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Research Article

Fuzzy Linguistic Protoforms to Summarize Heart Rate Streams of Patients with Ischemic Heart Disease

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Cardiac rehabilitation is a key program which significantly decreases mortality rates in high-risk patients with ischemic heart disease. Due to the huge lack of accessibility to such programs at Health Centers, outdoor-based programs for cardiac rehabilitation have been proposed as an excellent tool to improve accessibility for patients at Health Centers. These outdoor-based programs make use of wrist-worn devices for real-time monitoring of rehabilitation sessions based on clinical guidelines. In this way, a greater number of patients can fortunately gain access to the rehabilitation program. However, this advantage also means that the cardiac rehabilitation team has to monitor a greater number of sessions due to the increase of the number of benefited patients, so the team members spend a lot of time analyzing each patient's sessions. In this paper, we present a methodology to evaluate heart rate streams of patients with ischemic heart disease using a linguistic approach. This innovative methodology manages relevant linguistic descriptions (protoforms) for the cardiac rehabilitation team to identify sessions with interest indicators by means of linguistic summaries. Therefore, the analysis process is automated in a comprehensible way, offering short linguistic descriptions to the cardiac rehabilitation team, who promptly provide feedback to their patients. In order to show the great efficiency and effectiveness of the proposed methodology, we have used and applied this methodology to real data provided by patients of an outdoor cardiac rehabilitation program run by the Health Council of the Andalusian Health Service (Spain).

1. Introduction

The Internet of Things (IoT) paradigm [1] is based on the idea of multiple devices located around the world working to acquire information and store it in order to subsequently process and analyze this data with the aim of providing intelligent services [2].

In this context, there are open challenges to extract rich information from the huge amount of data from sensor sources in large-scale deployment within the revolutionary paradigm of IoT. To alleviate this limitation, multiple areas in the research field have been involved, such as data filtering, data aggregation, semantic analysis, and information utilization [3].

On the other hand, linguistic descriptions of data generate natural language texts [4] that convey the most relevant information contained, and sometimes hidden, in the data. Protoforms and fuzzy logic, which were proposed by Zadeh [5, 6] as a useful knowledge model for reasoning [7], summarization [8], and aggregation [9] of data under uncertainty, are modeled by fuzzy sets whose degree of truth to fuzzy sets is defined by membership functions.

Both IoT paradigm and fuzzy linguistic models have been successfully proposed for managing uncertainty and vagueness in an interpretable way, which is a key issue to obtain high performance and results [10]. So, the use of protoforms and fuzzy logic has provided brilliant results in IoT systems in multiple areas with sensor data streams, such as weather forecasting [11], predicting of demand for urgent care in smart cities [12], fever medication control [13], visual scenes [14], or monitoring of patients with preeclampsia in wearable devices [15]. So, the fuzzy logic has been demonstrated as a useful tool to deal with the uncertainty in the complex Internet of Things systems. In the green multimodal routing problem to improve the reliability of the routes, the fuzzy logic was proposed to model the uncertainty in a piecewise linear function to represent the road traffic congestion [16]. The approaches for activity recognition based on fuzzy logic have provided excellent results in the optimization of the configuration of a heterogeneous architecture of sensors [17]. In the context of robot manipulators' systems with multiple sensors and actuators, a fuzzy control scheme with adaptation algorithms has been proposed to manage the uncertainty of the information [18]. To predict the available maximal data transfer rate of single-pair high-speed digital subscriber line connections from measured frequency dependent electrical parameters of wire pairs, an approach based on fuzzy logic has been proposed [19].

This paper falls in the research field of linguistic descriptions of data with protoforms and fuzzy logic applied to e-health solutions based on complex IoT systems, specifically, in cardiovascular diseases, which represent the main health problem in developed countries according to the World Health Organization (WHO) [20] and where fuzzy logic has been shown to work as an effective modeling tool in cardiac rehabilitation [21, 22].

In the health field, secondary prevention programs and Cardiac Rehabilitation Units (CRU) have been developed in several countries [23, 24], having been proven as the most effective tool to improve prognosis. Cardiac rehabilitation (CR) is defined as the sum of activities required to influence the underlying cause of heart disease favorably, as well as to ensure the best physical, social, and mental condition of patients, enabling them to occupy a normal place in society by their own means [25].

In previous work [22], an outdoor cardiac rehabilitation program (CRP) for patients was embedded in a wrist-worn device with a heart rate sensor for personalized care. Outdoor CRPs have increased the accessibility of cardiac rehabilitation programs due to the fact that they overcome several limitations, such as lack of time, commodities, geographical area, and access to health services [24, 26]. In [22], an outdoor program was designed and supervised remotely by the cardiac rehabilitation team by means of a wearable mobile-cloud platform for collecting and synchronizing data between patients and the cardiac rehabilitation team. To do so, a linguistic approach based on fuzzy logic was proposed in order to model the cardiac rehabilitation protocol and the expert knowledge from the cardiac rehabilitation team.

A great impact under the outdoor CRP is that the number of benefited patients has drastically increased. This positive fact, however, means that the health team must monitor a greater number of sessions. In this way, the team members spend a lot of time analyzing the patients' sessions in a home-based CRP and they are overwhelmed with the huge amounts of information generated by each patient's wrist-worn device.

In order to solve this limitation, in this paper we present a methodology that generates textual information, summaries, from the heart rate streams of patients with ischemic heart disease by means of protoforms and fuzzy logic. So, this paper presents a methodology to summarize patients' rehabilitation sessions, offering understandable information for the cardiac rehabilitation team. The key points of the proposed methodology are the following:

(i) To allow the cardiac rehabilitation team to supervise a huge number of sessions and patients by means of linguistic summaries, which integrate an intuitive representation

(ii) To model a proposed methodology where the linguistic summaries are focused on rich expressiveness, including linguistic temporal terms and linguistic quantifiers by means of linguistic aggregation operators

(iii) To provide a flexible linguistic methodology where the cardiac rehabilitation team intuitively defines the key interest indicators using protoforms based on expert knowledge in order to recover and dynamically select the rehabilitation sessions that suit and match the expert criteria

The proposed methodology is applied to real data provided by several patients of a cardiac rehabilitation program run by the Health Council of the Andalusian Health Service (Spain) in order to show its efficiency and effectiveness.

The remainder of this paper is structured as follows: Section 2 presents the novel methodology to generate linguistic summaries of the rehabilitation sessions from the heart rate streams of patients with ischemic heart disease for the cardiac rehabilitation team. Section 3 presents a case study to show the utility and applicability of the proposed methodology with real data from the rehabilitation sessions of three patients freely participating in cardiac rehabilitation programs provided by the Health Council of the Andalusian Health Service in the Region of Jaén (Spain). Finally, some concluding remarks are pointed out in conjunction with future works.

2. Methodology

In this section, we describe a methodology to generate linguistic summaries of the rehabilitation sessions (RSs) from the heart rate streams (HRS) of patients with ischemic heart disease that follow an outdoor CRP. The HRS data are collected from real patients wearing a high-quality wrist-worn device, which has improved the quality of heart rate measurements and their health applications [27]. It is noteworthy that the linguistic modeling developed in this work has been defined by health experts in the CRP to summarize the sessions with interest indicators in cardiac rehabilitation.

For this purpose, we will present data processing of HRS from rehabilitation sessions through three stages. In the first stage, the raw data from heart rate streams is initially preprocessed using a previous approach [22]. In this stage, a fuzzy model is proposed to monitor the heart rate under a linguistic approach in real time by means of three representative terms and their membership functions, *low*, *adequate*, and *high*, as well as short-term fuzzy temporal windows (FTWs). The linguistic terms are computed in real time within the wrist-worn device in order to advise patients

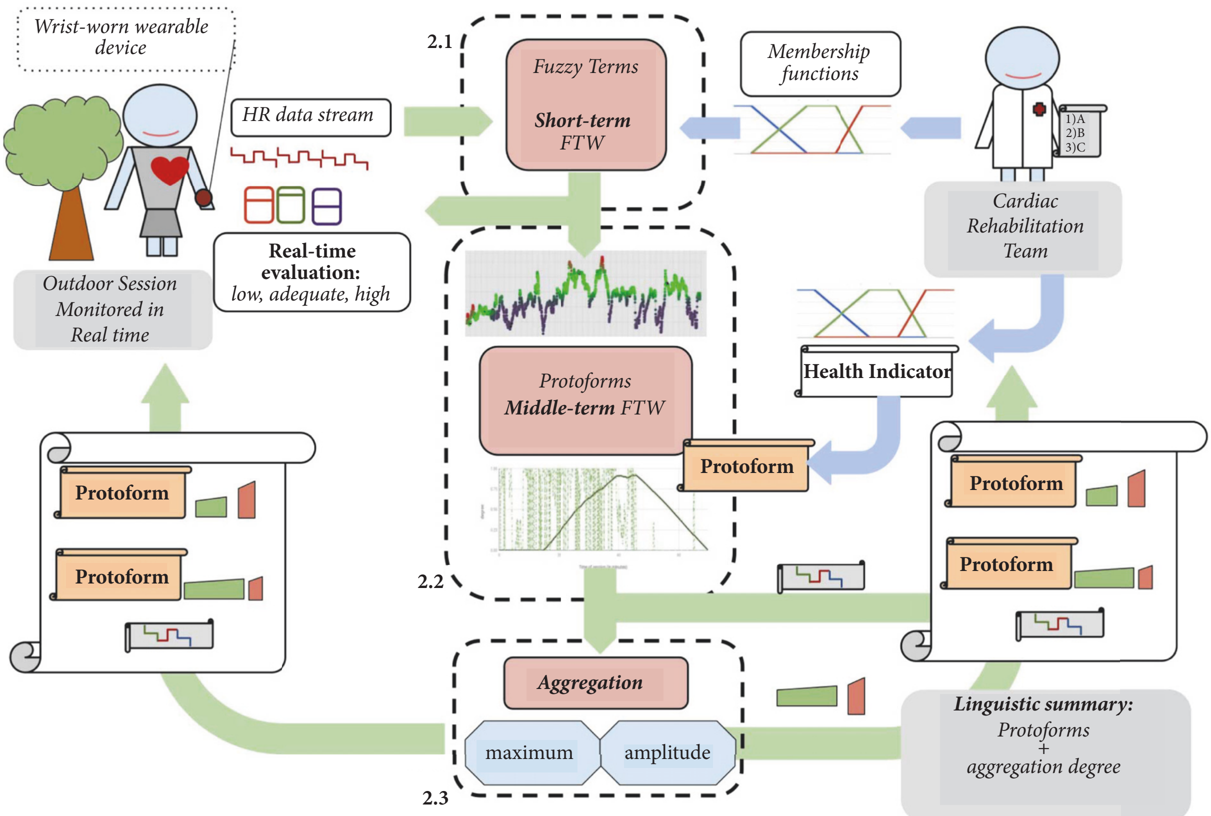


Figure 1: Architecture of the proposed methodology. First (2.1), the heart rate streams generated by wearable devices are computed to determine if the adherence to rehabilitation programs is adequate in real time. Second (2.2), linguistic summaries identify health indicators defined by experts using protoforms. Third (2.3), a final aggregation degree from the protoforms describes the rehabilitation sessions of the patients.

while they are undergoing the rehabilitation session. The main key points are briefly described in Section 2.1.

In this work, we present a methodology to compute linguistic summaries identifying key health indicators using expert knowledge from the linguistic terms computed in the first stage. Unlike the first stage, which is computed in real time while the patient is doing the exercise and without the complete information of the session, the summaries are computed for the cardiac rehabilitation team in a centralized way to evaluate a huge number of patients and sessions. For this purpose, we present a flexible model which selects and recovers the sessions according to expert criteria linguistically and intuitively.

So, in the second stage, we integrate an interpretable approach for the cardiac rehabilitation team that models knowledge linguistically. To do so, we use ad hoc protoforms. Protoforms are a general form of linguistic data summery [8].

The protoforms proposed in this methodology define linguistic summaries from the HRS of the sessions by means of long-term fuzzy temporal windows and fuzzy quantifiers, which provide rich expressiveness in the model.

In the third stage, we present the computing of a unique aggregation degree for the protoforms which describes the rehabilitation sessions summarizing the relevance and impact of the protoforms, during the complete rehabilitation sessions. To do so, two semantics of aggregation operators are described: *maximum* and *amplitude*.

In Figure 1, we describe the three stages of the proposed methodology to summarize the rehabilitation sessions of patients by means of protoforms using a linguistic approach.

2.1. Real-Time Monitoring of Heart Rate Streams. In work [22], real-time monitoring of heart rate streams was proposed. First, three intuitive terms *low*, *adequate*, and *high* defined with three membership functions by means of fuzzy sets were proposed in order to describe the variable *heart rate* (HR).

The HR was measured by a pair value $\overline{s_i} = \{s_i, t_i\}$, where s_i represents a given value in the HRS and t_i is its time stamp. Hence, the HRS of a session is composed of a set of measured values $S_{HRS} = \{\overline{s_0}, \ldots, \overline{s_i}, \ldots, \overline{s_n}\}$, which are collected by the heart rate sensor of the wearable device.

Second, the fuzzification of HR, $\overline{hr_i}$, is determined by (i) Optimal Heart Rate Training Zones (OHRTZ) where the patient must develop the sessions, which is represented as a discrete HR range within $[r^*_+, r^*_-]$, and (ii) the Ventilatory Thresholds $[VT_1, VT_2]$, which represent the aerobic-anaerobic thresholds for the performance of an efficient and safe physical activity. So, the terms *low*, *adequate*, and *high* of the variable HR s_i are described by a fuzzy set characterized by a membership function whose shape corresponds to the trapezoidal functions (TS, TR, and TL are described in Abbreviations) of

$$\begin{aligned} \mu_{adequate}\left(S_{HRS}\right) &= TS\left(S_{HRS}\right)\left[VT_1, r_-^*, r_+^*, VT_2\right], \\ &\qquad VT_1 < r_-^* < r_+^* < VT_2 \\ \mu_{high}\left(S_{HRS}\right) &= TR\left(S_{HRS}\right)\left[r_+^*, VT_2\right], \quad VT_2 > r_+^* \\ \mu_{low}\left(S_{HRS}\right) &= TL\left(S_{HRS}\right)\left[VT_1, r_-^*\right], \quad VT_1 < r_-^*. \end{aligned} \tag{1}$$

In Figure 2, we show the representation of the fuzzy sets (membership functions) of HR for the three linguistic terms: *low*, *adequate*, and *high*. As noted in work [22], the thresholds of TS $r_+^*(t_i), r_-^*(t_i), VT_1(t_i), VT_2(t_i)$ could progressively increase from basal state-defining time-dependent terms, but for the sake of simplicity here we write r_+^*, r_-^*, VT_1, VT_2.

Third, a fuzzy temporal window [28, 29] to model the HRS was proposed in order to weight fuzzy linguistic terms based on fuzzy temporal linguistic terms and provide flexibility in the presence of eventual signal loss or variance in the sample rate. The FTWs are described straightforwardly according to the distance of the current time t_0 to a given time stamp t_i as $\Delta t_i = t_i - t_0$. In this work, the use of FTWs is introduced also to describe temporal evaluation of long and middle terms in S_{HRS}.

Fourth, the degrees of the fuzzy linguistic terms $V = \{V_{low}, V_{adequate}, V_{high}\}$ are weighted by the degree of their time stamps evaluated by the FTW T_k:

$$\begin{aligned} V_r \cap T_k\left(\overline{s_i}\right) &= V_r\left(s_i\right) \cap T_k\left(\Delta t_i\right) \in [0,1] \\ V_r \cup T_k\left(S_{HRS}\right) &= \bigcup_{\overline{s_i} \in S_{HRS}} V_r \cap T_k\left(\overline{s_i}\right) \in [0,1]. \end{aligned} \tag{2}$$

A Fuzzy Weighted Average (FWA) [30] was proposed as an operation to model the t-norm and conorm:

$$\begin{aligned} V_r \cup T_k\left(S_{HRS}\right) &= \frac{1}{\sum_{t_i}^{S_{HRS}} T_k\left(\Delta t_i\right)} \sum_{t_i}^{S_{HRS}} T_k\left(\Delta t_i\right) V_r\left(s_i\right) \\ &\quad \times T_k\left(\Delta t_i\right), \in [0,1]. \end{aligned} \tag{3}$$

Under evaluation, the experts defined and selected the most adequate size for the FTWs T_{low}, $T_{adequate}$, and T_{high} in order to weight the terms V_{low}, $V_{adequate}$, and V_{high}, respectively. An embedded application in the wrist-worn device computes the degree of the terms *low*, *adequate*, and *high* in real time for each $\overline{s_i}$ using

$$\begin{aligned} low\left(\overline{s_i}\right) &= V_{low} \cup T_{low}\left(\overline{s_i}\right) \\ adequate\left(\overline{s_i}\right) &= V_{adequate} \cup T_{adequate}\left(\overline{s_i}\right) \\ high\left(\overline{s_i}\right) &= V_{high} \cup T_{high}\left(\overline{s_i}\right). \end{aligned} \tag{4}$$

Finally, the degrees of the fuzzy linguistic terms $low(\overline{s_i})$, $adequate(\overline{s_i})$, and $high(\overline{s_i})$ are computed within the wrist-band device: (i) showing a visually interpretable colored circle which represents whether the current HR $\overline{s_i}$ is *low*, *adequate*, or *high* during its FTWs and (ii) alerting the patient through sensor vibration in the wrist-band when $\overline{s_i}$ is computed as high $adequate(\overline{s_i}) < high(\overline{s_i})$ in its FTWs.

2.2. Fuzzy Linguistic Summaries of Cardiac Rehabilitation Sessions. In the previous section, we described the real-time evaluation of HR in a wrist-band application using a clinical-based protocol for monitoring and advising S_{HRS}. Here, we detail a methodology to generate linguistic summaries and identify key interest indicators using expert knowledge from the fuzzy linguistic terms computed in the wrist-worn devices, which describe the real-time adherence and performance of the patient in his/her HRS.

For this purpose, we start from the degree of the terms *adequate*, *low*, and *high* described in (4) for each $\overline{s_i}$ within the data stream S_{HRS}. An example is shown in Figure 3, where a timeline with a real HRS is plotted using gradual colors *blue*, *green*, and *red* based on the degree of the fuzzy linguistic terms $low(\overline{s_i})$, $adequate(\overline{s_i})$, and $high(\overline{s_i})$, respectively.

2.2.1. Protoforms for Describing Heart Rate Streams. The aim of the proposed methodology is to generate linguistic summaries from the rehabilitation sessions of patients. For this purpose, in this second stage we process the fuzzy linguistic terms $low(\overline{s_i})$, $adequate(\overline{s_i})$, and $high(\overline{s_i})$ from the data stream S_{HRS}, which are described in the previous section.

First, in order to integrate an interpretable and rich-expressive approach to model the expert knowledge linguistically, we introduce an ad hoc *protoform* P_o in the form of

$$P_o\left(\overline{s_i}\right) : \left(Q_k\ L_i\ T_j\right), \tag{5}$$

where

(i) L_i defines a fuzzy linguistic term to evaluate the data stream. Here, L_i is straightforwardly related to fuzzy terms $low(\overline{s_i})$, $adequate(\overline{s_i})$, and $high(\overline{s_i})$

(ii) T_j defines a fuzzy temporal term where the term L_i is aggregated. The use of FTWs, which were introduced in the previous section, is extended to generate linguistic summaries of middle-long temporal terms from S_{HRS}. The aggregation of L_i over T_j for a given $\overline{s_i}$ is computed by (4) as $L_i \cup T_j(\overline{s_i})$

(iii) Q_k defines a fuzzy quantifier (FQ) to evaluate the impact and fulfillment of the linguistic term L_i within the temporal window T_j [28]. A FQ applies a transformation $\mu_{Q_K} : [0,1] \longrightarrow [0,1]$ to the aggregated temporal degree of $\mu_{Q_K}(V_r \cup T_k(\overline{s_i}))$

The aim of modeling knowledge through protoforms is allowing the rehabilitation team to define key interest indicators using expert intuitive representations of temporal and quantification terms linguistically. An example of protoform is *most of the time* (Q_k) *HR is adequate* (L_i) *for around 40-60 minutes* (T_j). We note that the protoforms are suitable to linguistically describe the impact and fulfillment of a fuzzy linguistic term in a fuzzy temporal window in more detail,

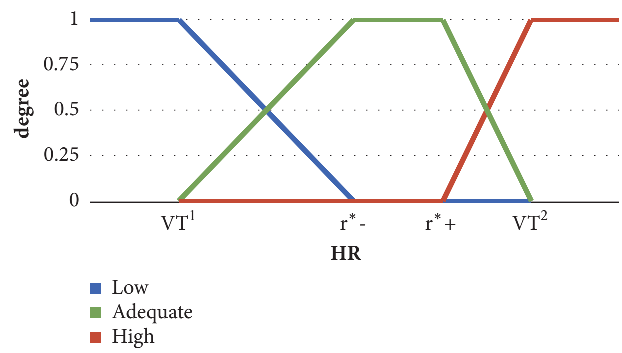


FIGURE 2: Representation with trapezoidal membership functions of the linguistic terms *low*, *adequate*, and *high* of the HR variable by means of Optimal Heart Rate Training Zones and Ventilatory Thresholds.

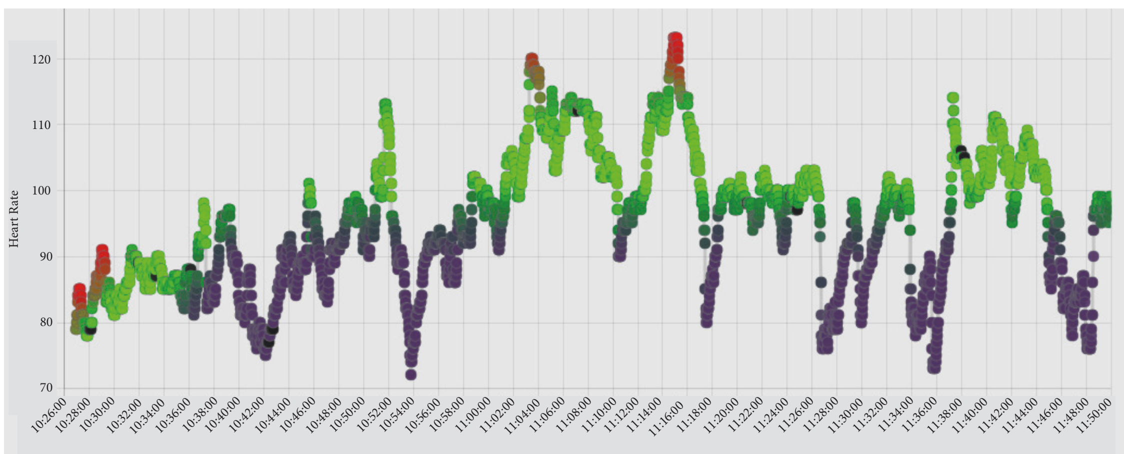


FIGURE 3: A timeline with a real HRS. The time and value configure the points in the chart $\overline{s_i} = \{s_i, t_i\}$, which are plotted using gradual colors *blue*, *green*, and *red* based on the degree of the terms $low(\overline{s_i})$, $adequate(\overline{s_i})$, and $high(\overline{s_i})$, respectively.

but subsequently they can be renamed to a shorter linguistic description, such as *adequate HR in session*.

In Figure 4, the protoform *most of the time HR is adequate for around 30-50 minutes* is shown, whose degree is represented with a real S_{HRS}. We also plot the degree of the protoform $P_o(\overline{s_i})$ in relation to the term $adequate(\overline{s_i})$ to describe the importance of the middle-long FTW and the FQ which enable the linguistic interpretability of the sentence *adequate heart rate in session* in the sensor stream. For the sake of simplicity, the shapes of the fuzzy membership functions of the example are detailed in Section 3.

Second, protoforms $P_o(\overline{s_i})$ can be combined using fuzzy logical operators to increase the linguistic capabilities of the model. So, we briefly introduce the following basic operations, which could be straightforwardly increased with advanced fuzzy operations in other contexts:

(i) Fuzzy negation operator, which is represented as the complement $\neg$ by the fuzzy function $\neg P_o(\overline{s_i}) = 1 - P_o(\overline{s_i})$

(ii) Fuzzy union operator, which is represented by the t-norm $P_o \wedge P_q(\overline{s_i}) = P_o(\overline{s_i}) \wedge P_q(\overline{s_i})$. The semantic function proposed for the fuzzy union operator is min: $P_o \wedge P_q(\overline{s_i}) = \min\{P_o(\overline{s_i}), P_q(\overline{s_i})\}$

(iii) Fuzzy intersection operator, which is represented by the conorm $P_o \vee P_q(\overline{s_i}) = P_o(\overline{s_i}) \vee P_q(\overline{s_i})$. The semantic function proposed for the fuzzy intersection operator is max: $P_o \wedge P_q(\overline{s_i}) = \max\{P_o(\overline{s_i}), P_q(\overline{s_i})\}$

2.2.2. Aggregation Operation for Protoforms. As we detailed in Section 2.2.1, protoforms are defined to represent the health indicators from the rehabilitation sessions linguistically by means of expert knowledge. Although the evaluation of protoforms is properly computed $P_o(\overline{s_i})$ throughout the data stream S_{HRS}, an aggregation degree is proposed here in order to summarize the relevance of a protoform.

First, in (6) we describe the aggregation operation $\cup(P_o)$ of the protoform P_o, which computes a single degree from the degree of protoform $P_o(\overline{s_i})$ over S_{HRS}, as

$$\cup(P_o) = \bigcup_{\overline{s_i}}^{S_{HRS}} P_o(\overline{s_i}) \in [0, 1]. \tag{6}$$

In order for the aggregated degree of the aggregation operator $\cup(P_o)$ to keep its semantic integrity with the degree of the protoforms, we define two properties which the aggregation operation should assess:

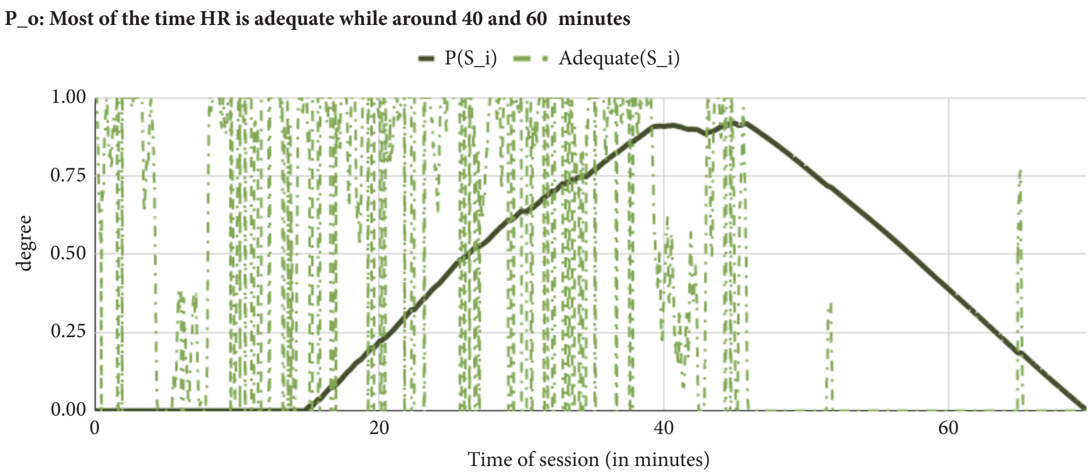


FIGURE 4: A timeline with the degrees of the term *adequate*($\overline{s_i}$) (dotted line) and protoform *most of the time HR is adequate for around 30-50 minutes* $P_o(\overline{s_i})$ (thick line) on a real HRS. Based on its semantics, the degree of the protoform increases when the patient has maintained an adequate HR for 30 minutes.

(i) $\cup_0$: zero-aggregation from zero stream. If the degree of the protoform is zero in the stream, the aggregation is zero: $P_o(\overline{s_i}) = 0, \forall \overline{s_i} \in S_{HRS} \longrightarrow \cup(P_o) = 0$. It is a necessary condition of boundary property [31]

(ii) $\cup_+$: positive-aggregation from nonzero stream. If there is a nonzero degree of the protoform in the stream, the aggregation is nonzero: $\exists P_o(\overline{s_i}) > 0, \overline{s_i} \in S_{HRS} \longrightarrow \cup(P_o) \neq 0 \longrightarrow \cup(P_o) > 0$

We note the two properties have been determined based on the semantics of the protoforms to describe the HRS, which guarantee that the number of instances which match to a given protoform $\cup(P_o) > 0$ is equal although the semantics of the aggregation varies. Specifically, the aggregation degree is zero if and only if the protoform is not representative in the session $\cup(P_o) = 0 \longleftrightarrow P_o(\overline{s_i}) = 0, \forall \overline{s_i} \in S_{HRS}$; in another case $\cup(P_o) > 0$. We note that, in different fields and other contexts, further properties in aggregation operators can be selected [32].

Second, we propose two semantics to aggregate the degree of the protoform $P_o(\overline{s_i})$: *maximum* and *amplitude*.

(i) On the first hand, the *maximum* aggregation operator $\max(P_o)$ computes the maximal value of degree of the protoform, which is shown in

$$\max(P_o) = \max\{P_o(\overline{s_0}), \dots, P_o(\overline{s_i})\} \in [0,1]. \quad (7)$$

The semantics of $\max(P_o)$ describes the maximal truth degree of the protoform providing an intuitive representation as aggregation, which has been widely used in fuzzy logic as aggregation of rules within Mamdani-type inference models [33]. It fulfills the properties of zero-aggregation $\cup_0$ and positive-aggregation $\cup_+$.

$$\begin{aligned} &\cup_0 : P_o(\overline{s_i}) = 0, \\ &\qquad \forall \overline{s_i} \in S_{HRS} \equiv \cup(P_o) = \max\{0,\dots,0\} = 0 \\ &\cup_+ : \exists P_o(\overline{s_i})^+ > 0, \\ &\qquad \overline{s_i} \in S_{HRS} \equiv \cup(P_o) = \max\{0, P_o(\overline{s_i})^+, \dots, 0\} > 0 \end{aligned} \quad (8)$$

(ii) On the other hand, the *amplitude* aggregation operator $|(P_o)|$ describes the persistence and presence of the protoform degree $P_o(\overline{s_i})$ throughout the rehabilitation session. For this purpose, the fuzzy quantification of the weight of the protoform $W(P_o)$ degree within the HRS is proposed in

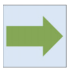
$$\begin{aligned} W(P_o) &= \frac{\sum_{\overline{s_i}}^{S_{HRS}} \overline{s_i}}{|S_{HRS}|} \\ |(P_o)| &= Q(W(P_o)) \in [0,1]. \end{aligned} \quad (9)$$

First, the weight of the protoform degree $W(P_o)$, which represents a suitable measure as fuzzy aggregation [34], is computed as the relation between the

TABLE 1: Membership functions for the terms adequate, low, and high μ_V and their respective FTWs μ_T.

Term	μ_V	μ_T
adequate	$TS(s_i)[VT_1, r_-^*, r_+^*, VT_2]$	$TL(\Delta t_i)[3s, 5s]$
low	$TL(s_i)[VT_1, r_-^*]$	$TL(\Delta t_i)[3s, 5s]$
high	$TR(s_i)[r_+^*, VT_2]$	$TL(\Delta t_i)[0s, 1s]$

Table 2: Textual description in natural language: short linguistic description and related protoforms.

Icon	Id	short linguistic descriptions	Protoforms: Q_k L_i T_j
	P_1	*Adequate HR in session*	*At least half of the time the HR is adequate for around 25-50 minutes*
	P_2	*High HR in session is worrying*	*Most of the time the HR is high for around 1-3 minutes*
	P_3	*Low HR intensity in session*	*Most of the time the HR is low for around 15-25 minutes*
	P_4	*Unstable HR progression in session*	*While a part of the time the HR is high in the last 2 minutes ∩ While a part of the time the HR was low 1-3 minutes ago*

sum of the degrees $\sum_{\overline{s_i}}^{S_{HRS}} \overline{s_i}$ regarding the norm (or size) of the complete HRS $|S_{HRS}|$. Second, a fuzzy quantification is provided by a FQ which transforms $W(P_o)$ into $|(P_o)| = Q(W(P_o))$ through the fuzzy membership function $\mu_Q : [0,1] \longrightarrow [0,1]$. For the sake of simplicity, we refer to μ_Q as Q.

To guarantee that $|(P_o)|$ fulfills the properties of zero-aggregation and positive-aggregation, the membership function Q of the FQ should assess the following properties: Q is a monotone function $x \leq y \longrightarrow Q(x) \leq Q(y)$, $Q(0) = 0$, and $\lim_{x \longrightarrow 0^+} Q(x) > 0$:

$$\begin{aligned}
\cup_0 : \quad & Q(0) = 0, \\
& P_o(\overline{s_i}) = 0, \\
& \forall \overline{s_i} \in S_{HRS} \equiv W(P_o) = 0 \longrightarrow \\
& Q(W(P_o)) = 0 \\
\cup_+ : \quad & \lim_{x \longrightarrow 0^+} Q(x) > 0, \\
& x \leq y \longrightarrow \\
& Q(x) \leq Q(y), \\
& \exists P_o(\overline{s_i}) > 0, \\
& \overline{s_i} \in S_{HRS} \equiv \sum_{\overline{s_i}}^{S_{HRS}} \overline{s_i} > 0 \longrightarrow \\
& W(P_o) > 0 \longrightarrow \\
& Q(W(P_o)) > 0
\end{aligned} \tag{10}$$

3. Case Study

In this section, we present a case study which illustrates the proposed methodology. The data of the rehabilitation sessions correspond to three patients who freely participated in the project *Monitoring of Patients with Ischemic Heart Disease within Outdoor Cardiac Rehabilitation Programs* of the Council of Health for the Andalusian Health Service in the Region of Jaén (Spain). They had a wrist-worn heart rate device (Polar M600) (https://www.polar.com/es/productos/sport/M600-GPS-smartwatch (accessed on 10/14/2018)), which collected the heart rate data during the rehabilitation sessions using a wearable application.

141 rehabilitation sessions were collected (48, 55, and 38, respectively, for the three patients) from April to August 2018. The duration of the sessions was defined and adapted to patient evolution by the cardiac rehabilitation team, varying from 30 minutes to 80 minutes. A total of 639.709 heart rate samples were collected.

3.1. Real-Time Monitoring of Heart Rate Streams. The application embedded in the wrist-worn heart rate device collected the heart rate of patients and advised patients during their outdoor rehabilitation sessions. For this purpose, the application computes fuzzy linguistic terms *adequate*, *high*, and *low* using a short FTW. Their membership functions were obtained from the previous work [22] and are described in Table 1.

The values of OHRTZ $[r_+^*, r_-^*]$ and Ventilatory Thresholds $[VT_1, VT_2]$ were adapted for each patient based on an initial controlled stress test at the Health Center.

3.2. Protoforms for Describing Heart Rate Streams. The protoforms enable the rehabilitation team to define key health indicators linguistically using expert knowledge. In Table 2, some examples of the protoforms defined by the cardiac rehabilitation team are described.

Next, the membership functions of the FTWs and FQs were straightforwardly defined by both the computer science team and cardiac rehabilitation team of the project. They have been defined by different shapes of trapezoidal membership functions, whose values are shown in Table 3.

In Figure 5, we show the computing of the protoform P_o in real rehabilitation sessions, including the degree of the term L_i. We note that the protoforms are also useful to determine the region of interests over S_{HRS} indicating the ranges where the truth degree is activated $P_o(\overline{s_i}) > 0$.

3.3. Aggregation Operation for Protoforms. In this section, we describe the results of the aggregation of the protoform in the HRS of patients, which determine a single and descriptive

Table 3: Trapezoidal membership functions for FTWs and FQ of protoforms.

Textual description in natural language	**Type**	μ_T / μ_Q
For around 25-50 minutes	T_j	$TL(\Delta t_i)[25m, 50m]$
For around 15-25 minutes	T_j	$TL(\Delta t_i)[15m, 25m]$
In the last 2 minutes	T_j	$TL(\Delta t_i)[1m, 2m]$
Around 1-3 minutes ago	T_j	$TS(\Delta t_i)[0m, 1m, 2m, 3m]$
At least half of the time	Q_k	$TR((V_r \cup T_k(\overline{s_i}))[0.25, 0.75]$
While a part of the time	Q_k	$TR((V_r \cup T_k(\overline{s_i}))[0.25, 0.5]$
Most of the time	Q_k	$TR((V_r \cup T_k(\overline{s_i}))[0.5, 1]$

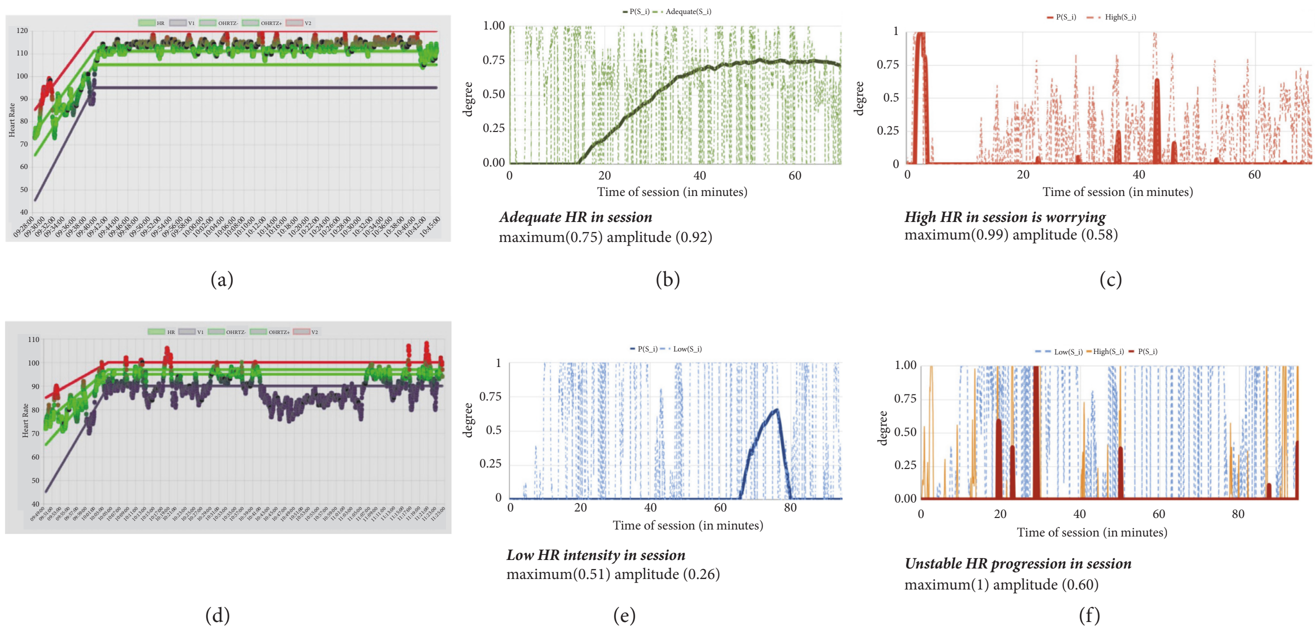


Figure 5: Example of the protoforms in real rehabilitation sessions. (a) A session with predominance of adequate adherence and some high rates. (b) The degree of the term *adequate* is represented by the dotted line; the degree of the protoform *at least half of the time the HR is adequate for around 25-50 minutes* is represented by the thick line. The aggregated degrees *maximum* and *amplitude* are shown in the bottom. (c) The degree of the term *high* is represented by the dotted line; the degree of the protoform *most of the time the HR is high for around 1-3 minutes* is represented by the thick line. The aggregated degrees *maximum* and *amplitude* are shown in the bottom. (d) A session with predominance of low adherence and some high rates. (e) The degree of the term *low* is represented by the dotted line; the degree of the protoform *most of the time the HR is low for around 15-25 minutes* is represented by the thick line. The aggregated degrees *maximum* and *amplitude* are shown in the bottom. (f) The degrees of the terms *high* and *low* are represented by the dotted line in yellow and blue, respectively; the degree of the protoform *part of the time the HR is high in the last 2 minutes* ∩ *part of the time the HR was low around 1-3 minutes ago* is represented by the thick line. The aggregated degrees *maximum* and *amplitude* are shown in the bottom.

degree from the protoform degree $P_o(\overline{s_i})$ over the heart rate stream S_{HRS}.

In this work, two semantics to aggregate the degree of the protoform are proposed: *maximum* $\max(P_o)$ and *amplitude* $|(P_o)|$, which represent the maximal truth degree of the protoform and the presence of the protoform throughout the rehabilitation session, respectively.

The *maximum* aggregation operator is not parametric. Conversely, the *amplitude* aggregation operator was modeled using expert knowledge of the computer science team as well as the cardiac rehabilitation team, who determined the membership function of the fuzzy quantifiers, μ_Q, for each protoform $|(P_o)|$, which are described in Table 4.

Table 4: Membership functions for fuzzy quantifiers of *amplitude* aggregation operator.

Textual description in natural language	μ_Q
adequate heart rate in session	$TR(W(P_o))[0, 0.5]$
the high rate is worrying	$TR(W(P_o))[0, 0.05]$
the session presents low intensity	$TR(W(P_o))[0, 0.25]$
the session has unstable rates	$TR(W(P_o))[0, 0.01]$

We note that (i) the membership functions are in the shape of $TR(x)[0, \alpha]$ to fulfill the properties of zero-aggregation and positive-aggregation described in

Table 5: Metrics in the aggregation of protoforms from rehabilitation sessions by patient (number of sessions N and percentage % from the RS $|RS|$ total in parentheses).

	Total	P_1	P_2	P_3	P_4
Patient	$\|RS\|$	N(%)	N(%)	N(%)	N(%)
1	49	2(4%)	24(48%)	49(100%)	9(18%)
2	56	36(64%)	44(78%)	48(85%)	26(46%)
3	39	36(92%)	35(90%)	4(10%)	7(18%)

Table 6: Metrics in the aggregation of protoforms from rehabilitation sessions (number of sessions and percentage from the total of RS in parentheses).

	P_1		P_2		P_3		P_4	
$\alpha - cut$	**max**	**amp**	**max**	**amp**	**max**	**amp**	**max**	**amp**
$\alpha = 0$	74(51%)	74(51%)	103(72%)	103(72%)	101(70%)	101(70%)	42(29%)	42(29%)
$\alpha = 0.5$	46(32%)	51(35%)	51(35%)	81(56%)	72(48%)	87(51%)	15(10%)	29(20%)
$\alpha = 0.9$	32(22%)	32(23%)	40(27%)	65(45%)	63(43%)	60(42%)	13(9%)	20(14%)

Section 2.2.2 and (ii) the quantification membership functions of protoforms where a high rate is involved relate short weights to relevant amplitudes, due to the fact that high rates are very significant and worrying in S_{HRS}. An example of real sessions and the aggregation operators *maximum* and *amplitude* for the proposed protoforms is presented in Figure 5.

In Table 5, we summarize the number and percentage of RS which have been recovered for each protoform $\cup(P_o) > 0$ and user. We note the descriptive summary which represents the aggregation in determining and differentiating the performance of the patients within the rehabilitation program.

We note, based on the properties described for the aggregation operators, that both *maximum* $\max(P_o)$ and *amplitude* $|(P_o)|$ recover same HRs.

Finally, as each aggregation operator determines a degree $\cup(P_o) \in [0, 1]$ which can be filtered by a threshold α using a straightforward $\alpha - cut$ in order to recover more descriptive and relevant sessions which match the protoform P_o, this intuitive value α can be also modified by experts when analyzing the summaries of the patients to filter and select the RSs. In Table 6, we present the number of RSs recovered by $\alpha - cut$ for each protoform in function of the values of $\alpha = \{0, 0.5, 0.9\}$.

4. Conclusions and Future Works

Complex IoT systems for e-health solutions allow us to reach a greater number of patients. However, this positive fact generates a vast amount of information that must be analyzed by the health team. This paper has been focused on the real-time monitoring of an outdoor cardiac rehabilitation program for patients with ischemic heart disease, which was designed and supervised remotely by the cardiac rehabilitation team. In this program, wearable wrist-worn devices with heart rate sensors integrating a high-quality protocol based on clinical guidelines were used to monitor the heart rate of patients in a personalized way. A wearable mobile-cloud platform was defined for collecting and synchronizing data between patients and the cardiac rehabilitation team to provide feedback.

The main motivation behind this work has been to provide the cardiology rehabilitation team with linguistic summaries of the rehabilitation sessions based on the heart rate streams of patients. In order to address this challenge, a methodology has been proposed in this paper, which is based on the use of the linguistic descriptions of data with protoforms and fuzzy logic of the heart rate streams of patients in order to provide linguistic summaries with rich expressiveness of interest indicators. So, the proposed methodology models short descriptions such as *the high rate is worrying in the session* or *the session presents low intensity*.

The proposed methodology enables (i) a fast analysis process to monitor a higher number of benefited patients and (ii) identification of sessions with interest indicators for the cardiac rehabilitation team to provide feedback. To do so, on the one hand, linguistic temporal terms and linguistic quantifiers have been used on linguistic aggregation operators in the heart rate streams of patients. On the other hand, flexible linguistic modeling has been defined in the proposed methodology where the cardiac rehabilitation team intuitively defines the key interest indicators using protoforms by means of expert knowledge in order to recover and dynamically select the rehabilitation sessions which suit and match the expert criteria.

In future works, the methodology will be extended to automatically generate linguistic recommendations for the patients in further sessions by means of machine learning techniques and based on the knowledge of the cardiac rehabilitation team.

Abbreviations

HR:	Heart rate
FQ:	Fuzzy quantifier
FTW:	Fuzzy temporal window
FWA:	Fuzzy Weighted Average
HRS:	Heart rate stream
OHRTZ:	Optimal Heart Rate Training Zones
RS:	Rehabilitation session
TS:	$TS(x)[l_1, l_2, l_3, l_4] = \{0, x \leq 0; (x - l_1)/(l_2 - l_1), l_1 \leq x \leq l_2; 1, l_2 \leq x \leq l_3; (l_4 - x)/(l_4 - l_3), l_3 \leq x \leq l_4; 0, l_4 \leq x\}$
TR:	$TR(x)[l_1, l_2] = \{1, x \leq l_1; (l_2 - x)/(l_2 - l_1), l_1 \leq x \leq l_2; 0, l_2 \leq x\}$
TL:	$TL(x)[l_1, l_2] = \{0, x \leq l_1; (x - l_1)/(l_2 - l_1), l_1 \leq x \leq l_2; 1, l_2 \leq x\}$.

Data Availability

The HR data from the rehabilitation sessions and the results presented in this work are available through the URL http://serezade.ujaen.es:8054/redcore-summary-data/ (accessed on 10/25/2018).

Conflicts of Interest

The authors declare no conflicts of interest.

Authors' Contributions

María Dolores Peláez-Aguilera developed the methodology and contributed materials and analysis tools; Javier Medina designed the methodology and wrote the paper; Macarena Espinilla analyzed and improved the methodology and wrote the paper; María Rosa Fernández Olmo defined the rehabilitation indicators.

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