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Decision-making for supplier selection problems based on QUALIFLEX technique using likelihood method in LIVIFS environment

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ABSTRACT

The notion of linguistic interval-valued intuitionistic fuzzy set (LIVIFS) is one of the best tools in order to deal with the qualitative decision making problems. Therefore, in this paper a linguistic interval-valued intuitionistic fuzzy (LIVIF) QUALIFLEX method with a likelihood-based comparison approach is proposed. First, the notion of likelihood of fuzzy preference relation (FPRs) to compare the linguistic interval valued intuitionistic fuzzy numbers (LIVIFNs). By employing a criterion-wise preference assessment of alternatives through the comparison of likelihoods, we introduce a novel QUALIFLEX-based model. This model aims to quantify the degree of concordance in the complete preference order for effective management of decisions involving multiple criteria. We demonstrate the practicality and applicability of the proposed methods through an illustrative example, specifically focusing on the context of Supplier Selection Problems. To validate the efficacy of the proposed methodology, a comparative analysis is performed against other existing methods.

1. Introduction

The advancement of the economy and society has shifted the dynamics of competition among enterprises. It is no longer a one-sided battle focused solely on price and quality; rather, it has become a competition centered around supply chains (Gokasar, Pamucar, Decevi, & Ding, 2023). At the origin of the supply chain, the supplier plays a pivotal role in its entirety (Rahimi, Kumar, Moomivand, & Yari, 2021; Sahoo, Tripathy, Pati, & Parida, 2023). Selecting the appropriate supplier (Gergin, Peker, & Gök Kısa, 2022; Gerogiannis, Kazantzi, & Anthopoulos, 2012) forms a solid foundation for the development of the enterprise. The process of evaluating and selecting suppliers is not merely the individual decision of purchasers; rather, it is a complex multi-attribute group decision-making problem Pamucar, Torkayesh, Deveci, and Simic (2022), Qahtan et al. (2023). Overall, the selection of suppliers is a intricate decision-making task that encompasses both objective (quantitative) and subjective (qualitative) evaluation criteria Rahnamay Bonab, Haseli, Rajabzadeh, Jafarzadeh Ghoushchi, Hajiaghaei-Keshteli, and Tomaskova (2023). Traditional decision-making tools and techniques are well-suited for handling quantitative criteria. In contrast, decision-making information related to ill-defined subjective criteria is inherently vague and poses a challenge. To overcome such a challenge, the notion of intuitionistic fuzzy set (IFS) was first initiated by Atanassov (1986) as a generalization of fuzzy sets (FSs) (Zadeh, 1965) and afterwards extended to the concept interval-valued intuitionistic fuzzy set (IVIFS) (Atanassov & Gargov, 1989), has demonstrated its effectiveness in addressing imprecise and vague information within ambiguous decision environments (Gou & Xu, 2017; Jamkhaneh & Garg, 2018; Luo, Xu, & Gou, 2018).

In spite of the different decision making approaches proposed to deal with the selection of suppliers (Jana, Garg, Pal, Sarkar, & Wei, 2023; Riaz, Athar Farid, Jana, Pal, & Sarkar, 2023; Shahrokhi, Bernard, & Shidpour, 2011; Song, Zhang, & Zhou, 2006; Wang & Lv, 2015; Wang, Wang, & Zhao, 2013), there is still a necessity to model and properly compute with qualitative information and its inherent uncertainty and vagueness together a more comprehensive MCDM method able to outrank alternatives in such decision contexts.

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Among the different MCDM methods that can be considered to accomplish the previous necessity, QUALIFLEX (qualitative flexible multiple criteria tool) method introduced by Paelinck (1977, 1978), it is a well-known outranking structure for solving MCDM models with crisp numbers, and one of the most sophisticated outranking decision making approach to deal with the real life decision making problems. Although, initially was proposed to deal with crisp numbers, several extensions have been proposed in the literature. Griffith, Paelinck, Griffith, and Paelinck (2011) considered the qualitative regression method (QUALIREG) based on the QUALIFLEX method. Chen and Tsui (2012) presented a model using IFSs to calculate the whole preference order's concordance level with permutation methods. In this way, they used to undertake cardinal or ordinal assessments of alternatives. Chen, Chang, and Lu (2013) also employed the QUALIFLEX approach to relate optimism and pessimism in an IFS decision environment. An interval type-2 fuzzy environment has also been included in the QUALIFLEX method (Mendel, 2007). Chen et al. (2013) used a type-2 fuzzy structure and considered an expanded QUALIFLEX approach for dealing with MCDM problems in the presence of interval type-2 trapezoidal fuzzy numbers (Chen, 2013). Even though, in decision making problems, the usefulness and applicability of the QUALIFLEX approach have been thoroughly explored, and the integration of QUALIFLEX method to the IVIF decision environment has been successfully applied, Chen (2014) presented a QUALIFLEX method with likelihood-based comparisons for solving MCDM problems based on IVIFS. But there are still important aspects to explore and improve because for instance, IVIFS are not suitable in order to deal with the qualitative information. Thereby, to deal with this type of information LIVIFS, the proposal of Deveci, Pamucar, Gokasar, Delen, and Martinez (2022), Garg and Kumar (2019a) is more fixable and suitable.

Therefore, this paper aims at introducing a new QUALIFLEX technique for solving MCDM problems using likelihood-based comparisons in a LIVIFSs environment. The key aspect of such an outranking approach involves assessing all possible alternatives in pairs, utilizing likelihood-based preference functions established on LIVIFSs. Subsequently, the preference functions are leveraged through measures of concordance and discordance to derive both partial and complete rankings for the alternatives.

Consequently, the main novelties introduced by the proposal, of a LIVIFS QUALIFLEX approach for MCDM problems that is complemented by a likelihood-based comparison procedure, are the below ones:

- To develop new QUALIFLEX technique for solving MCDM problems using likelihood-based comparisons in a LIVIFSs environment.
- To establish a new model of outranking, i.e., under the LIVIFS environment, QUALIFLEX technique, which requires likelihood-based comparisons for addressing MCDM results.
- To define the concepts of lower and upper likelihood concepts for FPRs between LIVIFNs and a likelihood measure for FPR in LIVIF situations.
- To calculate the concordance/discordance index, we develop a likelihood-based comparison idea. Furthermore, to employ incomplete or partial information, this research considers different kinds of preference arrangement decision-makers. For each permutation. We determine the optimal criteria weight vector and the optimal value for concordance/discordance index options by solving a linear programming model for consistent weighted data and conflicting weighted data. We obtain the above values by solving an integrated nonlinear programming model. We finally sort out the permutation having the maximal index for complete concordance/discordance and achieve the needed alternatives ranking order.

Eventually, the method will be applied to a supplier selection scenario to validate and show its validity and soundness.

The rest of the article is arranged as follows. In Section 2, some of the concepts of LIVIFSs are briefly provided, an MCDM problem is formulated based on LIVIFSs. The likelihood of FPRs in the LIVIFS environment is discussed in Section 3. Section 4 establishes a likelihood-based QUALIFLEX method for handling decision making difficulties with incomplete preference results under the LIVIF environment. Furthermore, in the absence of appropriate weight information, this part creates a linear programming model to determine the criterion weights. In Section 5, we look at the proposed method's viability and application, and we put it to the test in a scenario where the best supplier is chosen. In Section 5, we compare and contrast the suggested method to the IFS QUALIFLEX method and the widely utilized TOPSIS approach. Lastly, Section 6 provides sensitivity analysis along with conclusions and gives directions for future research.

2. Preliminaries

Here some vital operations and definitions of LIVIFSs theory are concisely discussed in this section. This section also includes a decision making based on LIVIFSs. The evaluations of alternative assessments in MCDM can be given using LIVIFSs because the decision-makers procedures are subject to their judgments.

2.1. Fundamental ideas of LIVIFSs theory

Definition 1 (*Garg & Kumar, 2019b*). Let $S_{[0,l]} = \{s_k \mid s_0 \leq s_l\}$ be denote a continuous linguistic term set (where $s_0 \leq s_k \leq s_l$ and l is any positive integer, and for each pair $s_\theta, s_\phi \in S_{[0,l]}$, $s_\theta > s_\phi$ iff $\theta > \phi$). A LIVIFS $\widetilde{A}$ in a finite universe of discourse X is defined as

$$\widetilde{A} = \{\langle x, s_\theta(x), s_\phi(x)\rangle | x \in X\}. \tag{1}$$

where $s_\theta(x) = [s_\theta^-(x), s_\theta^+(x)]$ and $s_\phi(x) = [s_\phi^-(x), s_\phi^+(x)]$ are subsets of $[s_0, s_l]$ and known as MD and NMD of x to the set $\widetilde{A}$ and for every $x \in X$, $s_\theta^+(x) + s_\phi^+(x) \leq s_l$ (i.e., $\theta^+(x) + \phi^+(x) \leq l$). Therefore, the following can be expressed as $\widetilde{A}$:

$$\widetilde{A} = \{\langle x, [s_\theta^-(x), s_\theta^+(x)], [s_\phi^-(x), s_\phi^+(x)]\rangle | x \in X\} \tag{2}$$

Definition 2 (*Garg & Kumar, 2019b*). The linguistic intuitionist index (degree of indeterminacy) of x to $\widetilde{A}$ is computed as

$$s_\pi(x) = [s_\pi^-(x), s_\pi^+(x)] = [l - s_\theta^+(x) - s_\phi^+(x), l - s_\theta^-(x) - s_\phi^-(x)] \tag{3}$$

The given LIVIFS $\widetilde{A}$ reduces to an ordinary LIFS, if $s^-_\theta(x) = s^+_\theta(x)$ and $s^-_\phi(x) = s^+_\phi(x)$. Further for convenience, the set of all LIVIFSs in X is denoted by LIVIFS(X).

Garg and Kumar (2019a) defined the LIVIFNs. Let $\widetilde{A}_x$ denote a LIVIFN which is defined as:

$$\widetilde{A}_x = (s_{\theta_{\widetilde{A}}}(x), s_{\phi_{\widetilde{A}}}(x)) = ([s^-_{\theta_{\widetilde{A}}}(x), s^+_{\theta_{\widetilde{A}}}(x)], [s^-_{\phi_{\widetilde{A}}}(x), s^+_{\phi_{\widetilde{A}}}(x)]). \tag{4}$$

Definition 3 (*Xian, Dong, Liu, & Jing, 2018*). Let $\alpha_1 = ([s_{\theta_1}, s_{\phi_1}], [s_{\xi_1}, s_{\psi_1}])$, $\alpha_2 = ([s_{\theta_2}, s_{\phi_2}], [s_{\xi_2}, s_{\psi_2}])$ be two LIVIFNs, then

(a) If $\theta_1 = \theta_2$, $\phi_1 = \phi_2$, $\xi_1 = \xi_2$, $\psi_1 = \psi_2$, then $\alpha_1 = \alpha_2$;
(b) If $\theta_1 \leq \theta_2, \phi_1 \leq \phi_2$ and $\xi_1 \geq \xi_2, \psi_1 \geq \psi_2$, then $\alpha_1 \leq \alpha_2$;
(c) the negation (Complementation) of α_1 is defined as $\alpha_1^c = ([s_{\xi_1}, s_{\psi_1}], [s_{\theta_1}, s_{\phi_1}])$.

2.2. Decision environment defined on LIVIFSs

Suppose to an MCDM problem. LIVIFSs represent the alternative evaluations ratings. Let denote the feasible alternatives by $\mathscr{A}_1, \mathscr{A}_2, \ldots,$ and $\mathscr{A}_m$ from which a DM can select, here m is a number of choices. Let $\mathscr{A} = \{\mathscr{A}_1, \mathscr{A}_2, \ldots, \mathscr{A}_m\}$ represent an choice set and $c_1, c_2, \ldots$ and c_n set for criteria which calculate the performances of options, where n is a criteria numbers. The criterion set can be divided into two sets, C_b and C_c, where C_b treats as benefit criteria and C_c represents a set of cost criteria, $C_b \cap C_c = \phi$ and $C_b \cup C_c = C$. Let $\widetilde{A}^b_{ij}$ and $\widetilde{A}^c_{ij}$ denote the ratings of alternative $\mathscr{A}_i \in \mathscr{A}$ (where $i = 1, 2, \ldots, m$) for the criteria $c_j \in C_b$ and C_c (where $j = 1, 2, \ldots, n$), respectively. Thus, $\widetilde{A}^b_{ij}$ and $\widetilde{A}^c_{ij}$ can be symbolized as the following:

$$\widetilde{A}^b_{ij} = (s^b_{\theta_{ij}}, s^b_{\phi_{ij}}) = ([s^{b-}_{\theta_{ij}}, s^{b+}_{\theta_{ij}}], [s^{b-}_{\phi_{ij}}, s^{b+}_{\phi_{ij}}]) \text{ for } c_j \in C_b \tag{5}$$

and

$$\widetilde{A}^c_{ij} = (s^c_{\theta_{ij}}, s^c_{\phi_{ij}}) = ([s^{c-}_{\theta_{ij}}, s^{c+}_{\theta_{ij}}], [s^{c-}_{\phi_{ij}}, s^{c+}_{\phi_{ij}}]) \text{ for } c_j \in C_c \tag{6}$$

where $s^b_{\theta_{ij}} = [s^{b-}_{\theta_{ij}}, s^{b+}_{\theta_{ij}}]$ and $s^c_{\theta_{ij}} = [s^{c-}_{\theta_{ij}}, s^{c+}_{\theta_{ij}}]$ denote the intervals of membership degree (degree of satisfaction) of alternative $\mathscr{A}_i$ for criteria c_j and $s^b_{\phi_{ij}} = [s^{b-}_{\phi_{ij}}, s^{b+}_{\phi_{ij}}]$ and $s^c_{\phi_{ij}} = [s^{c-}_{\phi_{ij}}, s^{c+}_{\phi_{ij}}]$ represent the intervals of MD (NMD) of alternative $\mathscr{A}_i$ for criteria c_j specified by the decision-maker.

To maintain the regularity, for criteria with the identical desired direction, we can take the complement of $\widetilde{A}^c_{ij}$ to handle the cost criteria as a benefit criteria. Let the LIVIFN $\widetilde{A}_{ij}$ represent the rating of the choice $\mathscr{A}_i \in A$ regarding criteria $c_j \in C$, and let

$$\widetilde{A}_{ij} = \begin{cases} \widetilde{A}^b_{ij}(= (s^b_{\theta_{ij}}, s^b_{\phi_{ij}})) \text{ when } c_i \in C_b \\ (\widetilde{A}^c_{ij})^c(= (s^c_{\phi_{ij}}, s^c_{\theta_{ij}})) \text{ when } c_i \in C_c \end{cases} \tag{7}$$

Therefore, the rating of alternative $\mathscr{A}_i$ regarding criterion c_j can be denoted as the following:

$$\widetilde{A}_{ij} = (s_{\theta_{ij}}, s_{\phi_{ij}}) = ([s^-_{\theta_{ij}}, s^+_{\theta_{ij}}], [s^-_{\phi_{ij}}, s^+_{\phi_{ij}}]) \tag{8}$$

where

$$([s^-_{\theta_{ij}}, s^+_{\theta_{ij}}], [s^-_{\phi_{ij}}, s^+_{\phi_{ij}}]) = \begin{cases} ([s^{b-}_{\theta_{ij}}, s^{b+}_{\theta_{ij}}], [s^{b-}_{\phi_{ij}}, s^{b+}_{\phi_{ij}}]) \text{ when } c_j \in C_b \\ ([s^{c-}_{\phi_{ij}}, s^{c+}_{\phi_{ij}}], [s^{c-}_{\theta_{ij}}, s^{c+}_{\theta_{ij}}]) \text{ when } c_j \in C_c \end{cases} \tag{9}$$

For every alternative $\mathscr{A}_i$ and criteria c_j, the hesitation interval of $\widetilde{A}_{ij}$ is calculated as

$$s_{\pi_{ij}} = [s^-_{\pi_{ij}}, s^+_{\pi_{ij}}] = [l - s^+_{\theta_{ij}} - s^+_{\phi_{ij}}, l - s^-_{\theta_{ij}} - s^-_{\phi_{ij}}] \tag{10}$$

The LIVIFS can denote the features for the alternative $\mathscr{A}_i$ in the manner shown below:

$$\begin{aligned} \widetilde{A}_i &= \{\langle c_1, (s_{\theta_{i1}}, s_{\phi_{i1}})\rangle, \langle c_2, (s_{\theta_{i2}}, s_{\phi_{i2}})\rangle, \ldots, \langle c_n, (s_{\theta_{in}}, s_{\phi_{in}})\rangle\} \\ &= \{\langle c_j, ([s^-_{\theta_{ij}}, s^+_{\theta_{ij}}], [s^-_{\phi_{ij}}, s^+_{\phi_{ij}}])\rangle \mid c_j \in C, j = 1, 2, \ldots, n\}, i = 1, 2, \ldots, m. \end{aligned} \tag{11}$$

3. Likelihood of LPRs between LIVIFNs

Chen (2014) proposed the notion of likelihood approach for FPRs between IVIFNs in the context of IVIFS. We propose, to extend the likelihood idea for FPRs between LIVIFNs in the context of IVIFSs in a decision environment.

Consider the two LIVIFNs, $\widetilde{A}_{\beta j}$ and $\widetilde{A}_{\beta\star j}$ signify the values of choices $\mathscr{A}_\beta$ and $\mathscr{A}_{\beta\star}$, respectively, with respect to criterion c_j, and

$$\widetilde{A}_{\beta j} = ([s^-_{\theta_{\beta j}}, s^+_{\theta_{\beta j}}], [s^-_{\phi_{\beta j}}, s^+_{\phi_{\beta j}}]), \tag{12}$$

and

$$\widetilde{A}_{\beta^\star_j} = ([s^-_{\theta_{\beta\star j}}, s^+_{\theta_{\beta\star j}}], [s^-_{\phi_{\beta\star j}}, s^+_{\phi_{\beta\star j}}]). \tag{13}$$

Suppose an event "$\mathscr{A}_{\beta j} \geq \mathscr{A}_{\beta\star j}$" indicates the "option $\mathscr{A}_\beta$, with respect to criterion c_j is not inferior to option $\mathscr{A}_{\beta\star}$". For calculating the probability of the event "$\mathscr{A}_{\beta j} \geq \mathscr{A}_{\beta\star j}$", we make use of concept of LIFS preference relation $\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta\star j}$, which is correspondingly written as $s^-_{\theta_{\beta j}} \geqslant s^-_{\theta_{\beta\star j}}$, $s^+_{\theta_{\beta j}} \geqslant s^+_{\theta_{\beta\star j}}$, $s^-_{\phi_{\beta j}} \leqslant s^-_{\phi_{\beta\star j}}$ and $s^+_{\phi_{\beta j}} \leqslant s^+_{\phi_{\beta\star j}}$ according to the inclusion relation of the LIVIFSs. Let for the LIVIFSs, $\mathcal{L}(\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta\star j})$ used the likelihood fuzzy preference relation (LFPR) $\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta\star j}$. We calculate $\mathcal{L}(\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta\star j,})$ lower LFPR and upper LFPR $\mathcal{L}^-(\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta\star j,})$ and $\mathcal{L}^+(\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta\star j,})$, respectively, of the relation $\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta\star j,}$.

Definition 4. Let $\widetilde{A}_{\beta j} = ([s^-_{\theta_{\beta j}}, s^+_{\theta_{\beta j}}], [s^-_{\phi_{\beta j}}, s^+_{\phi_{\beta j}}])$ and $\widetilde{A}_{\beta\star j} = ([s^-_{\theta_{\beta\star j}}, s^+_{\theta_{\beta\star j}}], [s^-_{\phi_{\beta\star j}}, s^+_{\phi_{\beta\star j}}])$ be any two LIVIFNs defined on C, where $0 \le s^+_{\theta_{\beta j}} + s^+_{\phi_{\beta j}} \le l$ and $0 \le s^+_{\theta_{\beta\star j}} + s^+_{\phi_{\beta\star j}} \le l$. The lower likelihood $\mathcal{L}^-(\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta\star j,})$ of FPR $\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta\star j,}$ on LIVIFSs is defined as

$$\mathcal{L}^-(\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta\star j,}) = \max\left\{ l - \max\left\{ l.\frac{(l - s^-_{\phi_{\beta\star j}}) - s^-_{\theta_{\beta j}}}{(l - s^-_{\theta_{\beta j}} - s^+_{\phi_{\beta j}}) + (l - s^+_{\theta_{\beta\star j}} - s^-_{\phi_{\beta\star j}})}, 0 \right\}, 0 \right\} \tag{14}$$

The upper LFPR $\mathcal{L}^+(\widetilde{A}_{\beta j} \ge \widetilde{A}_{\beta * j})$ of a FPR $\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta\star j}$on LIVIFSs is defined as

$$\mathcal{L}^+(\widetilde{A}_{\beta j} \ge \widetilde{A}_{\beta * j}) = \max\left\{ l - \max\left\{ l.\frac{(l - s^+_{\phi_{\beta\star j}}) - s^+_{\theta_{\beta j}}}{(l - s^+_{\theta_{\beta j}} - s^-_{\phi_{\beta j}}) + (l - s^-_{\theta_{\beta\star j}} - s^+_{\phi_{\beta\star j}})}, 0 \right\}, 0 \right\} \tag{15}$$

Property. Let $\widetilde{A}_{\beta j} = ([s^-_{\theta_{\beta j}}, s^+_{\theta_{\beta j}}], [s^-_{\phi_{\beta j}}, s^+_{\phi_{\beta j}}])$ and $\widetilde{A}_{\beta\star j} = ([s^-_{\theta_{\beta\star j}}, s^+_{\theta_{\beta\star j}}], [s^-_{\phi_{\beta\star j}}, s^+_{\phi_{\beta\star j}}])$ be any two LIVIFNs defined on C. The lower LFPR $\mathcal{L}^-(\widetilde{A}_{\beta j} \ge \widetilde{A}_{\beta\star j})$ and upper LFPR $\mathcal{L}^+(\widetilde{A}_{\beta j} \ge \widetilde{A}_{\beta * j})$ of FPR $\widetilde{A}_{\beta j} \ge \widetilde{A}_{\beta * j}$, satisfy the following properties:

$(LFPR.1)$ $\quad 0 \le \mathcal{L}^-(\widetilde{A}_{\beta j} \ge \widetilde{A}_{\beta * j}) \le l$;
$(LFPR.2)$ $\quad 0 \le \mathcal{L}^+(\widetilde{A}_{\beta j} \ge \widetilde{A}_{\beta * j}) \le l$;
$(LFPR1.3)$ $\quad \mathcal{L}^-(\widetilde{A}_{\beta j} \ge \widetilde{A}_{\beta * j}) \le \mathcal{L}^+(\widetilde{A}_{\beta j} \ge \widetilde{A}_{\beta * j})$;
$(LFPR1.4)$ $\quad \mathcal{L}^-(\widetilde{A}_{\beta j} \ge \widetilde{A}_{\beta * j}) + \mathcal{L}^+(\widetilde{A}_{\beta * j} \ge \widetilde{A}_{\beta j}) = l$

Proof. We will only prove (LFPR1.4). Regarding to the situations of $l - s^-_{\phi_{\beta * j}} \le s^-_{\theta_{\beta j}}$ and $l - s^+_{\phi_{\beta j}} \le s^+_{\theta_{\beta * j}}$, we combine these two inequalities and obtain $s^-_{\theta_{\beta j}} + s^+_{\phi_{\beta j}} + s^+_{\theta_{\beta * j}} + s^-_{\phi_{\beta * j}} \geqslant 2l$. But, this result is not sanctioned because of the postulates $s^+_{\theta_{\beta j}} + s^+_{\phi_{\beta j}} \le l$ and $s^+_{\theta_{\beta * j}} + s^+_{\phi_{\beta * j}} \le l$. So the discussion of situation $l - s^-_{\phi_{\beta * j}} \le s^-_{\theta_{\beta j}}$ and $l - s^+_{\phi_{\beta j}} \le s^+_{\theta_{\beta * j}}$ is unnecessary. Therefore, only three cases are considered in this proof, comprise with: (a) $l - s^-_{\phi_{\beta * j}} \geqslant s^-_{\theta_{\beta * j}}$ and $l - s^+_{\phi_{\beta j}} \geqslant s^+_{\theta_{\beta * j}}$; (b) $l - s^-_{\phi_{\beta * j}} \geqslant s^-_{\theta_{\beta j}}$ and $l - s^+_{\phi_{\beta j}} \le s^+_{\theta_{\beta * j}}$; and (c) $l - s^-_{\phi_{\beta * j}} \le s^-_{\theta_{\beta j}}$ and $l - s^+_{\phi_{\beta j}} \geqslant s^+_{\theta_{\beta * j}}$.

For case (a), since $l - s^-_{\theta_{\beta j}} - s^+_{\phi_{\beta j}} \geqslant 0$, $l - s^+_{\theta_{\beta * j}} - s^-_{\phi_{\beta * j}} \geqslant 0$, and $(l - s^-_{\phi_{\beta * j}}) - s^-_{\theta_{\beta j}} \geqslant 0$, we know that

$$\begin{aligned} &\max\left\{ l.\frac{(l - s^-_{\phi_{\beta * j}}) - s^-_{\theta_{\beta j}}}{(l - s^-_{\theta_{\beta j}} - s^+_{\phi_{\beta j}}) + (l - s^+_{\theta_{\beta * j}} - s^-_{\phi_{\beta * j}})}, 0 \right\} \\ &= l.\frac{(l - s^-_{\phi_{\beta * j}}) - s^-_{\theta_{\beta j}}}{(l - s^-_{\theta_{\beta j}} - s^+_{\phi_{\beta j}}) + (l - s^+_{\theta_{\beta * j}} - s^-_{\phi_{\beta * j}})} \end{aligned}$$

Moreover, since $l - s^+_{\phi_{\beta j}} \geqslant s^+_{\theta_{\beta * j}}$,

$$\begin{aligned} &l - l.\frac{(l - s^-_{\phi_{\beta * j}}) - s^-_{\theta_{\beta j}}}{(l - s^-_{\theta_{\beta j}} - s^+_{\phi_{\beta j}}) + (l - s^+_{\theta_{\beta * j}} - s^-_{\phi_{\beta * j}})} \\ &= l.\frac{l - s^+_{\phi_{\beta j}} - s^+_{\theta_{\beta * j}}}{(l - s^-_{\theta_{\beta j}} - s^+_{\phi_{\beta j}}) + (l - s^+_{\theta_{\beta * j}} - s^-_{\phi_{\beta * j}})} \geqslant 0. \end{aligned}$$

Thus, we attain

$$\begin{aligned} &\mathcal{L}^-(\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta\star j}) \\ &= l.\frac{l - s^+_{\phi_{\beta j}} - s^+_{\theta_{\beta * j}}}{(l - s^-_{\theta_{\beta j}} - s^+_{\phi_{\beta j}}) + (l - s^+_{\theta_{\beta * j}} - s^-_{\phi_{\beta * j}})}. \end{aligned}$$

Similarly, we obtain

$$\begin{aligned} &\mathcal{L}^+(\widetilde{A}_{\beta\star j} \geqslant \widetilde{A}_{\beta j}) \\ &= l.\frac{l - s^-_{\phi_{\beta * j}} - s^-_{\theta_{\beta j}}}{(l - s^+_{\theta_{\beta * j}} - s^-_{\phi_{\beta * j}}) + (l - s^-_{\theta_{\beta j}} - s^+_{\phi_{\beta j}})}. \end{aligned}$$

It can be easily understand from Case (a) that $\mathcal{L}^-(\widetilde{A}_{\beta j} \ge \widetilde{A}_{\beta * j}) + \mathcal{L}^+(\widetilde{A}_{\beta * j} \ge \widetilde{A}_{\beta j}) = l$. According to the situation in Case (b), we possess

$$\begin{aligned} &l - \max\left\{ l.\frac{(l - s^-_{\phi_{\beta * j}}) - s^-_{\theta_{\beta j}}}{(l - s^-_{\theta_{\beta j}} - s^+_{\phi_{\beta j}}) + (l - s^+_{\theta_{\beta * j}} - s^-_{\phi_{\beta * j}})}, 0 \right\} \\ &= l.\frac{l - s^+_{\phi_{\beta j}} - s^+_{\theta_{\beta * j}}}{(l - s^-_{\theta_{\beta j}} - s^+_{\phi_{\beta j}}) + (l - s^+_{\theta_{\beta * j}} - s^-_{\phi_{\beta * j}})}. \end{aligned}$$

Since in Case (b) $l - s^+_{\phi_{\beta j}} \leq s^+_{\theta_{\beta^* j}}$, we get $\mathcal{L}^-(\widetilde{A}_{\beta j} \geq \widetilde{A}_{\beta^* j}) = 0$. Moreover,

$$\max\left\{ l.\frac{(l - s^+_{\phi_{\beta j}}) - s^+_{\theta_{\beta^* j}}}{(l - s^+_{\theta_{\beta^* j}} - s^-_{\phi_{\beta^* j}}) + (l - s^-_{\theta_{\beta j}} - s^+_{\phi_{\beta j}})}, 0 \right\} = 0.$$

Which implies, $\mathcal{L}^+(\widetilde{A}_{\beta^* j} \geq \widetilde{A}_{\beta j}) = l$, and thus it is proved, that in Case (b) $\mathcal{L}^-(\widetilde{A}_{\beta j} \geq \widetilde{A}_{\beta^* j}) + \mathcal{L}^+(\widetilde{A}_{\beta^* j} \geq \widetilde{A}_{\beta j}) = l$. Now Consider the condition $l - s^-_{\phi_{\beta^* j}} \leq s^-_{\theta_{\beta j}}$ of Case (c), we get

$$l.\frac{(l - s^-_{\phi_{\beta^* j}}) - s^-_{\theta_{\beta j}}}{(l - s^-_{\theta_{\beta j}} - s^+_{\phi_{\beta j}}) + (l - s^+_{\theta_{\beta^* j}} - s^-_{\phi_{\beta^* j}})} \leq 0.$$

Therefore, $\mathcal{L}^-(\widetilde{A}_{\beta j} \geq \widetilde{A}_{\beta^* j}) = l$. Further, for condition $l - s^+_{\theta_{\beta j}} \geqslant s^+_{\theta_{\beta^* j}}$,

$$l.\frac{(l - s^+_{\phi_{\beta j}}) - s^+_{\theta_{\beta^* j}}}{(l - s^+_{\theta_{\beta^* j}} - s^-_{\phi_{\beta^* j}}) + (l - s^-_{\theta_{\beta j}} - s^+_{\phi_{\beta j}})} \geqslant 0.$$

To apply the situation that $l - s^-_{\phi_{\beta^* j}} \leq s^-_{\theta_{\beta j}}$, we get

$$\begin{aligned} \mathcal{L}^+(\widetilde{A}_{\beta^* j} \geq \widetilde{A}_{\beta j}) &= \max\left\{ l - l.\frac{(l - s^+_{\phi_{\beta j}}) - s^+_{\theta_{\beta^* j}}}{(l - s^+_{\theta_{\beta^* j}} - s^-_{\phi_{\beta^* j}}) + (l - s^-_{\theta_{\beta j}} - s^+_{\phi_{\beta j}})}, 0 \right\} \\ &= \max\left\{ l.\frac{l - s^-_{\phi_{\beta^* j}} - s^-_{\theta_{\beta j}}}{(l - s^+_{\theta_{\beta^* j}} - s^-_{\phi_{\beta^* j}}) + (l - s^-_{\theta_{\beta j}} - s^+_{\phi_{\beta j}})}, 0 \right\} = 0. \end{aligned}$$

Therefore, it is easily proved that $\mathcal{L}^-(\widetilde{A}_{\beta j} \geq \widetilde{A}_{\beta^* j}) + \mathcal{L}^+(\widetilde{A}_{\beta^* j} \geq \widetilde{A}_{\beta j}) = l$ in Case(c). Hence, we proved that (LFPR1.4) is valid. □

Example 1. Consider that the evaluations of two hotels (B_1 and B_2) in Karachi with respect to the criteria of good service (c_1), and $l = 8$, are given by the following:

$$\widetilde{B}_{11} = ([s^-_{\theta_{11}}, s^+_{\theta_{11}}], [s^-_{\phi_{11}}, s^+_{\phi_{11}}]) = ([s_3, s_5], [s_1, s_3]), \text{ and}$$
$$\widetilde{B}_{21} = ([s^-_{\theta_{21}}, s^+_{\theta_{21}}], [s^-_{\phi_{21}}, s^+_{\phi_{21}}]) = ([s_2, s_4], [s_2, s_3]).$$

Using (15) and (16), we get

$$\begin{aligned} \mathcal{L}^-(\widetilde{B}_{11} \geq \widetilde{B}_{21}) &= \max\left\{ l - \max\left\{ l.\frac{(l - s^-_{\phi_{21}}) - s^-_{\theta_{11}}}{(l - s^-_{\theta_{11}} - s^+_{\phi 11}) + (l - s^+_{\theta_{21}} - s^-_{\phi_{21}})}, 0 \right\}, 0 \right\} \\ &= \max\left\{ 8 - \max\left\{ 8.\frac{(8-2)-3}{(8-3-3)+(8-4-2)}, 0 \right\}, 0 \right\} \\ &= 2 \end{aligned}$$

$$\begin{aligned} \mathcal{L}^+(\widetilde{B}_{11} \geq \widetilde{B}_{21}) &= \max\left\{ l - \max\left\{ l.\frac{(l - s^+_{\phi_{21}}) - s^+_{\theta_{11}}}{(l - s^+_{\theta_{11}} - s^-_{\phi_{11}}) + (l - s^-_{\theta_{21}} - s^+_{\phi_{21}})}, 0 \right\}, 0 \right\} \\ &= \max\left\{ 8 - \max\left\{ 8.\frac{(8-3)-5}{(8-5-1)+(8-2-3)}, 0 \right\}, 0 \right\} \\ &= 8 \end{aligned}$$

It is clear that $\mathcal{L}^-(\widetilde{B}_{11} \geq \widetilde{B}_{21}) \leq \mathcal{L}^+(\widetilde{B}_{11} \geq \widetilde{B}_{21})$. Moreover,

$$\begin{aligned} \mathcal{L}^+(\widetilde{B}_{21} \geq \widetilde{B}_{11}) &= \max\left\{ 8 - \max\left\{ 8.\frac{(8-3)-4}{(8-4-2)+(8-3-3)}, 0 \right\}, 0 \right\} \\ &= 6 \end{aligned}$$

Thus, we have $\mathcal{L}^-(\widetilde{B}_{11} \geq \widetilde{B}_{21}) + \mathcal{L}^+(\widetilde{B}_{21} \geq \widetilde{B}_{11}) = 8 = l$.

Definition 5. Let $\widetilde{A}_{\beta j} = ([s^-_{\theta_{\beta j}}, s^+_{\theta_{\beta j}}], [s^-_{\phi_{\beta j}}, s^+_{\phi_{\beta j}}])$ and $\widetilde{A}_{\beta^* j} = ([s^-_{\theta_{\beta^* j}}, s^+_{\theta_{\beta^* j}}], [s^-_{\phi_{\beta^* j}}, s^+_{\phi_{\beta^* j}}])$ be any two LIVIFNs defined on C. The likelihood $L(\widetilde{A}_{\beta j} \geq \widetilde{A}_{\beta^* j})$ of a FPR $\widetilde{A}_{\beta j} \geq \widetilde{A}_{\beta^* j}$ on the LIVIFSs is defined as follows:

$$\mathcal{L}(\widetilde{A}_{\beta j} \geq \widetilde{A}_{\beta^* j}) = \frac{1}{2}(\mathcal{L}^-(\widetilde{A}_{\beta j} \geq \widetilde{A}_{\beta^* j}) + \mathcal{L}^+(\widetilde{A}_{\beta j} \geq \widetilde{A}_{\beta^* j})) \tag{16}$$

which means that, $\mathscr{A}_\beta$ is not inferior to alternative $\mathscr{A}_{\beta^\star}$ with respect to criterion $c_j \in C$ to the degree of $\mathcal{L}(\widetilde{A}_{\beta j} \geq \widetilde{A}_{\beta^* j})$.

Corollary 1. *Let $\widetilde{A}_{\beta j} = ([s^-_{\theta_{\beta j}}, s^+_{\theta_{\beta j}}], [s^-_{\phi_{\beta j}}, s^+_{\phi_{\beta j}}])$ and $\widetilde{A}_{\beta^\star j} = ([s^-_{\theta_{\beta^* j}}, s^+_{\theta_{\beta^* j}}], [s^-_{\phi_{\beta^* j}}, s^+_{\phi_{\beta^* j}}])$ be any two LIVIFNs defined on C. The Likelihood $\mathcal{L}(\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta^\star j})$ of $\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta^\star j}$ satisfies the following properties:*

(LFPR2.1) $0 \leq \mathcal{L}(\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta^\star j}) \leq l$;
(LFPR2.2) $\mathcal{L}(\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta^\star j}) = 0$; if $l - s^-_{\phi_{\beta j}} \leq s^-_{\theta_{\beta^\star j}}$;
(LFPR2.3) $\mathcal{L}(\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta^\star j}) = l$ if $s^-_{\theta_{\beta j}} \geqslant l - s^-_{\phi_{\beta^\star j}}$;
(LFPR2.4) $\mathcal{L}(\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta^\star j}) + \mathcal{L}(\widetilde{A}_{\beta j} \leqslant \widetilde{A}_{\beta^\star j}) = l$;
(LFPR2.5) $\mathcal{L}(\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta^\star j}) = \mathcal{L}(\widetilde{A}_{\beta j} \leqslant \widetilde{A}_{\beta^\star j}) = \frac{l}{2}$ if $\mathcal{L}(\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta^\star j}) = \mathcal{L}(\widetilde{A}_{\beta j} \leqslant \widetilde{A}_{\beta^\star j})$;
(LFPR2.6) $\mathcal{L}(\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta^\star j}) \geqslant \frac{l}{2}$ if $\mathcal{L}(\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta^{\star\star} j}) \geqslant \frac{l}{2}$ and $\mathcal{L}(\widetilde{A}_{\beta^{\star\star} j} \geqslant \widetilde{A}_{\beta^\star j}) \geqslant \frac{l}{2}$.

Proof. We will only prove (LFPR2.6). Suppose to the contrary, $\mathcal{L}(\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta^{\star\star} j}) \geqslant \frac{l}{2}$ and $\mathcal{L}(\widetilde{A}_{\beta^{\star\star} j} \geqslant \widetilde{A}_{\beta^\star j}) \geqslant \frac{l}{2}$ but not $\mathcal{L}(\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta^\star j}) \geqslant \frac{l}{2}$. Then,

$$\mathcal{L}(\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta^\star j}) < \frac{l}{2} \tag{17}$$

If $l - s^+_{\theta_{\beta j}} - s^+_{\phi_{\beta^{\star\star} j}} < 0$, we have $\mathcal{L}^+(\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta^{\star\star} j}) = l$. Following to (LFPR1.1) in Property 1, we have $L^-(\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta^{\star\star} j}) \geqslant 0$, and therefore, $\mathcal{L}(\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta^{\star\star} j}) \geqslant \frac{l}{2}$. As opposed, if $l - s^+_{\theta_{\beta j}} - s^+_{\phi_{\beta^{\star\star} j}} \geqslant 0$, then $\mathcal{L}(\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta^{\star\star} j}) \geqslant \frac{l}{2}$ which implies that $\frac{1}{2}(\mathcal{L}^-(\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta^{\star\star} j}) + \mathcal{L}^+(\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta^{\star\star} j})) \geqslant \frac{l}{2}$. Thus, $L^-(\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta^{\star\star} j}) + \mathcal{L}^+(\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta^{\star\star} j}) \geqslant l$. Since $\mathcal{L}^-(\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta^{\star\star} j}) \leq \mathcal{L}^+(\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta^{\star\star} j})$ by utilizing (P1.3) in Property 1, by necessary situation $\mathcal{L}(\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta^{\star\star} j}) \geqslant \frac{l}{2}$ is as follows:

$\mathcal{L}^+(\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta^{\star\star} j}) \geqslant \frac{l}{2}$. Since, $l - s^+_{\theta_{\beta j}} - s^+_{\phi_{\beta^{\star\star} j}} \geqslant 0$; hence,

$$l - l.\frac{(l - s^+_{\phi_{\beta^{\star\star} j}}) - s^+_{\theta_{\beta j}}}{(l - s^+_{\theta_{\beta j}} - s^-_{\phi_{\beta j}}) + (l - s^-_{\theta_{\beta^{\star\star} j}} - s^+_{\phi_{\beta^{\star\star} j}})} \geqslant \frac{l}{2}.$$

It follows that

$$l.\frac{l - s^-_{\phi_{\beta j}} - s^-_{\theta_{\beta^{\star\star} j}}}{(l - s^-_{\phi_{\beta j}} - s^-_{\theta_{\beta^{\star\star} j}}) + (l - s^+_{\theta_{\beta j}} - s^+_{\phi_{\beta^{\star\star} j}})} \geqslant \frac{l}{2}.$$

Thus we get

$$0 \leq l - s^+_{\theta_{\beta j}} - s^+_{\phi_{\beta^{\star\star} j}} \leq l - s^-_{\phi_{\beta j}} - s^-_{\theta_{\beta^{\star\star} j}} \tag{18}$$

If $l - s^+_{\phi_{\beta^* j}} - s^+_{\theta_{\beta^{\star\star} j}} < 0$, we have $L^+(\widetilde{A}_{\beta^{\star\star} j} \geqslant \widetilde{A}_{\beta^\star j}) = l$. Implies that $\mathcal{L}(\widetilde{A}_{\beta^{\star\star} j} \geqslant \widetilde{A}_{\beta^\star j}) \geqslant \frac{l}{2}$ because $\mathcal{L}^-(\widetilde{A}_{\beta^{\star\star} j} \geqslant \widetilde{A}_{\beta^\star j}) \geqslant 0$. As opposed, if $l - s^+_{\phi_{\beta^* j}} - s^+_{\theta_{\beta^{\star\star} j}} \geqslant 0$, the given supposition that $L(\widetilde{A}_{\beta^{\star\star} j} \geqslant \widetilde{A}_{\beta^\star j}) \geqslant \frac{l}{2}$ shows that $\frac{1}{2}.(\mathcal{L}^-(\widetilde{A}_{\beta^{\star\star} j} \geqslant \widetilde{A}_{\beta^\star j}) + \mathcal{L}^+(\widetilde{A}_{\beta^{\star\star} j} \geqslant \widetilde{A}_{\beta^\star j})) \geqslant \frac{l}{2}$ and $\mathcal{L}^+(\widetilde{A}_{\beta^{\star\star} j} \geqslant \widetilde{A}_{\beta^\star j}) \geqslant \frac{l}{2}$. Similarly, we can obtain

$$0 \leq l - s^+_{\theta_{\beta^{\star\star} j}} - s^+_{\phi_{\beta^* j}} \leq l - s^-_{\phi_{\beta^{\star\star} j}} - s^-_{\theta_{\beta^\star j}} \tag{19}$$

Supposition that $\mathcal{L}(\widetilde{A}_{\beta j} \geq \widetilde{A}_{\beta^{\star\star} j}) \geqslant \frac{l}{2}$. Following to (P2.4) in Property 2, we have $\mathcal{L}(\widetilde{A}_{\beta j} \geq \widetilde{A}_{\beta^{\star\star} j}) + L(\widetilde{A}_{\beta j} \leq \widetilde{A}_{\beta^{\star\star} j}) = l$. Because $L(\widetilde{A}_{\beta j} \geq \widetilde{A}_{\beta^{\star\star} j}) \geqslant \frac{l}{2}$, we attain $\mathcal{L}(\widetilde{A}_{\beta j} \leq \widetilde{A}_{\beta^{\star\star} j}) = \mathcal{L}(\widetilde{A}_{\beta^{\star\star} j} \geqslant \widetilde{A}_{\beta j}) \leq \frac{l}{2}$. When $l - s^+_{\phi_{\beta j}} - s^+_{\theta_{\beta^{\star\star} j}} < 0$, also we have $\mathcal{L}^+(\widetilde{A}_{\beta^{\star\star} j} \geqslant \widetilde{A}_{\beta j}) = l$, which disagree with $L(\widetilde{A}_{\beta^{\star\star} j} \geqslant \widetilde{A}_{\beta j}) \leq \frac{l}{2}$. Hence, it is logical that $l - s^+_{\phi_{\beta j}} - s^+_{\theta_{\beta^{\star\star} j}} \geqslant 0$. Thus, the sufficient status that $\mathcal{L}(\widetilde{A}_{\beta^{\star\star} j} \geqslant \widetilde{A}_{\beta j}) \leq \frac{l}{2}$ is asserted as follows: $\mathcal{L}^+(\widetilde{A}_{\beta^{\star\star} j} \geqslant \widetilde{A}_{\beta j}) \leq \frac{l}{2}$. It obeys that

$$l - l.\frac{(l - s^+_{\phi_{\beta j}}) - s^+_{\theta_{\beta^{\star\star} j}}}{(l - s^+_{\theta_{\beta^{\star\star} j}} - s^-_{\phi_{\beta^{\star\star} j}}) + (l - s^-_{\theta_{\beta j}} - s^+_{\phi_{\beta j}})} \leq \frac{l}{2}. \quad \square$$

Thus, we get

$$l.\frac{l - s^-_{\theta_{\beta j}} - s^-_{\phi_{\beta^{\star\star} j}}}{(l - s^-_{\theta_{\beta j}} - s^-_{\phi_{\beta^{\star\star} j}}) + (l - s^+_{\theta_{\beta^{\star\star} j}} - s^+_{\phi_{\beta j}})} \leq \frac{l}{2}.$$

Because $l - s^-_{\theta_{\beta j}} - s^-_{\phi_{\beta^{\star\star} j}} \geqslant l - s^+_{\theta_{\beta j}} - s^+_{\phi_{\beta^{\star\star} j}} \geqslant 0$, we have

$$0 \leq l - s^-_{\theta_{\beta j}} - s^-_{\phi_{\beta^{\star\star} j}} \leq l - s^+_{\theta_{\beta^{\star\star} j}} - s^+_{\phi_{\beta j}} \tag{20}$$

Supposition that $L(\widetilde{A}_{\beta^{\star\star} j} \geqslant \widetilde{A}_{\beta^\star j}) \geqslant \frac{l}{2}$. We get $\mathcal{L}(\widetilde{A}_{\beta^\star j} \geqslant \widetilde{A}_{\beta^{\star\star} j}) \leq \frac{l}{2}$ because $\mathcal{L}(\widetilde{A}_{\beta^{\star\star} j} \geqslant \widetilde{A}_{\beta^\star j}) + \mathcal{L}(\widetilde{A}_{\beta^\star j} \geqslant \widetilde{A}_{\beta^{\star\star} j}) = l$. If $l - s^+_{\phi_{\beta^{\star\star} j}} - s^+_{\theta_{\beta^\star j}} < 0$, then $L^+(\widetilde{A}_{\beta^\star j} \geqslant \widetilde{A}_{\beta^{\star\star} j}) = l$, which disagree with $L(\widetilde{A}_{\beta^\star j} \geqslant \widetilde{A}_{\beta^{\star\star} j}) \leq \frac{l}{2}$. Hence, the state $l - s^+_{\phi_{\beta^{\star\star} j}} - s^+_{\theta_{\beta^\star j}} \geqslant 0$ is well founded. The condition sufficient of $L(\widetilde{A}_{\beta^\star j} \geqslant \widetilde{A}_{\beta^{\star\star} j}) \leq \frac{l}{2}$ is as follows:

$L^+(\widetilde{A}_{\beta^\star j} \geqslant \widetilde{A}_{\beta^{\star\star} j}) \leq \frac{l}{2}$. Similarly, we obtain

$$0 \leq l - s^-_{\theta_{\beta^{\star\star} j}} - s^-_{\phi_{\beta^\star j}} \leq l - s^+_{\theta_{\beta^\star j}} - s^+_{\phi_{\beta^{\star\star} j}}.$$

Summating the inequalities from (19)–(21), we get

$$\begin{aligned} 0 &\leq l - s^+_{\theta_{\beta j}} - s^+_{\phi_{\beta^{\star\star} j}} + l - s^+_{\theta_{\beta^{\star\star} j}} - s^+_{\phi_{\beta^\star j}} + l - s^-_{\theta_{\beta^{\star\star} j}} - s^-_{\phi_{\beta^\star j}} \\ &\leq l - s^-_{\phi_{\beta j}} - s^-_{\theta_{\beta^{\star\star} j}} + l - s^-_{\phi_{\beta^{\star\star} j}} - s^-_{\theta_{\beta^\star j}} + \end{aligned}$$

$$l - s^{+}_{\theta_{\beta\star\star j}} - s^{+}_{\phi_{\beta j}} + l - s^{+}_{\theta_{\beta\star j}} - s^{+}_{\phi_{\beta\star\star j}}.$$

Accordingly

$$l - s^{+}_{\theta_{\beta j}} - s^{+}_{\phi_{\beta\star j}} + l - s^{-}_{\theta_{\beta j}} - s^{-}_{\phi_{\beta\star j}} \leq l - s^{-}_{\phi_{\beta j}} - s^{-}_{\theta_{\beta\star j}} + l - s^{+}_{\phi_{\beta j}} - s^{+}_{\theta_{\beta\star j}} \tag{21}$$

Note that

$$l - s^{+}_{\theta_{\beta j}} - s^{+}_{\phi_{\beta\star j}} + l - s^{-}_{\theta_{\beta j}} - s^{-}_{\phi_{\beta\star j}} \geq l - s^{+}_{\theta_{\beta j}} - s^{+}_{\phi_{\beta\star j}} + l - s^{+}_{\theta_{\beta j}} - s^{+}_{\phi_{\beta\star j}} = 2.(l - s^{+}_{\theta_{\beta j}} - s^{+}_{\phi_{\beta\star j}})$$

and

$$l - s^{-}_{\phi_{\beta j}} - s^{-}_{\theta_{\beta\star j}} + l - s^{+}_{\phi_{\beta j}} - s^{+}_{\theta_{\beta\star j}} \leq l - s^{-}_{\phi_{\beta j}} - s^{-}_{\theta_{\beta\star j}} + l - s^{-}_{\phi_{\beta j}} - s^{-}_{\theta_{\beta\star j}} = 2.(l - s^{-}_{\phi_{\beta j}} - s^{-}_{\theta_{\beta\star j}}).$$

Hence, the inequality in (22) creates

$$2.(l - s^{+}_{\theta_{\beta j}} - s^{+}_{\phi_{\beta\star j}}) \leq 2.(l - s^{-}_{\phi_{\beta j}} - s^{-}_{\theta_{\beta\star j}}).$$

Or equivalently $l - s^{+}_{\theta_{\beta j}} - s^{+}_{\phi_{\beta\star j}} \leq l - s^{-}_{\phi_{\beta j}} - s^{-}_{\theta_{\beta\star j}}$.

If $l - s^{+}_{\theta_{\beta j}} - s^{+}_{\phi_{\beta\star j}} \geq 0$, then we have $(l - s^{-}_{\phi_{\beta j}} - s^{-}_{\theta_{\beta\star j}}) + (l - s^{+}_{\theta_{\beta j}} - s^{+}_{\phi_{\beta\star j}}) \leq 2.(l - s^{-}_{\phi_{\beta j}} - s^{-}_{\theta_{\beta\star j}})$. The above inequality can be rewritten as follows:

$$\frac{l - s^{-}_{\phi_{\beta j}} - s^{-}_{\theta_{\beta\star j}}}{(l - s^{-}_{\phi_{\beta j}} - s^{-}_{\theta_{\beta\star j}}) + (l - s^{+}_{\theta_{\beta j}} - s^{+}_{\phi_{\beta\star j}})} \geq \frac{1}{2}$$

Or equivalently,

$$l - l.\frac{(l - s^{+}_{\phi_{\beta\star j}}) - s^{+}_{\theta_{\beta j}}}{(l - s^{+}_{\theta_{\beta j}} - s^{-}_{\phi_{\beta j}}) + (l - s^{-}_{\theta_{\beta\star j}} - s^{+}_{\phi_{\beta\star j}})} \geq \frac{l}{2},$$

which implies that $L^{+}(\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta\star j}) \geq \frac{l}{2}$. If $l - s^{+}_{\theta_{\beta j}} - s^{+}_{\phi_{\beta\star j}} < 0$, then we have $L^{+}(\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta\star j}) = l$. Therefore, the necessary condition that $L(\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta\star j}) \geq \frac{l}{2}$ is satisfied.

Conversely, if $s^{+}_{\phi_{\beta j}} + s^{+}_{\theta_{\beta\star j}} \leq s^{-}_{\theta_{\beta j}} + s^{-}_{\phi_{\beta\star j}}$, we have $l - s^{-}_{\phi_{\beta\star j}} - s^{-}_{\theta_{\beta j}} \leq l - s^{+}_{\theta_{\beta\star j}} - s^{+}_{\phi_{\beta j}}$. Then, it follows that

$$2.(l - s^{-}_{\phi_{\beta\star j}} - s^{-}_{\theta_{\beta j}}) \leq (l - s^{-}_{\phi_{\beta\star j}} - s^{-}_{\theta_{\beta j}}) + (l - s^{+}_{\theta_{\beta\star j}} - s^{+}_{\phi_{\beta j}}).$$

Thus, the above inequality can be rewritten as follows:

$$\frac{l - s^{-}_{\phi_{\beta\star j}} - s^{-}_{\theta_{\beta j}}}{(l - s^{-}_{\phi_{\beta\star j}} - s^{-}_{\theta_{\beta j}}) + (l - s^{+}_{\theta_{\beta\star j}} - s^{+}_{\phi_{\beta j}})} \leq \frac{1}{2}.$$

Or equivalently,

$$l - l.\frac{(l - s^{+}_{\phi_{\beta j}}) - s^{+}_{\theta_{\beta\star j}}}{(l - s^{+}_{\theta_{\beta\star j}} - s^{-}_{\phi_{\beta\star j}}) + (l - s^{-}_{\theta_{\beta j}} - s^{+}_{\phi_{\beta j}})} \leq \frac{l}{2},$$

which produce that $\mathcal{L}^{+}(\widetilde{A}_{\beta\star j} \geqslant \widetilde{A}_{\beta j}) \leq \frac{l}{2}$. Thus, follows sufficient condition as $\mathcal{L}(\widetilde{A}_{\beta\star j} \geqslant \widetilde{A}_{\beta j}) \leq \frac{l}{2}$ is fulfilled. When $s^{+}_{\phi_{\beta j}} + s^{+}_{\theta_{\beta\star j}} > s^{-}_{\theta_{\beta j}} + s^{-}_{\phi_{\beta\star j}}$, the condition necessary as $\mathcal{L}(\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta\star j}) \geq \frac{l}{2}$ is satisfied still. It is shown that $\mathcal{L}(\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta\star j}) \geq \frac{l}{2}$; which contradict Eq. (19). Hence, (LFPR2.6) is valid.

Example 2. Once again Consider, Example 1, the likelihood $\mathcal{L}(\widetilde{B}_{11} \geq \widetilde{B}_{21})$ of a FPR $\widetilde{B}_{11} \geq \widetilde{B}_{21}$ is calculated as follows:

$$\mathcal{L}(\widetilde{B}_{11} \geq \widetilde{B}_{21}) = \frac{1}{2}(L^{-}(\widetilde{B}_{11} \geq \widetilde{B}_{21}) + \mathcal{L}^{+}(\widetilde{B}_{21} \geq \widetilde{B}_{11})) = \frac{1}{2}(2 + 8) = 5.$$

Suppose the evaluation of the third hotel (B_3) on good service(c_1) is given by

$$\widetilde{B}_{31} = ([s^{-}_{\theta_{31}}, s^{+}_{\theta_{31}}], [s^{-}_{\phi_{31}}, s^{+}_{\phi_{31}}]) = ([s_4, s_6], [s_1, s_2])$$

Using (15), (16) and (21), we get $\mathcal{L}(\widetilde{B}_{31} \geq \widetilde{B}_{11}) = 5$ and $\mathcal{L}(\widetilde{B}_{31} \geq \widetilde{B}_{21}) = 6$. We note that $\mathcal{L}(\widetilde{B}_{31} \geq \widetilde{B}_{11}) \geq \frac{l}{2}$ and $\mathcal{L}(\widetilde{B}_{11} \geq \widetilde{B}_{21}) \geq \frac{l}{2}$. It follows that $L(\widetilde{B}_{31} \geq \widetilde{B}_{21}) \geq \frac{l}{2}$.

4. Linguistic IVIF QUALIFLEX method

This section compares LIVIFN rating values and uses the concept of Likelihood of FPR to provide a QUALIFLEX technique using a linear programming model for solving MCDM issues in a LIVIFS environment with partial preference data.

4.1. Proposed method

Here, LIVIFN decision matrix $\widetilde{D}_l$ in (22), which hands over to m options on n criteria when $m!$ permutations of the ordering of the options exist. The LIVIF decision matrix $\widetilde{D}_l$ can be succinctly denoted as follows:

$$\widetilde{D}_l = \begin{array}{c|cccc} & c_1 & c_2 & \dots & c_n \\ \hline \mathscr{A}_1 & \widetilde{A}_{11} & \widetilde{A}_{12} & \dots & \widetilde{A}_{1n} \\ \mathscr{A}_2 & \widetilde{A}_{21} & \widetilde{A}_{22} & \dots & \widetilde{A}_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathscr{A}_m & \widetilde{A}_{m1} & \widetilde{A}_{m2} & \dots & \widetilde{A}_{mn} \end{array} \tag{22}$$

Assume P_t denotes the tth permutation:

$$P_t = (\dots, A_\beta, \dots, A_{\beta\star}, \dots), \text{ for } t = 1, 2, \dots, m!, \tag{23}$$

Concordance occur when $\mathscr{A}_\beta$ ranked is greater than or equal to $\mathscr{A}_{\beta\star}$. If $\mathscr{A}_\beta$ and $\mathscr{A}_{\beta\star}$ concordance takes place when the two pre-orders are ranked similarly. Discordance happens if they hold opposite demand positions in the two pre-orders.

Since the LIVIFN ratings $\widetilde{A}_{\beta j}$ and $\widetilde{A}_{\beta^* j}$ of the alternatives $\mathscr{A}_\beta$ and $\mathscr{A}_{\beta\star}$, respectively, are represented as $\widetilde{A}_{\beta j} = ([s^-_{\theta_{\beta j}}, s^+_{\theta_{\beta j}}], [s^-_{\phi_{\beta j}}, s^+_{\phi_{\beta j}}])$ and $\widetilde{A}_{\beta^* j} = ([s^-_{\theta_{\beta^* j}}, s^+_{\theta_{\beta^* j}}], [s^-_{\phi_{\beta^* j}}, s^+_{\phi_{\beta^* j}}])$, regarding to each criterion $c_j \in C$. As described above, the likelihood $\mathcal{L}(\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta\star j})$ of the LIVIFNs $\widetilde{A}_{\beta j}$ and $\widetilde{A}_{\beta^* j}$ has many significant characteristics, some of which are already covered in Properties 1 and 2. The likelihood of the FPR linkages between the LIVIFN ratings can then be used to build a comparison. Because of the ranking results of LIVIFNs, where likelihood-based comparison for the computation of the concordance/discordance index.

We computed $\mathcal{L}(\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta\star j})$ for each pair of options $(\mathscr{A}_\beta, \mathscr{A}_{\beta\star})$ $(\mathscr{A}_\beta, \mathscr{A}_{\beta\star} \in \mathscr{A})$ to conduct a comparison between $\widetilde{A}_{\beta j}$ and $\widetilde{A}_{\beta^* j}$. Follows (LFPR2.5) of Property 2, if $\mathcal{L}(\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta\star j}) = L(\widetilde{A}_{\beta j} \leq \widetilde{A}_{\beta\star j})$, indicates that $L(\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta\star j}) = L(\widetilde{A}_{\beta j} \leq \widetilde{A}_{\beta\star j}) = \frac{l}{2}$. Thus, for every pair of options $(\mathscr{A}_\beta, \mathscr{A}_{\beta\star})$ at the level of pre-order as per for $c_j \in C$ and the ordering analogous to P_t, the concordance/discordance index $I^t_j(\mathscr{A}_\beta, \mathscr{A}_{\beta\star})$ is expressed as follows:

$$I^t_j(\mathscr{A}_\beta, \mathscr{A}_{\beta\star}) = \mathcal{L}(\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta\star j}) - \frac{l}{2}, \tag{24}$$

Implies $I^t_j(\mathcal{A}_\beta, \mathcal{A}_{\beta\star}) \in [-\frac{l}{2}, \frac{l}{2}]$. The concordance exists If $\mathcal{L}(\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta\star j}) > \frac{l}{2}$, and we obtain $I^t_j(\mathscr{A}_\beta, \mathscr{A}_{\beta\star}) > 0$. If $\mathcal{L}(\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta\star j}) = \frac{l}{2}$, exaequo exists and $I^t_j(\mathscr{A}_\beta, \mathscr{A}_{\beta\star}) = 0$. If $\mathcal{L}(\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta\star j}) < \frac{l}{2}$ discordance occurs, and we attain $I^t_j(\mathscr{A}_\beta, \mathscr{A}_{\beta\star}) < 0$. Moreover, for the options at the level of pre-order for $c_j \in C$ and the ranking analogous to P_t, the concordance/discordance index I^t_j is defined as:

$$I^t_j = \sum_{A_\beta, A_{\beta\star} \in A} I^t_j(A_\beta, A_{\beta\star}). \tag{25}$$

While for the pair of alternatives $(A_\beta, A_{\beta\star})$ in P_t the index value $I^t_j(A_\beta, A_{\beta\star})$, according to the criterion c_j, can be entertained as an evaluation value. In practical application, there is no objection on allocating unbalanced importance to each criteria. Let the importance weight of each criterion $c_j \in C$ corresponding to the permutation P_t is denoted by w^t_j, which satisfies the normalize conditions $w^t_j \in [0,1] (j = 1, 2, \dots, n)$ and $\sum_{j=1}^n w^t_j = 1$. Let represent the set of all weight vectors by ρ_0, and

$$\rho_0 = \left\{ (w^t_1, w^t_2, \dots, w^t_n) | w^t_j \geq 0 (j = 1, 2, \dots, n), \sum_{j=1}^n w^t_j = 1 \right\} \tag{26}$$

We can be used primary basic ranking forms [18, 19] for the construction of incomplete data on the criterion weights given by the DM. We apply the five basic ranking forms to handle incomplete data on the criterion weights for a decision-making problem containing incomplete weight information.

(i) weak ranking:

$$\rho_1 = \left\{ (w^t_1, w^t_2, \dots, w^t_n) \in \rho_0 | w^t_{j_1} \geq w^t_{j_2} \text{ for all } j_1 \in \Gamma_1 \text{ and } j_2 \in \Lambda_1 \right\}, \tag{27}$$

where Γ_1 and Λ_1 are disjoint and subsets of the subscript index set $N = \{1, 2, \dots, n\}$ of all criteria.

(ii) strict ranking:

$$\rho_2 = \left\{ (w^t_1, w^t_2, \dots, w^t_n) \in \rho_0 | w^t_{j_1} - w^t_{j_2} \geq \psi_{j_1 j_2} \text{ for all } j_1 \in \Gamma_2 \text{ and } j_2 \in \Lambda_2 \right\} \tag{28}$$

where $\psi_{j_1 j_2}$ which satisfies the condition $\psi_{j_1 j_2} > 0$, is a constant, and Γ_2 and Λ_2 are disjoint subsets of N.

(iii) ranking of difference (or strength of preference):

$$\rho_3 = \left\{ (w^t_1, w^t_2, \dots, w^t_n) \in \rho_0 | w^t_{j_1} - w^t_{j_2} \geq w^t_{j_2} - w^t_{j_3} \text{ for all } j_1 \in \Gamma_3,\ j_2 \in \Lambda_3 \text{ and } j_3 \in \eta_3 \right\} \tag{29}$$

where Γ_3, Λ_3 and η_3 are disjoint, and $\Gamma_3, \Lambda_3, \eta_3 \subset N$.

(iv) The interval bound:

$$\rho_4 = \left\{ (w^t_1, w^t_2, \dots, w^t_n) \in \rho_0 | \sigma_{j_1} + \varepsilon_{j_1} \geq w^t_{j_1} \geq \sigma_{j_1} \text{ for all } j_1 \in \Gamma_4 \right\}, \tag{30}$$

where $\sigma_{j_1} \geq 0$ and $\varepsilon_{j_1} \geq 0$ along with the condition $0 \leq \sigma_{j_1} \leq \sigma_{j_1} + \varepsilon_{j_1} \leq 1$ are constants and $\Gamma_4 \subset N$.

(v) ratio bound (or ranking with multiples):

$$\rho_5 = \left\{ (w^t_1, w^t_2, \dots, w^t_n) \in \rho_0 | w^t_{j_1} \geq \sigma_{j_1 j_2} . w^t_{j2} \text{ for all } j_1 \in \Gamma_5 \text{ and } j_2 \in \Lambda_5 \right\}, \tag{31}$$

and the requirement is satisfied by $\sigma_{j_{12}}$, and $0 \leq \sigma_{j_{12}} \leq 1$ where Γ_5 and Λ_5 are disjoint subsets of N. Assume that ρ is a collection of the weights of the criteria that are known, and

$$\rho = \rho_1 \cup \rho_2 \cup \rho_3 \cup \rho_4 \cup \rho_5. \tag{32}$$

With the given conditions in ρ, for each pair of alternatives $(\mathscr{A}_\beta, \mathscr{A}_{\beta^\star})$ $(\mathscr{A}_\beta, \mathscr{A}_{\beta^\star} \in \mathscr{A})$ the ranking corresponding to P_t and the weighted concordance/discordance index at the level of the pre-order with regard to the n criteria in C $I^t(\mathscr{A}_\beta, \mathscr{A}_{\beta^\star})$ is expressed as:

$$I^t(\mathscr{A}_\beta, \mathscr{A}_{\beta^\star}) = \sum_{j=1}^{n} I^t_j(\mathscr{A}_\beta, \mathscr{A}_{\beta^\star}).w^t_j, \tag{33}$$

where $(w^t_1, w^t_2, \dots, w^t_n) \in \rho$.

The comprehensive concordance/discordance index I^t for the permutation P_t by combining I^t_j and $I^t(\mathscr{A}_\beta, \mathscr{A}_{\beta^\star})$ becomes

$$I^t = \sum_{j=1}^{n} \sum_{\mathscr{A}_\beta, \mathscr{A}_{\beta^\star} \in \mathscr{A}} I^t_j(\mathscr{A}_\beta, \mathscr{A}_{\beta^\star}).w^t_j. \tag{34}$$

The arithmetic weighted sum of the anchor value $(\frac{l}{2})$ and the likelihood of an FPR in a tied scenario serves as the evaluation criterion for the hypothesis for the ranking of the options.

The optimal weight values, for each $P_t = (t = 1, 2, \dots, m!)$ can be computed by the following linear programming model (LPM):

$$(M_1) \quad \max \begin{cases} I^t = \sum_{j=1}^{n} \sum_{\mathscr{A}_\beta, \mathscr{A}_{\beta^\star} \in \mathscr{A}}^{n} I^t_j(\mathscr{A}_\beta, \mathscr{A}_{\beta^\star}).w^t_j \\ \text{s.t. } (w^t_1, w^t_2, \dots, w^t_n) \in \rho, \\ \text{for each } t = 1, 2, \dots, m!. \end{cases} \tag{35}$$

After solving the LPM (M_1) each of the solutions produces an optimal weight vector $\bar{w}^t = (\bar{w}^t_1, \bar{w}^t_2, \dots, \bar{w}^t_n)$ and an optimal objective value $\bar{I}^t$ for each $t = 1, 2, \dots, m!$. There exist $m!$ of the choices, so $m!$ LPMs must be solved. In general, these $m!$ models are capable of producing many optimal results. To put it another way, not every permutation results in the same ideal weight vectors. The permutation with the best value out of all the $\bar{I}^t$ values is then chosen. The chosen permutation can be used to determine the best priority order for the options in the following phase.

In the presence of uncertainty, the decision-maker may render conflicting judgments regarding the importance of the criteria and preferences. There are no such solutions that would satisfy all of the ρ conditions in that situation. So, using goal programming, we create a multi-objective nonlinear programming model to solve the issues with inconsistent information. By introducing a number of non-negative deviation variables, the conditions in ρ are changed to ρ^*, as shown below:

$$\rho^* = \begin{cases} (w^t_1, w^t_2, \dots, w^t_n) \in \rho_0 | w^t_{j_1} + e^-_{(i)j_1j_2} \geq w^t_{j_2} \text{ forall } j_1 \in \Gamma_1 \text{ and } j_2 \in \Lambda_1; \\ w^t_{j_1} - w^t_{j_2} + e^-_{(ii)j_1j_2} \geq \psi_{j_1j_2} \text{ for all } j_1 \in \Gamma_2 \text{ and } j_2 \in \Lambda_2; \\ w^t_{j_1} - 2w^t_{j_2} + w^t_{j_3} + e^-_{(iii)j_1j_2j_3} \geq 0 \text{ for all } j_1 \in \Gamma_3, j_2 \in \Lambda_3 \text{ and } j_3 \in \eta_3; \\ w^t_{j_1} + e^-_{(iv)j_1} \geq \sigma_{j_1}, w^t_{j_1} - e^+_{(iv)j_1} \leq \sigma_{j_1} + \varepsilon_{j_1} \text{ for all } j_1 \in \Gamma_4; \\ \frac{w^t_{j_1}}{w^t_{j2}} + e^-_{(v)j_1j_2} \geq \sigma_{j_1j_2} \text{ for all } j_1 \in \Gamma_5 \text{ and } j_2 \in \Lambda_5. \end{cases} \tag{36}$$

For the case of inconsistent preference information, the bi-objective NLP is designed as follows:

$$[M2] \quad \max \left\{ I^t = \sum_{j=1}^{n} \sum_{A_\beta, A_{\beta^\star} \in A} I^t_j(A_\beta, A_{\beta^\star}).w^t_j \right\}$$

$$\min \left\{ \sum_{j_1,j_2,j_3 \in N}^{n} (e^-_{(i)j_1j_2} + e^-_{(ii)j_1j_2} + e^-_{(iii)j_1j_2j_3} + e^-_{(iv)j_1} + e^+_{(iv)j_1} + e^-_{(v)j_1j_2}) \right. \tag{37}$$

$$s.t. \begin{cases} (w^t_1, w^t_2, \dots, w^t_n) \in \rho^* \\ e^-_{(i)j_1j_2} \geq 0 \quad j_1 \in \Gamma_1 \text{ and } j_2 \in \Lambda_1, \\ e^-_{(ii)j_1j_2} \geq 0 \quad j_1 \in \Gamma_2 \text{ and } j_2 \in \Lambda_2, \\ e^-_{(iii)j_1j_2j_3} \geq 0 \quad j_1 \in \Gamma_3, j_2 \in \Lambda_3 \text{ and } j_3 \in \eta_3, \\ e^-_{(iv)j_1} \geq 0, e^+_{(iv)j_1} \geq 0 \quad j_1 \in \Gamma_4, \\ e^-_{(v)j_1j_2} \geq 0 \quad j_1 \in \Gamma_5 \text{ and } j_2 \in \Lambda_5, \end{cases}$$

Using the max–min operator, the model [M2], for each $t = 1, 2, \dots, m!$, may be incorporated into the following single-objective NLP:

$$[M3] \quad \max \alpha \tag{38}$$

$$s.t \begin{cases} \sum_{j=1}^{n} \sum_{\mathscr{A}_\beta, \mathscr{A}_{\beta^\star} \in \mathscr{A}} I^t_j(\mathscr{A}_\beta, \mathscr{A}_{\beta^\star}).w^t_j \geq \alpha, \\ -\sum_{j_1,j_2,j_3 \in N}^{n} (e^-_{(i)j_1j_2} + e^-_{(ii)j_1j_2} + e^-_{(iii)j_1j_2j_3} + e^-_{(iv)j_1} + e^+_{(iv)j_1} + e^-_{(v)j_1j_2}) \geq \alpha, \\ (w^t_1, w^t_2, \dots, w^t_n) \in \rho^* \\ e^-_{(i)j_1j_2} \geq 0 \quad j_1 \in \Gamma_1 \; and \; j_2 \in \Lambda_1, \\ e^-_{(ii)j_1j_2} \geq 0 \quad j_1 \in \Gamma_2 \; and \; j_2 \in \Lambda_2, \\ e^-_{(iii)j_1j_2j_3} \geq 0 \quad j_1 \in \Gamma_3, j_2 \in \Lambda_3 \; and \; j_3 \in \eta_3, \\ e^-_{(iv)j_1} \geq 0, e^+_{(iv)j_1} \geq 0 \quad j_1 \in \Gamma_4, \\ e^-_{(v)j_1j_2} \geq 0 \quad j_1 \in \Gamma_5 \; and \; j_2 \in \Lambda_5, \end{cases}$$

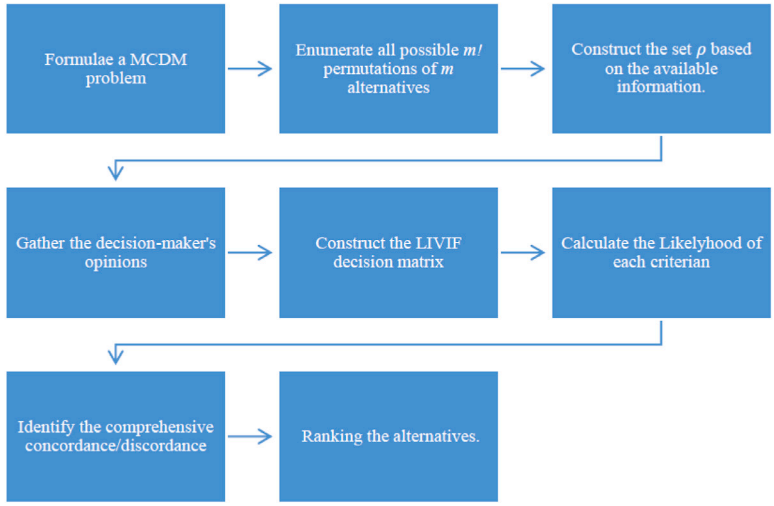

Fig. 1. Flow chart of the proposed method.

Each solution of the NLPM [M3] for each permutation t, where $t = 1, 2, \ldots, m!$, gives vector of optimal weight $\bar{w}^t = (\bar{w}_1^t, \bar{w}_2^t, \ldots, \bar{w}_n^t)$, and the optimal deviation values $\bar{\bar{e}}^{--}_{(i)j_1j_2}, \bar{e}^{--}_{(ii)j_1j_2}, \bar{e}^{--}_{(iii)j_1j_2j_3}, \bar{e}^{--}_{(iv)j_1}, \bar{e}^{-+}_{(iv)j_1}$ and $\bar{e}^{--}_{(v)j_1j_2}(j_1, j_2, j_3 \in N)$ for each $t = 1, 2, \ldots, m!$. The correlate comprehensive concordance/discordance index $\overset{I}{t}$ may then be obtained for the permutation P_t. When all of the $m!$ integrated NLP problem have been resolved, the best way to rank the options is to compare the $\bar{I}^t$ values of each permutation.

4.2. Computational complexity

Consider the MCDM problem having consistent and incomplete preference information. The vector for optimal weight and comprehensive concordance/discordance can be find out by index for all m! using the LPM [M1]. Let the number of conditions in ρ is denoted by Υ. And the LPM [M1] has Υ constraints with n decision variables (consist of $w_1^t, w_2^t, \ldots, w_n^t$). The simplex method can be used to solve the model [M1], where computational complexity degree is shallowed. Since permutations rapidly increases with an increase in the number of choices. Still, the complexity of the model [M1] concerning each P_t is relatively easy to solve. Additionally, the number of decision variables and constraints for each optimization model remains the same, i.e. n and Υ, respectively. Thus, the computational complexity of the individual LPMs in [M1] cannot be affected due to the number of permutations.

We can employ the integrated [M3] model to obtain the optimum solutions for all $m!$ permutations. Suppose that there are $\Upsilon^\star$ several conditions in ρ^*, then the number of deviation variables is also $\Upsilon^\star$ in ρ^*. So in the NLPM [M3], the total decision variables are $(n+\Upsilon^\star)$ (including $w_1^t, w_2^t, \ldots, w_n^t$, and all deviation variables), and several constraints are $\Upsilon^\star$. As compare to model [M1], the model [M3] is more complex; however finding its solution is not difficult because we can quickly obtain the optimal solutions using powerful computer hardware and software. The decision variables and constraints in [M3] that correspond to each P_t is the same for any number of permutations if the criteria and weight conditions remain unchanged for a given MCDM problems. Therefore, the increase in the number of permutations does not change the model's computational complexity [M3].

4.3. Proposed algorithm

The new *algorithm* is known as "The LIVIF QUALIFLEX outranking approach connected with likelihood-based comparison method for resolving an MCDM problem" undergoing incomplete information can be obtained as (see the graphical flowchart in Fig. 1):

Algorithm:

Step 1: Formulate a MCDM problem: Generate feasible alternatives ($A = \{A_1, A_2, \ldots, A_n\}$) and specify the evaluation criteria ($C = \{c_1, c_2, \ldots, c_n\}$).

Step 2: List all the possible $m!$ permutations of m alternatives, which must be tested. Let $P_t(t = 1, 2, \ldots, m!)$ denote the tth permutation.

Step 3: Determine the decision-maker's preferences regarding criteria by assessing weak order, strict order, difference order, interval bound, or ratio bound, in order to gain knowledge of the criterion weights. Formulate set ρ based on the available information.

Step 4: Conduct a survey of the decision-maker's viewpoints to acquire evaluative ratings for the alternatives concerning each criterion. i.e., the LIVIN ratings $\widetilde{A}_{kj}^b$ and $\widetilde{A}_{kj}^c$, for the benefit and cost criteria, respectively. Later, convert these evaluative ratings into $\widetilde{A}_{kj}$ for each $\mathscr{A}_k \in \mathscr{A}$ and $c_j \in C$ to construct the LIVIF decision matrix $\widetilde{D}_l$.

Step 5: Calculate $\mathcal{L}(\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta^\star j})$ using each $c_j \in C$ and each pair of options $(\mathscr{A}_\beta, \mathscr{A}_{\beta^\star})$ where $\mathscr{A}_\beta, \mathscr{A}_{\beta^\star} \in \mathscr{A}$.

Step 6: Using each pair of choices $(\mathscr{A}_\beta, \mathscr{A}_{\beta^\star})$ in permutation P_t evaluate the concordance/discordance index $I_j^t(\mathscr{A}_\beta, \mathscr{A}_{\beta^\star})$, concerning each criterion based on $\mathcal{L}(\widetilde{A}_{\beta j} \geqslant \widetilde{A}_{\beta^\star j})$.

Step 7: For each permutation P_t specify the concordance/discordance index I^t. Then, construct a LPM [M1] with consistent weight information, or the NLPM [M3] with inconsistent weighted data for each P_t.

Step 8: Solve [M1] or [M3] for each P_t to obtain the vector of optimal weight $\bar{w}^t$ and the optimal concordance/discordance index $\bar{I}^t$. The order of options in the permutation with the optimal $\bar{I}^t$ value is the best options.

5. Case instance and discussions

The following case instance, which was adapted from Ilyas, Carpitella, and Zoubir (2021), assume an MCDM problem for selection of the most appropriate supplier using an execution of the proposed methods.

5.1. Instance of the algorithm

The company faced the collapse of its major activities in the north of Morocco for a period of six weeks due to supply chain interference caused by COVID-19 (Ilyas et al., 2021) without having recognized a comprehensive plan to handle the issue. The organization is currently concentrating on reviewing its prior suppliers in an effort to make progress from the current predicament. The case study, which was developed from Ilyas et al. (2021), examines the issue of how to choose the best provider to aid the organization in such a circumstance. This study examine four suppliers, including Supplier 1 ($\mathscr{A}_1$), Supplier 2 ($\mathscr{A}_2$), Supplier 3 ($\mathscr{A}_3$) and Supplier 4 ($\mathscr{A}_4$). The criteria for formatting the suppliers include price/cost (c_1), experience (c_2), punctuality (c_3), quality (c_4), delivery performance and reliability (c_5) and reputation (c_6). In this problem, c_1 designate the cost criteria, while all the remaining variables represent the benefit criteria. So the evaluation criteria set is indicated by $C = \{c_1, c_2, \dots, c_6\}$ with $C_b = \{c_2, c_3, c_4, c_5, c_6\}$ and $C_c = \{c_1\}$. In the suppliers selection problem, four suppliers are available, and $\mathscr{A} = \{\mathscr{A}_1, \mathscr{A}_2, \mathscr{A}_3, \mathscr{A}_4\}$ represent the set of all candidate suppliers, the three experts were proposed in the evaluation to make use of LIVIFNs. The available linguistic variables are extremely low (s_0), very low (s_1), low (s_2), slightly low (s_3), medium (s_4), slightly high (s_5), high (s_6), very high (s_7), and extremely high (s_8). The evaluation results of the three experts are listed in the Tables 1–3.

Step 2: By utilizing the linguistic intuitionist fuzzy weighted averaging operator (Ma, Zhu, Ponnambalam, & Zhang, 2019) with known experts weights $0.2429, 0.5142$ and 0.2429 respectively we obtain the aggregated matrix as presented in Table 4.

In step 2: Using (32), we create 4! (=24) permutations of the ranking of the alternatives which must be tested and are expressed in the following:

$$
\begin{aligned}
&P_1 = (\mathscr{A}_1, \mathscr{A}_2, \mathscr{A}_3, \mathscr{A}_4),\ P_2 = (\mathscr{A}_1, \mathscr{A}_2, \mathscr{A}_4, \mathscr{A}_3), P_3 = (\mathscr{A}_1, \mathscr{A}_3, \mathscr{A}_2, \mathscr{A}_4), P_4 = (\mathscr{A}_1, \mathscr{A}_3, \mathscr{A}_4, \mathscr{A}_2),\\
&P_5 = (\mathscr{A}_1, \mathscr{A}_4, \mathscr{A}_2, \mathscr{A}_3),\ P_6 = (\mathscr{A}_1, \mathscr{A}_4, \mathscr{A}_3, \mathscr{A}_2),\ P_7 = (\mathscr{A}_2, \mathscr{A}_1, \mathscr{A}_3, \mathscr{A}_4), P_8 = (\mathscr{A}_2, \mathscr{A}_1, \mathscr{A}_4, \mathscr{A}_3),\\
&P_9 = (\mathscr{A}_2, \mathscr{A}_3, \mathscr{A}_1, \mathscr{A}_4), P_{10} = (\mathscr{A}_2, \mathscr{A}_3, \mathscr{A}_4, \mathscr{A}_1), P_{11} = (\mathscr{A}_2, \mathscr{A}_4, \mathscr{A}_1, \mathscr{A}_3), P_{12} = (\mathscr{A}_2, \mathscr{A}_4, \mathscr{A}_3, \mathscr{A}_1),\\
&P_{13} = (\mathscr{A}_3, \mathscr{A}_1, \mathscr{A}_2, \mathscr{A}_4), P_{14} = (\mathscr{A}_3, \mathscr{A}_1, \mathscr{A}_4, \mathscr{A}_2), P_{15} = (\mathscr{A}_3, \mathscr{A}_2, \mathscr{A}_1, \mathscr{A}_4), P_{16} = (\mathscr{A}_3, \mathscr{A}_2, \mathscr{A}_4, A_1),\\
&P_{17} = (\mathscr{A}_3, \mathscr{A}_4, \mathscr{A}_1, \mathscr{A}_2), P_{18} = (\mathscr{A}_3, \mathscr{A}_4, \mathscr{A}_2, \mathscr{A}_1), P_{19} = (\mathscr{A}_4, \mathscr{A}_1, \mathscr{A}_2, \mathscr{A}_3), P_{20} = (\mathscr{A}_4, \mathscr{A}_1, \mathscr{A}_3, \mathscr{A}_2),\\
&P_{21} = (\mathscr{A}_4, \mathscr{A}_2, \mathscr{A}_1, \mathscr{A}_3), P_{22} = (\mathscr{A}_4, \mathscr{A}_2, \mathscr{A}_3, \mathscr{A}_1), P_{23} = (\mathscr{A}_4, \mathscr{A}_3, \mathscr{A}_1, \mathscr{A}_2), P_{24} = (\mathscr{A}_4, \mathscr{A}_3, \mathscr{A}_2, \mathscr{A}_1),
\end{aligned}
$$

Step 4: Using (35), let $\rho_0 = \left\{(w_1^t, w_2^t, \dots, w_6^t) | w_j^t \geq 0 (j = 1, 2, \dots, 6), \sum_{j=1}^{6} w_j^t = 1\right\}$. According to all criteria, the authorities have provided their choices, and the given data for the criterion weights are given by the following:

$$
\begin{aligned}
&\rho_1 = \{(w_1^t, w_2^t, \dots, w_6^t) \in \rho_0 | w_3^t \geq w_5^t\},\\
&\rho_2 = \{(w_1^t, w_2^t, \dots, w_6^t) \in \rho_0 | 0.12 \geq w_6^t - w_2^t \geq 0.08\},\\
&\rho_3 = \{(w_1^t, w_2^t, \dots, w_6^t) \in \rho_0 | w_5^t - w_2^t \geq w_2^t - w_4^t\},\\
&\rho_4 = \{(w_1^t, w_2^t, \dots, w_6^t) \in \rho_0 | 0.20 \geq w_1^t \geq 0.15, 0.16 \geq w_5^t \geq 0.11\},\\
&\rho_5 = \{(w_1^t, w_2^t, \dots, w_6^t) \in \rho_0 | w_2^t \geq 0.6 \cdot w_4^t\}.
\end{aligned}
$$

Step 5: Using (41), for the known criterion information of weights, the set ρ is given as follows: $\rho = \{(w_1^t, w_2^t, \dots, w_6^t) \in \rho_0 | w_3^t \geq w_5^t, 0.12 \geq w_6^t - w_2^t \geq 0.08, w_5^t - w_2^t \geq w_2^t - w_4^t, 0.20 \geq w_1^t \geq 0.15, 0.16 \geq w_5^t \geq 0.11, w_2^t \geq 0.6 \cdot w_4^t\}$. The step 4 involves the evaluation of the suppliers by the company based on the eight criteria, and converted the data into the LIVIF format. Using the provided ratings, we constructed the LIVIF matrix $\widetilde{D}_l$ in (12), as shown in Table 1.

Step 6: Using (15) and (16), we calculated the lower likelihood $\mathcal{L}^-(\widetilde{A}_{\beta j} \geq \widetilde{A}_{\beta^* j})$ and upper likelihood $\mathcal{L}^+(\widetilde{A}_{\beta j} \geq \widetilde{A}_{\beta^* j})$, for each criterion $c_j \in C$ and each pair of $(\mathscr{A}_\beta, \mathscr{A}_{\beta^\star})$ where $\mathscr{A}_\beta, \mathscr{A}_{\beta^\star} \in \mathscr{A}$. Next we calculated the likelihood value $L(\widetilde{A}_{\beta j} \geq \widetilde{A}_{\beta^* j})$ of the fuzzy preference relation $\widetilde{A}_{\beta j} \geq \widetilde{A}_{\beta^* j}$, the corresponding results are presented in Table 5.

Step 7: Using (25) we determine the concordance/discordance index $I_j^t(\mathscr{A}_\beta, \mathscr{A}_{\beta^\star})$ each pair of $(\mathscr{A}_\beta, \mathscr{A}_{\beta^\star})$ of options as per criterion c_j, to P_t, are provided in Table 6.

Step 8: using (43), for each permutation P_t we recognized the concordance/ discordance index, as given in Table 7, consider for the $P_6 = (\mathscr{A}_1, \mathscr{A}_4, \mathscr{A}_3, \mathscr{A}_2)$,the index I^6 is given as follows:

$$
\begin{aligned}
I^6 &= \sum_{j=1}^{6} \sum_{\mathscr{A}_\beta, \mathscr{A}_{\beta^\star} \in \mathscr{A}} I_j^6(\mathscr{A}_\beta, \mathscr{A}_{\beta^\star}).w_j^6\\
&= (-2.1279 - 0.1684 - 0.0920 + 5.9903 + 5.8873 + 0.0735) \cdot w_1^6\\
&\quad + (1.2131 - 0.9619 + 1.3596 - 1.6810 + 1.1009 + 2.0214) \cdot w_2^6\\
&\quad + (3.4790 + 1.1773 - 1.6977 + 1.4933 - 1.1998 - 2.5141) \cdot w_3^6\\
&\quad + (-1.7169 + 0.1002 - 2.4497 + 1.6639 - 1.0578 + 1.4678) \cdot w_4^6\\
&\quad + (1.0876 - 1.2809 + 2.1678 - 1.7756 + 0.3590 + 2.8315) \cdot w_5^6\\
&\quad + (1.3356 + 0.2681 + 1.5471 - 0.2858 + 0.3590 + 0.6054) \cdot w_6^6\\
&= 1.5628 w_1 + 3.0521 w_2 + 0.7380 w_3 - 5.9925 w_4 + 4.7560 w_5 + 3.8294 w_6
\end{aligned}
$$

Table 1
LIV-IF decision matrix $\widetilde{D}_l^1$.

	$\mathscr{A}_1$	$\mathscr{A}_2$	$\mathscr{A}_3$	$\mathscr{A}_4$
c_1	$([s_3, s_5][s_2, s_3])$	$([s_1, s_2][s_3, s_4])$	$([s_2, s_3][s_4, s_5])$	$([s_3, s_4][s_1, s_2])$
c_2	$([s_2, s_3][s_3, s_4])$	$([s_1, s_3][s_3, s_5])$	$([s_3, s_4][s_1, s_2])$	$([s_2, s_4][s_1, s_3])$
c_3	$([s_1, s_3][s_2, s_4])$	$([s_2, s_3][s_3, s_4])$	$([s_2, s_4][s_1, s_3])$	$([s_3, s_4][s_2, s_3])$
c_4	$([s_2, s_4][s_3, s_4])$	$([s_3, s_5][s_1, s_2])$	$([s_2, s_5][s_1, s_3])$	$([s_2, s_5][s_2, s_3])$
c_5	$([s_2, s_5][s_1, s_2])$	$([s_1, s_2][s_3, s_5])$	$([s_3, s_4][s_2, s_3])$	$([s_1, s_3][s_2, s_4])$
c_6	$([s_4, s_5][s_1, s_2])$	$([s_2, s_4][s_1, s_3])$	$([s_3, s_4][s_2, s_3])$	$([s_2, s_3][s_3, s_4])$

Table 2
LIV-IF decision matrix $\widetilde{D}_l^2$.

	$\mathscr{A}_1$	$\mathscr{A}_2$	$\mathscr{A}_3$	$\mathscr{A}_4$
c_1	$([s_1, s_2][s_3, s_4])$	$([s_2, s_3][s_3, s_5])$	$([s_2, s_3][s_3, s_4])$	$([s_3, s_4][s_1, s_2])$
c_2	$([s_3, s_4][s_2, s_3])$	$([s_1, s_2][s_4, s_5])$	$([s_3, s_4][s_2, s_3])$	$([s_1, s_3][s_2, s_5])$
c_3	$([s_3, s_5][s_1, s_2])$	$([s_5, s_6][s_1, s_2])$	$([s_2, s_3][s_3, s_4])$	$([s_2, s_5][s_1, s_2])$
c_4	$([s_1, s_2][s_4, s_5])$	$([s_4, s_5][s_1, s_3])$	$([s_1, s_2][s_5, s_6])$	$([s_3, s_4][s_1, s_3])$
c_5	$([s_2, s_4][s_3, s_4])$	$([s_1, s_2][s_3, s_4])$	$([s_4, s_6][s_1, s_2])$	$([s_1, s_3][s_2, s_4])$
c_6	$([s_3, s_5][s_1, s_3])$	$([s_2, s_3][s_3, s_4])$	$([s_3, s_4][s_1, s_2])$	$([s_2, s_4][s_1, s_3])$

Table 3
LIV-IF decision matrix $\widetilde{D}_l^3$.

	$\mathscr{A}_1$	$\mathscr{A}_2$	$\mathscr{A}_3$	$\mathscr{A}_4$
c_1	$([s_2, s_3][s_3, s_5])$	$([s_3, s_5][s_1, s_3])$	$([s_2, s_4][s_1, s_3])$	$([s_2, s_5][s_1, s_2])$
c_2	$([s_3, s_4][s_1, s_3])$	$([s_4, s_5][s_1, s_2])$	$([s_3, s_5][s_1, s_2])$	$([s_1, s_3][s_3, s_4])$
c_3	$([s_1, s_3][s_3, s_4])$	$([s_5, s_6][s_1, s_2])$	$([s_2, s_4][s_3, s_4])$	$([s_2, s_4][s_1, s_3])$
c_4	$([s_2, s_4][s_1, s_3])$	$([s_4, s_6][s_1, s_2])$	$([s_2, s_4][s_3, s_4])$	$([s_3, s_5][s_2, s_3])$
c_5	$([s_4, s_5][s_1, s_2])$	$([s_2, s_3][s_3, s_5])$	$([s_3, s_5][s_1, s_2])$	$([s_4, s_5][s_1, s_2])$
c_6	$([s_4, s_6][s_1, s_2])$	$([s_4, s_5][s_1, s_3])$	$([s_2, s_3][s_3, s_4])$	$([s_3, s_5][s_2, s_3])$

Table 4
Aggregated LIV-IF decision matrix.

	$\mathscr{A}_1$	$\mathscr{A}_2$	$\mathscr{A}_3$	$\mathscr{A}_4$
c_1	$([s_{1.7864}, s_{3.1494}][s_{2.7186}, s_{3.9378}])$	$([s_{2.0409}, s_{3.3835}][s_{2.2974}, s_{4.1835}])$	$([s_2, s_{3.2638}][s_{2.4636}, s_{3.9378}])$	$([s_{2.7736}, s_{4.2700}][s_1, s_2])$
c_2	$([s_{2.7736}, s_{3.7772}][s_{1.8650}, s_{3.2171}])$	$([s_{1.8897}, s_{3.1494}][s_{2.6636}, s_{4.0023}])$	$([s_3, s_{4.2700}][s_{1.4282}, s_{2.4636}])$	$([s_{1.2573}, s_{3.2638}][s_{1.8650}, s_{4.1835}])$
c_3	$([s_{2.1121}, s_{4.1550}][s_{1.5453}, s_{2.8007}])$	$([s_{4.4499}, s_{5.5014}][s_{1.3058}, s_{2.3667}])$	$([s_2, s_{3.5137}][s_{2.2974}, s_{3.7300}])$	$([s_{2.2599}, s_{4.5500}][s_{1.1834}, s_{2.4354}])$
c_4	$([s_{1.5051}, s_{3.0727}][s_{2.6636}, s_{4.1835}])$	$([s_{3.7772}, s_{5.2814}][s_1, s_{2.4636}])$	$([s_{1.5051}, s_{3.4053}][s_{2.9875}, s_{4.5947}])$	$([s_{2.7736}, s_{4.5217}][s_{1.4004}, s_3])$
c_5	$([s_{2.5628}, s_{4.5217}][s_{1.7593}, s_{2.8564}])$	$([s_{1.2573}, s_{2.2599}][s_3, s_{4.4580}])$	$([s_{3.5420}, s_{5.3883}][s_{1.1834}, s_{2.2070}])$	$([s_{1.8897}, s_{3.5835}][s_{1.6901}, s_{3.3802}])$
c_6	$([s_{3.5137}, s_{5.2814}][s_1, s_{2.4636}])$	$([s_{2.5628}, s_{3.8165}][s_{1.7593}, s_{3.4783}])$	$([s_{2.7736}, s_{3.7772}][s_{1.5453}, s_{2.6117}])$	$([s_{2.2599}, s_{4.0622}][s_{1.5453}, s_{3.2171}])$

Since there is no inconsistent weighted data as per authorities choices, applying [M1] to erect the LPM for each P_t. For the following LPM was constructed for P_6:

$$\max \begin{cases} I^6 = 1.5628w_1 + 3.0521w_2 + 0.7380w_3 - 5.9925w_4 + 4.7560w_5 + 3.8294w_6 \\ \text{subject to} \begin{Bmatrix} w_3^6 \geq w_5^6, 0.12 \geq w_6^6 - w_2^6 \geq 0.08, w_5^6 - w_2^6 \geq w_2^6 - w_4^6, \\ 0.20 \geq w_1^6 \geq 0.15, 0.16 \geq w_5^6 \geq 0.11, w_2^6 \geq 0.6 \cdot w_4^6 \\ w_1^6 + w_2^6 + w_3^6 + w_4^6 + w_5^6 + w_6^6 = 1, \\ w_j^6 \geq 0 \text{ for all } j. \end{Bmatrix} \end{cases}$$

Step 8: For each permutation P_t, we obtained the vector of optimal weight $\bar{w}^t$ and the optimal concordance/discordance index $\bar{I}^t$ by solving the LPM. For example, applying P_6, we observed that the optimal objective value is 1.3065 having weight vector that is $\bar{w}^6$ = (0.2, 0.13, 0.16, 0.1, 0.16, 0.25) optimal. Since it is found that $\bar{I}^{11}$(=3.0911) produce the maximal value, therefore the favorable of the candidate suppliers is $P_{11} = (\mathscr{A}_2, \mathscr{A}_4, \mathscr{A}_1, \mathscr{A}_3)$ with the optimal

Results of the comprehensive concordance/discordance indices

$$I^1 = -6.3394w_1 + 1.2978w_2 + 1.1792w_3 - 2.1403w_4 - 0.8070w_5 + 2.4722w_6$$
$$I^2 = -2.3588w_1 - 2.0642w_2 + 4.1658w_3 + 1.1875w_4 - 4.3582w_5 + 1.9006w_6$$
$$I^3 = -6.1924w_1 + 5.3406w_2 - 3.8490w_3 - 7.2047w_4 + 4.8560w_5 + 3.6830w_6$$
$$I^4 = -2.4178w_1 + 6.4141w_2 - 6.2486w_3 - 9.3203w_4 + 8.3072w_5 + 4.4010w_6$$
$$I^5 = 1.4158w_1 - 0.9907w_2 + 1.7662w_3 - 0.9281w_4 - 0.9070w_5 + 2.6186w_6$$
$$I^6 = 1.5628w_1 + 3.0521w_2 - 3.2620w_3 - 5.9925w_4 + 4.7560w_5 + 3.8294w_6$$
$$I^7 = -5.5645w_1 - 1.4214w_2 + 4.5746w_3 + 2.7591w_4 - 5.1426w_5 - 0.6220w_6$$
$$I^8 = -1.5839w_1 - 4.7834w_2 + 7.5612w_3 + 6.0869w_4 - 8.6938w_5 - 1.1936w_6$$

Table 5
Results of the likelihoods of the FPRs.

	c_1	c_2	c_3	c_4	c_5	c_6
$\mathcal{L}(\widetilde{A}_{1j} \geq \widetilde{A}_{2j})$	3.9080	5.3596	2.3023	1.5503	6.1678	5.5471
$\mathcal{L}(\widetilde{A}_{1j} \geq \widetilde{A}_{3j})$	3.8316	3.0381	5.1773	4.1002	2.7191	4.2681
$\mathcal{L}(\widetilde{A}_{1j} \geq \widetilde{A}_{4j})$	1.8721	5.2131	3.4790	2.2831	5.0876	5.3356
$\mathcal{L}(\widetilde{A}_{2j} \geq \widetilde{A}_{1j})$	4.6829	2.6404	5.6977	6.4497	1.8322	2.4529
$\mathcal{L}(\widetilde{A}_{2j} \geq \widetilde{A}_{3j})$	3.9265	1.9786	6.5141	6.5322	1.1685	3.3946
$\mathcal{L}(\widetilde{A}_{2j} \geq \widetilde{A}_{4j})$	2.1127	4.0274	5.1998	5.0578	2.2744	3.6410
$\mathcal{L}(\widetilde{A}_{3j} \geq \widetilde{A}_{1j})$	4.1684	4.9619	2.8227	3.8998	5.2809	2.6015
$\mathcal{L}(\widetilde{A}_{3j} \geq \widetilde{A}_{2j})$	4.0735	6.0214	1.4859	1.4678	6.8315	4.6054
$\mathcal{L}(\widetilde{A}_{3j} \geq \widetilde{A}_{4j})$	2.0097	5.6810	2.5067	2.3361	5.7756	4.2858
$\mathcal{L}(\widetilde{A}_{4j} \geq \widetilde{A}_{1j})$	6.1279	2.7869	4.5210	5.7169	2.9124	2.6644
$\mathcal{L}(\widetilde{A}_{4j} \geq \widetilde{A}_{2j})$	5.8873	5.1009	2.8002	2.9422	5.7256	4.3590
$\mathcal{L}(\widetilde{A}_{4j} \geq \widetilde{A}_{3j})$	5.9903	2.3190	5.4933	5.6639	2.2244	3.7142

Table 6
Results of the concordance/discordance indices.

	c_1	c_2	c_3	c_4	c_5	c_6
$I_j^l(\mathscr{A}_1, \mathscr{A}_2)$	−0.0920	1.3596	−1.6977	−2.4497	2.1678	1.5471
$I_j^l(\mathscr{A}_1, \mathscr{A}_3)$	−0.1684	−0.9619	1.1773	0.1002	−1.2809	0.2681
$I_j^l(\mathscr{A}_1, \mathscr{A}_4)$	−2.1279	1.2131	−0.5210	−1.7169	1.0876	1.3356
$I_j^l(\mathscr{A}_2, \mathscr{A}_1)$	0.6829	−1.3596	1.6977	2.4497	−2.1678	−1.5471
$I_j^l(\mathscr{A}_2, \mathscr{A}_3)$	−0.0735	−2.0214	2.5141	2.5322	−2.8315	−0.6054
$I_j^l(\mathscr{A}_2, \mathscr{A}_4)$	−1.8873	0.0274	1.1998	1.0578	−1.7256	−0.3590
$I_j^l(\mathscr{A}_3, \mathscr{A}_1)$	0.1684	0.9619	−1.1773	−0.1002	1.2809	−1.3985
$I_j^l(\mathscr{A}_3, \mathscr{A}_2)$	0.0735	2.0214	−2.5141	−2.5322	2.8315	0.6054
$I_j^l(\mathscr{A}_3, \mathscr{A}_4)$	−1.9903	1.6810	−1.4933	−1.6639	1.7756	0.2858
$I_j^l(\mathscr{A}_4, \mathscr{A}_1)$	2.1279	−1.2131	0.5210	1.7169	−1.0876	−1.3356
$I_j^l(\mathscr{A}_4, \mathscr{A}_2)$	1.8873	1.1009	−1.1998	−1.0578	1.7256	0.3590
$I_j^l(\mathscr{A}_4, \mathscr{A}_3)$	1.9903	−1.6810	1.4933	1.6639	−1.7756	−0.2858

Table 7
Comparison of values of comprehensive concordance/discordance indices.

	$\overline{I}^1$	$\overline{I}^2$	$\overline{I}^3$	$\overline{I}^4$
LIF-QULIFLEX	0.7946	2.1060	0.0487	0.5142
Proposed method	0.0082	1.4420	0.0595	0.8709
	$\overline{I}^5$	$\overline{I}^6$	$\overline{I}^7$	$\overline{I}^8$
LIF-QULIFLEX	1.5944	4.4541	1.6874	3.3429
Proposed method	1.3369	1.3064	0.7905	2.2760
	$\overline{I}^9$	$\overline{I}^{10}$	$\overline{I}^{11}$	$\overline{I}^{12}$
LIF-QULIFLEX	1.0041	2.4950	5.2004	4.9646
Proposed method	−0.0553	1.3436	3.0911	2.9643
	$\overline{I}^{13}$	$\overline{I}^{14}$	$\overline{I}^{15}$	$\overline{I}^{16}$
LIF-QULIFLEX	−0.1913	0.3225	0.0709	−0.4145
Proposed method	−0.0452	0.7662	−0.6390	−0.2378
	$\overline{I}^{17}$	$\overline{I}^{18}$	$\overline{I}^{19}$	$\overline{I}^{20}$
LIF-QULIFLEX	0.3654	−0.1913	2.3218	1.0255
Proposed method	0.7918	0.2583	1.8977	2.3359
	$\overline{I}^{21}$	$\overline{I}^{22}$	$\overline{I}^{23}$	$\overline{I}^{24}$
LIF-QULIFLEX	4.7298	4.4940	0.8337	1.7501
Proposed method	3.0643	2.0507	1.2504	1.5955

$$I^9 = -5.2277w_1 + 0.5024w_2 + 2.2200w_3 + 2.5587w_4 - 2.5808w_5 - 2.2886w_6$$
$$I^{10} = -0.9719w_1 - 1.9238w_2 + 3.2620w_3 + 5.9925w_4 - 4.7560w_5 - 4.9598w_6$$
$$I^{11} = 2.6719w_1 - 7.2096w_2 + 8.6032w_3 + 9.5207w_4 - 10.8690w_5 - 3.8648w_6$$
$$I^{12} = 3.0087w_1 - 5.2858w_2 + 6.2486w_3 + 9.3203w_4 - 8.3072w_5 - 5.5314w_6$$
$$I^{13} = -5.8556w_1 + 7.2644w_2 - 6.2036w_3 - 7.4051w_4 + 7.4178w_5 + 2.0164w_6$$
$$I^{14} = -2.0810w_1 + 8.3379w_2 - 8.6032w_3 - 9.5207w_4 + 10.8690w_5 + 2.7344w_6$$
$$I^{15} = -5.0807w_1 + 4.5452w_2 - 2.8082w_3 - 2.5057w_4 + 3.0822w_5 - 1.0778w_6$$
$$I^{16} = -0.8249w_1 + 2.1190w_2 - 1.7662w_3 + 0.9281w_4 + 0.9070w_5 - 3.7490w_6$$
$$I^{17} = 2.1748w_1 + 5.9117w_2 - 7.5612w_3 - 6.0869w_4 + 8.6938w_5 + 0.0632w_6$$

$$I^{18} = 2.9497w_1 + 3.1925w_2 - 4.1658w_3 - 1.1875w_4 + 4.3582w_5 - 3.0310w_6$$
$$I^{19} = 5.6716w_1 - 3.4169w_2 + 2.8082w_3 + 2.5057w_4 - 3.0822w_5 - 0.0526w_6$$
$$I^{20} = 5.8186w_1 + 0.6259w_2 - 2.2200w_3 + 2.5057w_4 + 2.5808w_5 + 1.1582w_6$$
$$I^{21} = 6.4465w_1 - 6.1361w_2 + 6.2036w_3 + 7.4051w_4 - 7.4178w_5 - 3.1468w_6$$
$$I^{22} = 6.7833w_1 - 4.2123w_2 - 4.2123w_3 + 7.2047w_4 - 4.8560w_5 - 4.8134w_6$$
$$I^{23} = 6.1554w_1 + 2.5497w_2 - 4.5746w_3 - 2.7591w_4 + 5.1426w_5 - 0.5084w_6$$
$$I^{24} = 6.9303w_1 - 0.1695w_2 - 1.1792w_3 + 2.1403w_4 + 0.8070w_5 - 3.6026w_6$$

weight vector $\bar{w}^{11} = (0.15, 0.066, 0.11, 0.418, 0.11, 0.146)$. Moreover the best supplier is supplier 2 (A_2).

In practical decision-making problems the incomplete preference information is more realistic, mostly in complex and uncertain circumstances. Because of this, on the basis of criterion significance our proposed method also allow the incomplete information. For this circumstances, the decision-maker can apply the five basic ranking forms in (27) – (31) to give his/her preferences for any criteria. For example in step 3, we cannot recognize the relation of w_4 with other criterion weights, like w_1, w_3, w_6, according to the preference information given in ρ. Despite of incomplete information, in step 7, the linear programming model can be used, to obtain the optimal weights for each permutation P_t. Thus, our proposed method is adjustable because it needs only partial information based on the five basic ranking forms and not compulsorily complete information.

5.2. Discussion of related inconsistent data

Here, addressed the issue of inconsistency preference in this work, which might arise when measuring data in terms of criteria importance. As a result, we may create deviation variables to regulate the conditions in ρ, and then we can create an integrated nonlinear programming model [M3] to deal with MCDM with incomplete and inconsistent weighted results.

Suppose to the same most suitable supplier selection problem. Let us assume that we add the condition $0.05 \geq w_5 - w_3 \geq 0.01$ to the set ρ_2. The updated form of the sets ρ_2 and ρ are given as:

$$\rho_2^{(new)} = \{(w_1^t, w_2^t, \ldots, w_6^t) \in \rho_0 | 0.12 \geq w_6^t - w_2^t \geq 0.08, 0.05 \geq w_5 - w_3 \geq 0.01\},$$

$$\rho^{(new)} = \{(w_1^t, w_2^t, \ldots, w_6^t) \in \rho_0 | w_3^t \geq w_5^t, 0.12 \geq w_6^t - w_2^t \geq 0.08, 0.05 \geq w_5 - w_3 \geq 0.01, w_5^t - w_2^t \geq w_2^t - w_4^t, 0.20 \geq w_1^t \geq 0.15, 0.16 \geq w_5^t \geq 0.11, w_2^t \geq 0.6 \cdot w_4^t\}$$

Since the conditions $w_3^t \geq w_5^t$ and $0.05 \geq w_5 - w_3 \geq 0.01$ in ρ_1 and $\rho_2^{(new)}$ respectively are in conflict, and therefore, the weighted data in $\rho^{(new)}$ is partially inconsistent. We used [M3] to build the integrated NLPM for each P_t due to the inconsistent weight results in the choice. The conditions in $\rho^{(new)}$ were moderated to $\rho^{*(new)}$ by incorporating selected deviation variables in it, as shown below:

$$\rho^{*(new)} = \{(w_1^t, w_2^t, \ldots, w_6^t) \in \rho_0 | w_3^t + e^-_{(\text{i})35} \geq w_5^t, w_6^t - w_2^t + e^-_{(\text{ii})62} \geq 0.08,$$
$$w_5^t - w_3^t + e^-_{(\text{ii})53} \geq 0.01, w_6^t - w_2^t - e^+_{(\text{ii})62} \leq 0.12, w_5^t - w_3^t - e^+_{(\text{ii})53} \leq 0.05,$$
$$w_5^t - 2w_2^t + w_4^t + e^-_{(\text{iii})524} \geq 0, w_1^t + e^-_{(\text{iv})1} \geq 0.15, w_5^t + e^-_{(\text{iv})5} \geq 0.11,$$
$$w_1^t - e^+_{(\text{iv})1} \leq 0.20, w_5^t - e^+_{(\text{iv})5} \leq 0.16, \frac{w_2^t}{w_4^t} + e^-_{(\text{v})24} \geq 0.6\},$$

where the deviation variables $e^-_{(\text{i})35}, e^-_{(\text{ii})62}, e^-_{(\text{ii})53}, e^+_{(\text{ii})62}, e^+_{(\text{ii})53}, e^-_{(\text{iii})524}, e^-_{(\text{iv})1}, e^-_{(\text{iv})5}, e^+_{(\text{iv})1}, e^+_{(\text{iv})5}, e^-_{(\text{v})24}$ are non-negative real numbers.

The integrated NLPM for P_6 was established as follows:

$$\max \gamma$$
$$\text{such that.} \left\{ \begin{array}{c} 1.5628w_1^6 + 3.0521w_2^6 - 3.2620w_3^6 - 5.9925w_4^6 + 4.7560w_5^6 + 3.8294w_6^6 \geq \gamma, \\ -(e^-_{(\text{i})35} + e^-_{(\text{ii})62} + e^-_{(\text{ii})53} + e^+_{(\text{ii})62} + e^+_{(\text{ii})53} + e^-_{(\text{iii})524} + e^-_{(\text{iv})1} + e^-_{(\text{iv})5} + \\ e^+_{(\text{iv})1} + e^+_{(\text{iv})5} + e^-_{(\text{v})24}) \geq \lambda, \\ w_3^6 + e^-_{(\text{i})35} \geq w_5^6, w_6^6 - w_2^6 + e^-_{(\text{ii})62} \geq 0.08, w_5^6 - w_3^6 + e^-_{(\text{ii})53} \geq 0.01, \\ w_6^6 - w_2^6 - e^+_{(\text{ii})62} \leq 0.12, w_5^6 - w_3^6 - e^+_{(\text{ii})53} \leq 0.05, w_5^6 - 2w_2^6 + w_4^6 + \\ e^-_{(\text{iii})524} \geq 0, w_1^6 + e^-_{(\text{iv})1} \geq 0.15, \\ w_5^6 + e^-_{(\text{iv})5} \geq 0.11, w_1^6 - e^+_{(\text{iv})1} \leq 0.20, w_5^6 - e^+_{(\text{iv})5} \leq 0.16, \frac{w_2^6}{w_4^6} + e^-_{(\text{v})24} \geq 0.6, \\ w_1^6 + w_2^6 + w_3^6 + w_4^6 + w_5^6 + w_6^6 = 1, w_j^6 \geq 0 \text{ for all } j. \\ e^-_{(\text{i})35}, e^-_{(\text{ii})62}, e^-_{(\text{ii})53}, e^+_{(\text{ii})62}, e^+_{(\text{ii})53}, e^-_{(\text{iii})524}, e^-_{(\text{iv})1}, e^-_{(\text{iv})5}, e^+_{(\text{iv})1}, e^+_{(\text{iv})5}, e^-_{(\text{v})24} \geq 0 \end{array} \right\}. \tag{39}$$

We solved the above NLPM with the help of LINGO 19.0 and obtained the optimal objective value, $\overline{\gamma} = -0.01$, with the weight vector that optimal, $\overline{w}^6 = (0.15, 0.1527, 0.1, 0.2545, 0.11, 0.2327)$, and $\overline{e}^-_{(\text{i})35} = 0.01$ and $\overline{e}^-_{(\text{ii})62} = \overline{e}^-_{(\text{ii})53} = \overline{e}^+_{(\text{ii})62} = \overline{e}^+_{(\text{ii})53} = \overline{e}^-_{(\text{iii})524} = \overline{e}^-_{(\text{iv})1} = \overline{e}^-_{(\text{iv})5} = \overline{e}^+_{(\text{iv})1} = \overline{e}^-_{(\text{iv})5} = \overline{e}^-_{(\text{v})24} = 0$, are the corresponding optimal deviation values where the concern concordance/discordance indexes is $\overline{I}^6 = 0.2634$. We calculated all the $\overline{I}^t$ values and found that $\overline{I}^{20}$ generated the maximum value 1.7360. From this, we can conclude that $P_{20} = \{\mathscr{A}_4, \mathscr{A}_1, \mathscr{A}_3, \mathscr{A}_2\}$ is the best order of the suppliers under inconsistent weight information, which is significantly different from the result obtained for consistent weight information. The cause of this difference is the distinct weight distribution to the six criteria, for choice structures that are both consistent and inconsistent.

5.3. Comparative analysis

We have done a comparative study with various other approaches, such as the LIF-QUALIFLEX and fuzzy-TOPSIS method, to validate the results of the proposed algorithm.

The LIF-QUALIFLEX approach is first considered. Since LIVIFN ratings to solve MCDM problem of finding the best supplier. For this we first convert the LIVIFN into LIFN ratings. Since the LIVIFN rating of a option $\mathscr{A}_i$ as per x_j criterion that proved as $\widetilde{A}_{ij} = (s_{\theta_{ij}}, s_{\phi_{ij}}) = ([s^-_{\theta_{ij}}, s^+_{\theta_{ij}}], [s^-_{\phi_{ij}}, s^+_{\phi_{ij}}])$. The LIFN rating $\overline{A}_{ij}$ is defined as:

$$\overline{A}_{ij} = (s_{\bar{\theta}_{ij}}, s_{\bar{\phi}_{ij}}) = \left(\frac{s^-_{\theta_{ij}} + s^+_{\theta_{ij}}}{2}, \frac{s^-_{\phi_{ij}} + s^+_{\phi_{ij}}}{2} \right) \tag{40}$$

The likelihood of a FPR $\overline{A}_{\beta j} \geq \overline{A}_{\beta^* j}$ is calculated in LIFS as follows:

$$L(\overline{A}_{\beta j} \geq \overline{A}_{\beta^* j}) = \max\left\{ l - \max\left\{ l.\frac{(l - \bar{s}_{\phi_{\beta^* j}}) - \bar{s}_{\theta_{\beta j}}}{(l - \bar{s}_{\theta_{\beta j}} - \bar{s}_{\phi_{\beta j}}) + (l - \bar{s}_{\theta_{\beta^* j}} - \bar{s}_{\phi_{\beta^* j}})}, 0 \right\}, 0 \right\} \tag{41}$$

Next, for each P_t we determined concordance/discordance index $\overline{I}^t$, which is shown in Table 7. It is follows that $\overline{I}^{11} (= 5.2004)$ gives the largest value, As a result, the following is the best order for the four suppliers: $\mathscr{A}_2 > \mathscr{A}_4 > \mathscr{A}_1 > \mathscr{A}_3$, and vector of optimal weight in this case is $\overline{w}^{11} = (0.15, 0.066, 0.11, 0.418, 0.11, 0.146)$. This ranking result is identical to that produced using the LIVIF-QUALIFLEX method for the four options. As a result, the proposed method and the LIF-QUALIFLEX method produce the same results for ranking of options. Thus, we may conclude that the proposed method can also be implemented in a LIF environment.

The fuzzy TOPSIS is a next comparative method. The TOPSIS technique determines the shortest distance between alternative and PIS is for positive ideal solution, as well as the greatest distance between the chosen option and NIS stand for negative ideal solution (Zhu, Shuai, & Zhang, 2020) where $\widetilde{A}^+$ and $\widetilde{A}^-$ denote the LIVIF-PIS and LIVIF-NIS respectively, and are defined as follows:

$$\widetilde{A}^+ = \{\langle x_j, [s_l, s_l], [s_0, s_0] \rangle \mid x_j \in X, j = 1, 2, \dots, n\}, \tag{42}$$

$$\widetilde{A}^- = \{\langle x_j, [s_0, s_0], [s_l, s_l] \rangle \mid x_j \in X, j = 1, 2, \dots, n\}. \tag{43}$$

The weighted distances (Hwang, Yoon, Hwang, & Yoon, 1981), of each alternative from the LIVIF-PIS and NIS are denoted by $d(\widetilde{A}_i, \widetilde{A}^+)$ and $d(\widetilde{A}_i, \widetilde{A}^-)$ respectively, and calculated as follows:

$$\begin{aligned} d(\widetilde{A}_i, \widetilde{A}^+) &= \left[\frac{1}{4.l} \sum_{j=1}^{n} w_j \left(\left| s^-_{\theta_{ij}} - l \right|^\eta + \left| s^+_{\theta_{ij}} - l \right|^\eta + \left| s^-_{\varphi_{ij}} - 0 \right|^\eta + \left| s^+_{\varphi_{ij}} - 0 \right|^\eta \right) \right]^{\frac{1}{\eta}} \\ &= \left[\frac{1}{4.l} \sum_{j=1}^{n} w_j \left((l - s^-_{\theta_{ij}})^\eta + (l - s^+_{\theta_{ij}})^\eta + (s^-_{\varphi_{ij}})^\eta + (s^+_{\varphi_{ij}})^\eta \right) \right]^{\frac{1}{\eta}} \end{aligned} \tag{44}$$

$$\begin{aligned} d(\widetilde{A}_i, \widetilde{A}^-) &= \left[\frac{1}{4.l} \sum_{j=1}^{n} w_j \left(\left| s^-_{\theta_{ij}} - 0 \right|^\eta + \left| s^+_{\theta_{ij}} - 0 \right|^\eta + \left| s^-_{\varphi_{ij}} - l \right|^\eta + \left| s^+_{\varphi_{ij}} - l \right|^\eta \right) \right]^{\frac{1}{\eta}} \\ &= \left[\frac{1}{4.l} \sum_{j=1}^{n} w_j \left((s^-_{\theta_{ij}})^\eta + (s^+_{\theta_{ij}})^\eta + (l - s^-_{\varphi_{ij}})^\eta + (l - s^+_{\varphi_{ij}})^\eta \right) \right]^{\frac{1}{\eta}} \end{aligned} \tag{45}$$

Here η is the distance parameter, if $\eta = 1$, then (44),(45) become the weighted Hamming distances. If $\eta = 2$, then they reduce to the weighted Euclidean distances.

In the TOPSIS method, each closeness coefficient CC_i of the characteristic $\widetilde{A}_i$ for the alternative A_i is defined by the formula:

$$CC_i = \frac{d(\widetilde{A}_i, \widetilde{A}^-)}{d(\widetilde{A}_i, \widetilde{A}^+) + d(\widetilde{A}_i, \widetilde{A}^-)}, \tag{46}$$

where $0 \leq CC_i \leq l$

Then, under the circumstance of incomplete weight information, we created a multiple-objective programming model as follows:

$$\begin{aligned} &[M4] \ max\{CC_1, CC_2, \dots, CC_m\} \\ &\quad s.t \ (w_1, w_2, \dots, w_n) \in \rho. \end{aligned}$$

Using the max–min operator, the model in [M4] can be integrated into the following single-objective programming model:

$$\begin{aligned} &[M5] \ \max \lambda \\ &s.t \ CC_i \geq \lambda, i = 1, 2, \dots, m, \\ &\quad (w_1, w_2, \dots, w_n) \in \rho \end{aligned}$$

Consider to the same problem of selection of four suppliers under incomplete choices information. We used [M5] with $\eta = 2$ (the weighted Euclidean distance) to create the following NLPM based on the LIVIF decision matrix $\widetilde{D}_l$ in Table 4:

$$\max \lambda$$
$$s.t. \left\{ \begin{array}{c} CC_i \geq \lambda, i = 1, 2, 3, 4, \\ w_3 \geq w_5, 0.12 \geq w_6 - w_2 \geq 0.08, w_5 - w_2 \geq w_2 - w_4, \\ 0.20 \geq w_1 \geq 0.15, 0.16 \geq w_5 \geq 0.11, w_2 \geq 0.6 \cdot w_4 \\ w_1 + w_2 + w_3 + w_4 + w_5 + w_6 = 1, \\ w_j \geq 0 \text{ for all } j. \end{array} \right\}$$

where

$$
\begin{aligned}
CC_1 = &(1.7970w_1 + 2.5773w_2 + 2.8256w_3 + 1.7109w_4 + 2.8880w_5 + 3.7465w_6)^{0.5}/\\
&[(2.6573w_1 + 1.8429w_2 + 1.8651w_3 + 3.4051w_4 + 1.6536w_5 + 1.0808w_6)^{0.5}\\
&+ (1.7970w_1 + 2.5773w_2 + 2.8256w_3 + 1.7109w_4 + 2.8880w_5 + 3.7465w_6)^{0.5}]
\end{aligned}
$$

$$
\begin{aligned}
CC_2 = &(1.9593w_1 + 1.8108w_2 + 3.9566w_3 + 3.8066w_4 + 1.3823w_5 + 2.5164w_6)^{0.5}/\\
&[(2.4875w_1 + 2.6243w_2 + 0.8173w_3 + 1.0091w_4 + 3.3527w_5 + 1.9455w_6)^{0.5}\\
&+ (1.9593w_1 + 1.8108w_2 + 3.9566w_3 + 3.8066w_4 + 1.3823w_5 + 2.5164w_6)^{0.5}]
\end{aligned}
$$

$$
\begin{aligned}
CC_3 = &(1.9314w_1 + 3.1585w_2 + 2.0968w_3 + 1.5807w_4 + 3.8001w_5 + 2.8955w_6)^{0.5}/\\
&[(2.5002w_1 + 1.4694w_2 + 2.3536w_3 + 2.9166w_4 + 1.0301w_5 + 1.6986w_6)^{0.5}\\
&+ (1.9314w_1 + 3.1585w_2 + 2.0968w_3 + 1.5807w_4 + 3.8001w_5 + 2.8955w_6)^{0.5}]
\end{aligned}
$$

$$
\begin{aligned}
CC_4 = &(3.4664w_1 + 2.0136w_2 + 3.2262w_3 + 3.0216w_4 + 2.4241w_5 + 2.6921w_6)^{0.5}/\\
&[(1.4446w_1 + 2.7773w_2 + 1.6307w_3 + 1.5742w_4 + 2.2226w_5 + 1.9122w_6)^{0.5}\\
&+ (3.4664w_1 + 2.0136w_2 + 3.2262w_3 + 3.0216w_4 + 2.4241w_5 + 2.6921w_6)^{0.5}].
\end{aligned}
$$

We solved the above NLPM for each closeness coefficient CC_i, which deliver the values $CC_1 = 0.5575$, $CC_2 = 0.6009$, $CC_3 = 0.5387$ and $CC_4 = 0.5775$. The optimal objective value is 0.6009 with the optimal weight vector $(0.15, 0, 0.66, 0, 0.11, 0.08)$. Hence, the best order of the four suppliers is $P_{11} = (\mathscr{A}_2, \mathscr{A}_4, \mathscr{A}_1, A_3)$.

We consider the case of the weighted Hamming distances with $\eta = 1$. The optimal value is $CC_2 = 0.6138$ with optimal weight vector $\overline{w}^2 = (0.15, 0, 0.66, 0, 0.11, 0.08)$. The corresponding other objective values are $CC_1 = 0.5702, CC_3 = 0.5440, CC_4 = 0.5914$. The best ranking is $P_{11} = (\mathscr{A}_2, \mathscr{A}_4, \mathscr{A}_1, \mathscr{A}_3)$. So in both cases (Euclidean distances and Hamming distances) we achieved the same result.

Hence we conclude that, the LIVIF-QUALIFLEX method, LIF-QUALIFLEX method and fuzzy-TOPSIS method produce the same order ranking for the alternatives. One the one hand, the proposed method proves to be more effective in addressing problems where the number of criteria significantly exceeding the number of alternatives. This is because the computational results are derived based on permutations of alternatives in the proposed method, resulting in the integration of all criteria into a specific concordance/discordance index. The proposed method is highly suitable for situations where the number of criteria significantly exceeds the number of alternatives. Certainly, real-world examples of such challenges encompass decision-making scenarios related to public or government policies, the management of energy or natural resources, high-risk decision activities, problems characterized by extensive stakeholder involvement, and other complex or large-scale decisions that require the evaluation of multiple criteria for a restricted number of alternatives. High-risk perceivers are frequently characterized as narrow categorizers, as they tend to restrict their choices to a few secure alternatives. In contrast, low-risk perceivers are often labeled as broad categorizers because they have a tendency to select from a much wider range of alternatives. In high-risk scenarios, the proposed QUALIFLEX-based model serves as a valuable analytical tool for navigating multiple criteria decision-making processes. Decision makers who are highly involved tend to employ a more extensive set of criteria for the meticulous evaluation of a limited number of alternatives. In contrast, those who are less involved utilize simpler decision criteria to assess a broader range of alternatives. Hence, it is highly fitting to employ the suggested QUALIFLEX-based model in situations characterized by a high level of involvement. In essence, the suggested QUALIFLEX-based method proves valuable for addressing complex group decision-making problems characterized by comprehensive criteria and a restricted set of alternatives within the LIVIFS context.

5.4. Sensitivity analysis

In this section we analyzed the proposed method by applying two types of test criterion. To apply the first test criteria we changed the values of the alternative A_1 with respect to all criterion as shown in the following table:

c_1	$[2.3451, 4][1.7186, 3.9378]$
c_2	$[2, 3.8745][1.8650, 2.5000]$
c_3	$[2.5, 4][1.5453, 2.8007]$
c_4	$[1.5051, 3.0727][2.6636, 4.1213]$
c_5	$[2.5643, 4.5217][1.7593, 2.8564]$
c_6	$[2, 5.2814][1, 2.4636]$

After solving the corresponding linear programming problems, we obtained the optimal value $\bar{I}^{11} = 3.18146$ with weight vector $\overline{w}_{11} = (0.15, 0.066, 0.11, 0.418, 0.11, 0.146)$ for the same permutation P_{11} as that of the original problem and hence the ranking of alternatives $(\mathscr{A}_2, \mathscr{A}_4, \mathscr{A}_1, \mathscr{A}_3)$ is the same as for the original problem. From this we can conclude that, by changing the information of one of the alternative, does not effect the original order of the alternatives. For the second test criteria we converted the given problem into three subproblems. In each subproblem we considered the set of three alternatives, i.e $\{\mathscr{A}_1, \mathscr{A}_2, \mathscr{A}_3\}$ for the first subproblem, $\{\mathscr{A}_1, \mathscr{A}_2, \mathscr{A}_4\}$ for the second and $\{\mathscr{A}_2, \mathscr{A}_3, \mathscr{A}_4\}$ for the third subproblem. In each case there are six permutations, each of which is distinct from those of the original problem. Considering the first subproblem, we apply the proposed algorithm as follows:

$P1 = (\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3)$, $P2 = (\mathcal{A}_1, \mathscr{A}_3, \mathscr{A}_2)$, $P3 = (\mathscr{A}_2, \mathscr{A}_1, \mathscr{A}_3)$, $P4 = (\mathscr{A}_2, \mathscr{A}_3, \mathscr{A}_1)$, $P5 = (\mathscr{A}_3, \mathscr{A}_1, \mathscr{A}_2)$, $P6 = (\mathscr{A}_3, \mathscr{A}_2, \mathscr{A}_1)$

Results of Likelihood of fuzzy preference relations

	c_1	c_2	c_3	c_4	c_5	c_6
$\mathcal{L}(\widetilde{A}_{1j} \geq \widetilde{A}_{2j})$	3.9080	5.3596	2.3023	1.5503	6.1678	5.5471
$\mathcal{L}(\widetilde{A}_{1j} \geq \widetilde{A}_{3j})$	3.8316	3.0381	5.1773	4.1002	2.7191	4.2681
$\mathcal{L}(\widetilde{A}_{2j} \geq \widetilde{A}_{1j})$	4.6829	2.6404	5.6977	6.4497	1.8322	2.4529
$\mathcal{L}(\widetilde{A}_{2j} \geq \widetilde{A}_{3j})$	3.9265	1.9786	6.5141	6.5322	1.1685	3.3946
$\mathcal{L}(\widetilde{A}_{3j} \geq \widetilde{A}_{1j})$	4.1684	4.9619	2.8227	3.8998	5.2809	2.6015
$\mathcal{L}(\widetilde{A}_{3j} \geq \widetilde{A}_{2j})$	4.0735	6.0214	1.4859	1.4678	6.8315	4.6054

Results for concordance/discordance indices

	c_1	c_2	c_3	c_4	c_5	c_6
$I_j^l(\mathscr{A}_1, \mathscr{A}_2)$	−0.0920	1.3596	−1.6977	−2.4497	2.1678	1.5471
$I_j^l(\mathscr{A}_1, \mathscr{A}_3)$	−0.1684	−0.9619	1.1773	0.1002	−1.2809	0.2681
$I_j^l(\mathscr{A}_2, \mathscr{A}_1)$	0.6829	−1.3596	1.6977	2.4497	−2.1678	−1.5471
$I_j^l(\mathscr{A}_2, \mathscr{A}_3)$	−0.0735	−2.0214	2.5141	2.5322	−2.8315	−0.6054
$I_j^l(\mathscr{A}_3, \mathscr{A}_1)$	0.1684	0.9619	−1.1773	−0.1002	1.2809	−1.3985
$I_j^l(\mathscr{A}_3, \mathscr{A}_2)$	0.0735	2.0214	−2.5141	−2.5322	2.8315	0.6054

Results of the comprehensive concordance/discordance indices

$$I^1 = -0.3339w_1 - 1.6237w_2 + 1.9937w_3 + 0.1827w_4 - 1.9446w_5 + 1.2098w_6$$
$$I^2 = -0.1869w_1 + 2.4191w_2 - 3.0345w_3 - 4.8817w_4 + 3.7184w_5 + 2.4206w_6$$
$$I^3 = 0.4410w_1 - 4.3429w_2 + 5.3891w_3 + 5.0821w_4 - 6.2802w_5 - 1.8844w_6$$
$$I^4 = 0.7778w_1 - 2.4191w_2 + 3.0345w_3 + 4.8817w_4 - 3.7184w_5 - 3.5510w_6$$
$$I^5 = 0.1499w_1 + 4.3429w_2 - 5.3891w_3 - 5.0821w_4 + 6.2802w_5 + 0.7540w_6$$
$$I^6 = 0.9248w_1 + 1.6237w_2 - 1.9937w_3 - 0.1827w_4 + 1.9446w_5 - 2.3402w_6$$

After solving all the corresponding LMP we obtained the maximal value $\bar{I}^3 = 1.65226$, with optimal weight vector $\overline{w}_3 = (0.15, 0.066, 0.506, 0.022, 0.11, 0.146)$. Similarly for the subproblem 2 and subproblem 3 we obtained the optimal values $\bar{I}^6 = 1.56186$ with weight vector $w_6 = (0.2, 0.066, 0.11, 0.368, 0.11, 0.146)$ and $\bar{I}^5 = 1.65819$, with weight vector $w_5 = (0.2, 0.066, 0.11, 0.368, 0.11, 0.146)$ respectively.

In the first case $\mathscr{A}_2$ is the most desirable alternative as in the original problem, but in the remaining cases the alternative $\mathscr{A}_4$ is the leading one.

5.5. Practical implication

In linguistic intuitionistic fuzzy set theory, a linguistic intuitionistic fuzzy number is defined by both a linguistic membership degree and linguistic non-membership degree. Whereas, in linguistic interval-valued intuitionistic fuzzy set theory, a linguistic interval-valued intuitionistic fuzzy number is defined by both linguistic interval-valued membership degree and linguistic interval-valued non-membership degree, in order to address more effectively the imperfections inherent in subjective human judgment, particularly when contrasted with uncertain environment. To assess the applicability of decision-making tools in an linguistic interval-valued intuitionistic fuzzy environment, a case study on supplier selection is conducted for validation purposes, the work attempt linguistic interval-valued intuitionistic fuzzy (LIVIF) QUALIFLEX approach with a likelihood-based comparison method, LIF-QUALIFLEX method and TOPSIS method (Zhu et al., 2020). The LIVIF-QUALIFLEX method, LIF-QUALIFLEX method and TOPSIS method produce the same order ranking for the alternatives. The consistency of these methods is demonstrated by the same ranking order of candidate suppliers obtained in the aforementioned three decision-making approaches. Practitioners are recommended to embrace the methodological pathways outlined here for the purpose of achieving effective supplier selection. Practitioners are encouraged to engage in group decision-making processes by incorporating subjective evaluation criteria within the linguistic interval-valued intuitionistic fuzzy domain. This approach helps address real-world decision-making problems effectively. The selection of experts for participation in decision-making groups should be done judiciously to ensure a thoughtful and informed decision-making process.

6. Conclusions

New decision-making techniques are proposed in this work that are based on likelihood comparisons and the QUALIFLEX method in a LIVIF scenario. We started with upper and lower likelihood before proposing likelihood for LIVIFN comparison. We spoke about several positive aspects of the suggested likelihood technique. In the LIFS environment, we presented the concordance/discordance index, and calculated the terms using the likelihood-based comparison principle. Additionally, using the likelihood-based comparison notion and the QUALIFLEX method, built a decision-making strategy. To demonstrate the use and efficacy of the suggested strategy, presented a real-world decision-making problem involving supplier selection in a LIVIFS context. Additionally, contrasted the suggested strategy with other ways, demonstrating that it is well suited to handle decision-making issues in a LIVIF scenario. Investigated is the proposed approach's sensitivity.

The contributions of the aforementioned research are outlined below.

1. Due to the advantages of LIVIFS, an attempt has been made to apply the likelihood-based QUALIFLEX method with linguistic interval-valued intuitionistic fuzzy information to address a supplier selection problem. The consistent ranking order of candidate suppliers obtained through the three decision support tools, namely the LIF-QUALIFLEX method, LIVIF-TOPSIS method, and LIVIF-QUALIFLEX method, supports their reliability in a linguistic interval-valued intuitionistic fuzzy setting. While a variety of decision support tools based on the concept of LIVIFS can be thoroughly articulated from existing literature, the application of these tools in the context of supplier selection has seldom been explored.

2. The unique characteristic of the decision support tools utilized in the current study lies in their incorporation of the importance (weight) assigned by decision-makers. In many decision-making approaches, decision-makers are often assumed to have equal weights, implying that their opinions are considered equally important.

3. Regarding incomplete and inconsistent preference information, this paper developed a linear programming model to determine the optimal weight vector and the optimal comprehensive concordance/discordance indices. This approach aims to obtain the priority order of the alternatives. Moreover, a comprehensive nonlinear programming model was formulated to tackle challenges associated with incomplete and inconsistent information regarding criterion importance.

4. The practicality and applicability of the proposed method were validated through its implementation in addressing the real-world problem of selecting an appropriate supplier. As illustrated in the comparative analysis, the proposed method does not necessitate complicated computation procedures but still produces a reasonable and credible solution.

The limitations of the aforementioned research are outlined below.

1. The study has presented a conceptual illustrative example, specifically an empirical case study, rather than a real-world application. It is essential to investigate the validity and accuracy of these decision-making modules.

2. Another concern is related to the operational feasibility of these methodologies. The availability of decision-making information and the uncertain data required for the application of these methodologies seem to pose potential barriers to operational feasibility.

3. Over time, decision-makers should be encouraged to gather this type of data by conducting discussions and surveys facilitated by the selected decision-making group. This practice is crucial not only for the application of these methodologies but also for making important managerial decisions for their organization.

Future research will extend the proposed likelihood-based QUALIFLEX method to render it suitable for a decision environment of linguistic interval valued Pythagorean fuzzy set (LIVPFS) and linguistic interval valued Q-rung orthopair fuzzy set (LIV-q-ROFS) respectively. LIVPFS and LIV-q-ROFS can be applied to work with circumstances that have a high degree of uncertainty. On the other hand, we will combine the granular computing techniques with our developed method to address practical MCDM problems, such as the evaluation of green supply chain initiatives.

CRediT authorship contribution statement

Chiranjibe Jana: Writing – original draft, Writing – review & editing, Conceptualization, Investigation, Validation, Methodology, Formal analysis. **Afra Siab:** Writing – original draft, Writing – review & editing, Investigation, Validation. **Muhammad Sajjad Ali Khan:** Formal analysis, Validation, Methodology. **Madhumangal Pal:** Validation, Formal analysis, Methodology, Supervision, Project administration, Funding acquisition. **Luis Martinez:** Writing – original draft, Conceptualization, Formal analysis, Supervision. **Muhammad Asif Jan:** Writing – original draft, Methodology, Supervision.

Declaration of competing interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

This manuscript is the authors' original work and has not been published nor has it been submitted simultaneously elsewhere.

All authors have checked the manuscript and have agreed to the submission.

Data availability

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