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Real Life Applications of Multiple Criteria Decision Making Techniques in Fuzzy Domain

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Nonlinear Scaled Preferences in Linguistic Multi-criteria Group Decision Making

Diego García-Zamora, Álvaro Labella, Rosa M. Rodríguez, and Luis Martínez

Abstract Even though the use of data-driven decision models has increased its popularity during recent years, the resolution of some decision making problems still relies on the use of the expertise of specialists in the corresponding area, leading to decision situations characterized by the uncertainty and vagueness of the available information which may also require to model hesitancy between multiple choices. Thus, new approaches have been defined to model decision makers' indecision by means of complex linguistic expressions such as Extended Comparative Linguistic Expressions with Symbolic Translation (ELICIT), based on the 2-tuple linguistic model. Nevertheless, this approach, alike many other linguistic models, uses linear scales to model decision makers' preferences. Recent studies show that humans do not measure the distances between values at different levels of the linear scale in the same way and better decisions are obtained when nonlinear scales are considered. Therefore, this chapter introduces a multi-criteria group decision making model based on fuzzy TOPSIS dealing with ELICIT information which considers the nonlinear scales provided by the recently defined extreme values amplifications. It provides flexibility to express decision makers' preferences and guarantees more reliable results than those obtained with the classical linear scaled preferences.

Keywords Linguistic group decision making · ELICIT information · TOPSIS method · Extreme values amplifications

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1 Introduction

Nowadays, the use of decision processes guided exclusively by data (data-driven) with quantitative modelling is widely extended, and the involvement of human experts, who usually manage qualitative information, has been either neglected or relegated to second place [5, 7]. However, by considering the commitment, cost and relevance of taking into account human stakeholders in economical, social or learning studies, the use of decision approaches guided by experts (expert-driven) is still essential [12, 16] in several fields which demand intelligent, efficient and effective decisions under uncertain conditions. Therefore, in multi-attribute decision making, it is key to consider multi-criteria group decision making (MCGDM) models which allow to make decisions supported by human knowledge.

To model the vagueness of expert knowledge, fuzzy logic and fuzzy linguistic approach were proposed to model the uncertainty by linguistic variables [18] close to the human beings' way of thinking. In this sense, decision makers (DMs) can give their preferences by linguistic terms such as "good", "secure" or "comfortable", arising the linguistic decision making (LDM) approach [9], which allows to accomplish linguistic operations. In the literature, there are different approaches in order to model linguistic information [15]. One of the most well-known linguistic approaches is the 2-tuple linguistic model [8] because it allows to make accurate computations with linguistic information by preserving the interpretability of the obtained results. However, this model allows DMs to elicit his/her knowledge with linguistic information that consists of single linguistic terms that in certain situations in which DMs doubt among several terms may be insufficient. To this regard, other approaches have defined complex linguistic expressions to model DMs' hesitancy, such as the comparative linguistic expressions (CLEs) [15], although the computational processes related to these expressions are limited from the precision and interpretation points of view. To get over these shortcomings, Labella et al. [6] introduced the Extended Comparative Linguistic Expressions with Symbolic Translation (ELICIT) based on the CLEs by incorporating some features of the 2-tuple linguistic model.

These linguistic approaches assume that the DMs who participate in a group decision making (GDM) problem give their preferences by following linear scales. However, recent proposals [3, 10] suggest that when people rate alternatives, we do not measure the distances between values at different levels of the linear scale in the same way. In order to provide more realistic preferences values, closer to the real opinion of the DMs, García-Zamora et al. [4] proposed extreme values amplifications (EVAs) which aim at remapping the original preferences into nonlinear scaled preferences which are more realistic from a psychological point of view.

Keeping in mind the previous issues, this chapter proposes a MCGDM model based on fuzzy TOPSIS [2] in which DMs provide their preferences by using ELICIT information which will be remapped according to a nonlinear scale in order to obtain more realistic results according to human way of thinking. The main features of this contribution are summarized below:

- A MCGDM model is proposed to guide decision situations which require the use of expert knowledge.
- The DMs' preferences will be represented by ELICIT information which allows to model uncertainty and hesitancy by using flexible linguistic expressions.
- The original DMs' preferences will be remapped by nonlinear scales to provide more realistic preferences from a psychological point of view.
- The fuzzy TOPSIS method is combined with nonlinear scaled ELICIT preferences to guarantee more reliable results than the obtained with the classical linear scaled preferences.
- The performance of the proposed method is shown in a real case study.

The remainder sections are summarized below. Section 2 revises some preliminary concepts to facilitate the understanding of the proposal. Section 3 introduces the main proposal which consists of a MCGDM model based on fuzzy TOPSIS with ELICIT information and nonlinear preferences. In Sect. 4, a real-world MCGDM problem is solved to illustrate the working of the model. Section 5 will point out some conclusions.

2 Preliminaries

This section is devoted to revise several basics regarding MCGDM with linguistic information. First, some notation about MCGDM is clarified. Then, 2-tuple linguistic model and ELICIT information, which are key to model complex linguistic preferences, are reviewed. Afterwards, it is analysed the use of nonlinear scales to remap the original preferences obtained from the experts into more realistic values. Finally, the fuzzy TOPSIS method is described.

2.1 Linguistic Decision Making

Human beings face decision situations that consist of evaluating different choices and deciding which is the best one. In GDM, several DMs with different perspectives try to achieve an agreed solution for a decision problem. The alternatives may be evaluated on several criteria, for instance, there are several criteria to consider when we buy a car, speed, security or price. Under these conditions, we talk about MCGDM problems.

Most of MCGDM problems are defined under uncertain environments whose main features are both the lack of information and precision. Modelling such a uncertainty by means of precise numeric assessments may be a really hard task for the DMs, thus they need to use an expression domain able to express uncertainty. Fuzzy modelling and the use of linguistic information have provided successful results arising the LDM. Particularly, the fuzzy linguistic approach has been one of

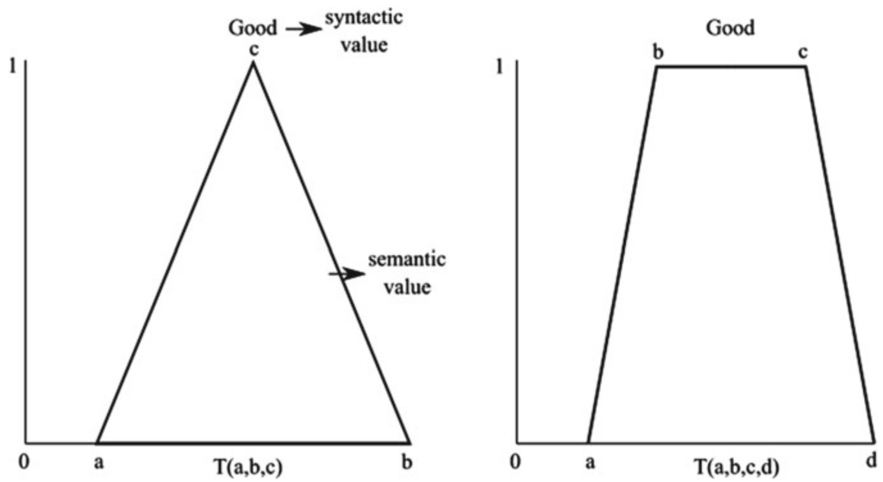

Fig. 1 Linguistic label

the most used methodologies to model uncertainty using means of *linguistic variables* [18] which are characterized by a syntactic value, represented by a common word in natural language, and a semantic value represented by a fuzzy set [19] defined by a membership function with different graphic representations. Triangular and trapezoidal representations are the most common representations (see Fig. 1).

2.2 2-Tuple Linguistic Model

The 2-tuple linguistic model [8] was proposed to improve the performance of classical linguistic computational models. Whereas the classical approaches presented either a lack of accuracy or understanding in their results, the 2-tuple linguistic approach models the linguistic values through a continuous fuzzy representation which allows to overcome such limitations.

Formally, the 2-tuple linguistic values are modelled by a tuple $(s_i, \alpha) \in \overline{S} := S \times [-0.5, 0.5[$ in which s_i represents a single linguistic term that belongs to a predefined linguistic term set $S = \{s_0, s_1, \ldots, s_g\}$ (for a certain even number $g \in \mathbb{N}$) and α refers to the concept symbolic translation, that is, a numerical value that represents the shifting of s_i fuzzy membership function. The symbolic translation concept changed the classical view of a linguistic term set understood as a set of discrete elements defined in a continuous domain which implied approximation processes to derive the results (see Fig. 2). Given a 2-tuple linguistic value $(s_i, \alpha) \in \overline{S}$, the possible values for the symbolic translation α are as follows:

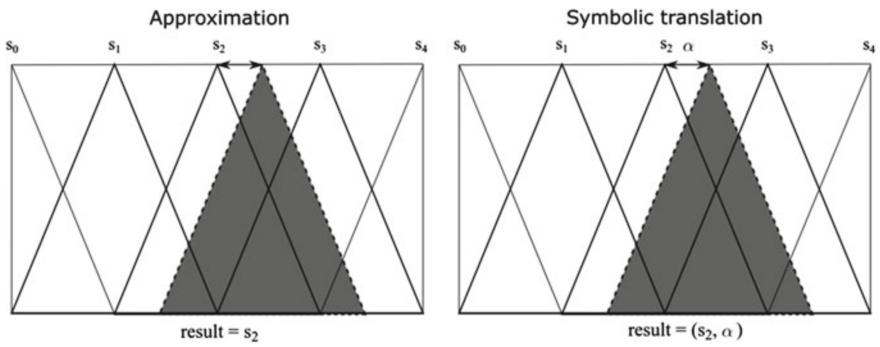

Fig. 2 Symbolic translation

$$\alpha \in \begin{cases} [-0.5, 0.5) & \text{if } s_i \in \{s_1, s_2, \ldots, s_{g-1}\} \\ [0, 0.5) & \text{if } s_i = s_0 \\ [-0.5, 0] & \text{if } s_i = s_g \end{cases}.$$

In addition, a 2-tuple linguistic value can be transformed into an equivalent numerical value $x \in [0, g]$, which makes the computations with 2-tuple linguistic values simple:

Proposition 1 *[8] Let $S = \{s_0, \ldots, s_g\}$ be a linguistic term set. Then, the function $\Delta^{-1} : \overline{S} \to [0, g]$ defined by*

$$\Delta_S^{-1}(s_i, \alpha) = i + \alpha, \ \forall\, (s_i, \alpha) \in \overline{S}$$

is a bijection whose inverse $\Delta_S : [0, g] \to \overline{S}$ is given by

$$\Delta_S(x) = (s_{round(x)}, x - round(x)) \ \forall\, x \in [0, g],$$

where $round(\cdot)$ is the function that assigns the closest integer number $i \in \{0, \ldots, g\}$.

2.3 Extended Comparative Linguistic Expressions with Symbolic Translation

Labella et al. [6] introduced the ELICIT information to overcome existing limitations related to linguistic preferences' modelling in GDM. These drawbacks concern two main aspects:

- Precision: accuracy in computations is key to obtain reliable results for GDM problems. Some proposals are limited by a discrete interpretation of the linguistic terms set or carry out transformation processes for the input information which lead to loss of information and precision in computations [14, 20].

- Understanding: results should be represented by a format which DMs are able to understand. Some proposals transform the initial linguistic DMs' preferences into other kinds of representations that are more complex from the point of view of comprehension [13, 17].

ELICIT information is based on the CLEs introduced by Rodríguez et al. [15]. These expressions are generated by a context-free grammar which models comparative linguistic structures closer to language human beings such as, *at least bad*, *at most fast* or *between expensive and rather expensive*. However, the CLEs computational processes introduced in the literature are limited by a discrete representation of the expression domain, which inevitably implies approximations to the real results, and consequently, loss of information. To overcome this limitation regarding precision, ELICIT information uses the symbolic translation concept of the 2-tuple linguistic model (see Sect. 2.2). In this way, the context-free grammar used to model CLEs is modified to replace the single linguistic terms that compose these expressions by 2-tuple linguistic values defined in a continuous domain:

Definition 1 [6] Let G_H be a context-free grammar and $S = \{s_0, \ldots, s_g\}$ a linguistic term set. The elements of $G_H = (V_N, V_T, I, P)$ are defined as follows:

$$
\begin{aligned}
&V_N = \{\langle\text{continuous primary term}\rangle, \langle\text{composite term}\rangle,\\
&\langle\text{unary relation}\rangle, \langle\text{binary relation}\rangle, \langle\text{conjunction}\rangle\}\\
&V_T = \{\text{at least, at most, between, and}, (s_0, \alpha)^{\gamma},\\
&(s_1, \alpha)^{\gamma}, \ldots, (s_g, \alpha)^{\gamma}\}\\
&I \in V_N\\
&P = \{I ::= \langle\text{continuous primary term}\rangle |\\
&\langle\text{composite term}\rangle\\
&\langle\text{composite term}\rangle ::= \langle\text{unary relation}\rangle\\
&\langle\text{continuous primary term}\rangle |\\
&\langle\text{binary relation}\rangle\langle\text{continuous primary term}\rangle\\
&\langle\text{conjunction}\rangle\langle\text{continuous primary term}\rangle\\
&\langle\text{continuous primary term}\rangle ::= (s_0, \alpha)^{\gamma} |\\
&(s_1, \alpha)^{\gamma} | \ldots | (s_g, \alpha)^{\gamma}\\
&\langle\text{unary relation}\rangle ::= \text{at least} | \text{at most}\\
&\langle\text{binary relation}\rangle ::= \text{between}\\
&\langle\text{conjunction}\rangle ::= \text{and}\}.
\end{aligned}
$$

Thus, this context-free grammar and the linguistic term set $S = \{$very cheap, cheap, indifferent, expensive, very expensive$\}$ can model linguistic expressions such as, at least $(\text{cheap}, 0.2)^{0.1}$, at most $(\text{expensive}, -0.1)^{0.12}$ or between $(\text{expensive}, 0)^{-0.11}$ and $(\text{very expensive}, 0.32)^{0}$. Notice the γ parameter is used to perform fuzzy computations with ELICIT information. To do so, the ELICIT expressions are transformed into trapezoidal fuzzy numbers by means the fuzzy envelope computation [6]:

Definition 2 [6] Let $H_S = \{\overline{s}_1, \overline{s}_2, \ldots, \overline{s}_m\} \subset \overline{S}$ be a set of ordered 2-tuple linguistic values. The fuzzy envelope of H_S, $env(H_S)$, is defined as the trapezoidal fuzzy number $T(a, b, c, d)$ where:

$$\begin{aligned} a &= \frac{\Delta_S^{-1}(\overline{s}_1) - \frac{1}{g}}{g}, \quad b = \frac{\Delta_S^{-1}(\overline{s}_1)}{g}, \\ c &= \frac{\Delta_S^{-1}(\overline{s}_m)}{g}, \quad d = \frac{\Delta_S^{-1}(\overline{s}_m) + \frac{1}{g}}{g}. \end{aligned} \tag{1}$$

2.4 Nonlinear Preferences in GDM: Extreme Values Amplifications

Even though classical GDM assumes that DMs give their preferences by using linear scales, several studies [3, 10] reveal that when nonlinear scales are used to remap their original preferences, better decisions are obtained. The reason behind this is a psychological fact: when people make judgements by rating alternatives, we are not completely precise expressing distances between values. For instance, when marking an exam, an A+ test will be required much more quality for getting an S than a C+ test for getting a B–.

In order to be able to model this human beings' behavior in consensus models for GDM, García-Zamora et al. [4] defined extreme values amplifications as those automorphisms on the unit interval which increase or decrease the distances between the most extreme values by preserving the symmetry of the remapped values respect 0.5. Formally:

Definition 3 (*Extreme Values Amplification*) [4] Let $D : [0, 1] \to [0, 1]$ be a function satisfying:

1. D is an automorphism on the interval $[0, 1]$,
2. D is a $\mathcal{C}^1$ function,
3. D satisfies $D(x) = 1 - D(1 - x)\ \forall\, x \in [0, 1]$,
4. $D'(0) > 1$ and $D'(1) > 1$,
5. D is concave in a neighbourhood of 0 and convex in a neighbourhood of 1.

D is called an EVA on the interval $[0, 1]$.

In addition, it was proved that a function satisfying the previous definition also verifies the following properties:

Theorem 1 *[4] Let $D : [0, 1] \to [0, 1]$ be an EVA on $[0, 1]$. Then:*

1. *The function $d_D : [0, 1] \times [0, 1] \to [0, 1]$ given by*

$$d_D(x, y) = |D(x) - D(y)| \; \forall \, x, y \in [0, 1],$$

   is a restricted dissimilarity [1] and the function $S_D : [0, 1] \times [0, 1] \to [0, 1]$ defined by

$$S_D(x, y) = 1 - |D(x) - D(y)| \; \forall \, x, y \in [0, 1],$$

   is a restricted equivalence function [1].
2. *Three intervals $I_1, \; I_2, \; I_3 \subset [0, 1]$ such that $0 \in I_1, \; 1 \in I_3$, and $I_1 < I_2 < I_3$ satisfying*

$$\begin{aligned} |D(y) - D(x)| &> |y - x| \; \forall \, x, y \in I_1 \; : \; x \neq y, \\ |D(y) - D(x)| &< |y - x| \; \forall \, x, y \in I_2 \; : \; x \neq y, \\ |D(y) - D(x)| &> |y - x| \; \forall \, x, y \in I_3 \; : \; x \neq y. \end{aligned}$$

   can be found.
3. *The graph of D is over the diagonal of the square $[0, 1] \times [0, 1]$ for values close enough to 0 and it is under the same diagonal for those values close enough to 1,*
4. *There are a neighbourhood U_0 containing 0 and a neighbourhood U_1 containing 1 such that for every $x, y \in U_0^\circ, \; x < y$, there exists $h_0 > 0$ such that the inequality $|D(x) - D(x - t)| \geq |D(y) - D(y - t)|$ holds for any $t \in [0, h_0]$ and for every $x, y \in U_1^\circ, \; x < y$, there exists $h_1 > 0$ such that the inequality $|D(x - t) - D(x)| \leq |D(y - t) - D(y)|$ holds for any $t \in [0, h_1]$.*

In other words:

1. EVAs remap the original linear scaled preferences elicited from DMs into non-linear scaled preferences,
2. EVAs amplify the distance between the extreme values, and reduce the distance between the intermediate ones,
3. EVAs have a concrete shape (see Fig. 3).

In this chapter, we will consider the EVA $m : [0, 1] \to [0, 1]$ defined by

$$m(x) = \begin{cases} \frac{1}{2} - \frac{1}{2}(1 - 2x)^2 & 0 \leq x < \frac{1}{2} \\ \frac{1}{2} + \frac{1}{2}(2x - 1)^2 & \frac{1}{2} \leq x \leq 1 \end{cases}, \tag{2}$$

due to its good performance in [4].

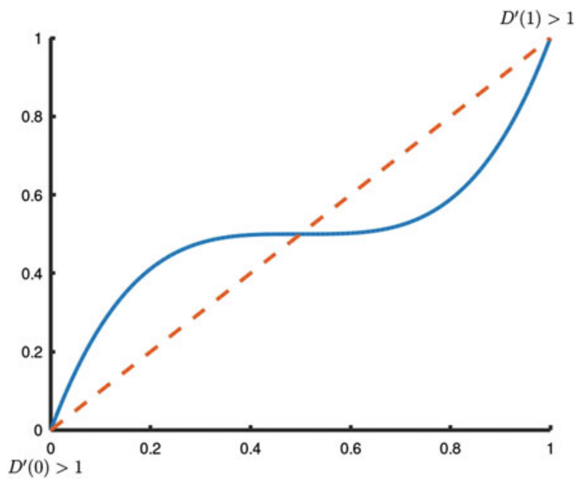

Fig. 3 Sketch of an EVA

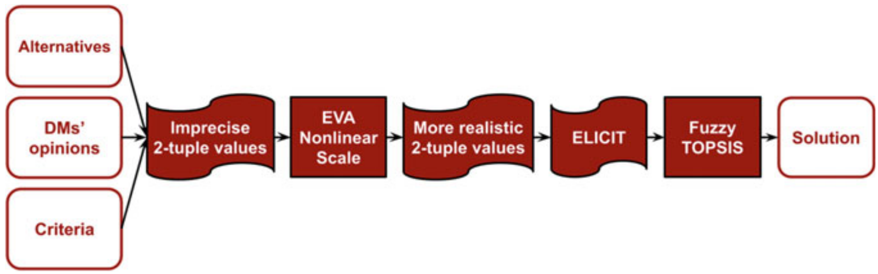

Fig. 4 Scheme of the proposed model

3 Linguistic Nonlinear Scales in Fuzzy TOPSIS

Here, nonlinear scales provided by EVAs are adapted for linguistic preferences. First, the notion of EVA is extended to 2-tuple linguistic values, and subsequently, to ELICIT information. Afterwards, this nonlinear scales are included in fuzzy TOPSIS method to solve a MCGDM problem in order to provide more realistic results from a psychological point of view. Fig. 4 shows the proposal's resolution scheme.

3.1 Nonlinear Preferences for Linguistic Information: 2-Tuple EVAs

This subsection is devoted to adapt the nonlinear scales provided by EVAs, formerly defined for numeric preferences, to linguistic environments (2-tuple and ELICIT information) in order to remap the original values of the DMs' preferences into new values which are more realistic from a psychological point of view [3, 10].

Let us consider a linguistic term set $S = \{s_0, s_1, \ldots, s_g\}$ and the associated set of 2-tuple linguistic values $\overline{S} = S \times [-0.5, 0.5]$. Formally, we aim at defining a nonlinear deformation $D_S : \overline{S} \to \overline{S}$ such that the distances between the extreme 2-tuple values are increased and the distances between intermediate 2-tuple values are reduced.

For the sake of clarity, given two nonempty compact intervals $[a, b], [\alpha, \beta] \subset \mathbb{R}$ we consider the standard affine transformation $h_{a,b}^{\alpha,\beta} : [a, b] \to [\alpha, \beta]$ defined by

$$h_{a,b}^{\alpha,\beta}(x) = \frac{x-a}{b-a}(\beta - \alpha) + \alpha \ \ \forall\, x \in [a, b].$$

In particular, we are interested in the one which transforms the interval [0, 1] into $[0, g]$, $h = h_{0,1}^{0,g} : [0, 1] \to [0, g]$ and in its inverse $h^{-1} = h_{0,g}^{0,1} : [0, g] \to [0, 1]$.

This nomenclature allows to easily define EVAs for 2-tuples values by using the original definition of EVA on [0, 1] [4]:

Definition 4 (*2-Tuple Extreme Value Amplification*) Let $D_S : \overline{S} \to \overline{S}$ be a mapping such that the function $D : [0, 1] \to [0, 1]$ defined by

$$D := h^{-1} \circ \Delta_S^{-1} \circ D_S \circ \Delta_S \circ h, \tag{3}$$

is an EVA on [0, 1]. Then, D_S will be called an EVA on the 2-tuple set $\overline{S}$.

Since both $h : [0, 1] \to [0, g]$ and $\Delta_S : [0, g] \to \overline{S}$ are bijections, the following result is straight-forward:

Proposition 2 *Let* $D : [0, 1] \to [0, 1]$ *be an EVA on* [0, 1]. *Then,* $D_S : \overline{S} \to \overline{S}$ *defined by*

$$\begin{aligned} D_S(s_i, \alpha) &= \Delta_S \circ h \circ D \circ h^{-1} \circ \Delta_S^{-1}(s_i, \alpha) \\ &= \left(s_{round(gD(\frac{i+\alpha}{g}))}, gD\left(\frac{i+\alpha}{g}\right) - round\left(gD\left(\frac{i+\alpha}{g}\right)\right)\right) \ \ \forall\, (s_i, \alpha) \in \overline{S}, \end{aligned}$$

is an EVA on $\overline{S}$.

This extension of the notion of EVA to the 2-tuple environment allows to preserve the properties summarised in Theorem 1. In general terms, the performance of 2-tuple EVAs can be summarised as follows:

- they remap 2-tuple values into 2-tuple values,
- the original preferences are deformed according to a nonlinear scale,
- the distances between 2-tuple values close to the extremes of the linguistic term set, namely s_0 and s_g, are amplified, whereas the distances between 2-tuple values close to the intermediate linguistic term, $s_{\frac{g}{2}}$, are reduced,

- the closer the distance between a point and the extreme values of the linguistic term set, the greater the amplification of distances is.

In addition, it should be highlighted that when using a 2-tuple EVA $D_S : \overline{S} \rightarrow \overline{S}$ there are just three fixed points, i.e. three 2-tuple values which are remapped on themselves, namely $(s_0, 0)$, $(s_{\frac{g}{2}}, 0)$ and $(s_g, 0)$. Furthermore, the deformation provided by 2-tuple EVAs maintains the symmetry towards the median value $(s_{\frac{g}{2}}, 0)$:

$$\Delta^{-1}(D_S(s_i, \alpha)) + \Delta^{-1}(D_S(s_{g-i}, -\alpha)) = g \ \forall\, i = 1, 2, \ldots, g, \ \forall\, \alpha \in [-0.5, 0.5[.$$

3.2 Nonlinear Preferences in ELICIT Information

Even though the 2-tuple linguistic model allows to represent linguistic information and make precise computations with it, this preference structure is not able to model DMs' opinions with some hesitation such as "between good and very good". Therefore, in order to consider richer linguistic expressions, the nonlinear scales provided by EVAs are adapted to ELICIT representation model, based on the 2-tuple linguistic model, which is able to model such hesitation.

In order to consider nonlinear scales in ELICIT information, it suffices to apply 2-tuple EVAs to the 2-tuple linguistic terms that define the ELICIT expression. Formally, given a linguistic term set $S = \{s_0, s_1, \ldots, s_g\}$ any input ELICIT information can be described as the fuzzy envelope of a set of certain 2-tuple linguistic ordered values $\overline{s}_1, \overline{s}_2, \ldots, \overline{s}_m \in \overline{S}$, $m \in \mathbb{N}$. Therefore, to adapt a 2-tuple EVA $D : \overline{S} \rightarrow \overline{S}$ to ELICIT information, we have just to consider the fuzzy envelope associated to $D(\overline{s}_1), D(\overline{s}_2), \ldots, D(\overline{s}_m)$.

3.3 Fuzzy TOPSIS with Linguistic Nonlinear Preferences

This section proposes a MCGDM model based on fuzzy TOPSIS [2] in which DMs' opinions are modelled through the linguistic nonlinear scales given by 2-tuple EVAs and the ELICIT linguistic approach. First, the DMs $E = \{dm_1, dm_2, \ldots, dm_m\}$ $m \in \mathbb{N}$ are asked to give their opinions about the alternatives $X = \{x_1, x_2, \ldots, x_n\}$ and criteria $C = \{c_1, c_2, \ldots, c_r\}$ by using linguistic information. To do so, a linguistic term set $S = \{s_0, s_1, \ldots, s_g\}$ is provided to the DMs whose opinions are elicited by using ELICIT expressions such as "s_1", "at most s_{g-1}" or "at least s_2". Afterwards, the fuzzy envelope for these linguistic expressions will be expressed in terms of linear scaled 2-tuple values, which will be remapped into nonlinear scaled 2-tuple values by using a 2-tuple EVA. Then, the nonlinear scaled preferences will be used to construct the trapezoidal fuzzy numbers corresponding to nonlinear scaled ELICIT information. Finally, the fuzzy TOPSIS method will be applied to obtain the solution for the MCGDM problem [11].

The complete proposal is developed in detail below:

1. Gathering preferences: the DMs $dm_1, dm_2, \ldots, dm_m$ give their linguistic ratings about the criteria importance $c_1, c_2, \ldots, c_r$ and about the alternatives $x_1, x_2, \ldots, x_n$ according to the given criteria by using CLEs.
2. Obtaining ELICIT information: the obtained CLEs are transformed into ELICIT information by assigning $\alpha = \gamma = 0$ for each linguistic term that composes the expression.
3. Remapping using nonlinear scales: the 2-tuple linguistic values associated to the ELICIT information obtained in the previous step are remapped by using an EVA (see Eq. 3). The resulting obtained values define a new nonlinear scaled ELICIT value.
4. Computing fuzzy envelopes: the ELICIT information obtained in the previous step is transformed into trapezoidal fuzzy numbers by using the fuzzy envelope (see Eq. 1). For each alternative x_i and each criteria c_j, the obtained trapezoidal fuzzy numbers are denoted by $\tilde{x}_{ij}^k = (a_{ij}^k, b_{ij}^k, c_{ij}^k, d_{ij}^k)$, and the opinions about the importance for each criteria c_j are denoted by the trapezoidal fuzzy numbers $\tilde{w}_j^k = (w_{j1}^k, w_{j2}^k, w_{j3}^k, w_{j4}^k)$.
5. Aggregating information: the obtained fuzzy numbers are aggregated into a collective opinion.

   - Collective fuzzy decision matrix $\tilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij}, d_{ij})$

$$\begin{aligned} a_{ij} &= \min_{k=1,2,\ldots,m} \{a_{ij}^k\}, \quad b_{ij} = \frac{1}{m}\sum_{k=1}^{m} b_{ij}^k, \\ c_{ij} &= \frac{1}{m}\sum_{k=1}^{m} c_{ij}^k, \quad d_{ij} = \max_{k=1,2,\ldots,m} \{d_{ij}^k\}. \end{aligned} \tag{4}$$

   - Collective fuzzy weights $\tilde{w}_j = (w_{j1}, w_{j2}, w_{j3}, w_{j4})$.

$$\begin{aligned} w_{j1} &= \min_{k=1,2,\ldots,m} \{w_{j1}^k\}, \quad w_{j2} = \frac{1}{m}\sum_{k=1}^{m} w_{j2}^k, \\ w_{j3} &= \frac{1}{m}\sum_{k=1}^{m} w_{j3}^k, \quad w_{j4} = \max_{k=1,2,\ldots,m} \{w_{j4}^k\}. \end{aligned} \tag{5}$$

6. Computing the normalized fuzzy decision matrix: a normalized fuzzy decision matrix $\tilde{R} = [\tilde{r}_{ij}]$ is built from the aggregated fuzzy information.

$$\begin{aligned} \tilde{r}_{ij} &= \left(\frac{a_{ij}}{d_j^+}, \frac{b_{ij}}{d_j^+}, \frac{c_{ij}}{d_j^+}, \frac{d_{ij}}{d_j^+}\right), \; d_j^+ = \max_i \{d_{ij}\} \text{ (benefit criteria)}, \\ \tilde{r}_{ij} &= \left(\frac{a_j^-}{d_{ij}}, \frac{a_j^-}{c_{ij}}, \frac{a_j^-}{b_{ij}}, \frac{a_j^-}{a_{ij}}\right), \; a_j^- = \min_i \{a_{ij}\} \text{ (cost criteria)}. \end{aligned} \tag{6}$$

7. Computing the weighted normalized decision matrix: the normalized decision ratings are multiplied by the aggregated criteria fuzzy weights.

$$\tilde{T} = [\tilde{t}_{ij}],\ \tilde{t}_{ij} = \tilde{r}_{ij} \times \tilde{w}_j. \tag{7}$$

Note that the multiplication between two trapezoidal fuzzy numbers is defined as $\tilde{A} \times \tilde{B} = (a_1a_2, b_1b_2, c_1c_2, d_1d_2)$

8. Deriving the fuzzy ideal solution and the fuzzy anti-ideal solution: the ideal (A^+) and anti-ideal (A^-) solutions are computed as follows:

$$A^+ = (\tilde{t}_1^+, \tilde{t}_2^+, \ldots, \tilde{t}_r^+),\ \tilde{t}_j^+ = \max_i \{\tilde{t}_{ij4}\}, \tag{8}$$

$$A^- = (\tilde{t}_1^-, \tilde{t}_2^-, \ldots, \tilde{t}_r^-),\ \tilde{t}_j^- = \min_i \{\tilde{t}_{ij1}\}. \tag{9}$$

9. Deriving distance from ideal and anti-ideal solution: for each alternative x_i the distance from the fuzzy ideal solution (δ_i^+) and the fuzzy anti-ideal solution (δ_i^-) is computed as follows:

$$\begin{aligned} \delta_i^+ &= \sum_{j=1}^{r} d(\tilde{t}_{ij}, \tilde{t}_j^+), \\ \delta_i^- &= \sum_{j=1}^{r} d(\tilde{t}_{ij}, \tilde{t}_j^-). \end{aligned} \tag{10}$$

where $d(\cdot)$ is a distance function between two trapezoidal fuzzy numbers defined as $d(\tilde{A}, \tilde{B}) = \sqrt{\frac{1}{4}[(a_1 - a_2)^2 + (b_1 - b_2)^2 + (c_1 - c_2)^2 + (d_1 - d_2)^2]}$.

10. Computing the closeness coefficient: for each alternative x_i, the closeness coefficient CC_i is derived as follows:

$$\mathrm{CC}_i = \frac{\delta_i^-}{\delta_i^- + \delta_i^+}. \tag{11}$$

11. Ranking the alternatives: the best alternative is the one with the highest CC_i.

The proposal is summarized in the following algorithm:

Algorithm 1: ELICIT fuzzy TOPSIS with nonlinear preferences.

Input : DMs ELICIT preferences
1 Obtain the 2-tuple representation for the ELICIT information;
2 Apply a 2-tuple EVA to the values obtained in the previous step;
3 Use the nonlinear scaled 2-tuple values to reconstruct ELICIT trapezoidal fuzzy numbers;
4 Apply fuzzy TOPSIS to the nonlinear scaled ELICIT information;
Output: Solution to the MCGDM problem.

Table 1 Initial linguistic DMs' preferences for criteria weights (bt stands for between)

	c_1	c_2	c_3	c_4
dm_1	F	F	F	I
dm_2	I	bt LI and F	VI	bt F and I
dm_3	I	I	VI	F
dm_4	VI	I	I	F

4 Case Study

This section is devoted to show the performance of the proposed model in a real-world problem.

Nowadays, the entertaining paradigm is changing for users. New streaming platforms offer lots of advantages before traditional cable TV services such as the possibility to decide what to watch at any time, month-to-month payments, non-permanency subscriptions or add-free contents. For these reasons, more and more people decide to choose one of these streaming platforms instead of traditional TV.

In this context, it is usual for families to face the harder part of this transition process: choosing the best streaming platform among the endless options. Initially, this could be a hard task for novice users of these kinds of services, since there are multiple factors to take into account in order to choose the most suitable alternative such as the cost, exclusive movies and series, variety of contents and frequency of new releases.

Therefore, here it is considered a GDM problem in which a family consisting of 4 members wants to subscribe to an online streaming platform according to the aforementioned criteria.

The $n = 4$ alternatives are $x_1 =$ Netflix, $x_2 =$ Amazon Prime Video, $x_3 =$ HBO and $x_4 =$ Disney+ and they have been rated according to $r = 4$ criteria, namely $c_1 =$ Cost, $c_2 =$ Exclusive movies and series, $c_3 =$ Variety of contents and $c_4 =$ Frequency of new releases. The linguistic expression domain for the rating of the criteria is

$$S_1 = \{\text{Not important (NI), A little important (LI),}\\ \text{Fair (F), Important (I), Very important (VI)}\}$$

and the linguistic terms for rating the alternatives according to the criteria are

$$S_2 = \{\text{Very bad (VB), Bad (B), Fair (F), Good (G), Very good (VG)}\}.$$

The initial DMs' preferences regarding the criteria importance and the rating of the alternatives are given, respectively, in Tables 1 and 2.

Table 2 Initial linguistic DMs' preferences for rating alternatives (bt stands for between)

		c_1	c_2	c_3	c_4
dm_1	x_1	G	G	F	VG
	x_2	VG	G	F	VG
	x_3	F	G	bt F and G	VG
	x_4	F	G	F	VG
dm_2	x_1	G	F	bt F and G	F
	x_2	VG	bt B and F	F	bt B and F
	x_3	bt F and G	bt F and G	F	F
	x_4	bt F and G	At least F	bt F and G	G
dm_3	x_1	G	G	bt F and G	F
	x_2	VG	B	B	F
	x_3	F	F	G	G
	x_4	F	B	F	B
dm_4	x_1	F	G	F	F
	x_2	G	B	B	B
	x_3	VB	B	B	B
	x_4	F	VG	VG	G

Table 3 DMs' preferences for rating alternatives in ELICIT (bt stands for between)

		c_1	c_2	c_3	c_4
dm_1	x_1	$(G, 0^0)$	$(G, 0)^0$	$(F, 0)^0$	$(\mathrm{VG}, 0)^0$
	x_2	$(\mathrm{VG}, 0)^0$	$(G, 0)^0$	$(F, 0)^0$	$(\mathrm{VG}, 0)^0$
	x_3	$(F, 0)^0$	$(G, 0)^0$	bt $(F, 0)^0$ and $(G, 0)^0$	$(\mathrm{VG}, 0)^0$
	x_4	$(F, 0)^0$	$(G, 0)^0$	$(F, 0)^0$	$(\mathrm{VG}, 0)^0$
dm_2	x_1	$(G, 0)^0$	$(F, 0)^0$	bt $(F, 0)^0$ and $(G, 0)^0$	$(F, 0)^0$
	x_2	$(\mathrm{VG}, 0)^0$	bt $(B, 0)^0$ and $(F, 0)^0$	$(F, 0)^0$	bt $(B, 0)^0$ and $(F, 0)^0$
	x_3	bt $(F, 0)^0$ and $(G, 0)^0$	bt $(F, 0)^0$ and $(G, 0)^0$	$(F, 0)^0$	$(F, 0)^0$
	x_4	bt $(F, 0)^0$ and $(G, 0)^0$	At least $(F, 0)^0$	bt $(F, 0)^0$ and $(G, 0)^0$	$(G, 0)^0$
dm_3	x_1	$(G, 0)^0$	$(G, 0)^0$	bt $(F, 0)^0$ and $(G, 0)^0$	$(F, 0)^0$
	x_2	$(\mathrm{VG}, 0)^0$	$(B, 0)^0$	$(B, 0)^0$	$(F, 0)^0$
	x_3	$(F, 0)^0$	$(F, 0)^0$	$(G, 0)^0$	$(G, 0)^0$
	x_4	$(F, 0)^0$	$(B, 0)^0$	$(F, 0)^0$	$(B, 0)^0$
dm_4	x_1	$(F, 0)^0$	$(G, 0)^0$	$(F, 0)^0$	$(F, 0)^0$
	x_2	$(G, 0)^0$	$(B, 0)^0$	$(B, 0)^0$	$(B, 0)^0$
	x_3	$(\mathrm{VB}, 0)^0$	$(B, 0)^0$	$(B, 0)^0$	$(B, 0)^0$
	x_4	$(F, 0)^0$	$(\mathrm{VG}, 0)^0$	$(\mathrm{VG}, 0)^0$	$(G, 0)^0$

Table 4 DMs' preferences for criteria weights in ELICIT (bt stands for between)

	c_1	c_2	c_3	c_4
dm_1	$(F, 0)^0$	$(F, 0)^0$	$(F, 0)^0$	$(I, 0)^0$
dm_2	$(I, 0)^0$	bt $(\mathrm{LI}, 0)^0$ and $(F, 0)^0$	$(VI, 0)^0$	bt $(F, 0)^0$ and $(I; 0)^0$
dm_3	$(I, 0)^0$	$(I, 0)^0$	$(VI, 0)^0$	$(F, 0)^0$
dm_4	$(VI, 0)^0$	$(I, 0)^0$	$(I, 0)^0$	$(F, 0)^0$

Table 5 Nonlinear DMs' preferences for rating alternatives (bt stands for between)

		c_1	c_2	c_3	c_4
dm_1	x_1	$(G, -0.5)^0$	$(G, -0.5)^0$	$(F, 0)^0$	$(\mathrm{VG}, 0)^0$
	x_2	$(\mathrm{VG}, 0)^0$	$(G, -0.5)^0$	$(F, 0)^0$	$(\mathrm{VG}, 0)^0$
	x_3	$(F, 0)^0$	$(G, -0.5)^0$	bt $(F, 0)^0$ and $(G, -0.5)^0$	$(\mathrm{VG}, 0)^0$
	x_4	$(F, 0)^0$	$(G, -0.5)^0$	$(F, 0)^0$	$(\mathrm{VG}, 0)^0$
dm_2	x_1	$(G, -0.5)^0$	$(F, 0)^0$	bt $(F, 0)^0$ and $(G, -0.5)^0$	$(F, 0)^0$
	x_2	$(\mathrm{VG}, 0)^0$	bt $(F, -0.5)^0$ and $(F, 0)^0$	$(F, 0)^0$	bt $(F, -0.5)^0$ and $(F, 0)^0$
	x_3	bt $(F, 0)^0$ and $(G, -0.5)^0$	bt $(F, 0)^0$ and $(G, -0.5)^0$	$(F, 0)^0$	$(F, 0)^0$
	x_4	bt $(F, 0)^0$ and $(G, -0.5)^0$	At least $(F, 0)^0$	bt $(F, 0)^0$ and $(G, -0.5)^0$	$(G, -0.5)^0$
dm_3	x_1	$(G, -0.5)^0$	$(G, -0.5)^0$	bt $(F, 0)^0$ and $(G, -0.5)^0$	$(F, 0)^0$
	x_2	$(\mathrm{VG}, 0)^0$	$(F, -0.5)^0$	$(F, -0.5)^0$	$(F, 0)^0$
	x_3	$(F, 0)^0$	$(F, 0)^0$	$(G, -0.5)^0$	$(G, -0.5)^0$
	x_4	$(F, 0)^0$	$(F, -0.5)^0$	$(F, 0)^0$	$(F, -0.5)^0$
dm_4	x_1	$(F, 0)^0$	$(G, -0.5)^0$	$(F, 0)^0$	$(F, 0)^0$
	x_2	$(G, -0.5)^0$	$(F, -0.5)^0$	$(F, -0.5)^0$	$(F, -0.5)^0$
	x_3	$(\mathrm{VB}, 0)^0$	$(F, -0.5)^0$	$(F, -0.5)^0$	$(F, -0.5)^0$
	x_4	$(F, 0)^0$	$(\mathrm{VG}, 0)^0$	$(\mathrm{VG}, 0)^0$	$(G, -0.5)^0$

Table 6 Nonlinear DMs' preferences for criteria weights (bt stands for between)

	c_1	c_2	c_3	c_4
dm_1	$(F, 0)^0$	$(F, 0)^0$	$(F, 0)^0$	$(I, -0.5)^0$
dm_2	$(I, -0.5)^0$	bt $(F, -0.5)^0$ and $(F, 0)^0$	$(\mathrm{VI}, 0)^0$	bt $(F, 0)^0$ and $(I, -0.5)^0$
dm_3	$(I, -0.5)^0$	$(I, -0.5)^0$	$(\mathrm{VI}, 0)^0$	$(F, 0)^0$
dm_4	$(\mathrm{VI}, 0)^0$	$(I, -0.5)^0$	$(I, -0.5)^0$	$(F, 0)^0$

Firstly, the CLEs are converted into ELICIT expressions (see Tables 3 and 4).

The ELICIT information is still represented by linear preference scales. Then, the preferences are remapped using nonlinear scales by using an EVA. Tables 5 and 6 give the nonlinear preferences by using the EVA $m(x)$ introduced in Eq. 2.

Before starting the fuzzy TOPSIS to obtain the ranking of the alternatives, the nonlinear ELICIT preferences are transformed into fuzzy numbers by using the fuzzy envelope computation (see Eq. 1). The respective fuzzy envelopes are given in Tables 7 and 8.

From this step, the fuzzy TOPSIS starts. First, all the DMs' preferences are aggregated to obtain a collective fuzzy decision matrix and the collective fuzzy criteria weights computed by means of Eqs. 4 and 5, respectively (see Tables 9 and 10).

Afterwards, the fuzzy decision matrix is normalized by using Eq. 6 (see Table 11).

The fuzzy normalized decision matrix is multiplied by the collective fuzzy weights in order to obtain the weighted fuzzy decision matrix (see Eq. 7), which is represented in Table 12.

Therefore, the fuzzy ideal solution (see Eq. 8) is

$$A^+ = (1, 0.875, 1, 0.875)$$

and the fuzzy anti-ideal solution (see Eq. 9) is

$$A^- = (0, 0.015625, 0.03125, 0.03125).$$

Now, the distances from the ideal and anti-ideal solution are computed by using Eq. 10 (see Table 13).

Finally, the closeness coefficient for each alternatives is computed by Eq. 11 (see Table 14).

According to the fuzzy TOPSIS, the best streaming platform is x_4 = Disney+.

Table 7 Fuzzy DMs' preferences

		c_1	c_2	c_3	c_4
dm_1	x_1	$T(0.375, 0.625, 0.625, 0.875)$	$T(0.375, 0.625, 0.625, 0.875)$	$T(0.25, 0.5, 0.5, 0.75)$	$T(0.75, 1, 1, 1)$
	x_2	$T(0.75, 1, 1, 1)$	$T(0.375, 0.625, 0.625, 0.875)$	$T(0.25, 0.5, 0.5, 0.75)$	$T(0.75, 1, 1, 1)$
	x_3	$T(0.25, 0.5, 0.5, 0.75)$	$T(0.375, 0.625, 0.625, 0.875)$	$T(0.25, 0.5, 0.625, 0.875)$	$T(0.75, 1, 1, 1)$
	x_4	$T(0.25, 0.5, 0.5, 0.75)$	$T(0.375, 0.625, 0.625, 0.875)$	$T(0.25, 0.5, 0.5, 0.75)$	$T(0.75, 1, 1, 1)$
dm_2	x_1	$T(0.375, 0.625, 0.625, 0.875)$	$T(0.25, 0.5, 0.5, 0.75)$	$T(0.25, 0.5, 0.625, 0.875)$	$T(0.25, 0.5, 0.5, 0.75)$
	x_2	$T(0.75, 1, 1, 1)$	$T(0.125, 0.375, 0.5, 0.75)$	$T(0.25, 0.5, 0.5, 0.75)$	$T(0.125, 0.375, 0.5, 0.75)$
	x_3	$T(0.25, 0.5, 0.625, 0.875)$	$T(0.25, 0.5, 0.625, 0.875)$	$T(0.25, 0.5, 0.5, 0.75)$	$T(0.25, 0.5, 0.5, 0.75)$
	x_4	$T(0.25, 0.5, 0.625, 0.875)$	$T(0.375, 0.625, 1, 1)$	$T(0.25, 0.5, 0.625, 0.875)$	$T(0.375, 0.625, 0.625, 0.875)$
dm_3	x_1	$T(0.375, 0.625, 0.625, 0.875)$	$T(0.375, 0.625, 0.625, 0.875)$	$T(0.25, 0.5, 0.625, 0.875)$	$T(0.25, 0.5, 0.5, 0.75)$
	x_2	$T(0.75, 1, 1, 1)$	$T(0.125, 0.375, 0.375, 0.625)$	$T(0.125, 0.375, 0.375, 0.625)$	$T(0.25, 0.5, 0.5, 0.75)$
	x_3	$T(0.25, 0.5, 0.5, 0.75)$	$T(0.25, 0.5, 0.5, 0.75)$	$T(0.375, 0.625, 0.625, 0.875)$	$T(0.375, 0.625, 0.625, 0.875)$
	x_4	$T(0.25, 0.5, 0.5, 0.75)$	$T(0.125, 0.375, 0.375, 0.625)$	$T(0.25, 0.5, 0.5, 0.75)$	$T(0.125, 0.375, 0.375, 0.625)$
dm_4	x_1	$T(0.25, 0.5, 0.5, 0.75)$	$T(0.375, 0.625, 0.625, 0.875)$	$T(0.25, 0.5, 0.5, 0.75)$	$T(0.25, 0.5, 0.5, 0.75)$
	x_2	$T(0.375, 0.625, 0.625, 0.875)$	$T(0.125, 0.375, 0.375, 0.625)$	$T(0.125, 0.375, 0.375, 0.625)$	$T(0.125, 0.375, 0.375, 0.625)$
	x_3	$T(0, 0, 0, 0.25)$	$T(0.125, 0.375, 0.375, 0.625)$	$T(0.125, 0.375, 0.375, 0.625)$	$T(0.125, 0.375, 0.375, 0.625)$
	x_4	$T(0.25, 0.5, 0.5, 0.75)$	$T(0.75, 1, 1, 1)$	$T(0.75, 1, 1, 1)$	$T(0.375, 0.625, 0.625, 0.875)$

Table 8 Fuzzy criteria weights

	c_1	c_2	c_3	c_4
dm_1	$T(0.25, 0.5, 0.5, 0.75)$	$T(0.25, 0.5, 0.5, 0.75)$	$T(0.25, 0.5, 0.5, 0.75)$	$T(0.375, 0.625, 0.625, 0.875)$
dm_2	$T(0.375, 0.625, 0.625, 0.875)$	$T(0.125, 0.375, 0.5, 0.75)$	$T(0.75, 1, 1, 1)$	$T(0.25, 0.5, 0.625, 0.875)$
dm_3	$T(0.375, 0.625, 0.625, 0.875)$	$T(0.375, 0.625, 0.625, 0.875)$	$T(0.75, 1, 1, 1)$	$T(0.25, 0.5, 0.5, 0.75)$
dm_4	$T(0.75, 1, 1, 1)$	$T(0.375, 0.625, 0.625, 0.875)$	$T(0.375, 0.625, 0.625, 0.875)$	$T(0.25, 0.5, 0.5, 0.75)$

Table 9 Fuzzy decision matrix

	c_1	c_2	c_3	c_4
x_1	$T(0.25, 0.59375, 0.59375, 0.875)$	$T(0.25, 0.59375, 0.59375, 0.875)$	$T(0.25, 0.5, 0.5625, 0.875)$	$T(0.25, 0.625, 0.625, 1)$
x_2	$T(0.375, 0.90625, 0.90625, 1)$	$T(0.125, 0.4375, 0.46875, 0.875)$	$T(0.125, 0.4375, 0.4375, 0.75)$	$T(0.125, 0.5625, 0.59375, 1)$
x_3	$T(0, 0.375, 0.40625, 0.875)$	$T(0.125, 0.5, 0.53125, 0.875)$	$T(0.125, 0.5, 0.53125, 0.875)$	$T(0.125, 0.625, 0.625, 1)$
x_4	$T(0.25, 0.5, 0.53125, 0.875)$	$T(0.125, 0.65625, 0.75, 1)$	$T(0.25, 0.625, 0.65625, 1)$	$T(0.125, 0.65625, 0.65625, 1)$

Table 10 Fuzzy collective weights

	c_1	c_2	c_3	c_4
Coll. W	$T(0.25, 0.6875, 0.6875, 1)$	$T(0.125, 0.53125, 0.5625, 0.875)$	$T(0.25, 0.78125, 0.78125, 1)$	$T(0.25, 0.53125, 0.5625, 0.875)$

Table 11 Normalized decision matrix

	c_1	c_2	c_3	c_4
x_1	$T(0.25, 0.59375, 0.59375, 0.875)$	$T(0.25, 0.59375, 0.59375, 0.875)$	$T(0.25, 0.5, 0.5625, 0.875)$	$T(0.25, 0.625, 0.625, 1)$
x_2	$T(0.375, 0.90625, 0.90625, 1)$	$T(0.125, 0.4375, 0.46875, 0.875)$	$T(0.125, 0.4375, 0.4375, 0.75)$	$T(0.125, 0.5625, 0.59375, 1)$
x_3	$T(0, 0.375, 0.40625, 0.875)$	$T(0.125, 0.5, 0.53125, 0.875)$	$T(0.125, 0.5, 0.53125, 0.875)$	$T(0.125, 0.625, 0.625, 1)$
x_4	$T(0.25, 0.5, 0.53125, 0.875)$	$T(0.125, 0.65625, 0.75, 1)$	$T(0.25, 0.625.0.65625, 1)$	$T(0.125, 0.65625, 0.65625, 1)$

Table 12 Weighted decision matrix

	c_1	c_2	c_3	c_4
x_1	$T(0.0625, 0.408, 0.408, 0.875)$	$T(0.031225, 0.3150.333, 0.765)$	$T(0.0625, 0.390, 0.439, 0.875)$	$T(0.0625, 0.332, 0.351, 0.875)$
x_2	$T(0.09375, 0.623, 0.623, 1)$	$T(0.015625, 0.232, 0.263, 0.765)$	$T(0.03125, 0.341, 0.341, 0.750)$	$T(0.03125, 0.298, 0.333)$
x_3	$T(0, 0.257, 0.279, 0.875)$	$T(0.015625, 0.265, 0.298, 0.765)$	$T(0.03125, 0.390, 0.415, 0.875)$	$T(0.03125, 0.332, 0.351, 0.875)$
x_4	$T(0.0625, 0.343, 0.365, 0.875)$	$T(0.015625, 0.348, 0.421, 0.875)$	$T(0.0625, 0.488, 0.512, 1)$	$T(0.03125, 0.348, 0.369, 0.875)$

Table 13 Distances to the ideal and anti-ideal solutions

Distances	δ_i^+	δ_i^+
x_1	2.390	1.937
x_2	2.406	1.966
x_3	2.536	1.869
x_4	2.354	2.077

Table 14 Closeness coefficients

Distances	CC_i
x_1	0.447
x_2	0.449
x_3	0.424
x_4	0.468

5 Conclusions

This proposal has introduced the use of nonlinear scaled preferences in linguistic MCGDM by defining an expert-driven fuzzy TOPSIS-based model which is able to consider expert knowledge in the decision process.

To do so, the DMs participating in the decision situation express their opinions by using CLEs, in order to allow them to express their hesitancy about certain ratings, which will be translated to ELICIT information. The corresponding trapezoidal fuzzy numbers of the ELICIT expressions are expressed in terms of 2-tuple values, which are remapped into nonlinear scaled 2-tuple linguistic values by using the novel notion of 2-tuple EVA. From the resulting nonlinear scaled 2-tuple values, new nonlinear scaled ELICIT values are obtained, which will be used in a fuzzy TOPSIS model to obtain more realistic results according to human psychology.

Finally, a real-world problem in which a family wants to decide which streaming platform is most suitable according to their preferences about several criteria has been shown.

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