Type-1 OWA Unbalanced Fuzzy Linguistic Aggregation Methodology: Application to Eurobonds Credit Risk Evaluation

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In decision making, a widely used methodology to manage unbalanced fuzzy linguistic information is the linguistic hierarchy (LH), which relies on a linguistic symbolic computational model based on ordinal 2-tuple linguistic representation. However, the ordinal 2-tuple linguistic approach does not exploit all advantages of Zadeh's fuzzy linguistic approach to model uncertainty because the membership function shapes are ignored. Furthermore, the LH methodology is an indirect approach that relies on the uniform distribution of symmetric linguistic assessments. These drawbacks are overcome by applying a fuzzy methodology based on the implementation of the type-1 ordered weighted average (T1OWA) operator. The T1OWA operator is not a symbolic operator and it allows to directly aggregate membership functions, which in practice means that the T1OWA methodology is suitable for both balanced and unbalanced linguistic contexts and with heterogeneous membership functions. Furthermore, the final output of the T1OWA methodology is always fuzzy and defined in the same domain of the original unbalanced fuzzy linguistic labels, which facilitates its interpretation via a visual joint representation. A case study is presented where the T1OWA operator methodology is used to assess the creditworthiness of European bonds based on real credit risk ratings of individual Eurozone member states modeled as unbalanced fuzzy linguistic labels.

1. INTRODUCTION

In most decision-making processes, there exists uncertainty concerning the suitability of each one of the alternatives to choose from. Mathematically, uncertainty has been tackled using precise numeric assessment values or using

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linguistic assessment values in both its representation and measurement. The second approach, though, happens when experts' sensations and feelings pervade the decision-making problem.

The fuzzy linguistic methodology, introduced by Zadeh in his seminal paper,[1] has proved to be useful in providing a mathematical structured framework to deal with decision-making problems with vagueness and imprecise pervading the information available, that is, when precise numeric assessments are not available but linguistic assessments are instead. For these type of decision-making problems, traditionally categorized as unstructured, can indeed be applied an structured methodology based on the implementation of Zadeh's concept of linguistic variable and its semantics to describe the meaning of each one of the elements of the considered universe of discourse, which is done using fuzzy sets membership functions. An important aspect to be taken into consideration within a linguistic methodology is the cardinality of the corresponding linguistic term set,[2] as the higher the cardinality is, the higher the uncertainty discrimination among the elements of the universe of discourse is achieved.

It is a common practice in decision-making problems with linguistic assessments to assume linguistic term sets (LTS) with uniform distribution of symmetric linguistic assessment on the discourse domain. Clearly this approach may be appropriate to problems where the distinction of uncertainty is proportional and equal among the set of linguistic terms, but not where this may not be the case. A typical example of this latter case is given in Ref. 3 for describing the UK educational grading system (see Figure 1). Clearly, the right-hand side of the scale has more terms than the left-hand side and consequently a triangular fuzzy set representation of the semantic of each assessment can only be captured with nonuniform distribution of nonsymmetric linguistic labels, which in the literature has been named as unbalanced linguistic representation of information.[3–5]

A methodology proposed in the literature and widely used to address decision-making problems with unbalanced linguistic information is the linguistic hierarchy (LH) methodology introduced in Ref. 6 and later applied to improve the precision in processes of computing with words in multigranular linguistic contexts in

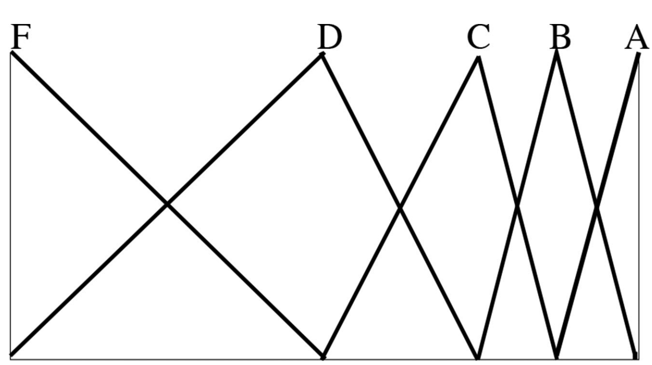


Figure 1. Semantic representation of the UK educational grading system.

Refs. 7–11. The aggregation of unbalanced linguistic information using the LH methodology was presented in Ref. 3. In summary, the LH methodology consists of building a representation structure with several levels, each one representing a different granularity set of uniform and symmetric linguistic terms that keeps the precedent level modal points in order to achieve a smooth transition between successive levels. Transformation functions are introduced to map linguistic labels of a level to linguistic labels at a different level without loss of information. Doing this, the unbalanced linguistic labels are mapped with its appropriate symmetric linguistic labels within the structure, and are transformed to a common domain with maximum granularity, which ultimately are aggregated using the 2-tuple linguistic computational model. Thus, the LH methodology deals with unbalanced linguistic information using an indirect approach via the already common and known uniform distribution of symmetric linguistic assessment on the universe of discourse.

The LH methodology relies on a linguistic symbolic computational model based on ordinal scales and indexes, the 2-tuple linguistic representation, and therefore it does not exploit the advantages of Zadeh's fuzzy linguistic approach. To avoid this issue, an alternative approach to process unbalanced linguistic information is possible by using the type-1 ordered weighted average (T1OWA) operator,[12] which was developed applying Zadeh's extension principle to Yager's OWA operator.[13] The T1OWA operator is not a symbolic operator; it allows to directly aggregate the whole linguistic terms because its computation involves the whole membership functions of the fuzzy sets used to appropriately represent the meaning of the linguistic terms, which in practice means that it can be suitable for both balanced and unbalanced linguistic contexts and with heterogeneous types of membership shapes (triangular, trapezoidal, Gaussian, etc.). As a consequence, the output of the T1OWA operator is of the same type than the linguistic terms, that is, a fuzzy set on the same universe of discourse. The T1OWA operator has been successfully applied to aggregate fuzzy linguistic information with fuzzy linguistic weights[14,15] and to address consensus reaching processes in multigranular fuzzy linguistic contexts.[16,17] Thus, the T1OWA operator is most appropriate to be implemented in decision-making problems where uncertainty is linked to fuzzy set theory rather than probability theory,[18–20] and in particular to contexts with unbalanced fuzzy linguistic information. This is the focus of the present paper, which aims to present a T1OWA-based methodology to deal with decision-making problems with unbalanced fuzzy linguistic information by using as example the study of credit risk on a potential Eurobonds rating based on real credit risks of Eurozone member states as opposed to previous effort based on mock examples.[14]

Credit risk evaluation of corporations or the debt issuance of a state or government is usually carried out by rating agencies, with the three big ones being Standard & Poor's (S&P), Moody's, and Fitch Group. Each agency utilizes its own methodology and criteria to measure the creditworthiness of corporations evaluated and its own scale based on a combination of letters, numbers, and/or positive and negative signs to assess the overall credit risk level of the corporation or state. Rating agencies rely on economic experts, mathematical models, or a combination of both to arrive at their final credit risk assessment. Financial information is obtained from both public and private institutions as well as from experts with great knowledge and experience. Although rating agencies work with information that is quite precise, it is obvious that there also exit economic factors outside their control that generate uncertainty regarding their recommendations and evaluations. The uncertainty that arises when experts analyze all the available economic information may make more difficult the precise assessment of credit risk. Indeed, credit risk assessments tend to include intuitions and feelings of experts that emanate from the mentioned uncertainty. Thus, there are well-founded grounds to support the use of fuzzy linguistic approaches in this context. The information can be associated with unbalanced linguistic labels whose meanings can be represented using fuzzy set membership functions.

The structure of the rest of the paper is as follows. Section 2 reviews succinctly the basic concept of a linguistic variable, its semantics, as well as the 2-tuple LH methodology. Section 3 presents a new fuzzy alternative to manage unbalanced fuzzy linguistic information based on the T1OWA operator. A detailed description of its expression for aggregating fuzzy sets is given in Section 3.2, while Section 3.3 presents an example of aggregation of unbalanced linguistic labels using the T1OWA operator and applied to assess the creditworthiness of European bonds based on real credit risk ratings of individual Eurozone member states. The paper concludes with Section 4 where conclusions are drawn.

2. UNBALANCED LINGUISTIC LABELS: THE INDIRECT ORDINAL 2-TUPLE LH APPROACH

In his seminal paper published in 1996,[21] Zadeh explicitly stated that the rationale for computing with words (CWW) might be supported by a necessity when numbers are not able to be used to quantify the imprecision of the information available, or by a tolerance of imprecision that allows for words instead of numbers, which might be costly to get. Later in Ref. 22 an additional rationale was added when words are simply used to summarize numerical information.

In CWW, the words are modeled into well-defined mathematical objects, which in turn are manipulated with sound mathematical computational tools. Indeed, words in CWW are considered labels of fuzzy sets with specified membership functions, which are computationally manipulated using fuzzy arithmetics, that is traditional mathematical arithmetics transformed via the extension principle.

Linguistic variables are employed extensively in applications of fuzzy logic, and they are formally represented as a 5-tuple $\langle L, T(L), U, S, M\rangle$[1], where (i) L is the name of the variable, (ii) $T(L)$ is a finite term set of (primary) labels or words (a collection of linguistic values), (iii) U is a universe of discourse or base variable, (iv) S is the syntactic rule that generates the terms in $T(L)$, and (v) M is a semantic rule that associates with each linguistic value X its meaning $M(X) : U \to [0, 1]$. Usually, $T(L)$ is denoted as L when there is no risk of confusion. A "compatibility function"[1] or semantic rule associates with each element of the base variable its compatibility with each linguistic value. This interpretation of the meaning of a linguistic label coincides with that of a fuzzy set, and as mentioned above linguistic labels are formally represented as fuzzy subsets of their base variable.

A very popular approach to represent and aggregate linguistic information is using a linguistic symbolic computational model based on indexes,[23–25] which is based on an ordinal interpretation of the linguistic label meaning. In Ref. 26, a more general symbolic approach was introduced: the 2–tuple linguistic model, which up to now has been used as the LH methodology computational model for unbalanced linguistic information. Sections 3.2 and 3.3 will present a fuzzy computational approach to unbalanced linguistic information based on the T1OWA operator.

2.1. Ordinal Linguistic Representation Using the 2-Tuple Linguistic Model

This linguistic model adds the concept of symbolic translation to the symbolic representation model based on indexes, which is used to represent the output of symbolic aggregation operators by means of a pair of values called linguistic 2–tuple: (s_i, α_i), with s_i being one of the original linguistic terms (i.e., $s_i \in S = \{s_0, ..., s_g\}$) and $\alpha_i \in [-.5, .5)$ is the symbolic translation. The aim of this representation structure is to achieve that the symbolic aggregation output is identical to the one obtained using the symbolic representation model based on indexes while at the same time preventing loss of information by making use of information previously discarded by such symbolic representation model.

Formally, let $\beta \in [0, g]$ be the result of a symbolic aggregation of the indexes of a set of labels in an LTS $S = \{s_0, ..., s_g\}$, and $i = round(\beta) \in \{0, \ldots, g\}$. The value $\alpha_i = \beta - i \in [-0.5, 0.5)$ is called a *symbolic translation*, and the pair of values (s_i, α_i) is called the *2–tuple linguistic representation model.* Thus, the following isomorphism can be established between the *2-tuple set* associated with S, $\langle S \rangle = S \times [-0.5, 0.5)$, and the closed interval $[0, g]$:

$$\Delta(\beta) = (s_i, \alpha), \quad \text{with} \quad \begin{cases} i = round(\beta), \\ \alpha = \beta - i. \end{cases}$$

The inverse function is $\Delta^{-1}(s_i, \alpha) = i + \alpha$, and the corresponding symbolic computational model was presented in Ref. 27.

2.2. The Ordinal 2-Tuple Linguistic Hierarchy

An LH may be seen as a hierarchy structure of different levels of LTS with different granularity, which are denoted as *l(t,n(t))* with *t* representing the LH level and *n(t)* the granularity of the linguistic term set at that level. Assumptions are that the cardinality of all LTS is odd, and graphically are represented using symmetrical and uniform distributed triangular membership functions on the domain [0,1] as Figure 2 shows. Both the process to build an LH and its computational model are explained below.

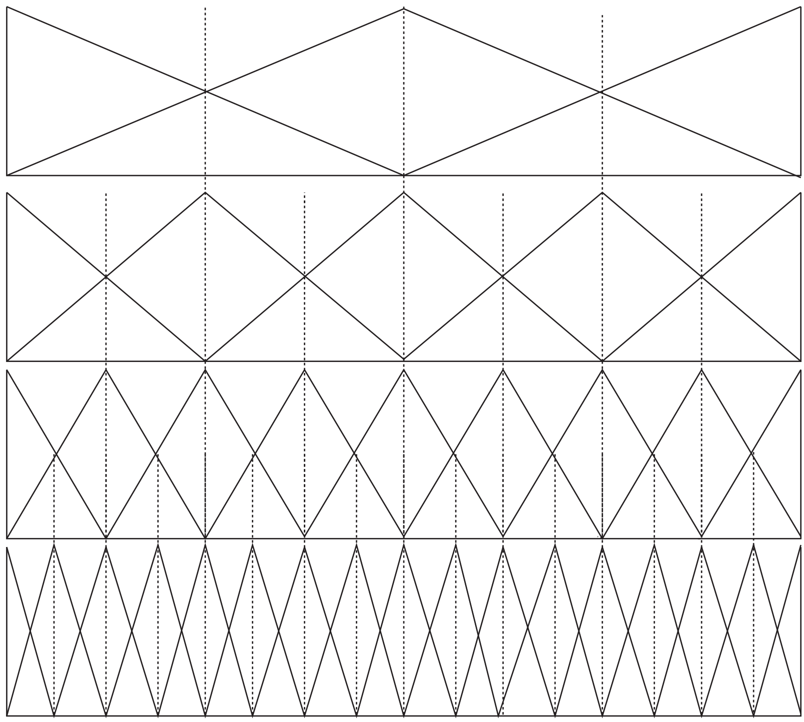


Figure 2. LH with four levels of granularity 3, 5, 9, and 17, respectively.

2.2.1. Building Linguistic Hierarchies

The granularity of each linguistic term set is increased from one level (t) to the next ($t+1$) using the following expression[7] and as illustrated in Figure 2:

$$l(t, n(t)) \rightarrow l(t+1, 2 \cdot n(t) - 1).$$

An issue associated with this approach is that the granularity of levels increases very rapidly, which has been partially resolved applying the least common multiple approach to all granularities of the LH as it was proposed in Refs. 28,29.

2.2.2. Computational Model

The LH computational model is based on the following symbolic 2-tuple transformation,[7]

$$TF_{t'}^{t} : l(t, n(t)) \longrightarrow l(t', n(t'))$$

$$TF_{t'}^{t}(s_i^{n(t)}, \alpha^{n(t)}) = \Delta\left(\frac{\Delta^{-1}(s_i^{n(t)}, \alpha^{n(t)}) \cdot (n(t') - 1)}{n(t) - 1}\right).$$

The aim of such transformation function is that linguistic terms, independently of its shape and semantic, can be mapped to a unique expression domain, and consequently be amenable to be manipulated with the 2-tuples computational model. Obviously, this approach disregard the membership functions and so linguistic labels are modeled via their corresponding symbolic ordinal 2-tuple representations and not treated as fuzzy sets.

Unbalanced linguistic information is managed within the LH methodology and 2-tuple computational model by first dividing the unbalanced linguistic term set into three term subsets, the one containing all labels below the central one (left lateral set), the one containing all the labels above the central one (right lateral set) and the one containing the central label (central set). In a second step, the granularities of the left lateral set and the right lateral set are compared against the (half) granularity value for each LH level so that the closest symmetrical and uniform distributed LH labels is found to represent the unbalanced linguistic information. After this mapping has been completed, the symbolic aggregation based on the 2-tuple computational model is applied to process the information, with its output being finally retranslated into the original unbalanced linguistic term set.

3. UNBALANCED FUZZY LINGUISTIC LABELS: THE DIRECT T1OWA APPROACH

In this section, a fuzzy approach to manage unbalanced fuzzy linguistic information will be presented based on the use of the T1OWA operator. The advantages of using this route are as follows: (i) it is fuzzy and not ordinal because the membership function characterizing the fuzzy linguistic labels is fully used in the computation process; (ii) the shape of the membership function is not restricted to be triangular type but could be of any type; (iii) there is no need to translate and retranslate unbalanced information using an indirect balanced framework, that is, it is a direct cardinal approach to dealing with unbalanced information compared to the indirect ordinal 2-tuple LH methodology; (iv) the final output will be a fuzzy set on the same domain than the original unbalanced fuzzy linguistic labels and it can be interpreted easily when compared with them graphically. If necessary, a defuzzification process could be applied, for example by computing the centroid of the solution fuzzy set or using the 2-tuple representation model. Anyway, it is proved that an equivalent set of values to the corresponding 2-tuple representation approach is obtained.

3.1. Fuzzy Linguistic Representation Model

The representation of linguistic information using fuzzy numbers, that is, convex normal fuzzy subsets of the real line, is commonly referred to as the cardinal representation in contrast to the ordinal representation covered above. In this framework, a linguistic label is characterized by a membership function on the unit interval [0,1] that maps each value in [0,1] to a degree of performance representing its compatibility with the linguistic assessment,[1] examples of which are shown in Figure 1.

It is not difficult to see that there exists a one-to-one mapping between the ordinal approach based on the 2-tuple representation of a term set of linguistic labels and the set of centroid elements of the fuzzy numbers used in the cardinal representation of the same term set of linguistic labels.[30] Indeed, denoting the centroid of the linguistic term $s_h \in S$ by $\boldsymbol{v}(s_h)$, the semantic of the linguistic labels underlies a ranking relation that implies $\boldsymbol{v}(l_h) > \boldsymbol{v}(l_k)$ when $h > k$. Without loss of generality, it can be assumed that $\boldsymbol{v}(s_0) = 0$ and $\boldsymbol{v}(s_g) = 1$, otherwise the centroids are replaced by the values $\frac{\boldsymbol{v}(s_h) - \boldsymbol{v}(s_0)}{\boldsymbol{v}(s_g) - \boldsymbol{v}(s_0)}$. Denoting the symbolic 2-tuple representation of s_h by $a_h = \Delta^{-1}((s_h, 0))$, the mapping

$$\delta\left(a_h\right) = \boldsymbol{v}(l_h), \tag{1}$$

is the restriction of a continuous and strictly increasing function $\delta : [0, s] \longrightarrow [0, 1]$ such that $\delta(0) = 0$ and $\delta(s) = 1$, that is, a bijective function δ exists and it can be used to derive the ordinal 2-tuple representation of a linguistic term set from the set of centroids of the fuzzy numbers used in a cardinal representation of the same linguistic term set. Obviously, it is not possible to derive a cardinal representation of a linguistic term set from an ordinal 2-tuple representation model. Furthermore, the type of membership function used in the cardinal representation model is not restricted to triangular type but could be trapezoidal or Gaussian type, that is, it could be of any type as long as it is convex and normal verifying that $\boldsymbol{v}(l_h) > \boldsymbol{v}(l_k)$ when $h > k$. Thus, the cardinal fuzzy approach to linguistic information representation is general, flexible, and appropriate to capture uncertainty, which is not the case with the ordinal approach.

3.2. The T1OWA Operator

In contrast to Yager's OWA operator[13] that is able to aggregate crisp numbers with crisp weights, the T1OWA operator was introduced in Ref. 12 to directly aggregate fuzzy sets with uncertainty weights. Thus, given a set $\{A^1, \ldots, A^n\}$ of type-1 fuzzy sets on $\mathbb{R}$ that are to be aggregated using the set of type-1 fuzzy weights sets defined on the domain of discourse [0,1], $\{W^1, \ldots, W^n\}$, the T1OWA operator output is a fuzzy set Y:

$$\Phi(A^1, \ldots, A^n) = Y$$

with membership function

$$\mu_Y(y) = \sup_{\substack{\sum_{k=1}^{n} \bar{w}_i a_{\sigma(i)} = y \\ w_i \in U, a_i \in X,}} \left(\mu_{W^1}(w_1) \wedge \cdots \wedge \mu_{W^n}(w_n) \wedge \mu_{A^1}(a_1) \wedge \cdots \wedge \mu_{A^n}(a_n)\right) \tag{2}$$

where $\bar{w}_i = \dfrac{w_i}{\sum_{i=1}^{n} w_i}$; σ is a permutation function such that $a_{\sigma(i)} \geq a_{\sigma(i+1)}, \ \forall i = 1, \cdots, n-1$.

Expression (2) has been proved to be too expensive from a computational point of view, which inevitably implied that its practical application in real world decision-making problems was curtailed. This issue, however, was overcome with the development of a fast approach to T1OWA operations based on the horizontal representation of a fuzzy sets via their corresponding family of crisp α-level sets, in what it is known as the representation theorem of fuzzy sets.[1]

For each $\alpha \in [0, 1]$, the α-level T1OWA operator to aggregates the α-level sets $\left\{A_\alpha^1, \cdots, A_\alpha^n\right\}$ with α-level weight sets $\left\{W_\alpha^1, \ldots, W_\alpha^n\right\}$ is given as:

$$\Phi_\alpha\left(A_\alpha^1, \cdots, A_\alpha^n\right) = \left\{ \frac{\sum_{i=1}^{n} w_i a_{\sigma(i)}}{\sum_{i=1}^{n} w_i} \middle| w_i \in W_\alpha^i,\ a_i \in A_\alpha^i, \forall i \right\}, \tag{3}$$

where $W_\alpha^i = \{w | \mu_{W_i}(w) \geq \alpha\}$, $A_\alpha^i = \{x | \mu_{A_i}(x) \geq \alpha\}$, and σ is a permutation function such that $a_{\sigma(i)} \geq a_{\sigma(i+1)},\ \forall\, i = 1, \cdots, n-1$.

According to the representation theorem of type-1 fuzzy sets, the following type-1, fuzzy set on $\mathbb{R}$ can be constructed:

$$G = \bigcup_{0<\alpha\leq 1} \alpha \Phi_\alpha\left(A_\alpha^1, \cdots, A_\alpha^n\right) \tag{4}$$

with membership function

$$\mu_G(x) = \bigvee_{\alpha: x \in \Phi_\alpha\left(A_\alpha^1, \cdots, A_\alpha^n\right)_\alpha} \alpha \tag{5}$$

Fuzzy sets Y and G, which apparently seem to be different, were proved in Ref. 14 to have the same membership functions and consequently are equal. This fundamental result is known as the *Representation Theorem of Type-1 OWA Operators*. Furthermore, this α-level approach was proved to be much faster than (2),[14] which implies that T1OWA aggregation are possible to be carried out in real-time decision-making problems. In particular, when the linguistic weights and the aggregated sets are fuzzy number, the α-level T1OWA operator produces closed intervals[14] and the computation of the T1OWA operator output according to (4), G, reduces to compute the left end-points and right end-points of closed intervals, which was solved in Ref. 14.[a]

3.3. Eurobonds Credit Risk Rating: Unbalanced Fuzzy Modeling and Aggregation Based on the T1OWA Operator

In this section, we present an example to evaluate credit risk of a potential issuance of European bonds. The aim of this example is not to carry out a rigorous

[a] An R package of the T1OWA is available at http://www.tech.dmu.ac.uk/~chiclana/type1owaR/.

study about the Eurobonds creditworthiness but to show as the combination of the unbalanced fuzzy linguistic approach with the T1OWA functionality can be successfully applied to model the uncertainty of real problems.

During the hardest and most difficult years of the present economic crisis, that is, 2010 and 2011, the European Commission and the European Central Bank considered the possibility of financing the public debt of Eurozone member states with a centralized common issuance of sovereign bonds among the Member States of the euro area, which are known as Eurobonds. As such, in November 2011 the European Commission published the *European Commission Green Paper on the Feasibility on Introducing Stability Bounds* to stimulate a political debate on the joint issuance of debt in the euro area to tackle the debt crisis, reduce the pressure on the debt issuances of some of the Eurozone member states, and enhance economic stability. Of the three approaches to the joint issuance of debt in the euro area, one considered that each Eurozone member state would cover part of their individual financing needs with national debt and the rest with Eurobonds. In this way, countries should tap financial markets on their own and consequently it could be possible to compare the credit rating of each member state with respect to the Eurobonds credit rating. The arrangement on how to guarantee Eurobonds between the state members was not made explicit on the green paper, although it seems reasonable to assume that each of the member states would be responsible for an amount proportional to their corresponding economy size within the global economic context. In any case, the proper assessment of the Eurobonds credit risk would be a crucial aspect to guarantee its success.

Potential investors considering their participation in debt issuances take into account the main credit rating information provided by agencies, such as Standard & Poor's, Moody's, or Fitch Group, that evaluate the capacity to meet financial obligations in full and on time of the corporations, states, or governments that issue the debt. Each agency utilizes its own methodology and criteria to evaluate the creditworthiness of corporations and produces a specific quality ranking, with a combination of letters and positive and negative symbols to represent the agency's final rating evaluation. In this paper, a case study is presented where the T1OWA operator is used to assess the creditworthiness of European bonds based on current and real credit risk ratings of individual Eurozone member states modeled as unbalanced fuzzy linguistic labels. To do this, S&P ratings,[b] its major rating descriptors shown in Table I, are used.

Some of the ratings in Table I are followed by a plus (+) or minus (−) sign to show relative standing within the major rating categories, which provides a finer discrimination or granularity. This is fully illustrated in Figure 3, where a mapping of the three big rating agencies rating scores is provided, which clearly illustrates their similarities.

S&P agency states that their opinions and assessments regarding the credit quality of a corporation are not to be interpreted as exact measurements of the chances that a particular debt issue will default, but as a relative estimation of

[b] Available information at http://www.standardandpoors.com/en_US/web/guest/ratings/ratings-criteria.

Table I. S&P major rating score descriptions.

Rating score	Semantic meaning
AAA	Extremely strong capacity to meet financial commitments
AA	Very strong capacity to meet financial commitments
A	Strong capacity to meet financial commitments, but somewhat susceptible to adverse economic conditions and changes in circumstances
BBB	Adequate capacity to meet financial commitments, but more subject to adverse economic conditions
BB	Less vulnerable in the near-term but faces major ongoing uncertainties to adverse business, financial and economic conditions
B	More vulnerable to adverse business, financial, and economic conditions but currently has the capacity to meet financial commitments
CCC	Currently vulnerable and dependent on favorable business, financial, and economic conditions to meet financial commitments
CC	Currently highly vulnerable
C	Currently highly vulnerable obligations and other defined circumstances
D	Payment default on financial commitments

the creditworthiness of a debt issuer within a dynamic risk context. Consequently, uncertainty due to the dynamic nature of risk is unavoidable, and the meaning of rating scores could be modeled appropriately using fuzzy sets with unsharp boundaries overlapping contiguous scores, that is, the fuzzy linguistic approach methodology is suitable to model agencies' ratings within this complex economic context.

The first issue to address when modeling S&P credit risk ratings using the fuzzy linguistic approach is to set the base variable domain. As mentioned before, this is normally set as being the unit interval [0,1], and as there is no evidence to suggest the contrary, it is adopted in what follows. In the document *Guide to credit rating essentials*,[31] S&P ratings are firstly divided in two main groups: (i) *Investment Grade* (IG) comprising ratings between AAA and BBB representing "relatively high levels of creditworthiness and credit quality," and (ii) *Speculative Grade* (SG) comprising ratings from BB to D to reflect "debt securities where the issuer currently has the ability to repay but faces significant uncertainties." Accordingly, the domain [0,1] is also first divided in two equal parts, with [0,0.5] for the SG term set, $SG = \{D, C, CC, CCC-, CCC, CCC+, B-, B, B+, BB-, BB, BB+\}$ and [0.5,1] for the IG term set, $IG = \{BBB - BBB, BBB+, A-, A, A+, AA-, AA, AA+, AAA\}$.

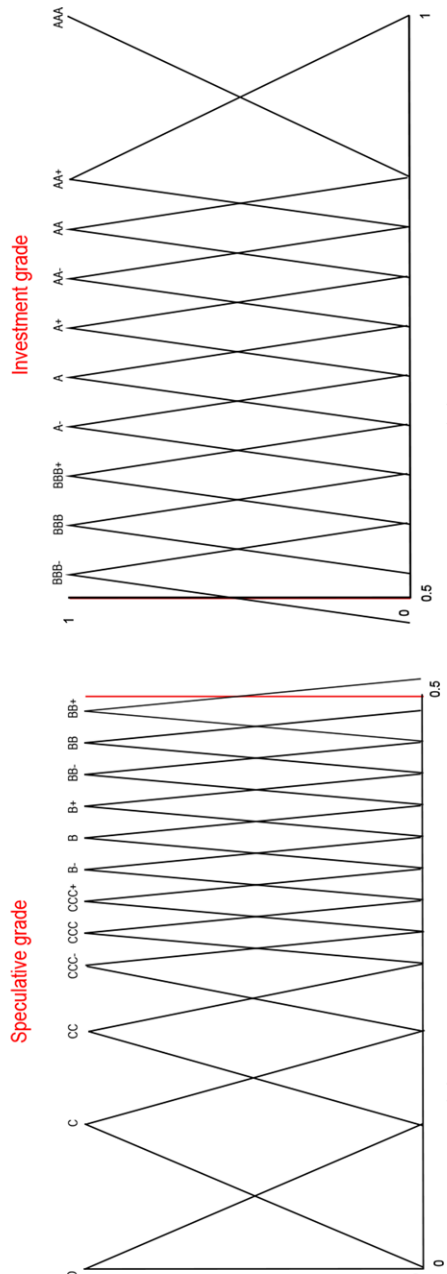
The second issue to address in the fuzzy linguistic methodology is whether to implement a balanced or unbalanced distribution of the labels. In the case study of our interest, we observe that the complete set of credit ratings $LTS = SG \cup IG$ consists of two subsets of different cardinality, and consequently their complete distribution within the domain [0,1] cannot be balanced or and/or symmetrical as they also have an even cardinality, and no mid-term label would exist. Additionally, as Figure 3 clearly illustrates, some of major categories described in Table I have different granularities. This asymmetric distribution of credit ratings is clearly appropriate to be modeled via unbalanced LTS.

Moody's		S&P		Fitch		
Long Term	Short Term	Long Term	Short Term	Long Term	Short Term	
Aaa	P-1	AAA	A-1+	AAA	A1+	Prime
Aa1		AA+		AA+		High grade
Aa2		AA		AA		
Aa3		AA-		AA-		
A1		A+	A-1	A+	A1	Upper medium grade
A2		A		A		
A3	P-2	A-	A-2	A-	A2	
Baa1		BBB+		BBB+		Lower medium grade
Baa2	P-3	BBB	A-3	BBB	A3	
Baa3		BBB-		BBB-		
Ba1	Not Prime	BB+	B	BB+	B	Non Investment grade speculative
Ba2		BB		BB		
Ba3		BB-		BB-		
B1		B+		B+		Highly Speculative
B2		B		B		
B3		B-		B-		
Caa1		CCC+	C	CCC	C	Substantial risks
Caa2		CCC				Extemely speculative
Caa3		CCC-				In default with little prospect for recovery
Ca		CC				In default with little prospect for recovery
/		D	/	DDD	/	In default
/				DD		
/				D		

Figure 3. Rating agencies' rating scores.

Using triangular membership functions to characterize fuzzy linguistic terms, Figure 4 depicts possible distributions of credit ratings for SG and IG categories, while Figure 5 depicts the complete unbalanced distribution of the complete set of credit ratings LTS. Clearly, different unbalanced representations are possible both in using different membership function types and different distributions of the credit ratings within their respective underlying domains.

A final issue to address in the proposed framework involves the determination of relative weights of all individual Eurozone member state sovereign debts that appropriately reflect their contribution in the aggregation process to derive the overall evaluation of creditworthiness and credit quality of European bonds issuance.



(a) Speculative grade

(b) Investment grade

Figure 4. Linguistic term sets grouped by main categories.

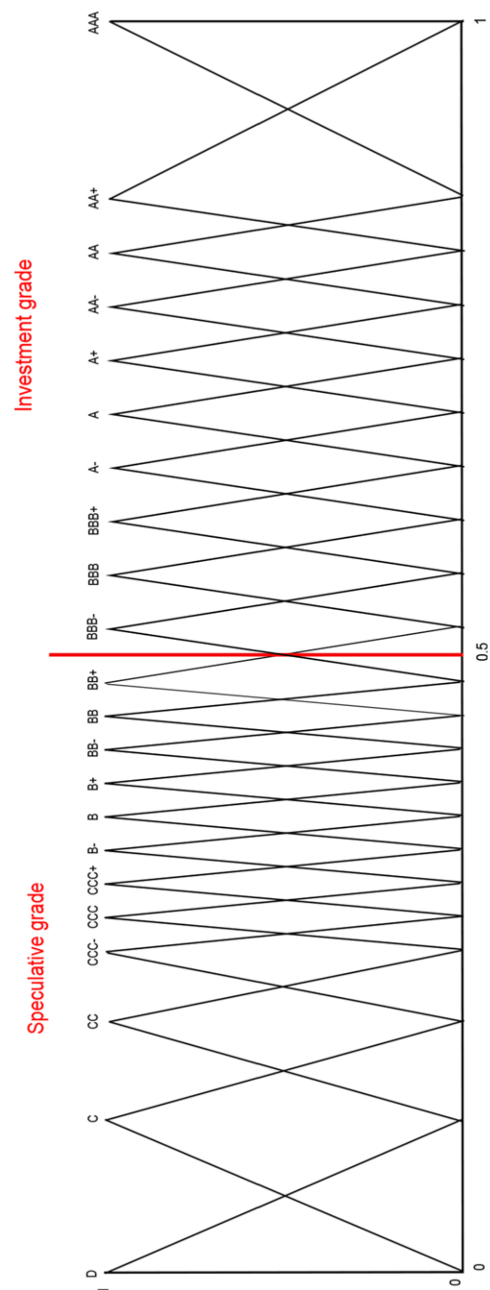


Figure 5. Linguistic term sets (LTS).

Table II. Credit rating and economy relative size for EU-28 counties.

State	S&P rating	GDP (million euros)	Relative weight (%)
Austria	AA+	322,594.6	2.39
Belgium	AA	395,262.0	2.92
Bulgaria	BB+	41,047.9	0.30
Croatia	BB	43,561.5	0.32
Cyprus	B+	18,118.9	0.13
Czech Republic	AA	157,284.8	1.16
Denmark	AAA	252,938.9	1.87
Estonia	AA	18,738.8	0.14
Finland	AA+	201,995.0	1.49
France	AA	2,113,687.0	15.63
Germany	AAA	2,809,480.0	20.78
Greece	B	182,438.3	1.35
Hungary	BB+	100,536.5	0.74
Ireland	A	174,791.3	1.29
Italy	BBB	1,609,462.2	11.90
Latvia	A	23,265.0	0.17
Lithuania	A	34,955.6	0.26
Luxembourg	AAA	45,288.1	0.33
Malta	BBB+	7,571.4	0.06
The Netherlands	AA+	642,851.0	4.75
Poland	A	395,962.4	2.93
Portugal	BB	171,211.1	1.27
Romania	BBB	144,282.2	1.07
Slovakia	A	73,593.2	0.54
Slovenia	A	36,144.0	0.27
Spain	BBB	1,049,181.0	7.76
Sweden	AAA	436,342.4	3.23
United Kingdom	AAA	2,017,193.8	14.92

It would not be correct to assign all sovereign debt issuances the same weight, mainly because individual member states contribution to European economy is not equal but related to their size in terms of gross domestic product at market prices (GDP) among others criteria. Thus, the normalized GDP value will reflect individual member states credit rating contribution to the overall European credit rating. Thus, the higher the GDP, the higher the weight in the aggregation process. Table II provides the relevant economic information, taken from Eurostat and S&P for the year 2013, of each one of the current 28 European Union countries.

The T1OWA operator result is shown in Figure 6 as a red dashed line in relation to the original set of 22 credit rating labels. Thus, the application of the T1OWA operator returns an output of the same type and with the same domain of the original information, which facilitates the decision making. In this case study, it is clearly that the T1OWA output overlaps in its majority with credit rating AA and also in part with credit rating AA-. In summary, the credit risk quality of potential European bonds in 2013 is within the general class IG; in particular, it is closest to AA and consequently it could had been considered as quite good and positive.

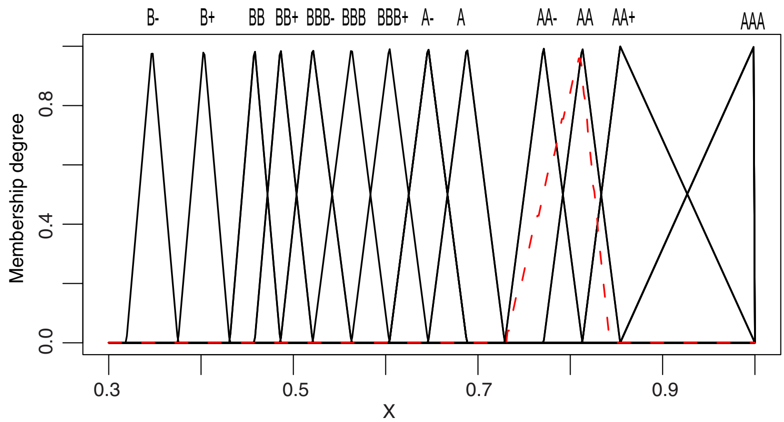


Figure 6. Aggregation result.

4. CONCLUSIONS

A fuzzy approach to directly fuse unbalanced linguistic information based on the T1OWA operator has been presented. In comparison to the existing approach to unbalanced linguistic information, the ordinal 2-tuple LH methodology, it is worth noting the following: (i) it allows for a soft interpretation of the linguistic information, implements, and makes use of the whole membership functions characterizing the linguistic label as fuzzy sets; (ii) the shape of the membership function is not restricted to be triangular type; (iii) there is no need to translate and retranslate unbalanced information as the 2-tuple LH methodology does; (iv) the final output is always fuzzy and defined in the same domain than the original unbalanced fuzzy linguistic labels, which facilitates its interpretation via their visual joint representation; (v) defuzzification could be applied if necessary, and indeed this process will always derive in an equivalent result to the 2-tuple LH methodology. The application of the T1OWA unbalanced fuzzy linguistic methodology has been illustrated in the evaluation of the creditworthiness and credit risk quality of a potential issuance of bonds at European Community level, which were the focus of many discussion within the EU during the hardest and most difficult years of the present economic crisis and that were known as Eurobonds. In the future, the T1OWA approach here presented will be compared with alternative linguistic tools that could be useful to manage unbalanced linguistic information, an example of which might derive from the work presented in Ref. 32.

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