

The use of Linguistic Information in Operational Research

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Abstract— The use of linguistic information has been applied successfully to many areas. In the literature we can find three different linguistic computational models for representing linguistic information and defining linguistic aggregation operators. In this contribution we shall review the use of the linguistic information in different areas of the Operational Research as decision-making, scheduling, management, etc. And finally, we shall make a comparative analysis of the results obtained combining information using the different linguistic computational models in operational research problems.

Keywords: Linguistic Information, Computing with Words, Operational Research

I. INTRODUCTION

The areas related to Operational Research present problems that deal with qualitative aspects. This type of aspects are difficult to assess by means of a precise numerical value, however the use of the fuzzy linguistic approach [37] has provided successful results for modelling these aspects.

The use of linguistic information implies processes of Computing with Words. In the specialized literature there exist different linguistic computational models to accomplish these operations:

1. *The approximative computational model based on the Extension Principle* [11]. This model uses the extension principle to make linguistic computations.
2. *The ordinal linguistic computational model* [10]. It makes direct computations on labels, using the ordinal structure of the linguistic term sets.
3. *The 2-tuple linguistic model* [19]. It uses the 2-tuple linguistic representation model and its characteristics to make linguistic computations.

In this contribution we review the use of the linguistic information in different areas of Operational Research as Decision-Making, Management and Scheduling. And afterwards, we shall make a comparative analysis of the results obtained by the above linguistic computational models over a decision-making problem.

In order to do so, this contribution is structured as follows: in Section II we shall review the fuzzy linguistic approach. In Section III we shall review the use of linguistic information in Operational Research areas. In Section IV a brief review of the different linguistic computational models is made. In Section V we shall make a comparative analysis of the results obtained by the above linguistic compu-

tational models over a decision-making problem. Finally, we shall pointed out some concluding remarks.

II. FUZZY LINGUISTIC APPROACH

Usually, we work in a quantitative setting, where the information is expressed by numerical values. However, many aspects of different activities in the real world cannot be assessed in a quantitative form, but rather in a qualitative one, i.e., with vague or imprecise knowledge. In that case a better approach may be to use linguistic assessments instead of numerical values. The fuzzy linguistic approach represents qualitative aspects as linguistic values by means of linguistic variables [37].

We have to choose the appropriate linguistic descriptors for the term set and their semantics. In the literature, several possibilities can be found [17]. One possibility of generating the linguistic term set consists of directly supplying the term set by considering all terms distributed on a scale on which a total order is defined [36]. For example, a set of seven terms S , could be:

$$S = \{s_0 : N, s_1 : VL, s_2 : L, s_3 : M, s_4 : H, s_5 : VH, s_6 : P\}$$

Usually, in these cases, it is required that in the linguistic term set there exist:

1. A negation operator: $\text{Neg}(s_i) = s_j$ such that $j = g-i$ ($g+1$ is the cardinality).
2. An order: $s_i \leq s_j \iff i \leq j$. Therefore, there exists a minimization and a maximization operator.

Here, we shall use as semantics of the linguistic terms triangular membership functions whose representation is achieved by a 3-tuple (a, b, c) , where b indicate the point in which the membership value is 1, with a and c indicating the left and right limits of the definition domain of the membership function [3]. An example may be:

$$\begin{aligned} P &= (.83, 1, 1) & VH &= (.67, .83, 1) & H &= (.5, .67, .83) \\ M &= (.33, .5, .67) & L &= (.17, .33, .5) & VL &= (0, .17, .33) \\ N &= (0, 0, .17). \end{aligned}$$

Other authors use a non-trapezoidal representation, e.g., Gaussian functions [4].

III. THE USE OF THE LINGUISTIC INFORMATION IN OPERATIONAL RESEARCH

The linguistic information has been used in those Operational Research problems dealing with qualitative aspects that are complex to assess by means of numerical precise values. Following we shall review in *Table I* different linguistic operational research problems solved in the literature.

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Operational Research

Applications	Topics	Ref.
Decision-Making	Linguistic Decision-Making	[9], [24], [36]
	Multicriteria Decision-Making	[6], [12]
	Group Decision-Making	[31], [16]
	Other Decision-Making Models	[5], [35], [32]
Management	Personnel Selection	[14], [21]
	Material Selection	[8], [15]
	Personnel Placement	[18], [29], [34]
	Risk in Software Development	[26], [27], [28]
Scheduling	Sequencing	[1], [2]
	Dispatching and Scheduling	[23], [25], [30], [33]

TABLE I
LINGUISTIC MODELING IN OPERATIONAL RESEARCH

IV. LINGUISTIC COMPUTATIONAL MODELS

Here we review the linguistic computational models for operating with linguistic information.

A. Linguistic Computational Model Based on the Extension Principle

The Extension Principle [13] generalizes crisp mathematical operations to fuzzy sets. The use of the Extension Principle increases the vagueness of the results. The results obtained by the fuzzy arithmetic are fuzzy numbers that usually do not match any linguistic term in the initial term set, so a linguistic approximation process is needed to express the result in the original expression domain[11]. In the literature we can find different linguistic approximation operators [3], [11].

A linguistic aggregation operation based on the Extension Principle can be expressed formally as:

$$S^n \xrightarrow{\tilde{F}} F(R) \xrightarrow{app_1} S$$

where \tilde{F} is an aggregation operator based on the Extension Principle, $app_1(\cdot)$ is a linguistic approximation process and S is the initial term set.

Another possibility is to express the results with fuzzy numbers this case is studied in [22].

B. Linguistic Computational Symbolic Model

A second approach used to operate on linguistic information is the symbolic one [10], that makes computations on the indexes of the linguistic labels. Usually it uses the ordered structure of the linguistic term sets, $S = \{s_0, \dots, s_g\}$ where $s_i < s_j$ iff $i < j$, to perform the computations. The intermediate results are numeric values, $\vartheta \in [0, g]$, which

must be approximated in each step of the process by means of an approximation function $app_2 : [0, g] \rightarrow \{0, \dots, g\}$ that obtains a numeric value, such that, it indicates the index of the associated linguistic term, $s_{app_2(\vartheta)} \in S$.

Formally, the symbolic aggregation is:

$$S^n \xrightarrow{C} [0, g] \xrightarrow{app_2} \{0, \dots, g\} \longrightarrow S$$

where C is a symbolic aggregation operator, $app_2(\cdot)$ is an approximation operation.

C. The 2-tuple Fuzzy Linguistic Representation Model

This model was presented in [19] and it is based on the concept of *Symbolic Translation*.

Definition 1. Let β be the result of an aggregation of the indexes of a set of labels assessed in a linguistic term set S , i.e., the result of a symbolic aggregation operation. $\beta \in [0, g]$, being $g+1$ the cardinality of S . Let $i = \text{round}(\beta)$ and $\alpha = \beta - i$ be two values, such that, $i \in [0, g]$ and $\alpha \in [-.5, .5)$ then α is called a *Symbolic Translation*.

Roughly speaking, the symbolic translation of a linguistic term, s_i , is a value in $[-.5, .5)$ that supports the "difference of information" between a counting of information $\beta \in [0, g]$ obtained after a symbolic aggregation operation and the closest value in $\{0, \dots, g\}$ that indicates the index of the closest linguistic term in S .

From this concept we shall develop a linguistic representation model which represents the linguistic information by means of 2-tuples (s_i, α_i) , $s_i \in S$ and $\alpha_i \in [-.5, .5)$:

This model defines transformation functions between numerical values and linguistic 2-tuples.

Definition 2. Let $S = \{s_0, \dots, s_g\}$ be a linguistic term set and $\beta \in [0, g]$ a value supporting the result of a symbolic

aggregation operation, then the 2-tuple that expresses the equivalent information to β is obtained with the following function:

$$\Delta : [0, g] \longrightarrow Sx[-0.5, 0.5]$$

$$\Delta(\beta) = \begin{cases} s_i & i = \text{round}(\beta) \\ \alpha = \beta - i & \alpha \in [-.5, .5) \end{cases}$$

where round is the usual round operation, s_i has the closest index label to " β " and " α " is the value of the symbolic translation.

Proposition 1. Let $S = \{s_0, \dots, s_g\}$ be a linguistic term set and (s_i, α) be a 2-tuple. There is always a Δ^{-1} function, such that, from a 2-tuple it returns its equivalent numerical value $\beta \in [0, g] \subset \mathcal{R}$.

Proof.

It is trivial, we consider the following function:

$$\Delta^{-1} : S \times [-.5, .5) \longrightarrow [0, g]$$

$$\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta$$

Remark 1: From definitions 1 and 2 and proposition 1, it is obvious that the conversion of a linguistic term into a linguistic 2-tuple consist of adding a value 0 as symbolic translation:

$$s_i \in S \implies (s_i, 0)$$

C.1 Linguistic computational model based on 2-tuples

Here we review the computational technique based on 2-tuples presented in [19], [20]:

1. Comparison of 2-tuples

The comparison of linguistic information represented by 2-tuples is carried out according to an ordinary lexicographic order.

Let (s_k, α_1) and (s_l, α_2) be two 2-tuples, with each one representing a counting of information:

- if $k < l$ then (s_k, α_1) is smaller than (s_l, α_2)
- if $k = l$ then
 1. if $\alpha_1 = \alpha_2$ then $(s_k, \alpha_1) = (s_l, \alpha_2)$
 2. if $\alpha_1 < \alpha_2$ then $(s_k, \alpha_1) < (s_l, \alpha_2)$
 3. if $\alpha_1 > \alpha_2$ then $(s_k, \alpha_1) > (s_l, \alpha_2)$

2. Aggregation of 2-tuples

In [19] we can find some 2-tuple aggregation operators, that are based on classical aggregation operators.

3. Negation operator of a 2-tuple

The negation operator over 2-tuples is defined as:

$$\text{Neg}((s_i, \alpha)) = \Delta(g - (\Delta^{-1}(s_i, \alpha)))$$

where $g + 1$ is the cardinality of S , $S = \{s_0, \dots, s_g\}$.

V. COMPARATIVE ANALYSIS

Here we shall solve a Decision Making problem with the computational models presented in section II and analyse the results.

A. Decision Making Problem

A distribution company needs to renew its computing system, so it contracts a consulting company to carry out a survey of the different possibilities existing on the market, to choose the best one for its needs. The alternatives are the following:

x_1	x_2	x_3	x_4
UNIX	WINDOWS-2000	AS/400	VMS

The consulting company has a group of four consultancy departments:

p_1	p_2	p_3	p_4
Cost analysis	Systems analysis	Risk analysis	Technology analysis

Each department provides a performance vector expressing its preferences for each alternative, in the linguistic term set $S = \{N, VL, L, M, H, VH, P\}$ (defined in section 2). In this case all the departments have the same degree of importance in the decision process.

		alternatives			
		x_1	x_2	x_3	x_4
experts	p_1	VL	M	M	L
	p_2	M	L	VL	H
	p_3	H	VL	M	M
	p_4	H	H	L	L

B. Decision Model

We shall use to solve this problem a decision process with the following steps:

- **Aggregation Process.** The values are aggregated to obtain a collective value for each alternative.
- **Exploitation Process.** Finally, rank the collective values, for obtaining the most appropriate alternative. In this paper, it will select alternatives with maximum collective value.

C. Linguistic Treatments

C.1 Computational Model based on the Extension Principle

A) Aggregation Process.

We shall compute a collective value for each alternative x_i using the arithmetic mean as aggregation operator, assuming equal importance for each expert ($w_i = 0.25$ $i = 1, \dots, 4$).

$$C_j = \left(\sum_{i=1}^m w_i \cdot a_{ij}, \sum_{i=1}^m w_i \cdot b_{ij}, \sum_{i=1}^m w_i \cdot c_{ij} \right)$$

These fuzzy sets obtained do not exactly match with any linguistic term in S , therefore we must apply to them a linguistic approximation process to obtain the results in the initial term set S . For selecting a term for the fuzzy set C_j , we shall use a linguistic approximation process ($\text{app}_1(\cdot)$) based on the euclidean distance:

$$d(s_i, C_j) = \sqrt{P_1(a_i - a_j)^2 + P_2(b_i - b_j)^2 + P_3(c_i - c_j)^2}$$

$$s_l = (a_l, b_l, c_l) \in S \quad C_j = (a_j, b_j, c_j)$$

where P_1, P_2, P_3 are weights representing the importance of a, b, c . So $app_1(C_j)$ chooses s_l^* , such that, $d(s_l^*, C_j) \leq d(s_l, C_j) \quad \forall s_l \in S$. This linguistic approximation process is applied to the above fuzzy sets, with $P_1 = 0.2, P_2 = 0.6, P_3 = 0.2$. The collective values obtained are:

$$\{M, M, L, M\}$$

B) Exploitation Process.

It obtains as solution set:

$$\{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_4\}$$

C.2 Symbolic Computational Model

A) Aggregation Process.

We shall use the Convex Combination [10] as aggregation operator.

Definition 3.[10] Let $A = \{a_1, \dots, a_m\}$ be a set of linguistic terms to be aggregated, the Convex Combination is defined in a recursive way as:

For $m=2$:

$$C^2\{\{w_1, 1-w_1\}, \{b_1, b_2\}\} = (w_1 \odot s_j) \oplus ((1-w_1) \odot s_i) = s_k, \quad s_j, s_i \in S, \quad (j \geq i)$$

such that, $k = \min\{g, i + \text{round}(w_1 \cdot (j - i))\}$, where $g + 1$ is the cardinality of S , $\text{round}(\cdot)$ is the usual round operation, and $b_1 = s_j, b_2 = s_i$.

For $m > 2$:

$$C^m\{w_k, b_k, k = 1, \dots, m\} = (w_1 \odot b_1) \oplus ((1-w_1) \odot C^{m-1}\{\eta_h, b_h, h = 2, \dots, m\}) = C^2\{\{w_1, 1-w_1\}, \{b_1, C^{m-1}\{\eta_h, b_h, h = 2, \dots, m\}\}\}$$

where $W = [w_1, \dots, w_m]$ is a weighting vector associated with A , such that, (i) $w_i \in [0, 1]$, and (ii) $\sum_i w_i = 1$; and $B = \{b_1, \dots, b_m\}$ is a vector, such that, $B = \{a_{\sigma(1)}, \dots, a_{\sigma(m)}\}$, where, $a_{\sigma(j)} \leq a_{\sigma(i)} \quad \forall i \leq j$, with σ being a permutation over the values a_i . $\eta_h = w_h / \sum_2^m w_k, h = 2, \dots, m$. Being \odot and \oplus the product of a number by a label and the addition of two labels respectively.

The weighting vector will be $\{.25, .25, .25, .25\}$, so the collective performance values are:

$$\{M, M, L, M\}$$

B) Exploitation Process.

It obtains as solution set:

$$\{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_4\}$$

C.3 2-tuple Computational Model

A) Aggregation Process.

We obtain a collective vector aggregating the performance values expressed by means of 2-tuples, using the arithmetic mean [19].

Definition 4.[19] Let $x = \{(r_1, \alpha_1), \dots, (r_n, \alpha_n)\}$ be a set of 2-tuples, the 2-tuple arithmetic mean \bar{x}^e is computed as,

$$\bar{x}^e\{(r_i, \alpha_i)\} = \Delta\left(\sum_{i=1}^n \frac{1}{n} \Delta^{-1}(r_i, \alpha_i)\right) = \Delta\left(\frac{1}{n} \sum_{i=1}^n \beta_i\right)$$

Obtaining the following collective vector:

$$\{(M, 0), (M, -.5), (L, .25), (M, -.25)\}$$

B) Exploitation Process.

It obtains as solution set:

$$\{\mathbf{x}_1\}$$

D. Comparative Analysis

Throughout this contribution we have solved a decision problem using three different methods, obtaining the following results:

	Dominance Degree				Sol. Set
	x_1	x_2	x_3	x_4	
EP	M	M	L	M	$\{x_1, x_2, x_4\}$
SM	M	M	L	M	$\{x_1, x_2, x_4\}$
2t	(M,0)	(M,-.5)	(L,.25)	(M,-.25)	$\{x_1\}$

TABLE II
RESULTS USING THE THREE METHODS

From *Table II* we notice that all the methods, even the method using the 2-tuple representation, obtain similar linguistic values for the dominance degrees which indicates the correctness of the results and hence of the methods. However, an important difference appears in the "Solution Set" column in *Table II*, the solution set obtained by the 2-tuple method is more precise (is a subset) than the sets obtained by the other ones. This is because collective values obtained by the methods of the first two rows of *Table II* are discrete, then when different alternatives have the same linguistic value as the dominance degree we cannot discern which is better than the others. Using the 2-tuple representation the collective values are continuous, therefore if several alternatives have the same linguistic term but a different value for the Symbolic Translation we can choose the best among them.

VI. CONCLUDING REMARKS

In this contribution we have reviewed the 2-tuple linguistic model and shown that this model improves the classical ones. Therefore, in those problems in which are combined the linguistic modelling and the *Operational Research* is possible to obtain better results using the 2-tuple linguistic preference modelling.

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