Improved score based decision making method by using fuzzy soft sets

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Popular decision making methods by using fuzzy soft sets belong to two main categories, namely, score-based methods and fuzzy choice values based methods. It is necessary an application background to choose which one is better, since each of them makes sense in specific decision making situations and both of them could be improved. Therefore, in this contribution we focus on improving the former one. To improve the score based method, it is provided a novel adjustable algorithm by using fuzzy soft set that introduces thresholds when comparing two membership function values and afterwards coming up with new concepts of scores.

Keywords: Soft set; fuzzy soft set; decision making; scores.

1. Introduction

Many real word problems take place under uncertainties. Fuzzy set theory,[1] rough set theory,[2] vague set theory[3] and many others are some well known mathematical tools that could be used to handle these uncertainties. Lacking of parameterized tools is a common limitation of all previous models.

Molodstov[4] put forward the notion of soft set theory, which considers alternatives from different parameters aspects which successfully overcomes such a limitation. After the appearance of soft set theory, the researches concentrate on three main directions: 1) the operations and algebraic structure of soft sets;[5] 2) the fusion of soft sets with other existing models for dealing with uncertainty, various generalized soft set models have been proposed by researchers;[6] and 3) the applications of soft sets (and generalized soft sets) in practice situations,[7–10] especially in decision making (DM).[11–14]

Fuzzy soft sets were constructed from the combination of fuzzy sets and soft sets. One of the most popular DM method by using fuzzy soft sets was put forward by Roy and Maji[15] and built upon the concept of scores of alternatives, that is denoted by classical score based method. In such a method, by computing row and column sums of a comparison table the scores of alternatives could be obtained. In this contribution, a DM method by using fuzzy soft sets on the basis of several new concepts of scores for alternatives will be proposed. Since we follow Roy and Maji's idea[15] of using scores to achieve the optimal choice, it could be viewed as an improvement of classical score based method. Different comparison thresholds in forms of values or fuzzy sets could be adopted during the computation process of scores according to either the willing of decision makers or demands of the circumstances, in this way the decision results will be adjustable.

The remainder is organized as follows: Section 2 makes a brief review on soft sets and fuzzy soft sets. Section 3 introduces some new concepts for scores of alternatives based on fuzzy soft sets, and afterwards puts forward a decision making algorithm. Conclusions are given in Section 4.

2. Preliminaries

This section reviews basic definitions of soft sets and fuzzy soft sets.

A soft set is defined as a mapping from a parameter set to the power set of universe:

Definition 2.1.[4] Let U be the universe set and $P(U)$ be the power set of U. Let E be the parameter set and $A \subseteq E$. A pair (F, A) is called a soft set over U, where F is a mapping $F : A \longrightarrow P(U)$.

For any $e \in A$, $F(e)$ is $e-$approximate elements of the soft set (F, A), then a soft set is a parameterized family of subsets of U.

A fuzzy set[1] F in the universe U is defined as $F = \{(x, \mu_F(x))/x \in$

$U, \mu_F(x) \in [0,1]\}$, where $\mu_F(x)$ indicates the membership degree of alternative x determined by a membership function F.

Fuzzy soft sets are fuzzy generalizations of soft sets:

Definition 2.2.[16] Let U be the universe set and $F(U)$ be all fuzzy sets on U. A pair (F, A) is called a fuzzy soft set over U, where F is a mapping $F : A \longrightarrow F(U)$.

For any $e \in A$, $F(e)$ is a fuzzy subset of U. A fuzzy soft set will degenerate to a soft set when $F(e)$ degenerates to a subset of U for all $e \in A$. If $F(e)$ is a crisp subset of U for any $e \in A$, then the fuzzy soft set (F, A) degenerates to a crisp soft set. Denote the membership degree of $x \in U$ with respect to $e \in A$ as $\mu_{F(e)}(x)$, then $F(e)$ can be represented by $F(e) = \{< x, \mu_{F(e)}(x) > |x \in U\}$.

3. Improved Score Based Decision Making Method by Using Fuzzy Soft Sets

Maji and Roy introduced the notion of scores for alternatives and a way to compute the scores when solving DM problems by using fuzzy soft sets.[15] However, the way for determining scores should not be unique, because making use of different scores, different decisions may meet the demands of different applications. In this section, a novel method for DM will be given by introducing several novel concepts of scores for alternatives.

In the following, all concepts are constructed under the background of DM. Let $U = \{x_1, x_2, \ldots, x_n\}$ be a universe set that consists of n alternatives, E be a parameter set, $A \subseteq E$ and $A = \{e_1, e_2, \ldots, e_m\}$.

First, a measure called "d-level score" is introduced:

Definition 3.1. Suppose that (F, A) is a fuzzy soft set over U. For a value $d \in [0, 1]$, d-level score of $x_i \in U$ with respect to $e_l \in A$ is defined as

$$S(x_i)(e_l)_d = R(x_i)(e_l)_d - T(x_i)(e_l)_d, \tag{1}$$

where $R(x_i)(e_l)_d = |x_j \in U : \mu_{F(e_l)}(x_i) - \mu_{F(e_l)}(x_j) \geq d|$ and $T(x_i)(e_l)_d = |x_j \in U : \mu_{F(e_l)}(x_j) - \mu_{F(e_l)}(x_i) \geq d|$.

Based on Eq. (1), the d-level score of x_i, denoted by S_i^d, is defined as

$$S_i^d = \sum_{l=1}^{m} S(x_i)(e_l)_d. \tag{2}$$

A tool called d-level score table could be constructed with rows labeled by parameters in A and columns corresponds to alternatives in U. For each entry position (i, j), there is a value $S(x_i)(e_l)_d$ computed by Eq. (1).

An example to illustrate d-level score table can be the following one:

Example 3.1. Let $U = \{x_1, x_2, \ldots, x_4\}$, $E = \{e_1, e_2, \ldots, e_6\}$ and (F, E) be a fuzzy soft set (see Table 1).

Table 1. Fuzzy soft set (F, E).

	e_1	e_2	e_3	e_4	e_5	e_6
x_1	0.6	0.8	0.1	0.4	0.2	0.9
x_2	0.9	0.2	0.4	0.3	0.7	0.6
x_3	0.6	0.7	0.4	0.8	0.1	0.2
x_4	0.2	0.3	0.1	0.3	0.4	0.1

Suppose that $d = 0.2$, then 0.2-level score table of (F, E) could be obtained (see Table 2).

Table 2. 0.2-level score table of (F, E).

	e_1	e_2	e_3	e_4	e_5	e_6
x_1	0	2	−2	−1	−2	3
x_2	3	−2	2	−1	3	1
x_3	0	2	2	3	−2	−2
x_4	−3	−2	−2	−1	1	−2

In detail, $S(x_2)(e_3)_{0.2} = R(x_2)(e_3)_{0.2} - T(x_2)(e_3)_{0.2} = 2 - 0 = 2$, then the value in entry position $(2, 3)$ is 2. Values in other entries of Table 2 can be computed in a similar way.

Actually, $d \in [0, 1]$ is a comparison threshold used to compare two membership values for computing the d-level score. The methods for determining the threshold value (TV) d can be various, since it should be chosen depending on the requirement of DM situation.

The importance degrees for various parameters may be also different in decision makers' consideration, therefore the comparison TVs could be different for different parameters, which could be done by applying a function (a fuzzy set) instead of a value during the comparison process. Consider the specific structure of a fuzzy soft set, if we choose a TV within interval $[0, 1]$ for each parameter in A, all thresholds with respect to all parameters could form a fuzzy set.

In the following, we apply a fuzzy set θ as the threshold to introduce a concept called level score corresponding to a fuzzy set:

Definition 3.2. Let (F, A) be a fuzzy soft set over U and $\theta : A \longrightarrow [0, 1]$ be a fuzzy set on A, called a comparison threshold fuzzy set (TFS). The level score of $x_i \in U$ on $e_l \in A$ corresponding to θ is denoted by $S(x_i)(e_l)_\theta$ and defined as

$$S(x_i)(e_l)_\theta = r(x_i)(e_l)_\theta - t(x_i)(e_l)_\theta \tag{3}$$

where $r(x_i)(e_l)_\theta = |x_j \in U : \mu_{F(e_l)}(x_i) - \mu_{F(e_l)}(x_j) \geq \theta(e_l)|$ and $t(x_i)(e_l)_\theta = |x_j \in U : \mu_{F(e_l)}(x_j) - \mu_{F(e_l)}(x_i) \geq \theta(e_l)|$. The level score of alternative x_i corresponding to θ is defined as

$$S_i^\theta = \sum_{l=1}^{m} S(x_i)(e_l)_\theta. \tag{4}$$

A tool called level score table corresponding to fuzzy set θ could be constructed with rows labeled by parameters in A and columns corresponding to alternatives in U. For each entry position (i, j), there is an input a value $S(x_i)(e_l)_\theta$ computed by Eq. (3). Obviously, the level score of alternative x_i corresponding to θ could be obtained by computing the row sums of the corresponding level score table.

For the comparison TFS θ, if $\theta(e_l) = t$ ($t \in [0, 1]$ is a constant) for all $e_l \in A$, then the level score corresponding to θ degenerates to a d-level score. In other words, level score table corresponding to a fuzzy set degenerates to a d-level score table.

Based on (F, A), several commonly used TFSs are defined: (1) Mid TFS θ_F^{mid}: $\theta_F^{mid}(e_l) = \frac{1}{|U|}(max_{x_i \in U}\mu_{F(e_l)}(x_i) - min_{x_i \in U}\mu_{F(e_l)}(x_i))$; (2) Min TFS θ_F^{min}: $\theta_F^{min}(e_l) = min_{\{x_i, x_j \in U\}}|\mu_{F(e_l)}(x_i) - \mu_{F(e_l)}(x_j)|$; (3) Max TFS θ_F^{max}: $\theta_F^{max}(e_l) = max_{\{x_i, x_j \in U\}}(\mu_{F(e_l)}(x_i) - \mu_{F(e_l)}(x_j))$ for $e_l \in A$.

Level scores based DM method is carried out according to the below algorithm:

Algorithm 3.1.
Step 1. *Collect assessments on alternatives with respect to parameters and present the assessments as a fuzzy soft set* (F, A).
Step 2. *Select a comparison TV* $d \in [0, 1]$ *(or chose a comparison TFS* θ*) for* (F, A).
Step 3. *Construct the d-level score table (or level score table corresponding to fuzzy set* θ*) of* (F, A).
Step 4. *For each alternative* x_i*, calculate the d-level score of* x_i*, i.e.* S_i^d *(or level score corresponding to* θ*, i.e.* S_i^θ*).*

Step 5. *Choose x_j as the optimal choice by $S_j^d = max_i S_i^d$ and denote the decision result by $D((F,A),t)$ (or select x_j as the optimal choice if $S_j^\theta = max_i S_i^\theta$ and denote the decision result by $D((F,A),\theta)$).*

The optimal choice can be one or several alternatives. If there are too many selected alternatives as the decision result in step 5, decision makers could go back to step 2 to change the comparison threshold in order to limit the number of optimal choices. The application of TVs in DM benefits from idea of Feng et al.[17]

Example 3.2. In a DM problem, suppose that the assessments provided by decision makers form a fuzzy soft set (F, E) with its tabular representation (Table 1) in Example 3.1. If the problem is handled by using Algorithm 3.1, and the comparison TV is chosen as $d = 0.2$. From the 0.2-level score table of (F, E) (Table 2), the 0.2-level scores of alternatives are $S_1^{0.2} = 0$, $S_2^{0.2} = 6$, $S_3^{0.2} = 3$, and $S_4^{0.2} = -9$. Then, the optimal choice is x_2.

Theorem 3.1. *If the TV $d = 0$, then the d-level score of each alternative computed by Eq. (1) is the same as its score by using classical score based method.*

By Theorem 3.1, it is shown that the decision result obtained from Algorithm 3.1 will be the same as the result obtained by using classical score based method when the TV degenerates to zero. However, compared with classical score based method, the introduction of TVs could not only reflect the quantity, but also the quality to which degree alternatives satisfy parameters when two membership function values need to be compared, which makes the decision result adjustable and increases the flexibility of the classical method.

Theorem 3.2. *Suppose that a DM problem could be handled by using a fuzzy soft set (F, A). Let $D((F,A),\theta)$ be the decision result obtained from Algorithm 3.1, where θ is a comparison TFS. If $\theta(e_l) > \theta_F^{max}(e_l)$ for all $e_l \in A$, then $D((F,A),\theta) = U$.*

4. Conclusion

Explorations on DM methods by using fuzzy soft sets contribute to the development of soft set theory. A novel adjustable method has been introduced in this contribution. The proposed algorithm has the potential to be extended to deal with uncertain situations in which generalization models of fuzzy soft sets are applied.

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