Extended Linguistic Hierarchies

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Abstract

In those problems dealing with linguistic information and multiple sources of information may happen that the sources have different degree of knowledge about the problem and could be suitable and necessary the use of different linguistic term sets with different granularity defining a multigranular linguistic context. Different approaches have been presented to deal with this type of context, being the linguistic hierarchies an structure quite interesting but presents a strong limitation about the term sets that can be used. In this contribution, we present the Extended Linguistic Hierarchies that break such a limitation and improve the management of multi-granular linguistic information.

1. Introduction

Real world problems can present quantitative or qualitative aspects. Those problems that present quantitative aspects are usually assessed by means of precise numerical values, on the other hand when the aspects are qualitative or there exists uncertainty related to the quantitative information it is better the use of a qualitative assessment. The use of the fuzzy linguistic approach [7] has obtained successful results in such a type of problems [1, 8], because it provides a direct way to model qualitative and uncertain information by means of linguistic variables.

The concept, *granularity of uncertainty*, plays a key role when we are dealing with linguistic information due to the fact that, it indicates the level of discrimination that the sources of information can use to express their knowledge, i.e., the cardinality of the term set [3]. Therefore, when there are multiple sources involve in a problem different ones might have different degree of knowledge about the aspects assessed and, could be suitable that each one can use terms sets with different granularity defining a multi-granular linguistic context.

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In the literature different approaches have been developed to deal with Multi-Granular Linguistic Information (*MGLI*) [2, 5]. These approaches manage the MGLI by conducting such an information in an unique linguistic term set in order to accomplish computing with words (CW) processes. Both approaches present advantages and disadvantages. The former may loss information in the CW processes and the latter so-called Linguistic Hierarchies (*LH*), presents a precise computational model but cannot use any linguistic term sets as, 5 and 7, that it is required in many problems.

The aim of this contribution is to present an Extension of the Linguistic Hierarchies, so-called Extended Linguistic Hierarchies (*ELH*) that keeps the advantages of the LH to deal with *MGLI* without loss of information and break the limitation about the terms that can be used in the multi-granular context.

In order to do that, the contribution is structured as follows. Section 2 introduces a linguistic background to understand the LH. Section 3 presents the Extended Linguistic Hierarchies, and finally we shall point out some concluding remarks in section 4.

2. Fuzzy Linguistic Approach

Many aspects of different activities in the real world cannot be assessed in a quantitative form, but rather in a qualitative one, i.e., with vague or imprecise knowledge. In that case a better approach may be to use linguistic assessments instead of numerical values. The fuzzy

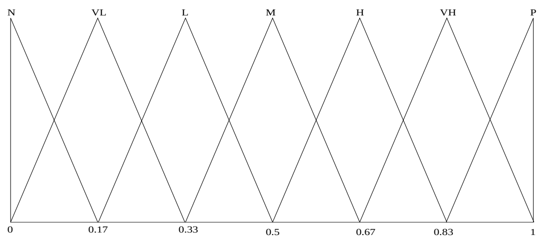

Figure 1. A Set of 7 Terms with its Semantic

linguistic approach represents qualitative aspects as linguistic values by means of linguistic variables [7].

In this approach it is necessary to choose the appropriate linguistic descriptors for the term set and their semantics, there exist different possibilities (further description see [3]). One possibility of generating the linguistic term set consists of directly supplying the term set by considering all terms distributed on a scale on which a total order is defined [6]. For example, a set of seven terms S, could be:

$$\{s_0 : N, s_1 : VL, s_2 : L, s_3 : M, s_4 : H, s_5 : VH, s_6 : P\}$$

Usually, in these cases, it is required that in the linguistic term set there exist:

1. A negation operator: $\text{Neg}(s_i) = s_j$ such that $j = g$-i (g+1 is the cardinality).
2. An order: $s_i \leq s_j \Longleftrightarrow i \leq j$. Therefore, there exists a min and a max operator.

The semantics of the terms are given by fuzzy numbers defined in the [0,1] interval, which are usually described by membership functions. For example, we might assign the following semantics to the set of seven terms (graphically, Fig.1):

$$\begin{array}{ll} P = (.83, 1, 1) & VH = (.67, .83, 1) \\ H = (.5, .67, .83) & M = (.33, .5, .67) \\ L = (.17, .33, .5) & VL = (0, .17, .33) \\ N = (0, 0, .17). & \end{array}$$

2.1. 2-tuple Linguistic Representation Model

This representation model was presented in [4] and it is the basis of the computational model for the LH. We then review this model in order to understand the LH. This model is based on symbolic methods and takes as the base of its representation the concept of Symbolic Translation.

Definition 1. *The Symbolic Translation of a linguistic term $s_i \in S = \{s_0, ..., s_g\}$ is a numerical value assessed in $[-.5, .5)$ that supports the "difference of information" between an amount of information $\beta \in [0, g]$ and the closest value in $\{0, ..., g\}$ that indicates the index of the closest linguistic term $s_i \in S$, being [0,g] the interval of granularity of S.*

From this concept the linguistic information is represented by means of 2-tuples (r_i, α_i), $r_i \in S$ and $\alpha_i \in [-.5, .5)$.

This model defines a set of functions between linguistic 2-tuples and numerical values.

Definition 2. *Let $S = \{s_0, \ldots, s_g\}$ be a set of linguistic terms. The* 2-tuple set associated with S *is defined as* $\langle S \rangle = S \times [-0.5, 0.5)$. *We define the function* $\Delta : [0, g] \longrightarrow \langle S \rangle$ *given by,*

$$\Delta(\beta) = (s_i, \alpha), \ \ with \ \begin{cases} i = \ round(\beta), \\ \alpha = \beta - i, \end{cases}$$

where round *assigns to* β *the integer number* $i \in \{0, 1, \ldots, g\}$ *closest to* β.

We note that Δ is bijective [4] and $\Delta^{-1} : \langle S \rangle \longrightarrow [0, g]$ is defined by $\Delta^{-1}(s_i, \alpha) = i + \alpha$. In this way, the 2-tuples of $\langle S \rangle$ will be identified with the numerical values in the interval $[0, g]$. This representation model has associated a computational model that was presented in [4].

2.2. Linguistic Hierarchies

We have mentioned that our objective in this contribution is to propose an approach to accomplish CW processes without loss of information with MGLI. In [5] was introduced an approach with this aim so-called Linguistic Hierarchies that carry out CW processes in a precise way but impose several limitations to the definition context. To achieve our objective we shall extend the LH, but first we will review the LH to understand its working.

A *linguistic hierarchy* is a set of levels, where each level is a linguistic term set with different granularity from the remaining of levels of the hierarchy. Each level belonging to a linguistic hierarchy is denoted as $\mathbf{l}(\mathbf{t}, \mathbf{n}(\mathbf{t}))$, t indicates the level of the hierarchy and *n(t)* indicates the granularity of the linguistic term set of the level t.

It's assumed that its levels contains linguistic terms sets with an odd number of terms and whose membership functions are triangular-shaped, symmetrical and uniformly distributed in $[0,1]$.

The levels belonging to a *LH* are ordered according to their granularity. A linguistic hierarchy, *LH*, is defined as the union of all levels t: $LH = \bigcup_t l(t,n(t))$. We are going to review the methodology to build a linguistic hierarchy and its computational model.

2.1.1. Building Linguistic Hierarchies. In the construction of a linguistic hierarchy its hierarchical order is given by the increase of the granularity of the linguistic term sets in each level.

Given a LH S, being a linguistic term set in the level t: $S = \{s_0,...,s_{n(t)-1}\}$, $s_k \in S, (k = 0,...,n(t)-1)$ a linguistic term of S. It's then denoted as, $S^{n(t)} = \{s_0^{n(t)},...,s_{n(t)-1}^{n(t)}\}$, because it belongs to level t and its granularity of uncertainty is $n(t)$.

A methodology to construct a *LH* was presented in [5] that imposed the following rules, so-called *linguistic hierarchy basic rules*:

1. To preserve all *former modal points* of the membership functions of each linguistic term from one level to the following one.

2. To make *smooth transitions between successive levels*. The aim is to build a new linguistic term set, $S^{n(t+1)}$. A new linguistic term will be added between each pair of terms belonging to the term set of the previous level t. To carry out this insertion, we shall reduce the support of the linguistic labels in order to keep place for the new one located in the middle of them.

Generally, a linguistic term set of level $t+1$ is obtained from its predecessor as:

$$l(t,n(t)) \to l(t+1, 2\cdot n(t)-1)$$

2.2.2. Transformation Functions among Levels of a Linguistic Hierarchy. In order to carry out CW processes, dealing with MGLI assessed in a LH, without loss of information in [5]was presented a transformation function, $TF_{t'}^t$, that permits to transform labels between levels without loss of information in order to conduct the MGLI in one expression domain:

$$TF_{t'}^t : l(t,n(t)) \longrightarrow l(t',n(t'))$$

$$TF_{t'}^t(s_i^{n(t)},\alpha^{n(t)}) = \Delta\left(\frac{\Delta^{-1}(s_i^{n(t)},\alpha^{n(t)})\cdot(n(t')-1)}{n(t)-1}\right) \tag{1}$$

$TF_{t'}^t$ is a one-to-one function between levels of the LH [5].

3. Extended Linguistic Hierarchies

It's clear that the hierarchy basic rules has some limitations to deal with MGLI without loss of information. Due to those assumptions the model does not allow to deal with contexts using the term sets with 5,7 and 9 labels that are quite common and necessary in many problems. In this section, we present a new methodology to construct linguistic hierarchies, so-called *Extended Linguistic Hierarchies* (*ELH*), and its computational model, such that they keep the precision in CW and allow the use of any linguistic term set. To do so, we shall follow the same scheme that the LH, first we present how to build an ELH and then its computational model.

3.1. Extended Building Linguistic Hierarchies

The reason that the LH keeps the information in CW is due to the basic rule 1 that keeps all former modal points from one level to another. The rule 2 just proposes the easiest way to keep these former modal points among all the levels being possible to transform the information between any two levels without loss of information.

Due to the fact that, we want to deal with levels that do not have to keep the former modal points from the before ones. We propose an extended linguistic hierarchy (ELH) with t_k levels where it is not necessary to keep the former points among each other. However, this ELH follows only one basic rule:

1. Given an ordered set of linguistic term sets by their granularity, with t_k sets $\{S^{n(t_1)},...,S^{n(t_k)}\}$, where, $S^{n(t_j)}$, may have any granularity. This set will be an ELH if and only if there exists a term set at a

new additional level t_{k+1} that keeps all the former modal points of all t_k sets.

Therefore to construct an ELH, first, it should be fixed the term sets that define the MGLI context, $\{S^{n(t_1)},...,S^{n(t_j)},...,S^{n(t_k)}\}, j = 1,...,k$ being $n(t_1)$ the smallest granularity and $n(t_k)$ the biggest granularity. For convenience, set $\delta_j = (n(t_j)) - 1$.

Lemma 1. *Let $S^{n(t_j)}$ be a linguistic term set. Then the former modal points set of the level t_j is $FP_{t_j} = \{fp^i_{t_j},...,fp^i_{t_j},...,fp^i_{t_j}\}, i = 0,...,2*\delta_j$, where each former modal point $fp^i_{t_j}$ is located at: $\frac{i}{(2*\delta_j)} \in [0,1]$.*

Theorem 1. Given an ordered set of linguistic term sets by their granularity, with t_k sets $\{S^{n(t_1)},...,S^{n(t_k)}\}$, where $S^{n(t_j)}$ may have any granularity. A new level, t_{k+1}, with the largest granularity can be defined as: $n(t_{k+1}) = (\prod_k \delta_j)+1$. Then the term set $S^{n(t_{k+1})}$ keeps all the former modal points of the other term sets, $S^{n(t_j)}, j = 1,...,k$

Proof.

The number of former modal points of the level t_{k+1} is:

$$(2*\delta_{k+1}) + 1,$$

According to Lemma 1, the former modal points $fp^i_{t_j}$ of the level t_j is located at: $\frac{i}{(2*\delta_j)} \in [0,1], i = 0,...,2*\delta_j, j = 1,...,k+1$. Because $n(t_{k+1}) > n(t_k)$, and

$$n(t_{k+1}) = (\prod_k \delta_j) + 1 \Rightarrow \delta_{k+1} = \prod_k \delta_j.$$

Notice that $2*\delta_{k+1}$ is multiplied by $2*\delta_j, j = 1,...,k$, hence the term set $S^{n(t_{k+1})}$ keeps all the former modal points of the other term sets, $S^{n(t_j)}, j = 1,...,k$. Once t_{k+1} has been fixed, the ELH will be the union of the levels, that defines the context problem and the last level that keeps all the former points and in which the processes of CW will be carried out to avoid any loss of information.

$$ELH = \bigcup(l(t_j,n(t_j))) \ which \ j = 1...k+1$$

Table 1 shows the granularity needed in the level t_{k+1} depending on the $n(t_j)$ defined in the previous levels and *Figure 2* shows the graphical examples of the extended linguistic hierarchy presented in Table 1. We can observe that the last level (t_{k+1}) contains all the former modal points of the membership functions of each linguistic term at the previous level from t_1 to t_k. In *Figure 2* the granularity of the last level, $n(t_3)$, is $n(t_{k+1}) = (\prod_k \delta_j) + 1, j = 1,2$, being $\delta_1 = 4$ and $\delta_2 = 6$. Then, $n(t_3) = 6*4+1 = 24+1 = 25$.

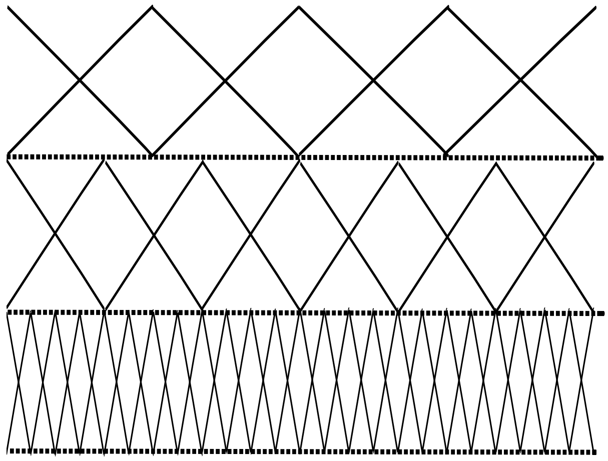

Figure 2. ELH of 5, 7, and 25 labels

3.2. Computational Model

Due to the fact that in the LH all the levels have to keep the former points of the predecessor. The transformations between any level can be carried out without loss of information. Nevertheless in the ELH this is not true, therefore to keep the information in the transformations, $TF^t_{t'}$ (see Equation. 1, one of the levels (t or t') must be t_{k+1}, this way guarantees the transformation between any level and the level t_{k+1} (and vice versa) of an extended linguistic hierarchy is carried out without loss of information.

A computational process with MGLI in an ELH is defined as follows:

- First, the labels $s_i^{n(t_j)}$ are transformed into the labels in the level t_{k+1}.

$$(s_j^{n(t_j)}, \alpha) \Rightarrow TF^{t_j}_{t_{k+1}}(s_j^{n(t_j)}, \alpha) = (s_k^{n(t_{k+1})}, \alpha'),$$

Example: Here, we show how the transformation functions act over the extended linguistic hierarchy, $LH = \bigcup l(1,5), l(2,7), l(3,25)$, whose term

Table 1. Example of ELH

ELH	level	$Granularity$	δ_j	$Number of Former Modal Points$
$l(t,n(t))$	t	$n(t)$	$n(t)-1$	$(2*\delta_j)+1$
$l(1,5)$	t_1	5	4	9
$l(2,7)$	t_2	7	6	13
$l(3,25)$	t_3	25	24	49

sets are:

$$\begin{array}{ll} l(1,5) & \{s_0^5, s_1^5, s_2^5, s_3^5, s_4^5\} \\ l(2,7) & \{s_0^7, s_1^7, s_2^7, s_3^7, s_4^7, s_5^7, s_6^7\} \\ l(3,25) & \{s_0^{25}, s_1^{25}, \ldots, s_{23}^{25}, s_{24}^{25}\} \end{array}$$

The transformations between terms of the different levels are carried out as:

$$TF_3^1(s_1^5,0) = \Delta^{-1}(\frac{\Delta(s_1^5,0)\cdot(25-1)}{5-1}) =$$

$$\Delta^{-1}(6) = (s_6^{25},0)$$

$$TF_3^2(s_3^7,0) = \Delta^{-1}(\frac{\Delta(s_2^7,0)\cdot(25-1)}{7-1}) =$$

$$\Delta^{-1}(8) = (s_8^{25},0)$$

The 2-tuple computational model is used to make the computations with the linguistic 2-tuples expressed in the term set, $S^{n(t_{k+1})}$. Obtaining results expressed by means of linguistic 2-tuples assessed in the same level, t_{k+1}.

Example: Using the 2-tuple mean operator [4] to aggregate the 2-tuples, whose expression is:

$$\bar{x} = \Delta(\frac{\sum_{i=1}^{n}\Delta^{-1}(s_i,\alpha_i)}{n}) \qquad (2)$$

The collective value obtained is:

$$\bar{x} = \Delta(\frac{\Delta^{-1}(s_6^{25},0)+\Delta^{-1}(s_8^{25},0)}{2}) =$$

$$\Delta(\frac{6+8}{2}) = \Delta(7) = (s_7^{25},0)$$

- Once the results have been obtained in the level t_{k+1} by means of linguistic 2-tuples, we can express them in the initial expression levels of the ELH by means of the transformation:

$$TF_{t_j}^{t_{k+1}}(s_f^{n(t_{k+1})},\alpha_f) = (s_k^{n(t_j)},\alpha),$$

Example: The collective value, $(s_7^{25},0)$, can be expressed in any linguistic term of the linguistic hierarchy:

$$TF_1^3(s_7^{25},0) = \Delta^{-1}(\frac{\Delta(s_7^{25},0)\cdot(5-1)}{25-1}) =$$

$$\Delta^{-1}(1.16) = (s_1^5,0.16)$$

$$TF_2^3(s_7^{25},0) = \Delta^{-1}(\frac{\Delta(s_7^{25},0)\cdot(7-1)}{25-1}) =$$

$$\Delta^{-1}(1.75) = (s_2^7,-0.25)$$

4. Concluding Remarks

The use of linguistic information is common in problems dealing with qualitative and/or uncertain information. In problems with multiple sources of information it may happen that different sources have different degree of knowledge need different term sets. In the literature there exist different proposals to deal with this type of information so-called Multi-Granular Linguistic Information. However these proposals have some limitations in the given number of term set in order to deal with MGLI. We have presented a framework based on Extended Linguistic Hierarchies that allows to deal with MGLI in a precise way using any linguistic term set.

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