A BELIEF LINGUISTIC RULE BASED INFERENCE METHODOLOGY FOR HANDLING DECISION MAKING PROBLEM IN QUALITATIVE NATURE*

ALBERTO CALZADA[1], J. LIU[2†], R.M. RODRIGUEZ[1] and L. MARTINEZ[1]

[1]*School of Computing, Dept. of Computer Science, University of Jaén, Jáen, Spain*

[2]*School of Computing and Mathematics, University of Ulster, Northern Ireland, UK*

This paper focuses on an inference methodology based on a belief linguistic rule base (B-LRB) which is a typical framework of a recently developed belief rule-base inference methodology (called RIMER), and highlights its distinct feature and advantage. It is called linguistic rule-base instead of fuzzy rule-base because the use of membership functions associated to the linguistic terms is unnecessary or it does not play a key role. The feature of B-LRB, the ways to generate a B-LRB, as well as the inference procedure based on B-LRB are specified along with an illustrate example at the end to show how it works and its applicability and feasibility.

1. Introduction

A new methodology has been proposed recently [1] for modeling a hybrid rule-base using a belief structure and for inference in the belief rule-based system using the evidential reasoning (ER) approach [2]. The methodology is referred to as a belief Rule-base Inference Methodology using the Evidential Reasoning approach – RIMER, where a rule-base is designed with belief degrees embedded in all possible consequents of a rule, called *belief rule-base*, is used to capture nonlinear causal relationships as well as uncertainty. The inference of a rule-based system is implemented using the ER approach. The special application framework of RIMER into fuzzy belief rule-base has been investigated and applied to safety analysis [3], along with the optimization model of RIMER [4].

In the paper, we focus on a special type of belief rule base in the framework of RIMER, i.e., a linguistic rule-base with belief structure, called *Belief Linguistic Rule Base* (B-LRB). The inference of a B-LRB is still implemented using the ER approach as in the RIMER. B-LRB based inference methodology provides an alternative method to modify and overcome limitations of traditional

---

* This work is partially supported by the research project TIN2009-08286 and P08-TIC-3548.

† The correspondence author, email: j.liu@ulster.ac.uk

fuzzy rule base approach, where the use of membership functions to the linguistic terms is unnecessary or the membership function does not play a key role, so the burden of quantifying a qualitative concept is eliminated and the systems can be simplified. Tthis is particular useful in the situation where the parameters involved have a non-probabilistic character related to imprecision of meanings or the parameters are in very qualitative nature which are difficult or infeasible to quantify them; or only very limited data are available (or even unavailable) on system parameters (e.g. at initial design stages or for a system with a high level of innovation, or consider the cost of a nuclear accident, or settlements in liability insurance, usually reached out of court and kept secret) in which only subjective expert judgments are available, hence the construction of rule-base and the evaluation mainly involves human expertise and knowledge.

The rest of this paper is organized as follows. RIMER approach is briefly reviewed in Section 2. The belief linguistic rule base framework along with its construction and its inference procedure is proposed in Section 3, following an illustration example in Section 4. Conclusions are drawn in Section 5.

2. Outline of RIMER

The *RIMER* approach is summarized and see [1] for more details. Suppose a belief rule-base is given by $R=\{R_1,\ldots, R_L\}$ with the k^{th} rule represented:

R_k: *IF U* is A^k *THEN D* with belief degrees β^k, with a rule weight θ_k and attribute weights

$$\delta_{k1},\ \ldots,\ \delta_{kT_k} \tag{1}$$

where U represents the antecedent attribute vector $(U_1,\ldots, U_{T_k})$, A^k the packet antecedents $\{A_1^k,\ldots, A_{T_k}^k\}$, and A_i^k $(i=1,\ldots, T_k)$ the referential value of the i^{th} antecedent attribute in the k^{th} rule; T_k is the number of antecedent attributes used in the k^{th} rule. Suppose T is the total number of antecedent attributes used in the rule base, D the consequent vector $(D_1,\ldots, D_N)$, and β^k the vector of the belief degrees $(\beta_{1k},\ldots, \beta_{Nk})$ for $k\in\{1,\ldots, L\}$, and β_{ik} the belief degree to which D_i is believed to be the consequent if in the k^{th} packet rule the input satisfies the packet antecedents A^k. θ_k is the relative weight of the k^{th} rule and δ_{kTk} the relative weights of the T_k antecedent attributes used in the k^{th} rule. L is the number of all the packet rules in the rule-base. If $\sum_{i=1}^{N}\beta_{ik}=1$, the k^{th} packet rule is said to be complete; otherwise, it is incomplete. Rule (1) is referred to as *a belief rule*.

Once given an input, the activation weight w_k for A^k, which measures the degree to which the k^{th} rule is weighted and activated, is calculated by:

$$w_k=\theta_k*\prod_{i=1}^{T_k}\left(\alpha_i^k\right)^{\bar{\delta}_i} / \sum_{i=1}^{L}[\theta_i*\prod_{l=1}^{T_k}\left(\alpha_l^i\right)^{\bar{\delta}_l}] \text{ with } \bar{\delta}_i=\delta_i / \max_{i=1,\ldots,T_k}\{\delta_i\} \tag{2}$$

where it is assumed that $\theta_k \in [0, 1]$ (k=1,..., L) and $\delta_i \in [0, 1]$ (i=1,..., T_k). α_i^k (i=1,..., T_k), called the *individual matching degree*, is the degree of belief to which the input for U_i belongs to A_i^k of the i^{th} individual antecedent in the k^{th} rule, and $\alpha_i^k \in [0, 1]$. α_i^k could be generated using various ways depending on the nature of an antecedent attribute which are discussed in Section 3.

Having determined the activation weight of each rule in the rule base, the ER approach [2] can be directly applied to combine the rules and generate final conclusions. The final conclusion generated by aggregating all activated rules by the actual input vector $\boldsymbol{I}$= $\{I_i; i=1, 2,..., d\}$ can be represented as follows

$$S(\boldsymbol{I})=\{(D_j,\ \beta_j), j=1,\ldots, N\} \tag{3}$$

The result in Eq. (3) reads that if the input is given by $\boldsymbol{I}$, then the consequent is D_1 to a degree β_1,..., and D_N to a degree β_N. Using this analytical ER algorithm [4], the overall combined degree of belief β_j in D_j is generated as follows:

$$\beta_j = \frac{\mu \times \left[\prod_{k=1}^{L} (w_k \beta_{j,k} + 1 - w_k \sum_{i=1}^{N} \beta_{i,k}) - \prod_{k=1}^{L} (1 - w_k \sum_{i=1}^{N} \beta_{i,k}) \right]}{1 - \mu \times \left[\prod_{k=1}^{L} (1 - w_k) \right]}, j=1,\ldots, N \tag{4}$$

where $\mu = \left[\sum_{j=1}^{N} \prod_{k=1}^{L} (w_k \beta_{j,k} + 1 - w_k \sum_{i=1}^{N} \beta_{i,k}) - (N-1) \prod_{k=1}^{L} (1 - w_k \sum_{i=1}^{N} \beta_{i,k}) \right]^{-1}$.

The final result is still a belief distribution on estimate, which gives a panoramic view about estimate for a given input.

3. Belief Linguistic Rule-based Inference Scheme

The belief linguistic rule-base (B-LRB) is defined following the same definition as in (1), however, $A^k = \{ A_1^k, \ldots, A_{T_k}^k \}$ are all linguistic terms which can be either quantitative or qualitative in nature, D is the consequent vector (D_1,..., D_N) which are all linguistic terms as well, in this particular application, are supposed to be qualitative in nature. In terms of the inference scheme, the following two necessary and crucial steps reflect the distinct features of the proposed method compared with traditional fuzzy rule-based approach.

3.1. *Determine the individual matching degree*

Qualitative parameters in a traditional fuzzy model are assessed using a subjective scale against which a range of linguistic values is mapped in domains defined by the model builder. In general, quantifying qualitative parameter may cause either the loss of information or inaccurate inference results due to the improper quantification approach. It is natural that qualitative attributes are assessed using human judgments, which are subjective in nature and are inevitably associated with uncertainties. This subjective assessment can be taken as an alternative solution due to the lack of information, e.g. when neither the membership function of each linguistic term nor numerical forms of the input is available at all, and is especially useful for qualitative attribute assessments, which sometimes is totally subjective. Hence a qualitative attribute could be directly assessed to a distribution using linguistic terms with the degrees of belief based on subjective judgments. In other words, α_{ij} can be assigned directly by the decision maker using his subjective judgments for each A_{ij}. If ε_{ij} is the degree of belief assigned to the association of A_{ij}, then $\alpha_{ij} = \varepsilon_{ij}$. In an assessment of qualitative parameter, for example, an expert may provide the following assessment: 30% sure that a parameter is at the medium level and 70% sure that it is at the high level.

Based on the above matching techniques, an input can be represented as a belief distribution, which can provide a panoramic view about the status of an attribute. The main advantage of doing so is that subjective judgments with uncertainty, whether complete or incomplete, can be consistently modeled under the unified framework without loss of their original features.

3.2. *Generating B-LRB*

Due to the qualitative nature of linguistic rule base, the causal relationship between IF part and THEN part is not possible to be obtained due to lack of complete and accurate information regarding the training set, e.g., new system. Such a situation typically arises when learning examples are generated by one or several experts, whose subjective evaluation of consequence may be tainted with imprecision and uncertainty. For each expert may not always be able to classify clearly of consequence class with full certainty. He or she may, however, be able to assess the "likelihood" that a certain phenomenon is present in the data. Hence, an approach to obtain B-LRB is to collect the opinions of several experts, and consider for each example the empirical distribution of expert opinions about its class belief. Belief distribution in Eq. (1) can be obtained in several different ways [5], including:

(1) direct elicitation from an expert, who is asked to quantify by a real number between 0 and 1 the degree of belief that IF part A_k belongs to each of the N evaluation grade $D_1,\ldots, D_N$. However, people find words more comfortable than numbers probably because the vagueness of words captures the uncertainty they feel about their probability assessment. Since, in addition, directly assessed numbers tend to be biased, various indirect elicitation methods have been developed. Here an alternative is suggested, i.e.,

(2) from an empirical distribution of expert opinions, using the relative frequency: $\beta_{ik}=u_{ik}/M$, where u_{ik} denotes the number of experts (out of M) who assigned IF part A_k (k=1,..., L) into the evaluation D_i (i=1,..., N) in THEN part. Alternatively, we could use possibilistic histograms [6] to generate a relative frequency as follows: $\beta_{ik}=u_{ik}/\Sigma_{t=1,\ldots,N}\, u_{tk},$ where $u_{ik}=(1/\mathrm{M})\ \Sigma_{j=1,\ldots,N} \min\,(u_{ik,}\, u_{jk})$.

4. An Illustrative Example

A simple example may further clarify the rationale of our approach. Consider a decision problem in evaluating consumer trustworthiness in Internet marketing [7], where trust has been shown as a predominantly important concept for establishing customer relationship on the web and is affected by many factors or attributes (e.g., about 10 factors). Practically the customers are well conversant with the natural languages about the factors effecting trust. Therefore the values of the factors become fuzzy because of its simplicity and flexibility, though vagueness is inherent. The decision on Trust effect can be described as: Not at all trustworthy, Below Moderate trustworthy, Moderate trustworthy, Somewhat trustworthy, Fully trustworthy. Suppose a trivial fuzzy rule base with the following rule to be explicitly put as hypothesis for online purchasing:

- IF the web source is somewhat reliable & advertising rate is very good & customer is fully satisfied & good services after sale & price is average & quality is very good & promptness is very good & risk is fully reliable & promises kept always & technically knowhow is somewhat THEN somewhat trustworthy (5)

Although there are still factors which could be quantified, like Price, we mainly concern about factors which are difficult to be quantified, e.g., "Reliability of Source" (How the source is reliable that affects consumer trust) and Satisfaction (How the customers already purchased using Internet are satisfied. It could be classified as the following cases: one or more factors are in qualitative nature in IF part (suppose the values in other parameters are same), what is the decision on Trust effect? Consequently, we always have to handle the causal relationship between the qualitative factors with the trust effect. Information as per the questionnaires considering all the above factors, as a sample, 50 students (customers) were personally interviewed and their responses were collected to construct the B-LRB. Using the approach in Section 3.2 about empirical distribution of expert opinions, we may have linguistic rules with belief degrees for multiple possible consequent terms based on (5) suppose the same IF part, for example,

THEN the decision on Trust effect is {(Not at all trustworthy, 0), (Below Moderate trustworthy, 0.05), (Moderate trustworthy, 0.13), (Somewhat trustworthy, 0.72), (Fully trustworthy, 0.1)}.

where THEN part is a belief distribution representation for the Trust effect, representing that we are 72% sure that the Trust effect is high, and 13% sure that the Trust effect is Moderate trustworthy, 10% sure that the Trust effect is Fully trustworthy, and 5% sure that the Trust effect is Below Moderate trustworthy The beliefs in the rule-base are used to characterize the Trust effect in a more rational and realistic way. Once the B-LRB is constructed in the similar way, the inference can be implemented following the procedure in Section 2.

5. Conclusions

This paper present and amplify *an inference methodology based on a belief linguistic rule base* (B-LRB) based on a recently developed RIMER approach. The key insight of the method is it enables the inference system more flexible to better emulate subjective human evaluation processes when numerical data is not available. B-LRB could be constructed and the inference could be implemented without necessity of any quantification transformation so that it can reduce or avoid any loss or distortion of information. Some distinct features to fit with application are highlighted and illustrated.

References

1. J.B. Yang, J. Liu J, J. Wang J, H-S. Sii, H-W. Wang, Belief rule-base inference methodology using the evidential reasoning approach - RIMER, *Transactions on Systems, Man, and Cybernetics, Part A*. **36(2)**: 266-285 (2006).
2. J.B. Yang and D.L. Xu, On the evidential reasoning algorithm for multiple attribute decision analysis under uncertainty, *Transactions on Systems, Man, and Cybernetics, Part A*. **32(3)**: 289-304 (2002).
3. J. Liu, J.B. Yang, J. Wang, H.S. Sii and Y.M. Wang, Fuzzy rule-based evidential reasoning approach for safety analysis, *I. J. of General Systems*, **33(2-3)**, 183-204 (2004).
4. J.B. Yang, J. Liu, D.L. Xu, J. Wang, H.W. Wang, Optimization models for training belief-rule-based systems, *IEEE Transactions on Systems, Man, and Cybernetics-Part A*, **37(4)**: 569-585 (2007).
5. S. Renooi, Probability elicitation for belief networks: issues to consider, *The Knowledge Engineering Review*. **16(3)**: 255-269 (2001).
6. D. Dubois, H. Prade, Unfair coins and necessity measures: towards a possibilistic interpretation of histograms, *Fuzzy Sets and Systems*. **10(1)**: 15-20 (1983).
7. C. Chakraborty and D. Chakraborty, Fuzzy rule base for consumer trustworthiness in Internet marketing: An interactive fuzzy rule classification approach, *Intelligent Data Analysis*. **11**: 339-353 (2007).