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The 18th International Conference on Intelligent Systems and Knowledge Engineering

Dynamic Knowledge Discovery under the Linguistic Concept Weighted Network Formal Context Based on Three-way Decision

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***Abstract*—Under uncertain environment, how to deal with data with complex relationships has been a hot topic of research. Network formal context was proposed by the combination of an adjacency matrix and formal context, which is a useful tool for knowledge discovery and rule extraction. In this paper, we propose the concept of linguistic concept weighted network formal context (LCWNFC), which not only handles the fuzzy relationship between objects, but also solves the problem that people are used to expressing information by linguistic values. In order to describe formal concept based on LCWNFC more detailed and complete, we further define two object induced weighted network linguistic three-way concepts (weighted network OEL-concepts) and discuss related properties. In different cognition and environments, the criteria for determining whether there exist relationships between objects are different. Subsequently, two dynamic knowledge discovery algorithms are given based on LCWNFC, together with examples to illustrate the rationality and effectiveness of the proposed algorithms.**

Keywords—Three-way concept analysis, Weighted network OEL-concepts, Dynamic knowledge discovery

I. Introduction

Formal concept analysis (FCA), proposed by the German mathematician Wille in 1982[1], is a useful tool for data processing and rule extraction. As an important order theory method for data processing and visualization based on the formal context, FCA could obtain implication and potential concept knowledge through the relationship between objects and attributes, which has been widely used in the fields of knowledge discovery [2][3], data mining [4][5], machine learning [6]–[8], feature selection [9]-[11] and so on. However, the classical formal concept essentially considers the thinking of two-way decision. It is known to us that two-way decision model only considers two cases of acceptance and rejection. However, people are also hesitating when processing some data in real life.

It is well-known that there are three situations of decision in real world problems, which are acceptance, rejection and noncommitment respectively. Based on this thinking, Yao[12] proposed the three-way decision theory, which is an extension of the common two-way decision theory. Furthermore, Qi et al. [13] proposed three-way concept analysis (3WCA) by combining the idea of three-way decision theory with FCA. Since 3WCA can describe knowledge more completely and accurately, it has attracted many researchers and yielded a number of research results. Both FCA and 3WCA focused on relationship between objects and attributes. However, there are some relationships between objects in real world. How to deal with object-object data is a topic worthy of discussion. Li et al. [14][15] proposed network formal context (NFC), which can describe both object-object relationships and object-attribute data. They also proposed knowledge discovery and updating algorithms and introduced network rule extraction algorithm. However, the relationship between objects may be fuzzy and uncertain in real life. The value describing the relationship of objects may be a fuzzy value instead of 0 or 1.

In addition, in the classical formal context, we can use 0 or 1 to express whether there is a relationship between objects and attributes. However, in daily life, due to the fuzziness of human thinking, people are used to expressing information by linguistic values. Zadeh proposed fuzzy sets [16] and linguistic variables

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[17] that can be used to process such information. Herrera et al. [18] proposed linguistic-term set (LTS) to describe all the discrete linguistic variables. Xu [19] studied the aggregation operators based on the probabilistic linguistic information. Zou et al. [20] proposed linguistic concept formal context, which can process data described in linguistic values in real life. Cui et al. [21] proposed property-oriented linguistic concept lattice to do reasoning and rule extraction researches, which can improve the accuracy of reasoning results and reduce the loss of information.

The purpose of this paper is to propose a new formal context: LCWNFC, which cannot only deal with fuzzy relationship between objects, but also express people thinking information by linguistic values. Hence, we propose the concept of LCWNFC. Then we define two object induced weighted network linguistic three-way concepts by considering global connectivity and local connectivity of network. We also discuss and certify related properties. Because in different field and environments, the criteria for judging whether there exist relationships between objects are different. We further propose two dynamic knowledge discovery algorithms based on LCWNFC. Furthermore, we illustrate the algorithms' rationality and effectiveness by a real application.

The remainder of this paper is organized as follows. In Section II, we briefly review some basic concepts required for this paper. In Section III, we propose weighted network OEL-concepts based on LCWNFC. In Section IV, we propose dynamic knowledge discovery algorithm based on LCWNFC and give examples to illustrate the rationality and effectiveness of the algorithms. In Section V, we give the conclusions and directions for future work.

II. Preliminaries

In this section, we review some basic notion, such as linguistic terms set, FCA, NFC and so on.

Definition 1. ([18]) Let $S = \{s_t | \mathrm{t} = 0,1,2, \dots, \mathrm{g}\}$ be a finite linguistic term set (LTS) consisiting of $\mathrm{g}+1$ linguistic terms and g is the positive integer. Each linguistic term s_t must have the following characteristics.

(1) There is a negation operator: $neg(s_i) = s_j, j = \mathrm{g} - i$;
(2) The set is ordered: $s_i \geq s_j \Leftrightarrow i \geq j$;
(3) There is a max operator: $s_i \geq s_j \Leftrightarrow \max(s_i, s_j) = s_i$;
(4) There is a min operator: $s_i \leq s_j \Leftrightarrow \min(s_i, s_j) = s_i$.

For example, a set of five linguistic terms S could be given as $S = \{s_0{:}\,none, s_1{:}\,low, s_2{:}\,medium, s_3{:}\,high, s_4{:}\,perfect\}$.

Definition 2. ([19]) Let $S = \{s_t | \mathrm{t} = 0,1,2, \dots, \mathrm{g}\}$ be a LTS, The discrete term set S is extended to a continuous linguistic term set $\bar{S} = \{s_\alpha | s_0 \leq s_\alpha \leq s_h, \alpha \in [0, h]\}$, where $h(h > \mathrm{g})$ is a sufficiently large positive integer. Consider any linguistic terms s_i, $s_j \in \bar{S}$, and $\lambda, \lambda_1, \lambda_2 \in [0,1]$, some operation rules are as follows.

(1) $s_i \oplus s_j = s_j \oplus s_i = s_{i+j}$;
(2) $s_i \otimes s_j = s_j \otimes s_i = s_{i*j}$;
(3) $\lambda s_i = s_{\lambda i}$;
(4) $(\lambda_1 + \lambda_2) s_i = \lambda_1 s_i \oplus \lambda_2 s_i$;
(5) $\lambda(s_i \oplus s_j) = \lambda s_i \oplus \lambda s_j$.

Definition 3. ([1]) A formal context is a triple (U, A, I), where $U = \{x_1, x_2, x_3, \dots, x_n\}$ is a finite and nonempty object set, $A = \{a_1, a_2, \dots, a_n\}$ is a finite and nonempty attribute set, and $I \subseteq U \times A$ is a binary relation. Here, $(x, a) \in I$ denotes that the object x possesses the attributes a and $(x, a) \notin I$ denotes that the object x doesn't possess the attributes a.

Definition 4. ([14]) A network formal context is a quadruple (U, A, AD, I), where $U = \{x_1, x_2, x_3, \dots, x_n\}$ is a finite and nonempty object set, $A = \{a_1, a_2, \dots, a_n\}$ is a finite and nonempty attribute set, $I \subseteq U \times A$ is a binary relation, and the $n \times n$ matrix $AD = (a_{ij})_{n \times n}$ is called an adjacency matrix with n vertices of the network, where

$$a_{ij} = \begin{cases} 1, \text{if the objects } x_i, x_j \text{ have an edge or } \mathrm{i} = \mathrm{j}; \\ 0, \text{otherwise.} \end{cases}$$

where the existence of edges between objects in the network formal context indicates that there are relationships between objects. This relationship can be determined from a variety of sources, such as expert judgement, the trajectory is used to determine whether the objects are in contact during infectious disease tracking, and so on.

Definition 5. ([20]) A linguistic concept formal context is defined as a triple (U, L_{s_a}, I), where $U = \{x_1, x_2, x_3, \dots, x_n\}$ is a finite and nonempty object set, $L_{s_a} = \{l_{-\tau}^1, \dots, l_0^1, \dots, l_\tau^1, l_{-\tau}^2, \dots, l_0^2, \dots, l_\tau^2, \dots, l_{-\tau}^n, \dots, l_0^n, \dots, l_\tau^n\}$ is a finite and nonempty linguistic concept set, and $I \subseteq U \times L_{s_a}$ is a binary relation. Here, $(x, l_{s_a}^i) \in I$ indicates that the object x can be described by a linguistic concept $l_{s_a}^i$, $(x, l_{s_a}^i) \notin I$ indicates that the object x cannot be described by a linguistic concept $l_{s_a}^i$.

Definition 6. ([20]) Let (U, L_{s_a}, I) be a linguistic concept formal context. For any subset $X \subseteq U$ and $B_{s_a} \subseteq L_{s_a}$, the positive operators $\Delta : P(U) \to P(L_{s_a})$ and $\Delta : P(L_{s_a}) \to P(U)$ are defined by

$$X^{\Delta} = \{l_{s_a}^i \in L_{s_a} | x I l_{s_a}^i, \forall x \in X\},$$
$${B_{s_a}}^{\Delta} = \{x \in U | x I l_{s_a}^i, \forall l_{s_a}^i \in B_{s_a}\}.$$

where X^{Δ} denotes a set of linguistic concepts possessed by all the objects in X, and ${B_{s_a}}^{\Delta}$ denotes a set of objects having all the linguistic concept in B_{s_a}.

In order to combine three-way decision theory with our research, we propose related operators as follows.

Definition 7. Let (U, L_{s_a}, I) be a linguistic concept formal context and $I^c \subseteq (U \times L_{s_a}) - I$. For any subset $X \subseteq U$ and $Y \subseteq U$, the negative operators $\nabla : P(U) \to P(L_{s_a})$ and $\nabla : P(L_{s_a}) \to P(U)$ are defined by

$$X^{\nabla} = \{l_{s_a}^i \in L_{s_a} | x I^c l_{s_a}^i, \forall x \in X\},$$
$${B_{s_a}}^{\nabla} = \{x \in U | x I^c l_{s_a}^i, \forall l_{s_a}^i \in B_{s_a}\}.$$

Definition 8. Let (U, L_{s_a}, I) be a linguistic concept formal context. For any $X \subseteq U$ and $Y_1, Y_2 \subseteq L_{s_a}$, the object-induced linguistic three-way operators $\lhd L$ and $L \rhd$ are defined by

$$X^{\lhd L} = (X^{\Delta}, X^{\nabla}),$$
$$(Y_1, Y_2)^{L \rhd} = \{x \in U | x \in {Y_1}^{\Delta} \; and \; x \in {Y_2}^{\nabla}\} = {Y_1}^{\Delta} \cap {Y_2}^{\nabla}.$$

We abbreviate them as OEL-operators.

Definition 9. Let (U, L_{s_a}, I) be a linguistic concept formal context. For any $X \subseteq U$ and $Y_1, Y_2 \subseteq L_{s_a}$, if $X^{\lhd L} = (X^{\Delta}, X^{\nabla}) = (Y_1, Y_2)$ $(Y_1, Y_2)^{L\rhd} = {Y_1}^{\Delta} \cap {Y_2}^{\nabla} = X$, then $(X, (Y_1, Y_2))$ is called object-induced linguistic three-way concept, or OEL-concept for short.

Here X is the extension and (Y_1, Y_2) is the intension of the OEL-concept $(X, (Y_1, Y_2))$, respectively. The set of all the OEL-concepts of the linguistic concept formal context (U, L_{s_a}, I) is denoted as $OEL(U, L_{s_a}, I)$.

This paper mainly focuses on OEL-concept, so attribute-induced linguistic three-way concept is neglected here due to the duality.

III. Linguistic Concept Weighted Network Formal Context

In this section, we will propose the notions of linguistic concept weighted network formal context. Then we discuss the related properties.

Definition 10. A linguistic concept weighted network formal context is defined as a quadruple (U, L_{s_a}, WAD, I), where $U = \{x_1, x_2, x_3, \dots, x_n\}$ is a finite and nonempty object set, $L_{s_a} = \{l^1_{-\tau}, \dots, l^1_0, \dots, l^1_\tau, l^2_{-\tau}, \dots, l^2_0, \dots, l^2_\tau, \dots, l^n_{-\tau}, \dots, l^n_0, \dots, l^n_\tau\}$ is a finite and nonempty linguistic concept set, $I \subseteq U \times L_{s_a}$ is a binary relation, and the $n \times n$ matrix $WAD = (a_{ij})_{n\times n}$ is called the weighted adjacency matrix with n vertices of the network, where

$$a_{ij} = \begin{cases} 1, \text{if } i = j; \\ \omega, \text{if the objects } x_i, x_j \text{ have an edge} \\ \text{and } \omega \text{ is the weight}, 0 < \omega \leq 1; \\ 0, \text{otherwise.} \end{cases}$$

The values of weighted adjacency matrix are values between [0,1] instead of 0 or 1, which indicate a fuzzy case of whether or not there exist a relationship between objects. In our work, a_{ij} is called the relation coefficient, or RC for short.

Example 1. The education of college students has always been a hot topic of concern in China. How to evaluate students in schools has always been a relatively important issue. Table I is a linguistic concept weighted network formal context (U, L_{s_a}, WAD, I), where $U = \{x_1, x_2, x_3, \dots, x_5\}$ represents 5 students, $L = \{l^1, l^2, l^3, l^4\} = \{scores\ of\ major\ courses, scores\ of\ all\ courses, counselor\ evalution, competition\ situation\}$ represents a set of attributes describing the four aspects, $s_a = \{s_{-1}, s_0, s_1\} = \{poor, ordinary, excelLent\}$ is the linguistic items that describe different aspects, and $L_{s_a} = \{l^1_{-1}, l^1_0, l^1_1, l^2_{-1}, l^2_0, l^2_1, l^3_{-1}, l^3_0, l^3_1, l^4_{-1}, l^4_0, l^4_1\}$ is a linguistic concept set. If $a_{12} = 0.8$ in the weighted adjacency matrix WAD it indicates that there exists contact between x_1 and x_2 and the relation coefficient is 0.8. And Table I's graphical representations can be found in Fig. 1.

TABLE I. A LCWNFC(U, L_{s_a}, WAD, I) OF EXAMPLE 1.

	x_1	x_2	x_3	x_4	x_5	l^1_{-1}	l^1_0	l^1_1	l^2_{-1}	l^2_0	l^2_1	l^3_{-1}	l^3_0	l^3_1	l^4_{-1}	l^4_0	l^4_1
x_1	1	1	0.8	0.6	0.6	0	1	0	1	0	0	1	0	0	0	0	1
x_2	1	1	0.8	0.6	0.6	0	0	1	0	0	1	0	0	1	1	0	0
x_3	0.8	0.8	1	0.6	0.6	0	1	0	0	1	0	0	1	0	0	1	0
x_4	0.6	0.6	0.6	1	1	0	1	0	1	0	0	1	0	0	1	0	0
x_5	0.6	0.6	0.6	1	1	0	0	1	1	0	0	1	0	0	1	0	0

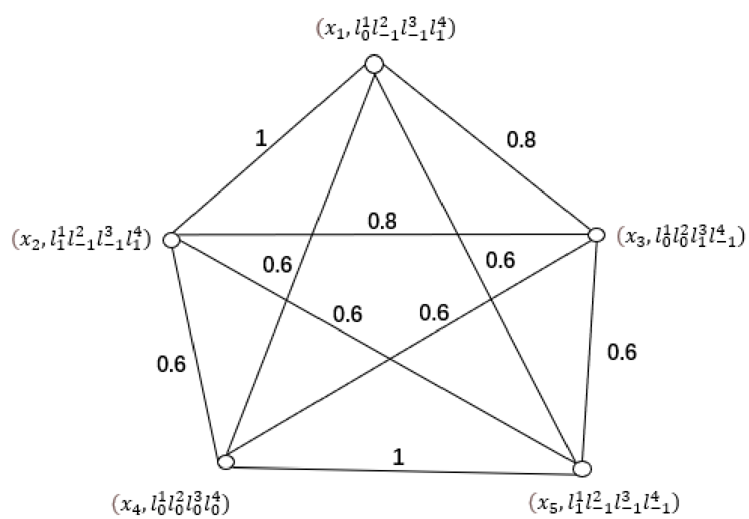

Fig. 1. The graphical representation of Table 1.

Definition 11. Let (U, L_{s_a}, WAD, I) be a linguistic concept weighted network formal context, $X \subseteq U$, $\lambda \in [0,1]$. For any $x_i, x_j \in X$ are called to be connected if $a_{ij} \geq \lambda$. X is called to be connected under λ if the sub-network composed of all the objects contained in X is connected under λ.

Definition 12. Let (U, L_{s_a}, WAD, I) be a linguistic concept weighted network formal context, $\lambda \in [0,1]$. For any $X \subseteq U$ and $Y_1, Y_2 \subseteq L_{s_a}$. If $X^{\lhd L} = (X^{\Delta}, X^{\nabla}) = (Y_1, Y_2)$, $(Y_1, Y_2)^{L\rhd} = {Y_1}^{\Delta} \cap {Y_2}^{\nabla} = X$ and X is connected under λ, then the order pair $(X, (Y_1, Y_2))$ is called an object induced weighted global network linguistic three-way concept under λ or $OWG_{\lambda} - concept$ for short. The set of all $OWG_{\lambda} - concepts$ in (U, L_{s_a}, WAD, I) is denoted as $N_{OWG_{\lambda}}(U, L_{s_a}, WAD, I)$.

Definition 13. Let (U, L_{s_a}, WAD, I) be a linguistic concept weighted network formal context, $\lambda \in [0,1]$. For any $X \subseteq U$ and $Y_1, Y_2 \subseteq L_{s_a}$. If $X^{\lhd L} = (X^{\Delta}, X^{\nabla}) = (Y_1, Y_2)$, X is connected under λ and there is no connection under λ between $x \in (Y_1, Y_2)^{L\rhd} - X$ and X, then the order pair $(X, (Y_1, Y_2))$ is called an object induced weighted local network linguistic three-way concept under λ or $OWL_{\lambda} - concept$ for short. The set of all $OWL_{\lambda} - concepts$ in (U, L_{s_a}, WAD, I) is denoted as $N_{OWL_{\lambda}}(U, L_{s_a}, WAD, I)$.

Both $OWG_{\lambda} - concept$ and $OWL_{\lambda} - concept$ are called to be an object induced weighted network linguistic three-way concept or weighted network OEL-concepts.

Property 1. Let (U, L_{s_a}, WAD, I) be a LCWNFC. Then properties of $OEL(U, L_{s_a}, I)$, $N_{OWG_{\lambda}}(U, L_{s_a}, WAD, I)$, $N_{OWL_{\lambda}}(U, L_{s_a}, WAD, I)$ are as follows.

(1). $N_{OWG_{\lambda}}(U, L_{s_a}, WAD, I) \subseteq OEL(U, L_{s_a}, I)$;

(2). $N_{OWG_{\lambda}}(U, L_{s_a}, WAD, I) \subseteq N_{OWL_{\lambda}}(U, L_{s_a}, AD, I)$;

(3). $N_{OWL_{\lambda}}(U, L_{s_a}, AD, I)$ and $OEL(U, L_{s_a}, I)$ are not directly relationship;

(4). $N_{OWG_0}(U, L_{s_a}, WAD, I) = OEL(U, L_{s_a}, I)$;

(5). $N_{OWG_1}(U, L_{s_a}, WAD, I) \subseteq N_{OWG_{\lambda}}(U, L_{s_a}, WAD, I) \subseteq N_{OWG_0}(U, L_{s_a}, WAD, I)$;

(6). $N_{OWL_1}(U, L_{s_a}, WAD, I) \subseteq N_{OWL_{\lambda}}(U, L_{s_a}, WAD, I) \subseteq N_{OWL_0}(U, L_{s_a}, WAD, I)$.

Proof. We can obtain $N_{OWG_{\lambda}}(U, L_{s_a}, WAD, I) \subseteq OEL(U, L_{s_a}, I)$ easily according to Definition 9 and Definition 12. And we prove the other properties as follows.

(2) According to Definition 12 and Definition 13, the conditions of $OWL_{\lambda} - concept$ are more relaxed than those of $OWG_{\lambda} - concept$. In other words, an $OWG_{\lambda} - concept$ must be an $OWL_{\lambda} - concepts$, but not vice versa. We can obtain that an $OWG_{\lambda} - concept$ $(X, (Y_1, Y_2))$ satisfies $X^{\lhd L} = (X^{\Delta}, X^{\nabla}) = (Y_1, Y_2)$, $(Y_1, Y_2)^{L\rhd} = {Y_1}^{\Delta} \cap {Y_2}^{\nabla} = X$ and X is connected under λ. As result, there must be no connected under λ between $x \in (Y_1, Y_2)^{L\rhd} - X$ and X, hence it satisfies the conditions of an OWL_{λ} −concept.

(3) According to Definition 9 and Definition 13, we can obtain that an $OWL_{\lambda} - concept$ $(X, (Y_1, Y_2))$ satisfies $X^{\lhd L} = (X^{\Delta}, X^{\nabla}) = (Y_1, Y_2)$, X is connected under λ and there is no connection under λ between $x \in (Y_1, Y_2)^{L\rhd} - X$ and X, and an $OEL - concept$ satisfies $X^{\lhd L} = (X^{\Delta}, X^{\nabla}) = (Y_1, Y_2)$, $(Y_1, Y_2)^{L\rhd} = {Y_1}^{\Delta} \cap {Y_2}^{\nabla} = X$. We can find both $OEL - concept$ and $OWL_{\lambda} - concept$ need to satisfy $X^{\lhd L} = (X^{\Delta}, X^{\nabla}) = (Y_1, Y_2)$. If $(Y_1, Y_2)^{L\rhd} = {Y_1}^{\Delta} \cap {Y_2}^{\nabla} = X$, $(X, (Y_1, Y_2))$ is an $OEL - concept$. It could be an $OWL_{\lambda} - concept$. But from what remains of their condition, we can't find further their relationship.

(4) According to Definition 9 and Definition 12, $OWG_{\lambda} - concepts$ are composed of the $OEL - concepts$ whose extents are connected under λ. When $\lambda = 0$, each $X \subseteq U$ is connected. So $N_{OGW_0}(U, L_{s_a}, WAD, I) = OELL(U, L_{s_a}, I)$.

(5) According to Definition 11 and Definition 12, we can obtain that an $OWG_{\lambda} - concept$ $(X, (Y_1, Y_2))$ satisfies $X^{\lhd L} = (X^{\Delta}, X^{\nabla}) = (Y_1, Y_2)$, $(Y_1, Y_2)^{L\rhd} = {Y_1}^{\Delta} \cap {Y_2}^{\nabla} = X$ and X is connected under λ. The smaller the value of λ, the more objects that satisfy the connection, the more $OWL_{\lambda} - concepts$. So $N_{OWG_1}(U, L_{s_a}, WAD, I) \subseteq N_{OWG_{\lambda}}(U, L_{s_a}, WAD, I) \subseteq N_{OWG_0}(U, L_{s_a}, WAD, I)$.

(6) The proof is similar to that of (5).

IV. Dynamic Knowledge Discovery Based on Linguistic Concept Weighted Network Formal Context

Dynamic knowledge discovery based on weighted network $OEL - concepts$ is to get useful knowledge from a LCWNFC. Compared with the $OEL - concepts$, a weighted network $OEL - concept$ considers the fuzzy relationships between objects, which can be viewed as knowledge discovery of connectivity nodes with weight.

In different cognition and environments, the criteria for determining whether there exist relationships between objects are different. We can obtain different $OWG_{\lambda} - concepts$ and $OWL_{\lambda} - concepts$ by setting different λ for different cognition and environments. Algorithms 1 and 2 give $N_{OWG_{\lambda}}(U, L_{s_a}, WADs, I)$ and $N_{OWL_{\lambda}}(U, L_{s_a}, WADs, I)$ dynamic knowledge discovery methods respectively.

Algorithms 1: Dynamic knowledge discovery of $N_{OWG_{\lambda}}(U, L_{s_a}, WAD, I)$

Input: A linguistic concept weighted network formal context (U, L_{s_a}, WAD, I) and λ

Output: $N_{OWG_{\lambda}}(U, L_{s_a}, WAD, I)$

1. For $X \subseteq 2^U$ do
2. if X is connected under λ then
3. let $X = X^{\lhd LL\rhd}$, $(Y_1, Y_2) = X^{\lhd L}$
4. else
5. compute a maximal connected subset X_1 of X, let $X = X_1$, and go back to Step 2
6. end if
7. if $Y_1 = \emptyset$ and $Y_2 = \emptyset$ then
8. return $(X, (Y_1, Y_2)) = \emptyset$
9. else if $X^{\lhd LL\rhd}$ is connected then

let $X = X^{\lhd LL \rhd}$, $(Y_1, Y_2) = X^{\lhd L}$ and return $(X, (Y_1, Y_2))$
else
return $(X, (Y_1, Y_2)) = \emptyset$
end if
end for

Example 2. We discuss $OGW_\lambda - concepts$ by setting different λ from the data of Table I as follows.

(1). Where λ =1, $N_{OWG_1}(U, L_{s_a}, WAD, I)$ of Table I are as follows.

$(x_1, (l_0^1 l_{-1}^2 l_{-1}^3 l_1^4, l_{-1}^1 l_1^1 l_0^2 l_1^2 l_0^3 l_1^3 l_0^4 l_{-1}^4)), (x_2, (l_1^1 l_1^2 l_1^3 l_{-1}^4, l_{-1}^1 l_0^1 l_{-1}^2 l_0^2 l_{-1}^3 l_0^3 l_0^4 l_1^4)), (x_3, (l_0^1 l_0^2 l_0^3 l_0^4, l_{-1}^1 l_1^1 l_{-1}^2 l_1^2 l_{-1}^3 l_1^3 l_{-1}^4 l_1^4)), (x_4, (l_0^1 l_{-1}^2 l_{-1}^3 l_{-1}^4, l_{-1}^1 l_1^1 l_0^2 l_1^2 l_0^3 l_1^3 l_0^4 l_1^4)), (x_5, (l_1^1 l_{-1}^2 l_{-1}^3 l_{-1}^4, l_{-1}^1 l_0^1 l_0^2 l_1^2 l_0^3 l_1^3 l_0^4 l_1^4)), (x_4 x_5, (l_{-1}^2 l_{-1}^3 l_{-1}^4, l_{-1}^1 l_0^2 l_1^2 l_0^3 l_1^3 l_0^4 l_1^4)).$

(2). Where λ =0.8, $N_{OWG_{0.8}}(U, L_{s_a}, WAD, I)$ of Table I are as follows.

$(x_1, (l_0^1 l_{-1}^2 l_{-1}^3 l_1^4, l_{-1}^1 l_1^1 l_0^2 l_1^2 l_0^3 l_1^3 l_0^4 l_{-1}^4)), (x_2, (l_1^1 l_1^2 l_1^3 l_{-1}^4, l_{-1}^1 l_0^1 l_{-1}^2 l_0^2 l_{-1}^3 l_0^3 l_0^4 l_1^4)), (x_3, (l_0^1 l_0^2 l_0^3 l_0^4, l_{-1}^1 l_1^1 l_{-1}^2 l_1^2 l_{-1}^3 l_1^3 l_{-1}^4 l_1^4)), (x_4, (l_0^1 l_{-1}^2 l_{-1}^3 l_{-1}^4, l_{-1}^1 l_0^1 l_0^2 l_1^2 l_0^3 l_1^3 l_0^4 l_1^4)), (x_5, (l_1^1 l_{-1}^2 l_{-1}^3 l_{-1}^4, l_{-1}^1 l_0^1 l_0^2 l_1^2 l_0^3 l_1^3 l_0^4 l_1^4)), (x_1 x_3, (l_0^1, l_{-1}^1 l_1^1 l_1^2 l_1^3 l_{-1}^4))(x_2 x_3, (\emptyset, l_{-1}^1 l_{-1}^2 l_{-1}^3 l_1^4)), (x_4 x_5, (l_{-1}^2 l_{-1}^3 l_{-1}^4, l_{-1}^1 l_0^2 l_1^2 l_0^3 l_1^3 l_0^4 l_1^4).$

Algorithms 2: Dynamic knowledge discovery of $N_{OWL_\lambda}(U, L_{s_a}, WAD, I)$

Input: A linguistic concept weighted network formal context (U, L_{s_a}, WAD, I) and λ

Output: $N_{OWL_\lambda}(U, L_{s_a}, WAD, I)$

1. for $X \subseteq 2^U$ do
2. if X is connected under λ then
3. if there is no connection between $x \in (Y_1, Y_2)^{L \rhd} - X$ and X then
4. let $(Y_1, Y_2) = X^{\lhd L}$
5. else
6. let $X = X \cup \{x\}$, $(Y_1, Y_2) = X^{\lhd L}$
7. end if
8. if $Y_1 = \emptyset$ and $Y_2 = \emptyset$ then
9. return $(X, (Y_1, Y_2)) = \emptyset$
10. else
11. return $(X, (Y_1, Y_2)$
12. end if
13. else
14. compute a maximal connected subset X_1 of X, let $X = X_1$, and go back to Step 2
15. end if
16. end for

Example 3. We discuss $OWL_\lambda - concepts$ by setting different λ from the data of Table I as follows.

(1). Where λ =1, $N_{OWL_1}(U, L_{s_a}, WAD, I)$ of Table I are as follows.

$(x_1, (l_0^1 l_{-1}^2 l_{-1}^3 l_1^4, l_{-1}^1 l_1^1 l_0^2 l_1^2 l_0^3 l_1^3 l_0^4 l_{-1}^4)), (x_2, (l_1^1 l_1^2 l_1^3 l_{-1}^4, l_{-1}^1 l_0^1 l_{-1}^2 l_0^2 l_{-1}^3 l_0^3 l_0^4 l_1^4)), (x_3, (l_0^1 l_0^2 l_0^3 l_0^4, l_{-1}^1 l_1^1 l_{-1}^2 l_1^2 l_{-1}^3 l_1^3 l_{-1}^4 l_1^4)), (x_4, (l_0^1 l_{-1}^2 l_{-1}^3 l_{-1}^4, l_{-1}^1 l_1^1 l_0^2 l_1^2 l_0^3 l_1^3 l_0^4 l_1^4)) , (x_5, (l_1^1 l_{-1}^2 l_{-1}^3 l_{-1}^4, l_{-1}^1 l_0^1 l_0^2 l_1^2 l_0^3 l_1^3 l_0^4 l_1^4)), (x_1 x_2, (\emptyset, l_{-1}^1 l_0^2 l_0^3 l_0^4)),(x_4 x_5, (l_{-1}^2 l_{-1}^3 l_{-1}^4, l_{-1}^1 l_0^2 l_1^2 l_0^3 l_1^3 l_0^4 l_1^4)).$

(2). Where λ =0.8, $N_{OWL_{0.8}}(U, L_{s_a}, WAD, I)$ of Table I are as follows.

$(x_1, (l_0^1 l_{-1}^2 l_{-1}^3 l_1^4, l_{-1}^1 l_1^1 l_0^2 l_1^2 l_0^3 l_1^3 l_0^4 l_{-1}^4)), (x_2, (l_1^1 l_1^2 l_1^3 l_{-1}^4, l_{-1}^1 l_0^1 l_{-1}^2 l_0^2 l_{-1}^3 l_0^3 l_0^4 l_1^4)), (x_3, (l_0^1 l_0^2 l_0^3 l_0^4, l_{-1}^1 l_1^1 l_{-1}^2 l_1^2 l_{-1}^3 l_1^3 l_{-1}^4 l_1^4)), (x_4, (l_0^1 l_{-1}^2 l_{-1}^3 l_{-1}^4, l_{-1}^1 l_0^1 l_0^2 l_1^2 l_0^3 l_1^3 l_0^4 l_1^4)), (x_5, (l_1^1 l_{-1}^2 l_{-1}^3 l_{-1}^4, l_{-1}^1 l_0^1 l_0^2 l_1^2 l_0^3 l_1^3 l_0^4 l_1^4)), (x_1 x_2, (\emptyset, l_{-1}^1 l_0^2 l_0^3 l_0^4))(x_1 x_3, (l_0^1, l_{-1}^1 l_1^1 l_1^2 l_1^3 l_{-1}^4)), (x_1 x_2 x_3, (\emptyset, l_{-1}^1)), (x_2 x_3, (\emptyset, l_{-1}^1 l_{-1}^2 l_{-1}^3 l_1^4)), (x_4 x_5, (l_{-1}^2 l_{-1}^3 l_{-1}^4, l_{-1}^1 l_0^2 l_1^2 l_0^3 l_1^3 l_0^4 l_1^4)).$

It can be easily observed that different $OWG_\lambda - concepts$ and $OWL_\lambda - concepts$ can be obtained by setting different λ.

V. Conclusion and Future Work

In the linguistic concept multi-weighted network formal context, it not only considers the degree of the influence between objects, but also uses linguistic value to describe the object-attribute information. In the present work, firstly, we propose LCWNFC, define the weighted network $OEL - concepts$ and discuss related properties. Finally, we propose dynamic knowledge discovery algorithms based on the LCWNFC and demonstrate the rationality and effectiveness of algorithms by an example.

This paper only studies the dynamic knowledge discovery under the LCWNFC based on three-way decision. In real life the relationship between objects may be affected by many factors and there is incomplete information between objects and attributes, how to describe and handle its data are our research work in the future.

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