Advances in Complex Decision Making

Using Machine Learning and Tools for Service-Oriented Computing

Edited by
Walayat Hussain
Honghao Gao
Fethi Rabhi
Luis Martínez

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Contents

5 Interval Type-2 Fuzzy Decision Analysis Framework Based on Online Textual Reviews

Xiao-Hong Pan and Shi-Fan He
Fuzhou University, Fujian, China
University of Jaén, Jaén, Spain

Diego García-Zamora and Luis Martínez
University of Jaén, Jaén, Spain

5.1 INTRODUCTION

According to the "wisdom of the crowd" theory, the decisions made by a large group of nonexpert decision-makers may be smarter than those made by a small group of experts [1, 2]. Although gathering such a large group of decision-makers could have been a complex task some years ago due to mobility costs or lack of space [3, 4], the development of social networks provides a huge source of nonexpert opinions about almost any topic [5, 6] that could be considered to make intelligent decisions [4]. Nevertheless, the data that can be found on the Web lacks a common structure [7]. In particular, the most usual format of unstructured information is the text expressed in natural language, but how to exploit the information contained in textual reviews to make decisions is still an open problem.

In this regard, one of the main challenges to be addressed is related to the management of the uncertainties derived from either vagueness (fuzziness) or incompleteness (ignorance) [8, 9]. A widely used classical representation scheme in decision making used to model the uncertainties inherent to incompleteness is the Linguistic Distribution Assessments (LDA) introduced by Zhang et al. [10, 11]. On the other hand, the management of uncertainties related to vagueness has been traditionally addressed with the fuzzy linguistic approach. In particular, the interval type-2 fuzzy sets (IT2FSs) proposed by Mendel et al. [12] are considered an effective tool for computing with words.

Therefore, this proposal introduces a multi-criteria decision analysis framework for Online Textual Reviews (OTRs) that models both imprecision and incompleteness by hybridizing IT2FSs and LDAs. First, we introduce a processing mechanism to transform unstructured online reviews into a distributed assessment framework. Then, an entropy-based interval type-2 fuzzy weights determination model is

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developed to capture the uncertainty of the criteria. Afterward, the Evidential Reasoning (ER) approach [13] is extended to aggregate the interval type-2 distributed information. Moreover, some rules are provided to generate the expected utility of each alternative. Finally, a minimax regret approach is defined to rank interval-valued expected utilities that improves existing interval-valued ranking approaches.

The remaining sections are summarized as follows. In Section 5.2, the basic notions necessary to understand the proposal are reviewed. Section 5.3 presents our multi-criteria decision-making framework for managing OTRs based on IT2FSs and LDAs. Section 5.4 shows the performance of our proposal when solving a multi-criteria decision problem. Finally, Section 5.5 highlights some conclusions.

5.2 PRELIMINARIES

To make the study self-contained, this section introduces some basic concepts about IT2FSs, the entropy-based weight determination model, the evidential reasoning approach, and the minimax regret approach.

5.2.1 Interval Type-2 Fuzzy Sets

The concept of IT2FSs was initially proposed by Mendel et al. [14, 15] on the basis of the type-2 fuzzy sets defined by Zadeh [16]. To date, the IT2FSs have been widely applied to deal with the uncertainty involved in decision-making processes [17–21]. Some basic theories and methods about IT2FSs are introduced below.

Definition 1

(T2FS [14, 15]). Suppose that $\mu_{\tilde{A}}(x,u)$ is the type-2 fuzzy membership function of type-2 fuzzy set (T2FS) $\tilde{A}$ and the T2FS $\tilde{A}$ can be expressed as:

$$\tilde{A} = \left\{\left((x,u), \mu_{\tilde{A}}(x,u)\right) \mid \forall x \in X, \forall u \in J_x \in [0,1]\right\}$$

where X is the universe of discourse of $\tilde{A}$, u is primary membership at $x \in X$, representing the belief degree of x belonging to A, and $\mu_{\tilde{A}}(x,u)$ denotes the membership of primary membership, that is, secondary membership. Moreover, the T2FS also can be denoted as follows:

$$\tilde{A} = \int_{x\in X} \int_{u\in J_x} \mu_{\tilde{A}}(x,u)/(x,u)$$

Definition 2

(IT2FS [14, 15]). If all the values of the secondary membership are equal to 1, that is, $\mu_{\tilde{A}}(x,u) = 1$, then $\tilde{A}$ is called IT2FS and can be expressed by the following mathematical formula:

$$\tilde{A} = \int_{x \in X} \int_{u \in J_x} 1/(x,u)$$

In which, x is the main variable and $J_x \in [0, 1]$ represents the primary membership at x.

It should be highlighted that different membership functions will lead to different interval type-2 fuzzy formats. In this regard, the trapezoidal IT2FSs proposed by Chen and Chang [22] is one of the most widely used IT2FSs in decision making.

Definition 3

(Trapezoidal [22]). The trapezoidal IT2FS defined in the universe of discourse X can be expressed in the following form:

$$\tilde{A} = \left(\mu_{\tilde{A}}^L, \mu_{\tilde{A}}^U\right) = \left[\left(a_1^L, a_2^L, a_3^L, a_4^L, h^L\left(\tilde{A}\right)\right), \left(a_1^U, a_2^U, a_3^U, a_4^U, h^U\left(\tilde{A}\right)\right)\right]$$

where a_1^L, a_2^L, a_3^L, a_4^L, a_1^U, a_2^U, a_3^U, and a_4^U are crisp values and satisfy $0 \le a_1^L \le a_2^L \le a_3^L \le a_4^L$, $0 \le a_1^U \le a_2^U \le a_3^U \le a_4^U$, $a_1^U \le a_1^L$ and $a_4^L \le a_4^U$; $h^L\left(\tilde{A}\right)$ and $h^U\left(\tilde{A}\right)$ denote the heights of trapezoidal IT2FS, satisfying $0 \le h^L\left(\tilde{A}\right) \le h^U\left(\tilde{A}\right) \le 1$. The membership functions of the trapezoidal IT2FS are defined in the following way (see Fig 5.1).

$$\mu_{\tilde{A}}^L(x) = \begin{cases} \dfrac{h^L\left(\tilde{A}\right)\cdot\left(x - a_1^L\right)}{a_2^L - a_1^L}, & a_1^L \le x < a_2^L \\ h^L\left(\tilde{A}\right), & a_2^L \le x < a_3^L \\ \dfrac{h^L\left(\tilde{A}\right)\cdot\left(a_4^L - x\right)}{a_4^L - a_3^L}, & a_3^L \le x < a_4^L \\ 0, & \text{otherwise} \end{cases}$$

$$\mu_{\tilde{A}}^U(x) = \begin{cases} \dfrac{h^U\left(\tilde{A}\right)\cdot\left(x - a_1^U\right)}{a_2^U - a_1^U}, & a_1^U \le x < a_2^U \\ h^U\left(\tilde{A}\right), & a_2^U \le x < a_3^U \\ \dfrac{h^U\left(\tilde{A}\right)\cdot\left(a_4^U - x\right)}{a_4^U - a_3^U}, & a_3^U \le x < a_4^U \\ 0, & \text{otherwise} \end{cases}$$

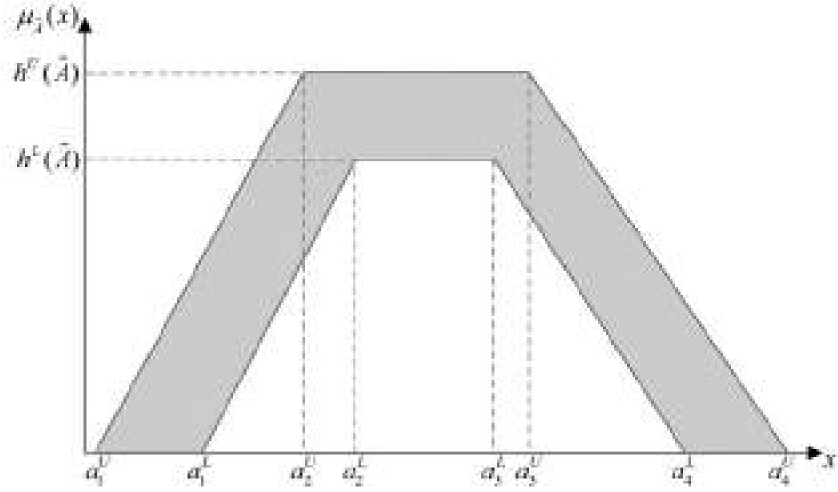

FIGURE 5.1 The interval type-2 fuzzy sets.

Definition 4

[23, 24] Suppose that $\tilde{A} = \left[\left(a_1^L, a_2^L, a_3^L, a_4^L, h^L(\tilde{A})\right), \left(a_1^U, a_2^U, a_3^U, a_4^U, h^U(\tilde{A})\right)\right]$ and $\tilde{B} = \left[\left(b_1^L, b_2^L, b_3^L, b_4^L, h^L(\tilde{B})\right), \left(b_1^U, b_2^U, b_3^U, b_4^U, h^U(\tilde{B})\right)\right]$ be two trapezoidal IT2FSs, then the arithmetic operations between $\tilde{A}$ and $\tilde{B}$ are defined as:

$$\tilde{A} + \tilde{B} = \begin{bmatrix} \left(a_1^L + b_1^L, a_2^L + b_2^L, a_3^L + b_3^L, a_4^L + b_4^L, \min\left\{h^L(\tilde{A}), h^L(\tilde{B})\right\}\right), \\ \left(a_1^U + b_1^U, a_2^U + b_2^U, a_3^U + b_3^U, a_4^U + b_4^U, \min\left\{h^U(\tilde{A}), h^U(\tilde{B})\right\}\right) \end{bmatrix}$$

$$\tilde{A} - \tilde{B} = \begin{bmatrix} \left(a_1^L - b_1^L, a_2^L - b_2^L, a_3^L - b_3^L, a_4^L - b_4^L, \min\left\{h^L(\tilde{A}), h^L(\tilde{B})\right\}\right), \\ \left(a_1^U - b_1^U, a_2^U - b_2^U, a_3^U - b_3^U, a_4^U - b_4^U, \min\left\{h^U(\tilde{A}), h^U(\tilde{B})\right\}\right) \end{bmatrix}$$

$$\lambda \cdot \tilde{A} = \left[\left(\lambda a_1^L, \lambda a_2^L, \lambda a_3^L, \lambda a_4^L, h^L(\tilde{A})\right), \left(\lambda a_1^U, \lambda a_2^U, \lambda a_3^U, \lambda a_4^U, h^U(\tilde{A})\right)\right], \lambda \geq 0$$

$$\tilde{A}^q = \left[\left(\left(a_1^L\right)^q, \left(a_2^L\right)^q, \left(a_3^L\right)^q, \left(a_4^L\right)^q, h^L(\tilde{A})\right), \left(\left(a_1^U\right)^q, \left(a_2^U\right)^q, \left(a_3^U\right)^q, \left(a_4^U\right)^q, h^U(\tilde{A})\right)\right], q \geq 0$$

5.2.2 Entropy-Based Weight Determination Model

The concept of entropy is derived from thermodynamics and it is a useful tool to measure the information contained in the data from an objective perspective [25]. Recently, entropy has also been applied to determine the criteria weights in multi-criteria decision-making problems [25–28]. The entropy-based weight determination model mainly consists of the following stages:

Stage 1: **Calculate the entropy of criterion C_j.** If the evaluation value of the alternative Z_i with respect to the criterion C_j is given by the crisp value $\bar{r}_{ij}$, then the entropy of criterion C_j can be calculated by:

$$E_j = -\frac{1}{\log n}\sum_{i=1}^{m}\hat{r}_{ij}\log\hat{r}_{ij},\ j = 1,2,\ldots,n.$$

where n represents the number of criteria, m is the number of alternatives, and $\hat{r}_{ij} := \frac{r_{ij}}{\sum_{i=1}^{m} r_{ij}}$.

Stage 2: **Calculate the dispersion of criterion C_j.** After obtaining the entropy, the dispersion of criterion C_j can be computed by:

$$\xi_j = 1 - E_j,\ j = 1,\ldots,n.$$

Stage 3: **Calculate the criteria weights.** Based on the dispersion of criterion C_j, we can obtain the weight of criterion C_j by the following equation:

$$w_j = \frac{\xi_j}{\sum_{j=1}^{n}\xi_j},\ j = 1,\ldots,n.$$

5.2.3 Evidential Reasoning Algorithm

The ER algorithm [29], which was developed on the basis of the Dempster–Shafer theory of evidence [30], is an effective tool for dealing with MCDM problems under uncertainty. This algorithm mainly consists of three stages: constructing the basic probability masses, fusing the basic probability masses under multiple criteria, and generating the expected utilities of alternatives.

The basic probability assignments (BPAs) are the basic information units in Dempster–Shafer theory of evidence:

Definition 5

(BPA [13]). Let $H = \{H_1, \ldots, H_N\}$ be a collectively exhaustive and mutually exclusive set of hypotheses or propositions, which is also called the frame of discernment. A BPA is a function $m : 2^H \to [0, 1]$ satisfying the following conditions:

$$m(\varnothing) = 0,$$

$$\sum_{A \subseteq H} m(A) = 1,$$

where $\varnothing$ denotes the empty set and 2^{Θ} stands for the power set of the frame of discernment:

$$2^{\Theta}=\left\{\varnothing,\{H_1\},\ldots,\{H_N\},\{H_1,H_2\},\ldots,\{H_1,H_N\},\{H_1,\ldots,H_{N-1}\},\Theta\right\}$$

Suppose that there are m alternatives $Z_1, \ldots, Z_m$, the basic probability assignments of Z_i under C_j supporting H_k can be generated by:

$$m_{k,j}(Z_i)=w_j\cdot\beta_{k,j}(Z_i),\ k=1,\ldots,N,\ j=1,\ldots,n$$
$$m_{H,j}(Z_i)=1-\sum_{k=1}^{N}m_{k,j}(Z_i)=1-w_j\cdot\sum_{k=1}^{N}\beta_{k,j}(Z_i),\ j=1,\ldots,n$$
$$\bar{m}_{H,j}(Z_i)=1-w_j,\ j=1,\ldots,n$$
$$\tilde{m}_{H,j}(Z_i)=w_j\cdot\left(1-\sum_{k=1}^{n}\beta_{k,j}(Z_i)\right),\ j=1,\ldots,n$$

where $\beta_{k,j}(Z_i)$ denotes the belief degree of alternative Z_i under criterion C_j belonging to grade H_k, which satisfies $\beta_{k,j}(Z_i)\geq 0$ and $\sum_{k=1}^{N}\beta_{k,j}(Z_i)\leq 1$; w_j is the weight of criterion C_j and satisfies $w_j\geq 0$ and $\sum_{j=1}^{n}w_j=1$.

These BPAs are usually aggregated according to the analytical ER algorithm [13]:

$$m_k(Z_i)=\rho\cdot\left[\prod_{j=1}^{n}\left(m_{k,j}(Z_i)+\bar{m}_{H,j}(Z_i)+\tilde{m}_{H,j}(Z_i)\right)-\prod_{j=1}^{n}\left(\bar{m}_{H,j}(Z_i)+\tilde{m}_{H,j}(Z_i)\right)\right],k=1,\ldots,N$$
$$\tilde{m}_H(Z_i)=\rho\cdot\left[\prod_{j=1}^{n}\left(\bar{m}_{H,j}(Z_i)+\tilde{m}_{H,j}(Z_i)\right)-\prod_{j=1}^{n}\bar{m}_{H,j}(Z_i)\right],k=1,\ldots,N$$
$$\bar{m}_H(Z_i)=\rho\cdot\left[\prod_{j=1}^{n}\bar{m}_{H,j}(Z_i)\right]$$
$$\rho=\left[\sum_{k=1}^{N}\prod_{j=1}^{n}\left(m_{k,j}(Z_i)+\bar{m}_{H,j}(Z_i)+\tilde{m}_{H,j}(Z_i)\right)-(N-1)\prod_{j=1}^{n}\left(\bar{m}_{H,j}(Z_i)+\tilde{m}_{H,j}(Z_i)\right)\right]^{-1}$$
$$\beta_k(Z_i)=\frac{m_k(Z_i)}{1-\bar{m}_H(Z_i)}$$
$$\beta_H(Z_i)=\frac{\tilde{m}_H(Z_i)}{1-\bar{m}_H(Z_i)}$$

where $\beta_k(Z_i)$ represents the belief degree of alternative Z_i belonging to H_k and $\beta_H(Z_i)$ is the unassigned belief degree.

Finally, the expected utilities of each alternative are generated according to the following equation.

$$u(Z_i) = \sum_{k=1}^{N} u(H_k) \cdot \beta_k(Z_i), i = 1, \ldots, m,$$

where $u(H_k)$ represents the utility of grade H_k.

If there is an unassigned belief degree, the expected utilities of alternatives are expressed by the interval values generated by [13]:

$$u^-(Z_i) = \sum_{k=1}^{N} \beta_k(Z_i) \cdot u(H_k) + \beta_H(Z_i) \cdot u(H_1), i = 1, \ldots, m$$

$$u^+(Z_i) = \sum_{k=1}^{N} \beta_k(Z_i) \cdot u(H_k) + \beta_H(Z_i) \cdot u(H_N), i = 1, \ldots, m$$

where $u^-(Z_i)$ and $u^+(Z_i)$ denote the lower and upper bounds of the expected utilities, respectively.

If there is no unassigned belief degree, that is, $\beta_H(Z_i) = 0$, then the expected utilities of alternatives are expressed by crisp values, that is, $u(Z_i) = u^-(Z_i) = u^+(Z_i)$.

5.2.4 Minimax Regret Approach

Due to the fact that the intervals cannot be directly compared and ranked, several interval-valued ranking approaches have been proposed such as the possibility-based ranking approach [31], the average-based ranking approach [32], the dominance degree-based ranking approach [33], and the fuzzy preference relationship-based ranking approach [34]. However, these approaches fail to distinguish the interval values with the same center but different widths. For this reason, Wang et al. [35] proposed a minimax regret approach to compare and rank interval values. The specific comparison process is provided below.

Definition 6

(Maximum regret degree [35]). Let $u(Z_i) = [u(Z_i)^-, u(Z_i)^+]$, $i = 1, \ldots, m$ be m interval-valued expected utilities. Then, the maximum regret degree of $u(Z_k)$ is defined as:

$$MR(u(Z_k)) = \max\left\{\max_{i \neq k}\left\{u(Z_i)^+\right\} - u(Z_k)^-, 0\right\}$$

In this regard, the interval-valued expected utility with the minimum maximum regret degree (i.e., minimax regret degree) will be selected as the optimal one. The complete ranking process of interval-valued expected utilities can be generated by the following elimination steps.

Step 1: Calculate the maximum regret degree of each interval-valued expected utility. The interval-valued expected utility with the minimum maximum regret degree is selected as the optimal one. Suppose that $u(Z_{k1})$ is determined as the optimal interval-valued expected utility, where $1 \leq k_1 \leq m$.

Step 2: Eliminate the interval-valued expected utility $u(Z_{k1})$ from further consideration and recalculate the maximum regret degrees of the rest interval-valued expected utilities. Suppose that $u(Z_{k2})$ has the minimum maximum regret degree, then $u(Z_{k2})$ is determined as the optimal interval-valued expected utility, where $1 \leq k_1 \neq k_2 \leq m$.

Step 3: Eliminate the interval-valued expected utility $u(Z_{k2})$ from further consideration and recalculate the maximum regret degrees of the remaining interval-valued expected utilities. Suppose that the interval-valued expected utility $u(Z_{k3})$ is selected as the optimal one, where $1 \leq k_1 \neq k_2 \neq k_3 \leq m$.

Step 4: Repeat the above eliminating process until only one interval-valued expected utility $u(Z_{km})$ is left; then, the complete ranking order of all interval-valued expected utilities are $u(Z_{k1}) \succ u(Z_{k2}) \succ \ldots \succ u(Z_{km})$, where "$\succ$" denotes "is superior to."

5.3 AN ONLINE REVIEW-BASED INTERVAL TYPE-2 FUZZY DECISION-MAKING METHOD

This section proposes an OTR-based interval type-2 fuzzy decision-making method. First, a new information processing mechanism is developed to translate OTRs into an interval type-2 fuzzy distributed structure. Second, an entropy-based interval type-2 fuzzy weight determination model is proposed to compute the criteria weights. Then, the ER algorithm is extended to fuse the interval type-2 fuzzy distributed information. Finally, an improved minimax regret approach is applied to compare and rank the alternatives.

5.3.1 Distributed Structure-Based Online Review Processing Mechanism

OTRs contain a lot of useful information related to users and products that may play a vital role in decision making [36]. The OTRs provided by people are usually characterized by unstructured text information and, consequently, distributed structures are a potential tool to tackle such unstructured text information [37]. Hence, this section proposes a new OTR-processing mechanism by combing the IT2FSs and distributed structure.

Suppose that N_i online reviews related to alternative Z_i are collected through the crawler tool based on a Scrappy framework. First, after a preliminary analysis, several aspects are selected as the decision criteria, which are denoted as $C_1, \ldots, C_n$. With the aid of sentiment analysis, the emotional grading of the online reviews under each criterion is then obtained. The number of online reviews under criterion C_j belonging to positive grading is Num − Pos(C_j), the number of online reviews belonging to negative grading is Num − Neg(C_j) and the number of online reviews belonging to neutral grading is Num − Neu(C_j). Afterward, the belief degree of alternative Z_i under criterion C_j belonging to the positive grading can be generated by:

$$\beta_{i,j}^{pos}(Z_i) = \frac{\text{Num} - \text{Pos}(C_j)}{N_i}$$

Similarly, the belief degree of alternative Z_i under criterion C_j belonging to the neutral grading and negative grading can be generated by:

$$\beta_{i,j}^{neu}(Z_i) = \frac{\text{Num} - \text{Neu}(C_j)}{N_i}$$

$$\beta_{i,j}^{neg}(Z_i) = \frac{\text{Num} - \text{Neg}(C_j)}{N_i}$$

As mentioned in Ref.[37], the outcomes of sentiment analysis include not only positive, negative, and neutral sentiments, but also ignorant sentiment. The belief degree of alternative Z_i under criterion C_j belonging to the ignorant grading can be calculated by:

$$\beta_{i,j}^{ign}(Z_i) = 1 - \beta_{i,j}^{pos}(Z_i) - \beta_{i,j}^{neu}(Z_i) - \beta_{i,j}^{neg}(Z_i)$$

The ignorant sentiment can be either positive, neutral, or negative [37]. When the ignorant sentiment is completely assigned to negative or neutral grading, the belief degree of alternative Z_i under criterion C_j belonging to the positive grading is still $\beta_{i,j}^{pos}(Z_i)$; when the ignorant sentiment is completely assigned to positive grading, the belief degree of alternative Z_i under criterion C_j belonging to the positive grading will be $\beta_{i,j}^{pos}(Z_i) + \beta_{i,j}^{ign}(Z_i)$. Hence, the belief degree of alternative Z_i under criterion C_j belonging to the positive grading can be characterized by the interval $\left[\beta_{i,j}^{pos}(Z_i), \beta_{i,j}^{pos}(Z_i) + \beta_{i,j}^{ign}(Z_i)\right]$. In the same way, the belief degrees of alternative Z_i under criterion C_j belonging to the negative grading and neutral grading is $\left[\beta_{i,j}^{neg}(Z_i), \beta_{i,j}^{neg}(Z_i) + \beta_{i,j}^{ign}(Z_i)\right]$ and $\left[\beta_{i,j}^{neu}(Z_i), \beta_{i,j}^{neu}(Z_i) + \beta_{i,j}^{ign}(Z_i)\right]$, respectively.

Through the proposed OTR-processing mechanism, the online reviews related to alternative Z_i can be transformed into distributed structure information with interval-valued belief degrees, that is, $H(Z_i) = \left\{\text{Pos}, \left[\beta_{i,j}^{pos}(Z_i), \beta_{i,j}^{pos}(Z_i) + \beta_{i,j}^{ign}(Z_i)\right]; \text{Neu}, \left[\beta_{i,j}^{neu}(Z_i), \beta_{i,j}^{neu}(Z_i) + \beta_{i,j}^{ign}(Z_i)\right]; \text{Neg}, \left[\beta_{i,j}^{neg}(Z_i), \beta_{i,j}^{neg}(Z_i) + \beta_{i,j}^{ign}(Z_i)\right]\right\}$, $j = 1, \ldots, n$. In which, Pos, Neu, and Neg are the abbreviations of the linguistic terms Positive, Neutral, and Negative. This distributed structure follows a Computing With Words (CWW) scheme in which the interval data driven-based CWW model [38] may be employed to perform the computations.

5.3.2 An Entropy-Based Interval Type-2 Fuzzy Weights Determination Model

In MCDM, distinct criteria usually have diverse effects on the decision results [39]. So far, many studies have focused on determining the weights of the criteria from different points of view [40–45]. However, most existing studies express the weights of the criteria using crisp values, which may fail to reflect the uncertainty of the criteria. To address this limitation, this subsection proposes an entropy-based interval type-2 fuzzy distributed weights determination model.

Suppose that there is an MCDM problem that consists of m alternatives and each alternative is influenced by n criteria. For the sake of simplicity, the alternatives and criteria are respectively denoted as $Z = \{Z_1, \ldots, Z_m\}$ and $C = \{C_1, \ldots, C_k\}$. The evaluation value of alternative Z_i with respect to criterion C_j is given by r_{ij}, where $\tilde{r}_{ij}$ is expressed by distributed information $H_{ij} = \left\{\left(S_{k,ij}, \left[\beta^-_{k,ij}(Z_i), \beta^+_{k,ij}(Z_i)\right]\right) \mid k = 1, \ldots, K\right\}$. While, $S_{1,ij}$, $S_{2,ij}$, and $S_{3,ij}$ are the linguistic terms representing positive grading, neutral grading, and negative grading, $\left[\beta^-_{k,ij}(Z_i), \beta^+_{k,ij}(Z_i)\right]$ is the interval-valued belief degree corresponding to the linguistic terms. The distributed information can be obtained by the aforementioned OTR-processing mechanism. The weights of the criteria are represented by the vector $w = (w_1, \ldots, w_n)$ and can be generated by the following processes:

1. **Calculate the entropy of criterion C_j.** The entropy of criterion C_j can be obtained by solving the following pair of nonlinear programming models:

$$\max/\min E_j = -\frac{1}{\ln n} \times \sum_{i=1}^{m} f_{ij} \times \ln f_{ij}$$

$$s.t. \begin{cases} f_{ij} = \dfrac{\text{Score}(H_{ij})}{\sum_{i=1}^{m} \text{Score}(H_{ij})}, & i = 1, \cdots, m,\ j = 1, \cdots, n \\ \text{Score}(H_{ij})^- \leq \text{Score}(H_{ij}) \leq \text{Score}(H_{ij})^+, & i = 1, \cdots, m,\ j = 1, \cdots, n \\ \text{Score}(H_{ij})^- \leq \text{Score}(H_{ij}) \leq \text{Score}(H_{ij})^+, & i = 1, \cdots, m,\ j = 1, \cdots, n \end{cases}$$

where $\text{Score}(H_{ij})$ is the score value of distributed information H_{ij} and can be calculated by:

$$\begin{aligned} \text{Score}(H_{ij}) &= \left[\text{Cen}(S_k)^-, \text{Cen}(S_k)^+\right] \cdot \left[\beta^-_{k,ij}(Z_i), \beta^+_{k,ij}(Z_i)\right] \\ &= \left[\text{Cen}(S_k)^- \cdot \beta^-_{k,ij}(Z_i), \text{Cen}(S_k)^+ \cdot \beta^+_{k,ij}(Z_i)\right] \end{aligned}$$

where H_{ij} denotes the distributed information of alternative Z_i with respect to criterion C_j; $\text{Cen}(S_k)^-$ and $\text{Cen}(S_k)^+$ represent the lower and upper bounds of the centroid of the IT2FS corresponding to linguistic term S_k.

Suppose that E_j^- and E_j^+ be the optimal solutions of the above pair of non-linear programming models, then the entropy of criterion C_j can be denoted by the interval $\left[E_j^-, E_j^+\right]$.

2. **Calculate the dispersion of criterion C_j.** After obtaining the entropy of criterion C_j, the dispersion of criterion C_j can be accordingly determined. Because the entropy is expressed by interval value, the dispersion should also be interval value and its lower and upper bounds can be calculated by:

$$\xi_j^- = 1 - E_j^+$$
$$\xi_j^+ = 1 - E_j^-$$

3. **Determine the weight of criterion C_j.** Based on the dispersion of criterion C_j, the weight of criterion C_j can be determined by:

$$w_j = \left[w_j^-, w_j^+\right] = \left[\frac{\xi_j^-}{\sum_{j=1}^{n} \xi_j^+}, \min\left\{\frac{\xi_j^+}{\sum_{j=1}^{n} \xi_j^-}, 1\right\}\right]$$

From the above analysis, we can observe that the criteria weights generated by the proposed weight determination model are characterized by interval values instead of crisp values.

5.3.3 An Evidential Reasoning-Based Information Fusion Approach

Evidential reasoning algorithm is a potentially effective tool to fuse the distributed information [13]. Thus, this subsection develops an evidential reasoning-based information fusion approach to fuse the interval type-2 fuzzy distributed information. The proposed approach mainly consists of three stages: construct the basic probability masses, fuse the basic probability masses, and generate the expected utilities of alternatives.

(1) **Transform the distributed information into basic probability masses**
After obtaining the belief degrees and criteria weights, the distributed information can be transformed into basic probability masses. In this chapter, both the belief degrees and criteria weights are expressed by interval values, so the basic probability masses are also characterized by interval values and can be calculated by:

$$m_{k,j}(Z_i) = \left[w_j^- \cdot \beta_{k,j}^-(Z_i), w_j^+ \cdot \beta_{k,j}^+(Z_i)\right] \tag{5.1}$$

$$\bar{m}_{\Theta,j}(Z_i) = \left[1 - w_j^+, 1 - w_j^-\right] \tag{5.2}$$

$$\tilde{m}_{\Theta,j}(Z_i) = \left[w_j^- \cdot \beta_{\Theta,j}^-(Z_i), w_j^+ \cdot \beta_{\Theta,j}^+(Z_i)\right] \tag{5.3}$$

$$\beta_{\tilde{\Theta},j}^{-}\left(Z_i\right)=\max\left(0,1-\sum_{k=1}^{N}\beta_{k,j}^{+}\left(Z_i\right)\right) \tag{5.4}$$

$$\beta_{\tilde{\Theta},j}^{+}\left(Z_i\right)=1-\sum_{k=1}^{N}\beta_{k,j}^{-}\left(Z_i\right) \tag{5.5}$$

In which, $\beta_{k,j}(Z_i)$ represents the belief degree of the evaluation value $\tilde{r}_{ij}$ belonging to grade H_k, $\bar{m}_j\left(\Theta\right)$ is the ignorance caused by criteria weights and $\tilde{m}_j\left(\Theta\right)$ denotes the ignorance caused by the uncertainty of evaluation value. Moreover, the basic probability masses should satisfy the following condition:

$$\sum_{k=1}^{N}m_{k,j}\left(Z_i\right)+\bar{m}_{\Theta,j}\left(Z_i\right)+\tilde{m}_{\Theta,j}\left(Z_i\right)=1$$

(2) **Fuse the basic probability masses under multiple criteria**

After transforming the distributed information into basic probability masses, the fused interval-valued belief degree of each alternative can be calculated by solving the following pair of nonlinear programming models:

$$\max/\min \beta_k\left(Z_i\right)=\frac{m_k\left(Z_i\right)}{1-\bar{m}_{\Theta}\left(Z_i\right)}$$

$$s.t.\begin{cases} m_k\left(Z_i\right)=\rho\cdot\left[\prod_{j=1}^{n}\left(m_{k,j}\left(Z_i\right)+\bar{m}_{\Theta,j}\left(Z_i\right)+\tilde{m}_{\Theta,j}\left(Z_i\right)\right)-\prod_{j=1}^{n}\left(\bar{m}_{\Theta,j}\left(Z_i\right)+\tilde{m}_{\Theta,j}\left(Z_i\right)\right)\right] \\ \tilde{m}_{\Theta}\left(Z_i\right)=\rho\cdot\left[\prod_{j=1}^{n}\left(\bar{m}_{\Theta,j}\left(Z_i\right)+\tilde{m}_{\Theta,j}\left(Z_i\right)\right)-\prod_{j=1}^{n}\bar{m}_{\Theta,j}\left(Z_i\right)\right] \\ \bar{m}_{\Theta}\left(Z_i\right)=\rho\cdot\left[\prod_{j=1}^{n}\bar{m}_{\Theta,j}\left(Z_i\right)\right] \\ \rho=\left[\sum_{k=1}^{N}\prod_{j=1}^{n}\left(m_{k,j}\left(Z_i\right)+\bar{m}_{\Theta,j}\left(Z_i\right)+\tilde{m}_{\Theta,j}\left(Z_i\right)\right)-\left(N-1\right)\cdot\prod_{j=1}^{n}\left(\bar{m}_{\Theta,j}\left(Z_i\right)+\tilde{m}_{\Theta,j}\right)\left(Z_i\right)\right]^{-1} \\ m_{k,j}^{-}\left(Z_i\right)\le m_{k,j}\left(Z_i\right)\le m_{k,j}^{+}\left(Z_i\right), k=1,\cdots,N \\ \bar{m}_{\Theta,j}^{-}\left(Z_i\right)\le \bar{m}_{\Theta,j}\left(Z_i\right)\le \bar{m}_{\Theta,j}^{+}\left(Z_i\right) \\ \tilde{m}_{\Theta,j}^{-}\left(Z_i\right)\le \tilde{m}_{\Theta,j}\left(Z_i\right)\le \tilde{m}_{\Theta,j}^{+}\left(Z_i\right) \\ \sum_{k=1}^{N}m_{k,j}\left(Z_i\right)+\bar{m}_{\Theta,j}\left(Z_i\right)+\tilde{m}_{\Theta,j}\left(Z_i\right)=1 \end{cases} \tag{5.6}$$

Suppose that the optimal solutions of the above pair of nonlinear programming models are $\beta_k(Z_i)^-$ and $\beta_k(Z_i)^+$, then the fused interval-valued belief degree of alternative Z_i is expressed by $[\beta_k(Z_i)^-, \beta_k(Z_i)^+]$, representing the belief degree of alternative Z_i belonging to grade H_k. Moreover, it should be highlighted that if the objective function is replaced with $\beta_\Theta(Z_i) = m_\Theta(Z_i)/(1-\bar{m}_\Theta(Z_i))$, then the optimal solutions of the nonlinear programming models will be the fused unassigned belief degree and can be denoted by $[\beta_\Theta(Z_i)^-, \beta_\Theta(Z_i)^+]$. Generate the expected utilities of alternatives.

Once the fused interval-valued belief degrees of alternatives are obtained, the expected utilities of alternatives can be calculated to compare and rank them. Because the belief degrees are expressed by interval values, the expected utilities should also be interval values. The following nonlinear programming models are constructed to calculate the lower and upper bounds of the expected utility, respectively.

The upper bound of the expected utility can be calculated by:

$$\max u(Z_i) = \sum_{k=1}^{N} \beta_k(Z_i)\cdot u(H_k) + \beta_\Theta(Z_i)\cdot u(H_N)$$

$$s.t.\begin{cases} \beta_k(Z_i) = \dfrac{m_k(Z_i)}{1-\bar{m}_\Theta(Z_i)} \\ m_k(Z_i) = \rho\cdot\left[\prod_{j=1}^{n}\left(m_{k,j}(Z_i)+\bar{m}_{\Theta,j}(Z_i)+\tilde{m}_{\Theta,j}(Z_i)\right)-\prod_{j=1}^{n}\left(\bar{m}_{\Theta,j}(Z_i)+\tilde{m}_{\Theta,j}(Z_i)\right)\right] \\ \tilde{m}_\Theta(Z_i) = \rho\cdot\left[\prod_{j=1}^{n}\left(\bar{m}_{\Theta,j}(Z_i)+\tilde{m}_{\Theta,j}(Z_i)\right)-\prod_{j=1}^{n}\bar{m}_{\Theta,j}(Z_i)\right] \\ \bar{m}_\Theta(Z_i) = \rho\cdot\left[\prod_{j=1}^{n}\bar{m}_{\Theta,j}(Z_i)\right] \\ \rho = \left[\sum_{k=1}^{N}\prod_{j=1}^{n}\left(m_{k,j}(Z_i)+\bar{m}_{\Theta,j}(Z_i)+\tilde{m}_{\Theta,j}(Z_i)\right)-(N-1)\cdot\prod_{j=1}^{n}\left(\bar{m}_{\Theta,j}(Z_i)+\tilde{m}_{\Theta,j}(Z_i)\right)\right]^{-1} \\ w_j^-\cdot\beta_{k,j}^-(Z_i) \le m_{k,j} \le w_j^+\cdot\beta_{k,j}^+(Z_i) \\ 1-w_j^+ \le \bar{m}_{\Theta,j}(Z_i) \le 1-w_j^- \\ \sum_{k=1}^{N} m_{k,j}(Z_i)+\bar{m}_{\Theta,j}(Z_i)+\tilde{m}_{\Theta,j}(Z_i) = 1 \end{cases}$$

(5.7)

Similarly, the lower bound of the expected utility can be calculated by:

$$\min u(Z_i)=\sum_{k=1}^{N}\beta_k(Z_i)\cdot u(H_k)+\beta_H(Z_i)\cdot u(H_1)$$

$$s.t.\begin{cases}\beta_k(Z_i)=\dfrac{m_k(Z_i)}{1-\bar{m}_H(Z_i)}\\ m_k(Z_i)=\rho\cdot\left[\prod_{j=1}^{n}\left(m_{k,j}(Z_i)+\bar{m}_{H,j}(Z_i)+\tilde{m}_{H,j}(Z_i)\right)-\prod_{j=1}^{n}\left(\bar{m}_{H,j}(Z_i)+\tilde{m}_{H,j}(Z_i)\right)\right]\\ \tilde{m}_H(Z_i)=\rho\cdot\left[\prod_{j=1}^{n}\left(\bar{m}_{H,j}(Z_i)+\tilde{m}_{H,j}(Z_i)\right)-\prod_{j=1}^{n}\bar{m}_{H,j}(Z_i)\right]\\ \bar{m}_H(Z_i)=\rho\cdot\left[\prod_{j=1}^{n}\bar{m}_{H,j}(Z_i)\right]\\ \rho=\left[\sum_{k=1}^{N}\prod_{j=1}^{n}\left(m_{k,j}(Z_i)+\bar{m}_{H,j}(Z_i)+\tilde{m}_{H,j}(Z_i)\right)-(N-1)\cdot\prod_{j=1}^{n}\left(\bar{m}_{H,j}(Z_i)+\tilde{m}_{H,j}(Z_i)\right)\right]^{-1}\\ w_j^-\cdot\beta_{k,j}^-(Z_i)\le m_{k,j}(Z_i)\le w_j^+\cdot\beta_{k,j}^+(Z_i)\\ 1-w_j^+\le\bar{m}_{H,j}(Z_i)\le 1-w_j^-\\ \sum_{k=1}^{N}m_{k,j}(Z_i)+\bar{m}_{H,j}(Z_i)+\tilde{m}_{H,j}(Z_i)=1\end{cases}\tag{5.8}$$

Suppose that the optimal solutions of the above two nonlinear programming models are denoted by $u(Z_i)^-$ and $u(Z_i)^+$, then the expected utility of the alternative Z_i can be represented by $[u(Z_i)^-, u(Z_i)^+]$.

5.3.4 An Improved Minimax Regret Approach

Since the obtained expected utilities are characterized by interval values and cannot be directly compared and ranked, this subsection proposes a new interval-valued ranking approach based on the minimax regret approach [31]. The specific implementation processes of the proposed approach are provided below.

Suppose that there are m interval-valued expected utilities $u(Z_i) = [u(Z_i)^-, u(Z_i)^+]$, $i = 1, \ldots, m$ and $u(Z_l) = [u(Z_l)^-, u(Z_l)^+]$ represents the final selected interval-valued expected utility. If $u(Z_l)^- < \max_{i\neq l}\left\{u(Z_i)^+\right\}$, the decision maker may feel regret/loss, and the maximum regret degree can be calculated by:

$$\mathrm{MR}\left(u(Z_l)\right)=\max\left\{\max_{i\neq l}u(Z_i)^+-u(Z_l)^-,0\right\}$$

It is evident that the interval-valued expected utility with the minimum maximum regret degree should be regarded as the optimal interval-valued expected utility. This rule can be embodied by the following mathematical expression:

$$\min_i\left\{\mathrm{MR}\left(u(Z_i)\right)\right\}=\min_i\left\{\max\left\{\max_{i\neq l}\left\{u(Z_l)^+\right\}-u(Z_i)^-,0\right\}\right\}$$

If there are several interval-valued expected utilities with the same minimum maximum regret degree, they can be further compared by the maximum secondary regret degree (MSR), which can be generated by the following rule.

Suppose that there are m' ($m' \leq m$) interval-valued expected utilities that have the same minimax regret degree and $u(Z_t) = [u(Z_t)^-, u(Z_t)^+]$ is the final selected interval-valued expected utility. The maximum secondary regret degree can be calculated by:

$$\mathrm{MSR}\left(u(Z_t)\right)=\max\left\{\max_{g\neq t}\left\{u(Z_g)^+\right\}-u(Z_t)^-,0\right\}$$

Obviously, the interval-valued expected utility with the minimum maximum secondary regret degree should be selected as the optimal interval-valued expected utility, which can be embodied by the following mathematical expression:

$$\min_t\left\{\mathrm{MSR}\left(u(Z_t)\right)\right\}=\min_t\left\{\max\left\{\max_{g\neq t}\left\{u(Z_g)^+\right\}-u(Z_t)^-,0\right\}\right\}$$

The proposed interval-valued ranking approach can be summarized as follows:

(i) If $\min\{\mathrm{MR}(u(Z_i))\} < \min\{\mathrm{MR}(u(Z_l))\}(i,\ l = 1, \ldots, m;\ l \neq i)$, then the interval-valued expected utility $u(Z_i)$ is better than $u(Z_l)$.

(ii) If $\min\{\mathrm{MPR}(u(Z_l))\} = \min\{\mathrm{MPR}(u(Z_t))\} < \min\{\mathrm{MPR}(u(Z_i))\}(l \neq t \neq i;\ t,\ l = 1, \ldots, m)$, then the maximum secondary regret degrees of $u(Z_l) = [u(Z_l)^-, u(Z_l)^+]$ and $u(Z_t) = [u(Z_t)^-, u(Z_t)^+]$ should be calculated to further distinguish $u(Z_l)$ and $u(Z_t)$, which will involve the following three situations.

Situation 1: If $\min\{\mathrm{MSR}(u(Z_l))\} < \min\{\mathrm{MSR}(u(Z_t))\}(l \neq t;\ l,\ t = 1, \ldots, m)$, then $u(Z_l)$ is better than $u(Z_t)$ and the ranking order of the three interval-valued expected utilities is $u(Z_l) \succ u(Z_t) \succ u(Z_i)$.

Situation 2: If $\min\{\mathrm{MSR}(u(Z_l))\} > \min\{\mathrm{MSR}(u(Z_t))\}(l \neq t;\ l,\ t = 1, \ldots, m)$, then $u(Z_l)$ is worse than $u(Z_t)$ and the ranking order of the three interval-valued expected utilities is $u(Z_t) \succ u(Z_l) \succ u(Z_i)$.

Situation 3: If $\min\{\mathrm{MSR}(u(Z_l))\} = \min\{\mathrm{MSR}(u(Z_t))\}(l \neq t;\ l,\ t = 1, \ldots, m)$, then $u(Z_l)$ is equal to $u(Z_t)$ and the ranking order of the three interval-valued expected utilities is $u(Z_t) \approx u(Z_l) \succ u(Z_i)$.

5.4 CASE STUDY

To illustrate the performance of the proposed decision method, a case study about NEVs purchasing is provided. The evaluation information related to NEVs purchasing is collected from OTRs. Afterward, the advantages of the proposed method are discussed and explained through comparative analysis.

5.4.1 Problem Description

With the intensification of the energy crisis and environmental pollution, NEVs have received more and more attention. China is the largest energy consumer, so it is particularly important to develop NEVs. In October 2020, the general office of the State Council issued the "new energy vehicle industry development plan (2021–2035)." The plan points out that the development of NEVs is the necessary way for China to move from a big automobile country to a powerful automobile country, and is a strategic measure to deal with climate change and promote green development. In the following 15 years, we should focus on developing NEV technology and improving the industrial layout of NEVs, and strive to make NEVs the mainstream of vehicle sales by 2035. The release of the new energy vehicle industry development plan (2021–2035) has brought a new wave of sales boom for NEVs. In the process of purchasing NEVs, many alternatives and conflicting criteria are often involved. To select the optimal alternative, it is necessary to evaluate the performance of different NEVs under these conflicting criteria. With the development of Internet 2.0 and the popularity of intelligent terminal equipment, automobile forums are becoming more and more popular. Through these automobile forums, people can express their opinions and feelings about various electric vehicles anytime and anywhere, providing an important data source for solving the evaluation problems of electric vehicles.

Automobile home (http://www.autohome.com.cn/) is a mainstream automobile Internet platform in China, providing consumers with a large amount of valuable OTR information. This subsection collects the online reviews related to NEVs on Automobile home through the crawler tool based on the scrapy framework [36]. Four popular NEVs (TESLA Model 3, NIO ES6, XPENG-P7, and BYD-Tang) are selected to demonstrate and explain this process. The collected online reviews are as follows: 546 online reviews on TESLA Model 3, 1273 online reviews on NIO ES6, 121 online reviews related to XPENG-P7, and 2718 online reviews on BYD-Tang. Five criteria (power, space, comfort, appearance, and cost performance) are derived from these collected online reviews. For the sake of simplicity, these four NEVs and five criteria are expressed by notations Z_1, Z_2, Z_3, Z_4, and C_1, C_2, C_3, C_4, C_5, respectively.

5.4.2 Implementation Process

We herein implement the proposed decision-making method to solve the above NEVs evaluation problem, which consists mainly of the following stages.

(1) Transform the unstructured online reviews into interval-valued distribution assessment vector.

TABLE 5.1
Linguistic Distributed Information Matrix with Interval-Valued Belief Degrees

	Z_1	Z_2	Z_3	Z_4
C_1	Pos, [0.172, 0.472];	Pos, [0.366, 0.566];	Pos, [0.236, 0.336];	Pos, [0.163, 0.463];
	Neu, [0.258, 0.558];	Neu, [0.258, 0.458];	Neu, [0.345, 0.445];	Neu, [0.108, 0.408];
	Neg, [0.163, 0.463]	Neg, [0.132, 0.332]	Neg, [0.232, 0.332]	Neg, [0.452, 0.852]
C_2	Pos, [0.328, 0.478];	Pos, [0.116, 0.366];	Pos, [0.108, 0.308];	Pos, [0.242, 0.342];
	Neu, [0.254, 0.404];	Neu, [0.232, 0.482];	Neu, [0.245, 0.445];	Neu, [0.341, 0.441];
	Neg, [0.162, 0.312]	Neg, [0.162, 0.412]	Neg, [0.267, 0.467]	Neg, [0.223, 0.323]
C_3	Pos, [0.118, 0.268];	Pos, [0.138, 0.338];	Pos, [0.285, 0.385];	Pos, [0.332, 0.432];
	Neu, [0.354, 0.504];	Neu, [0.246, 0.446];	Neu, [0.328, 0.428];	Neu, [0.452, 0.552];
	Neg, [0.262, 0.412]	Neg, [0.337, 0.537]	Neg, [0.336, 0.436]	Neg, [0.062, 0.162]
C_4	Pos, [0.262, 0.362];	Pos, [0.224, 0.374];	Pos, [0.268, 0.568];	Pos, [0.438, 0.588];
	Neu, [0.158, 0.258];	Neu, [0.208, 0.358];	Neu, [0.255, 0.555];	Neu, [0.183, 0.333];
	Neg, [0.363, 0.463]	Neg, [0.346, 0.496]	Neg, [0.062, 0.362]	Neg, [0.137, 0.387]
C_5	Pos, [0.345, 0.445];	Pos, [0.326, 0.426];	Pos, [0.342, 0.492];	Pos, [0.342, 0.542];
	Neu, [0.254, 0.354];	Neu, [0.247, 0.347];	Neu, [0.241, 0.391];	Neu, [0.287, 0.487];
	Neg, [0.265, 0.365]	Neg, [0.332, 0.432]	Neg, [0.068, 0.218]	Neg, [0.078, 0.278]

According to the proposed distributed structure-based online review processing mechanism, the collected online reviews are transformed into linguistic distributed information with interval-valued belief degrees, as shown in Table 5.1.

To follow a CWW scheme, the sentiment words (positive, neutral, and negative) in Table 5.1 are transformed into IT2FSs. The corresponding relationships between sentiment words and IT2FSs are provided below.

$$\text{Positive}:\left[(7.2,8.6,9.9.6,1);(8,8.8,8.8,9.2,0.9)\right];$$

$$\text{Neutral}:\left[(5.8,6.6,7.8,8.8,1);(6.2,7.4,7.4,8,0.9)\right];$$

$$\text{Negative}:\left[(0.8,3.6,5.8,7.2,1);(1.2,4.4,4.4,5.6,0.9)\right].$$

(2) Calculate the criteria weights of NEVs evaluation problem

First, based on the obtained linguistic distributed information matrix and IT2FSs, as shown in Table 5.2.

Second, the interval-valued entropy of each criterion can be calculated. The results are summarized in Table 5.3.

Then, the interval-valued dispersion of each criterion can be generated. The corresponding results are summarized in Table 5.4.

(3) Finally, the interval-valued criteria weights can be generated according to Eq. (5.3). The corresponding results are $w_1 = [0.733,1]$, $w_2 = [0.364,0.5]$, $w_3 = [0.25,0.333]$, $w_4 = [0.19,0.25]$, and $w_5 = [0.156,0.2]$. Fuse the interval type-2 fuzzy distributed information

TABLE 5.2
Score Value of Linguistic Distributed Information

	Z_1	Z_2	Z_3	Z_4
C_1	[1.36, 3.64]	[1.89, 3.43]	[1.86, 2.72]	[1.36, 3.89]
C_2	[1.81, 3]	[1.13, 3.05]	[1.28, 2.87]	[1.85, 2.71]
C_3	[1.58, 2.81]	[1.47, 3.08]	[2.1, 3]	[2.17, 3]
C_4	[1.65, 2.54]	[1.64, 2.89]	[1.5, 3.75]	[1.92, 3.27]
C_5	[2, 2.86]	[2.02, 2.9]	[1.69, 2.85]	[1.82, 3.35]

TABLE 5.3
Interval-Valued Entropy of Each Criterion

	C_1	C_2	C_3	C_4	C_5
E_j	[0.811, 0.861]	[0.808, 0.861]	[0.828, 0.861]	[0.825, 0.861]	[0.843, 0.861]

TABLE 5.4
Interval-Valued Dispersion of Each Criterion

	C_1	C_2	C_3	C_4	C_5
ξ_j	[0.139, 0.189]	[0.139, 0.192]	[0.139, 0.172]	[0.139, 0.175]	[0.139, 0.157]

First, according to Eqs. (5.1)–(5.5), the distributed information is transformed into basic probability masses. Due to space limitation, we only take the interval-valued belief degree of alternative Z_1 under criterion C_1 assigned to grade H_1 as an example. The results are provided below.

$$m_{1,1}(Z_1) = [0.027, 0.094]$$
$$\tilde{m}_{\theta,1}(Z_1) = [0, 0.08]$$
$$\bar{m}_{\theta,1} = [0, 0.267]$$

Then, the fused interval-valued belief degree of each alternative can be generated by Eq. (5.6) and the results are summarized in Table 5.5.

Finally, the interval-valued expected utilities of alternatives can be generated by Eqs. (5.7 and 5.8), the results are shown in Table 5.6.

(4) Compare and rank the interval-valued expected utilities

First, calculate the maximum regret degree of each interval-valued expected utility.

TABLE 5.5
Interval-Valued Belief Degree of Each Alternative

	Pos	Neu	Neg
Z_1	[0.125, 0.424]	[0.134, 0.387]	[0.142, 0.379]
Z_2	[0.112, 0.43]	[0.126, 0.389]	[0.134, 0.391]
Z_3	[0.106, 0.427]	[0.135, 0.401]	[0.123, 0.374]
Z_4	[0.132, 0.447]	[0.135, 0.401]	[0.123, 0.374]

TABLE 5.6
Interval-Valued Dispersion of Each Criterion

	Z_1	Z_2	Z_3	Z_4
$u(Z_i)$	[0.209, 0.303]	[0.197, 0.29]	[0.191, 0.285]	[0.2, 0.294]

$$MR\left(u\left(Z_1\right)\right)=\max\left[\max\{0.29,0.285,0.294\}-0.209,0\right]=0.085$$

$$MR\left(u\left(Z_2\right)\right)=\max\left[\max\{0.303,0.285,0.294\}-0.197,0\right]=0.106$$

$$MR\left(u\left(Z_3\right)\right)=\max\left[\max\{0.303,0.289,0.294\}-0.191,0\right]=0.112$$

$$MR\left(u\left(Z_4\right)\right)=\max\left[\max\{0.303,0.289,0.285\}-0.2,0\right]=0.103$$

From the results, we can find that Z_1, that is, TESLA Model 3 has the minimum maximum regret degree. Therefore, TESLA Model 3 is selected as the optimal alternative.

Then, calculate the maximum regret degree of each remaining interval-valued expected utility.

$$MR\left(u\left(Z_2\right)\right)=\max\left[\max\{0.285,0.294\}-0.197,0\right]=0.097$$

$$MR\left(u\left(Z_3\right)\right)=\max\left[\max\{0.289,0.294\}-0.191,0\right]=0.103$$

$$MR\left(u\left(Z_4\right)\right)=\max\left[\max\{0.289,0.285\}-0.2,0\right]=0.089$$

The alternative Z_4, that is, BYD-Tang is selected as the second alternative.

Finally, the maximum regret degrees of alternatives Z_2 and Z_3 are calculated.

$$MR\left(u\left(Z_2\right)\right)=\max\left[0.285-0.197,0\right]=0.088$$

$$MR\left(u\left(Z_3\right)\right)=\max\left[0.289-0.191,0\right]=0.098$$

From the above analysis, we can conclude that the ranking order of the four alternatives is $Z_1 \succ Z_4 \succ Z_3 \succ Z_2$.

5.5 CONCLUSIONS

This study proposed an interval type-2 decision analysis framework based on OTRs. First, considering the unstructured characteristics and individual differences of the text reviews, this study presents an interval type-2 distributed structure-based processing mechanism for the OTRs. Through this mechanism, the unstructured information is transformed into distributed information with interval-valued belief degrees. Second, to determine the criteria weights, an entropy-based interval type-2 fuzzy weights determination model is provided. The criteria weights are characterized by interval values, which can better describe the uncertainty of the criteria. Then, an ER-based information fusion approach is proposed to calculate the interval-valued expected utility of each alternative. Since the expected utilities are expressed by interval values, an improved minimax regret approach is developed to compare and rank the interval-valued expected utilities. Finally, a case study related to NEVs evaluation has been provided to show the effectiveness of the proposed method.

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