The fuzzy envelope for HFLTSs-based assessments provided by expert 2.

	A_1	A_2	A_3	A_4	A_5	A_6
X_1	[0, 0, 0.03, 0.33]	[0, 0, 0.03, 0.33]	[0.5, 0.67, 0.83, 1]	[0.67, 0.83, 0.83, 1]	[0.33, 0.5, 0.67, 0.83]	[0.5, 0.67, 0.83, 1]
X_2	[0.17, 0.33, 0.33, 0.5]	[0, 0.17, 0.17, 0.33]	[0.5, 0.85, 1, 1]	[0.67, 0.83, 0.83, 1]	[0.67, 0.97, 1, 1]	[0.33, 0.5, 0.67, 0.83]
X_3	[0, 0.17, 0.33, 0.5]	[0, 0.17, 0.17, 0.33]	[0, 0.17, 0.33, 0.5]	[0.83, 1, 1, 1]	[0.5, 0.67, 0.83, 1]	[0.67, 0.97, 1, 1]
X_4	[0, 0.17, 0.33, 0.5]	[0, 0, 0, 0.17]	[0.5, 0.67, 0.83, 1]	[0.67, 0.97, 1, 1]	[0.5, 0.67, 0.67, 0.83]	[0.5, 0.67, 0.83, 1]

Table 7
The fuzzy envelope for HFLTSs-based assessments provided by expert 3.

	A_1	A_2	A_3	A_4	A_5	A_6
X_1	[0, 0.17, 0.33, 0.5]	[0, 0, 0.03, 0.33]	[0, 0.3, 0.36, 0.67]	[0.17, 0.33, 0.5, 0.67]	[0.33, 0.5, 0.67, 0.83]	[0.5, 0.67, 0.83, 1]
X_2	[0.17, 0.33, 0.33, 0.5]	[0, 0.17, 0.33, 0.5]	[0.67, 0.97, 1, 1]	[0.5, 0.67, 0.83, 1]	[0.67, 0.97, 1, 1]	[0.67, 0.97, 1, 1]
X_3	[0.33, 0.5, 0.5, 0.67]	[0.5, 0.67, 0.67, 0.83]	[0.5, 0.67, 0.67, 0.83]	[0.67, 0.97, 1, 1]	[0, 0.17, 0.33, 0.5]	[0.17, 0.33, 0.5, 0.67]
X_4	[0.17, 0.47, 0.53, 0.83]	[0, 0, 0.03, 0.33]	[0.33, 0.5, 0.67, 0.83]	[0.5, 0.67, 0.83, 1]	[0.33, 0.5, 0.5, 0.67]	[0.67, 0.97, 1, 1]

Table 8
The cross-efficiency intervals of alternatives for different α-level.

Expert	Alternative	$\alpha = 0$	$\alpha = 0.2$	$\alpha = 0.4$	$\alpha = 0.6$	$\alpha = 0.8$	$\alpha = 1$
1	X1	[0,0.5]	[0.132,0.882]	[0.142,0.892]	[0.158,0.908]	[0.189,0.939]	[0.25,1]
	X2	[0,0.75]	[0.147,0.897]	[0.170,0.920]	[0.214,0.964]	[0.25,1]	[0.386,1]
	X3	[0.125,0.625]	[0.130,0.880]	[0.139,0.889]	[0.154,0.904]	[0.179,0.929]	[0.231,0.981]
	X4	[0.125,0.625]	[0.129,0.879]	[0.136,0.886]	[0.144,0.894]	[0.154,0.904]	[0.168,0.918]
2	X1	[0,0.5]	[0.022,0.761]	[0.063,0.780]	[0.143,0.815]	[0.318,0.875]	[0.368,0.875]
	X2	[0.125,0.375]	[0.128,0.866]	[0.133,0.856]	[0.139,0.847]	[0.147,0.738]	[0.049,0.225]
	X3	[0,0.25]	[0.125,0.858]	[0.127,0.840]	[0.131,0.824]	[0.137,0.778]	[0.043,0.218]
	X4	[0,0.5]	[0.147,0.875]	[0.175,0.875]	[0.195,0.875]	[0.220,0.875]	[0.164,0.875]
3	X1	[0.021,0.771]	[0.163,0.904]	[0.190,0.913]	[0.240,0.926]	[0.312,0.949]	[0.428,1]
	X2	[0.166,0.875]	[0.191,0.886]	[0.229,0.903]	[0.285,0.931]	[0.372,0.976]	[0.459,1]
	X3	[0.125,0.875]	[0.133,0.883]	[0.144,0.894]	[0.161,0.909]	[0.185,0.929]	[0.220,0.944]
	X4	[0.145,0.875]	[0.164,0.880]	[0.199,0.886]	[0.240,0.896]	[0.302,0.909]	[0.387,0.929]

Table 9
The weights of experts for different α-levels.

	$\alpha = 0$	$\alpha = 0.2$	$\alpha = 0.4$	$\alpha = 0.6$	$\alpha = 0.8$	$\alpha = 1$
1	0.3	0.254	0.256	0.263	0.265	0.256
2	0.442	0.492	0.483	0.478	0.468	0.485
3	0.258	0.254	0.26	0.259	0.266	0.258

intervals demonstrate a high level of acceptance. Across the range of α-levels, group consensus consistently exceeds 0.8. Specifically, at $\alpha = 0.8$, the lowest observed group consensus stands at 0.817, while at $\alpha = 0$, the highest group consensus reaches 0.895.

Fig. 10 illustrates the ranking of alternatives across various α-levels. It is evident from the figure that when α is set to 0, 0.2, and 0.4, the ranking of alternatives remains highly stable, with X_2 being identified as the optimal alternative and X_1 as the worst one. However, at $\alpha = 0.6$, there is a noticeable fluctuation in the ranking, followed by a return to stability as the α level continues to increase. More specifically, as α increases, we observe a significant improvement in the ranking of X_1, transitioning from the worst alternative to the most preferred one, which suggests that higher α-levels can enhance the performance optimization of X_1. At $\alpha = 0, 0.2, 0.4, 0.6$, X_2 consistently demonstrated top performance. However, as α continues to increase, its ranking begins to decline, indicating a diminishing effectiveness of this alternative. The ranking of X_3 exhibited minor fluctuations as α varied, transitioning from the second worst option to the worst one. However, in comparison to the significant fluctuations observed in the rankings of X_1 and X_2, the change in X_3's ranking is relatively small. Additionally, X_4 consistently remained a suboptimal solution across all α-levels, suggesting that its performance is not significantly affected by changes in α-levels.

4.3. Comparative analysis with other models

The proposed approach introduces fuzzy DEA cross-efficiency models specifically designed to address the GDM problem, incorporating hesitant fuzzy linguistic information. We will subsequently perform an extensive comparative analysis to contrast our method with other existing models, clearly illustrating its effectiveness and superiority.

4.3.1. Compare with fuzzy DEA cross-efficiency models

In this part, we will compare our approach with the fuzzy cross-efficiency models proposed by Liu and Lee (2021) and Yu et al. (2019). Both models are capable of evaluating the performance of DMUs using fuzzy inputs and outputs. The primary objective of this section is to assess the effectiveness of our proposed fuzzy DEA cross-efficiency model. The two methods chosen for comparison are not suitable for addressing the GDM problem, making them incompatible with our data format. To facilitate a fair comparison, we will utilize data from both methods to validate our fuzzy DEA cross-efficiency model. Following this, we will conduct a thorough analysis of the results to underscore the differences among fuzzy DEA cross-efficiency methods.

- Compare with Yu et al. (2019)
  In this research, Yu et al. (2019) utilized interval data to capture the dynamic nature of certain variables during the evaluation

Table 10
The collective cross-efficiency intervals of alternatives for different α-levels.

	$\alpha = 0$	$\alpha = 0.2$	$\alpha = 0.4$	$\alpha = 0.6$	$\alpha = 0.8$	$\alpha = 1$
X_1	[0.005,0.570]	[0.086,0.828]	[0.116,0.843]	[0.172,0.868]	[0.282,0.912]	[0.353,0.939]
X_2	[0.098,0.616]	[0.149,0.879]	[0.167,0.885]	[0.196,0.900]	[0.234,0.871]	[0.241,0.624]
X_3	[0.070,0.524]	[0.129,0.870]	[0.135,0.866]	[0.145,0.867]	[0.161,0.859]	[0.137,0.601]
X_4	[0.075,0.634]	[0.147,0.877]	[0.171,0.881]	[0.193,0.885]	[0.224,0.892]	[0.223,0.900]

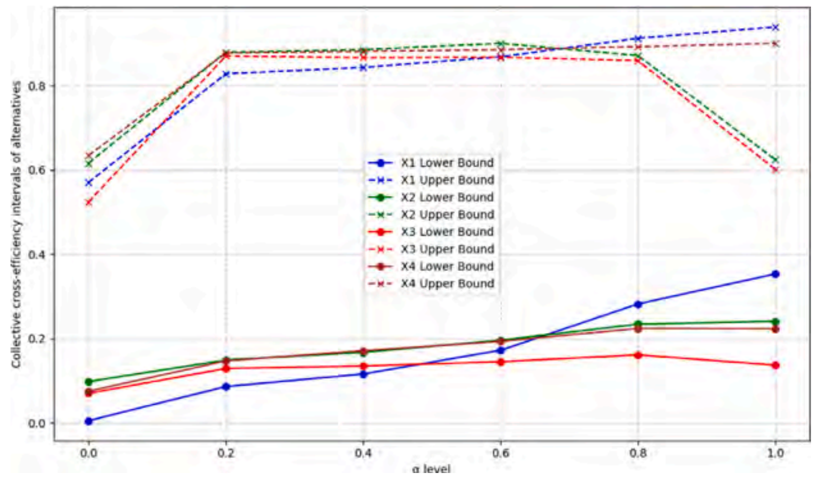


Fig. 8. The collective cross-efficiency intervals of alternatives for different α-levels.

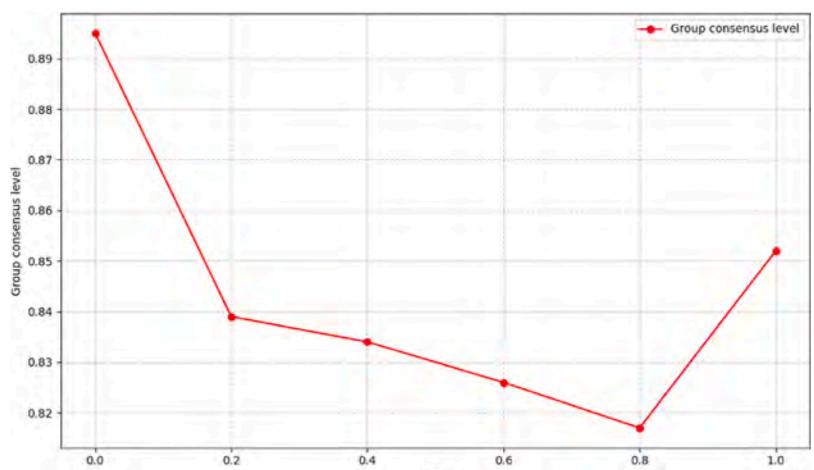


Fig. 9. The group consensus level for different α-levels.

phase, and then they developed a DEA cross-efficiency approach grounded in intervals. Furthermore, they employ stochastic multi-criteria acceptability analysis 2 to address the comprehensive ranking associated with interval efficiency. The comparative data originates from Gao et al. (2015) which worked on fuzzy evaluation of the seismic performance of reservoir dams during the Wenchuan earthquake. Fig. 11 shows a comparison between the proposed method and Yu et al. (2019)'s method in evaluating the performance of 19 reservoir dams, the rankings indicate that both methods are in agreement regarding the identification of the best and worst performing DMUs, DMU_9 and DMU_{15}, respectively. Additionally, for several of the DMUs placed in the intermediate range, specifically DMU_1, DMU_6, DMU_{16}, DMU_{17}, DMU_{18}, and DMU_{19}, there is a consistency in their rankings across both methods. For the remaining DMUs, there is a slight disparity in ranking between the two methods, with differences not exceeding two levels. Overall, Fig. 11 indicates a general alignment between the two methods in evaluating the performance of DMUs, with only minor differences in specific rankings, this indicates that our method is effective. Compared with the method proposed by Yu et al. (2019), our proposal has the following improvements:

(1) The proposed method can effectively solve GDM problems, a challenge that Yu et al.'s method cannot address, indicating that our approach further enriches the theoretical framework of GDM. The versatility of our proposal makes it suitable not only for decision-making scenarios involving precise numerical values but also for those in fuzzy environments. Therefore, our method demonstrates generality across various decision-making contexts.

(2) Our proposal ensures that the cross-efficiency of DMUs remains within the range of $[0, 1]$, preventing efficiency scores from exceeding 1. In contrast, Yu et al.'s method lacks these constraints, potentially leading to unrealistic cross-efficiency scores. This limitation imposes high requirements on the dataset, and even with precise input–output values, their method may still fail. Conversely, our proposed method guarantees robustness in evaluating alternatives throughout the decision-making process.

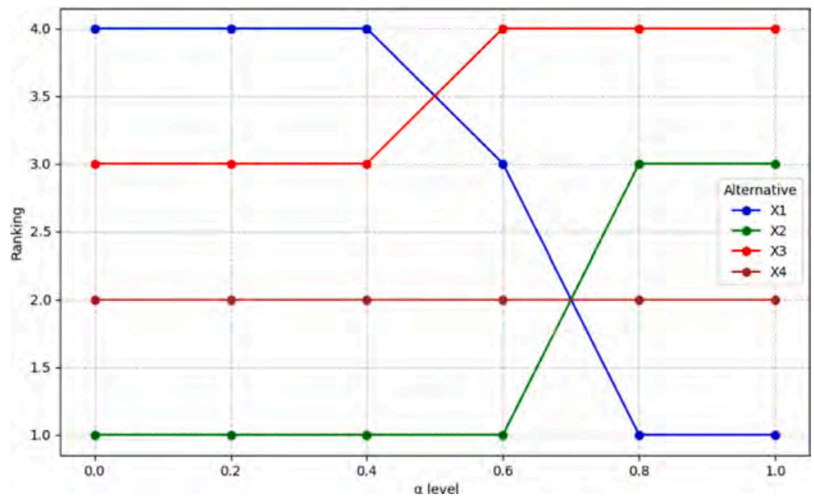


Fig. 10. The ranking of alternatives for different α-levels.

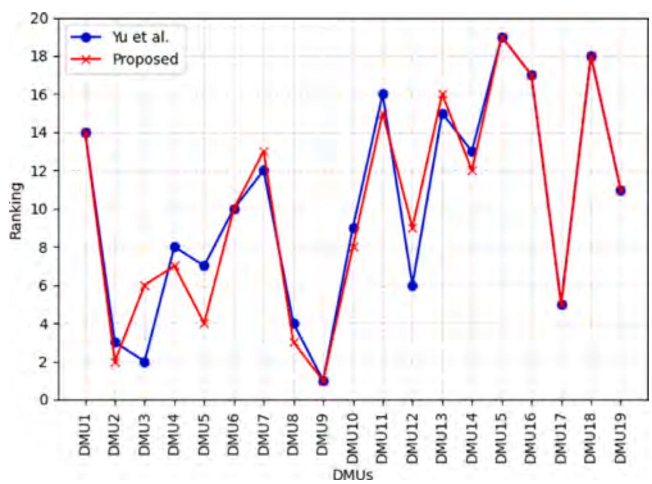


Fig. 11. The ranking of alternatives for the proposed method and Yu et al. (2019).

(3) Our model advances the method proposed by Yu et al. (2019) by treating all inputs and outputs of the DMU as fuzzy variables. The proposed method captures all possible values within their intervals, not just the endpoints. Consequently, it enhances the comprehensiveness of our method in the decision-making process.

- Compare with Liu and Lee (2021)

In this paper, Liu and Lee (2021) proposed a method for fuzzy cross-efficiency evaluation, eliminating the need for specifying weights for individual DMUs. By considering all possible weights simultaneously, their method computes fuzzy efficiency scores using an α-level-based approach. The comparative data is sourced from **Example 1** presented in Liu and Lee (2021), which involves the evaluation of five DMUs under two fuzzy inputs and two fuzzy outputs. We examined the ranking of alternatives according to our proposed method across different α-levels, and the ranking consistently remains unchanged: $DMU_5 > DMU_1 > DMU_3 > DMU_2 > DMU_4$. The ranking is shown in Fig. 12, which directly illustrates that the ranking results of the two methods are consistent.

To explore the differences between the two methods, Fig. 13 illustrates the upper and lower bounds of cross-efficiency scores for five DMUs, calculated by both methods across various α-levels. Regarding the lower bounds of cross-efficiency, the proposed method consistently derives higher scores than (Liu & Lee, 2021)'s method at all α-levels. Moreover, as α-levels increase, the lower bound cross-efficiency scores generally increase for both methods. For the upper bounds of cross-efficiency, there are no significant differences in the scores forDMU_1, DMU_3 and DMU_5 between the two methods. However, for DMU_2 and DMU_4, the proposed method generates lower scores than this compared method. Additionally, the upper bound scores for these DMUs generally decrease as α-levels increase for both methods. Furthermore, when the value of α is 1, the data processed by both models becomes crisp rather than fuzzy. In this scenario, the upper and lower bounds of cross-efficiency derived from both models align and produce identical results, demonstrating that both methods converge to the same outcome under precise input–output values. Comparing these two methods, it can be found that the proposed method is superior to this compared method in dealing with GDM problems. Besides, the research of Liu and Lee (2021) treated self-efficiency as a fuzzy variable, which leads to broader cross-efficiency intervals for alternatives, as illustrated in Fig. 13, this is not conducive to accurately reflecting the performance of alternatives. In contrast, our models determine the final self-efficiency by considering both the upper and lower bounds of self-efficiency, which aids in somewhat narrowing the cross-efficiency intervals. The ability to produce tighter intervals highlights the robustness and accuracy of our method, making it a more effective tool for GDM scenarios.

4.3.2. *Compare with group decision-making method*

This part provides a comparative analysis between our proposed method and the approach introduced by Wu et al. (2022), both of which utilize the same hesitant fuzzy linguistic information for the GDM problem. They formulated a group consensus optimization model based on assessments with HFLTSs to determine the weights of experts and then proposed a DEA-based TODIM method to calculate attribute weights and dominance degrees, thereby identifying the optimal alternative(s). Fig. 14(a) illustrates the weight assignments to experts by two

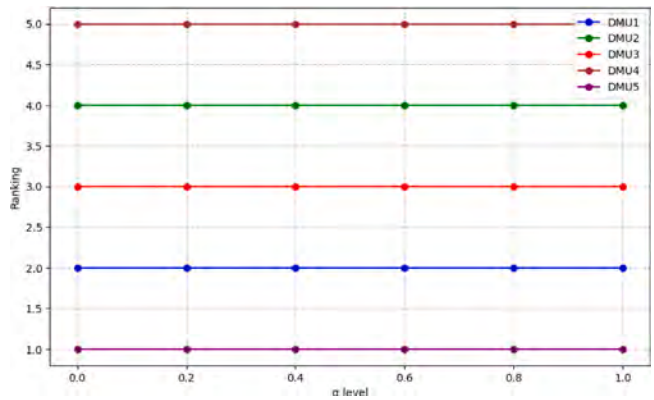


Fig. 12. The ranking of alternatives for the proposed method and Liu and Lee (2021).

different methods, revealing significant variations in the importance of the three experts between these two approaches. Specifically, the proposed method assigns the greatest importance to expert 2, while Wu et al. (2022) identifies expert 3 as the most crucial. Both methods identify X_2 as the optimal alternative and X_4 as the suboptimal one, however, there are slight discrepancies in the ranking of the other alternatives (refer to Fig. 14(b)). Compared to the method proposed by Wu et al. (2022), our proposal offers the following advantages:

(1) Both methods can use DEA models to reflect the importance of attributes. However, Wu et al.'s method employed the CCR model, which may result in multiple sets of optimal weights for attributes, leading to unnecessary confusion. Our approach overcomes the shortcomings of the CCR model by constructing a new DEA cross-efficiency method.

(2) There are differences in how linguistic-based input–output data are processed in the DEA model. Specifically, Wu et al.'s method converted linguistic data into crisp expected values using linguistic subscripts, which removes the inherent ambiguity of the original information. In contrast, our fuzzy cross-efficiency models use fuzzy intervals for inputs and outputs, considering all possible values within these intervals during the calculation process. This significantly enhances reliability and more accurately reflects the inherent fuzziness of decision-making.

(3) Our approach employs the maximum group consensus model to determine the weights of experts. Unlike the method proposed by Wu et al. (2022), which relies solely on expert evaluation information, our model utilizes individual fuzzy cross-efficiency intervals. As a result, the collective cross-efficiency intervals used for ranking reflect the consensus of both experts and alternatives.

5. Conclusions

In this study, to address the shortcomings of existing methods in extending DEA models to accommodate fuzzy linguistic information (Jin et al., 2022; Song et al., 2023), we have proposed the fuzzy DEA cross-efficiency models specifically designed to solve GDM problems. The proposal effectively integrates hesitant fuzzy linguistic evaluation information provided by DMs into the decision-making process, thereby reducing the loss of original information during calculations and enriching the methods and theoretical framework of GDM in fuzzy environments. The following are the improvements in implementing our proposed GDM framework:

- Transforming HFLTSs into corresponding fuzzy envelopes preserves the inherent fuzziness of the original linguistic information. Utilizing TrFNs for model operations in the decision-making process prevents information loss that would occur from the forced use of exact numerical values.
- The constructed fuzzy DEA cross-efficiency models encompass all values within the input–output intervals across different α-level sets, ensuring comprehensive and objective calculations of the optimal and worst cross-efficiency of alternatives.
- The development of the maximum group consensus model is employed to determine the weights of DMs, ensuring that the aggregated comprehensive cross-efficiency intervals are recognized by the group, thereby improving the acceptance of the final ranking of alternatives.

It is important to highlight that our proposal is highly adaptable to a diverse range of applications in GDM. These applications include but are not limited to, determining optimal locations for new energy vehicle charging stations, selecting sustainable green supply chains, managing environmental initiatives, and so forth. By incorporating qualitative attributes assessed by linguistic evaluations, our method significantly enhances the flexibility of decision-making processes across these fields. However, our current research still faces certain limitations. Specifically, while our proposal involves transforming HFLTSs into TrFNs to retain the inherent fuzziness of evaluation data, preserving the original information in the development of fuzzy cross-efficiency models based on α-levels remains a significant challenge. Additionally, the proposed method requires classifying the attributes of alternatives into either cost or benefit types. In some cases, this attribute classification may not be straightforward, complicating the application of our method. In the future, it would be valuable to explore strategies for minimizing information loss when using fuzzy DEA models to address GDM problems. Additionally, we plan to investigate the application of DEA methods to solve GDM problems under various attribute structures, including scenarios where attributes cannot be classified into distinct types. This research will aim to enhance the robustness and versatility of DEA models in diverse decision-making environments.

CRediT authorship contribution statement

Hui-Hui Song: Writing – review & editing, Writing – original draft, Validation, Software, Methodology, Conceptualization. **Ying-Ming Wang:** Writing – review & editing, Validation, Supervision, Funding acquisition. **Luis Martínez:** Writing – review & editing, Validation, Supervision, Conceptualization.

Data availability

Data will be made available on request.

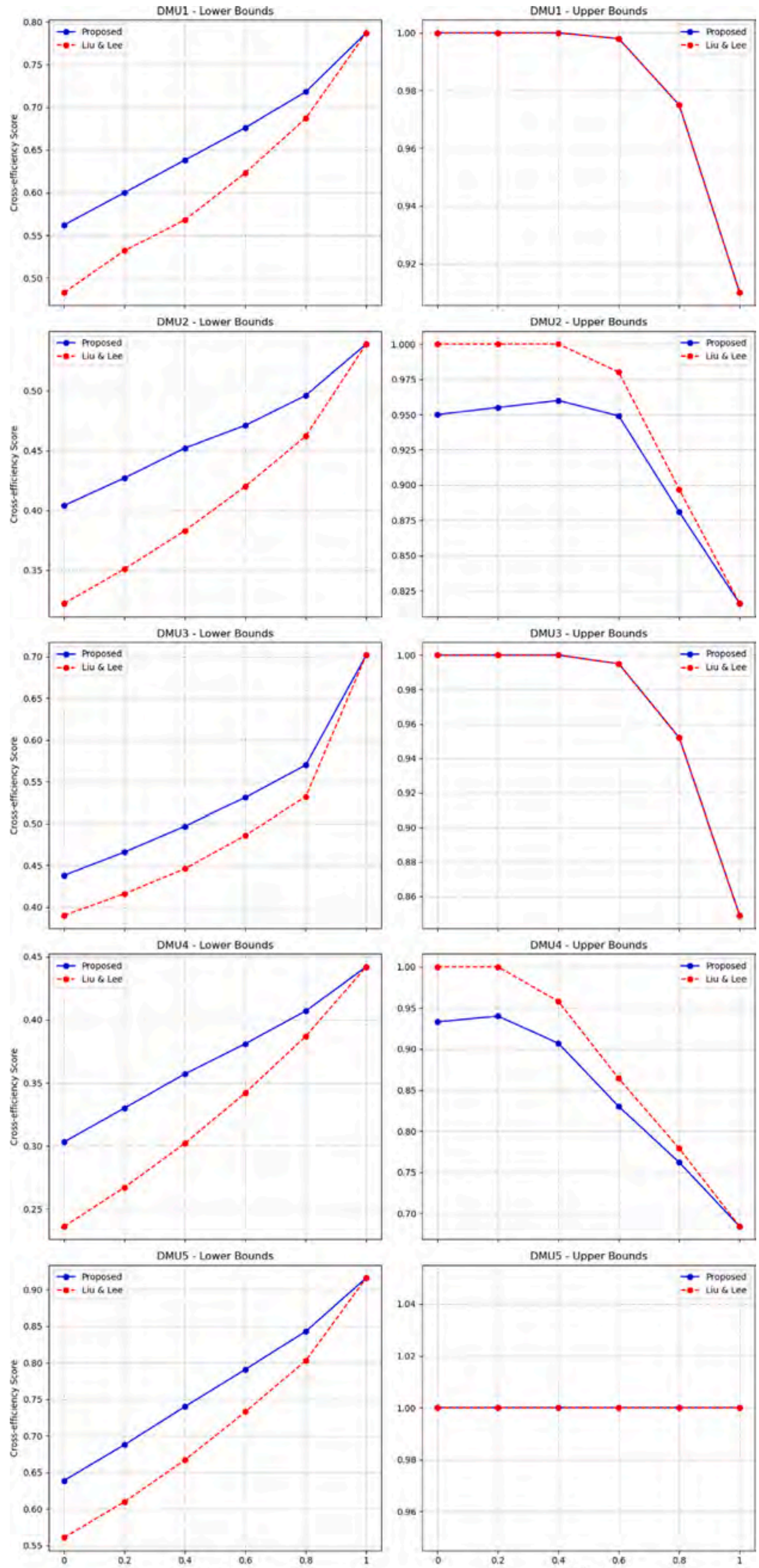
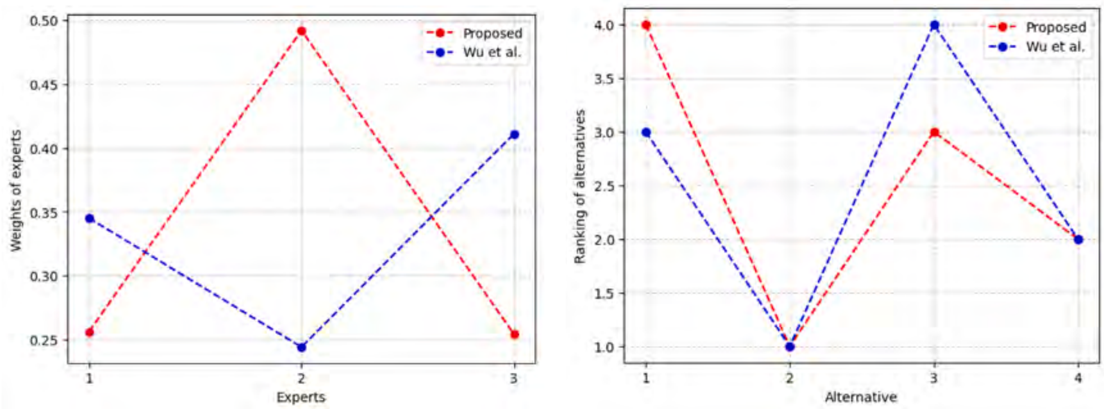


Fig. 13. Comparison of the cross-efficiency of DMUs between our method and Liu and Lee (2021).



(a) The weights of experts of two methods

(b) The ranking of alternatives of two methods

Fig. 14. Comparison between the proposed method and Wu et al. (2022).

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