COMAS: A Consensus Multi-agent Based System

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Abstract—In group decision making problems is common the necessity of achieving a consensus before making a decision. Many consensus reaching processes have been introduced in the literature but not many intelligent systems have finally been implemented to deal with such processes. In this contribution an initial prototype of a consensus support system supported on a multi-agent paradigm is presented, showing the system architecture (set of agents, behaviours and relationships) and a preliminary ontology to represent the problem knowledge and used for the agent communication.

Keywords-consensus; group decision making; multi-agent system;

I. INTRODUCTION

Group decision making (GDM) problems can be defined as decision situations where two or more decision-makers or experts try to achieve a common solution about a problem taking into account all opinions or preferences. Usually GDM problems have been solved carrying out a selection process where the best alternative/s is/are chosen without any previous aggrement [1]. Therefore it may happen that some experts consider that their preferences were not taken into account to obtain the solution, and hence they may disagree with that solution [2]. To avoid such a situation, it is advisable to carry out a consensus process where experts discuss and change their preferences to reach a sufficient agreement before making a decision.

Due to its importance in GDM, several consensus approaches have been proposed in the literature [3]–[5]. The consensus may be seen as the coincidence of all preferences of the members of a group, unusual situation in real GDM problems, or from a point of view more flexible, so-called "soft" consensus [6], as a mutual-agreement state among the members of a group where all opinions have been expressed and listened to the satisfaction of all members. This second interpretation of the consensus is what we consider in our works. Regarding the process to reach the consensus, this can be considered as a dynamic and iterative process consisting of several rounds, where experts express and discuss about their preferences on the alternatives of the problem. Traditionally this process is coordinated by a human moderator, who supervises all the process and evaluates the level of agreement achieved in each consensus round. If the agreement is not enough, the moderator encourages experts to change their preferences in an effort to make them closer in the next consensus round.

Some models based on classical software architectures have been proposed to automatize the consensus reaching processes [4], [7]. However, recently Kacprzyk and Zadrozny in [8] have introduced an initial theoretical approach for supporting consensus reaching processes by means of web intelligence, highlighting the use of the argumentation theory to generate machine-readable arguments [9]. Taking into account this new approach in this contribution we address the automation of consensus reaching processes from the point of view of the intelligent agent paradigm.

Intelligent agents have widely been studied in the last few years and may be defined as software entities capable of carrying out actions in an autonomous way to achieve a set of objectives by means of exchanging information with other agents and/or the environment [10]. As it seems logical, in any context or problem, an agent is not alone but different agents with different behaviors may be identified, constituting in this case a multi-agent system (MAS) [11], [12]. A key aspect in a MAS is its ontology, i.e., the set of concepts, terms and semantics necessary to represent the knowledge problem and used for the communication among agents [13]. Fortunately several organizations as FIPA[1] and OMG Agent PSIG[2] have joined their efforts towards a standardization of the agent technologies. So, agent development platforms such as JADE[3] and ZEUS[4] as well as ontology definition languages as OWL[5] (Web Ontology Language) and others have arisen in order to facilitate the implementation and development of MAS in different contexts.

MAS have been applied to different areas such as economy (e-commerce, auctions, bargaining theory, ...) [14], planning [15], industry [16] and recently works related to web services [17].

By considering the ideas presented in [8] and the consensus reaching models presented in [4], [7], this contribution

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[1]Foundation for Intelligent Physical Agents, http://www.fipa.org
[2]http://agent.omg.org
[3]Java Agent Development Environment, http://jade.tilab.com/
[4]Zeus agent building toolkit
[5]http://www.w3.org/TR/owl-features/

aims to implement a consensus support system based on a multi-agent architecture, called consensus multi-agent based system (COMAS) that facilitates, guides and automatizes consensus reaching processes by designing different intelligent agents that manage, supervise and control the consensus process. This system allows us to evaluate different consensus models and will be the basis for further developments in this field.

This contribution is organized as follows. In Section II, theoretical consensus reaching model implemented by COMAS is reviewed. In Section III, COMAS is presented introducing both the system architecture and the system ontologies. In Section IV, an example about the system performance is shown. Finally, in Section V some conclusions are drawn.

II. CONSENSUS REACHING MODEL DESCRIPTION

In the literature many theoretical models have been proposed to approach consensus processes [2], [18]–[20]. In this section we review main concepts of the consensus reaching model presented in [4], [7] in order to automate the consensus processes by assuming the moderator's tasks (see Fig. 1). This model will be implemented by COMAS.
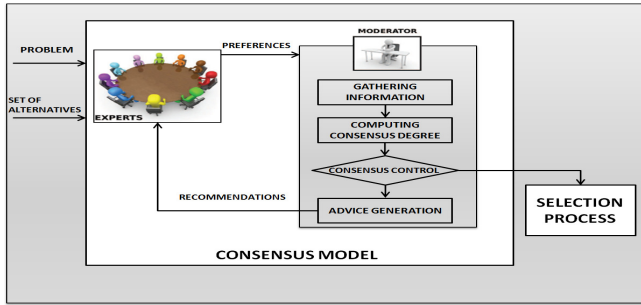

Figure 1. Consensus reaching model

Since the consensus reaching process is seen as an iterative process, in each round experts provide their preferences about the different alternatives of the GDM problem and the model computes the consensus degree achieved in that round. If it is not high enough, the model identifies those experts further from the collective opinion and recommends some changes to increase the level of agreement in the next round.

A detailed description of each phase is shown below:

1) **Gathering information.** Experts provide their opinions by mean of fuzzy preference relations. Fuzzy preference relations are classical structures to represent preferences in GDM problems [21]. So, given a GDM problem with a set of alternatives $X = \{x_1, x_2, \ldots, x_n\}$ ($n \geq 2$), and a group of experts, $E = \{e_1, e_2, \ldots, e_m\}$ ($m \geq 2$), each $e_i \in E$ expresses his/her preferences by means of a fuzzy preference relation $P_{e_i} \subset S \times S$, where its membership function

$\mu_{P_{e_i}}(x_l, x_k) = p_i^{lk} \in [0, 1]$ represents the preference degree of the alternative x_l over x_k according to the expert e_i.

2) **Computing consensus degree.** Once experts have provided their preferences, the model computes the level of agreement. Firstly, for each pair of experts e_i, e_j ($i < j$), a similarity matrix, $SM_{ij} = (sm_{ij}^{lk})$, is obtained by using the similarity measure proposed in [7],

$$sm_{ij}^{lk} = 1 - |(p_i^{lk} - p_j^{lk})| \qquad (1)$$

where sm_{ij}^{lk} is the similarity between the experts e_i and e_j on the pair of alternatives (x_l, x_k). Secondly, a consensus matrix, $CM = (cm^{lk})$, is calculated by aggregating all similarity matrices, where each aggregated pair of alternatives is computed as:

$$cm^{lk} = \phi(sm_{12}^{lk}, sm_{13}^{lk}, \ldots, sm_{1m}^{lk},$$
$$sm_{23}^{lk}, \ldots, sm_{2m}^{lk}, \ldots, sm_{(m-1)m}^{lk})$$

for $l, k \in \{1, \ldots, n\}$ and where ϕ is the arithmetic mean aggregation operator. Note that other aggregation operators may be used according to different consensus strategies such as we addressed in [22], where the consensus process behaviour with different aggregation operators was studied.

Finally, the agreement is computed at three different levels:

i) Pairs of alternatives,

$$cp^{lk} = cm^{lk}, \quad \forall l, k = 1, \ldots, n \quad \wedge \quad l \neq k,$$

where cp^{lk} represents the agreement on the pair of alternatives (x_l, x_k).

ii) Alternatives,

$$ca^l = \frac{\sum_{k=1, \ l \neq k}^{n} cp^{lk}}{(n-1)}, \qquad (2)$$

where ca^l represents the agreement on the alternative x_l.

iii) Preference relation,

$$cr = \frac{\sum_{l=1}^{n} ca^l}{n}, \qquad (3)$$

where cr represents the global agreement among all experts.

3) **Consensus control.** The agreement cr obtained in the previous phase is checked here. So, if cr is greater or equal than a consensus threshold given, this means that the wanted agreement has been achieved and the consensus reaching process finishes. Otherwise, the consensus process needs more discussion.

4) **Advice generation.** In this phase, the model suggests how experts should change their preferences in order

to increase the level of agreement in the following rounds. To do this, three tasks are carried out:

a) *Computing both the collective preference and experts' proximity values.* Firstly, a collective preference $\mathbf{P_{e_c}} = (p_c^{lk})$ is calculated by aggregating all experts' preference relations $\{\mathbf{P_{e_1}}, \ldots, \mathbf{P_{e_m}}\}$ at level of pairs of alternatives,

$$p_c^{lk} = \phi(p_1^{lk}, \ldots, p_m^{lk}). \qquad (4)$$

Afterwards, the model computes the proximity values between the preferences of each expert e_i and the collective preference on each pair (x_l, x_k),

$$pp_i^{lk} = 1 - \mid (p_i^{lk} - p_c^{lk}) \mid. \qquad (5)$$

These proximity values are used for identifying the furthest preferences from the collective preference and therefore the preferences that should be changed.

b) *Identification of preferences to be changed.* Different criteria may be used to choice the preferences to be changed by considering both the consensus degrees and proximity values [4], [7], [23]. Our model proposes to change the pairs of preferences of those alternatives whose level of agreement at level of alternative, ca^l, is not enough, obtaining the set of changes (*SC*),

$$SC = \{(x_l, x_k) \mid ca^l < cr \ \wedge \ cp^{lk} < cr\}.$$

for $l, k \in \{1, \ldots, n\}$. Once identified the pairs to be changed, the model identifies the experts who have to make the changes. So, the furthest experts to collective preference on the pairs $(x_l, x_k) \in SC$, will be required to modify their assessments. In order to identify them, a proximity threshold $\overline{pp}^{lk}$ for each pair belonging to *SC* is computed by aggregating all experts' proximities at level of pairs:

$$\overline{pp}^{lk} = \phi(pp_1^{lk}, \ldots, pp_m^{lk}). \qquad (6)$$

Now, each expert e_i whose $pp_i^{lk} \leq \overline{pp}^{lk}$ should modify his/her current assessments of the pairs $(x_l, x_k) \in SC$.

c) *Direction changes.* The model uses a set of direction rules to suggest the right direction of the changes in order to increase the level of agreement in the following round. For each preference to be changed, the model will suggest increasing or decreasing its current value. The direction changes are suggested taking into account experts' preferences, the collective preference and the following "direction rules" [4]:

DR.1. If $(p_i^{lk} - p_c^{lk}) < 0$, then the expert e_i should increase the assessment associated to the pair of alternatives (x_l, x_k).

DR.2. If $(p_i^{lk} - p_c^{lk}) > 0$, then the expert e_i should decrease the assessment associated to the pair of alternatives (x_l, x_k).

DR.3. If $(p_i^{lk} - p_c^{lk}) = 0$, then the expert e_i should not modify the assessment associated to the pair of alternatives (x_l, x_k).

III. A CONSENSUS MULTI-AGENT BASED SYSTEM (COMAS)

Any MAS implies the design of its architecture (i.e., set of necessary agents to fulfil the system tasks) and the definition of information pieces that will be used in the communication among agents.

In this section the architecture designed for COMAS and ontologies defined for the communication among agents are presented.

A. System architecture

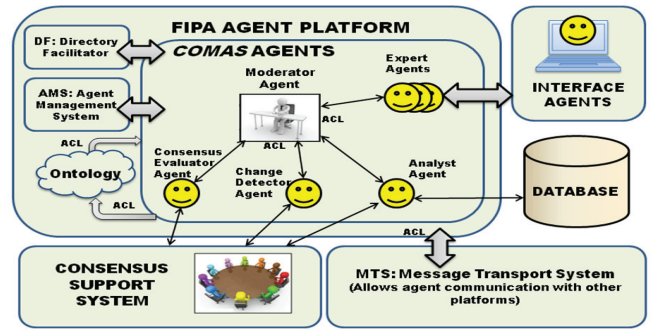The COMAS architecture based on FIPA standard is depicted in Fig. 2.

Figure 2.   COMAS architecture

As can be seen, several agents are proposed according to the different roles and tasks that can be identified in the consensus model reviewed in Section II. From the point of view of the relationships among agents, COMAS can be considered as cooperative MAS because each agent has its own role but all agents should work together in order to reach a common objective, i.e., the search for the consensus.

In short both the meaning and the behaviour of each agent are described below:

a) *Moderator agent:* It is the main agent of the system. Roughly speaking, it assumes the human moderator's tasks mentioned in the Section I, such as, requesting experts to participate in the consensus process, sending other agents notifications and suggestions, verifying the level of agreement, etc. It is responsible for checking the right development of the overall consensus process.
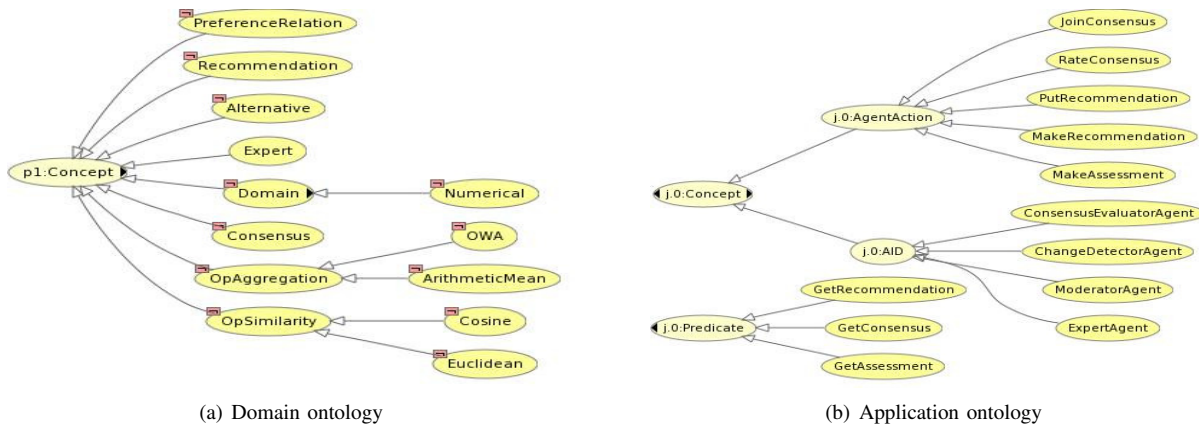
(a) Domain ontology   (b) Application ontology

Figure 3.   COMAS system ontologies

b) *Expert agent:* It represents the human expert and it acts in behalf of each expert who participates in the consensus problem. Once *moderator agent* requests the participation of the *expert agent* and this accepts it, the *expert agent* is responsible for giving its own preferences about the problem. Occasionally, it will receive recommendations to change its assessments in order to increase the level of agreement.

c) *Consensus evaluator agent:* This agent is in charge of computing the level of agreement achieved in each consensus round. From the experts' preferences, the *consensus evaluator agent* obtains the consensus degree cr and sends it to *moderator agent*.

d) *Change detector agent:* This agent is responsible for identifying the experts' preferences that should be changed and it also suggests the direction of the changes. This information is sent to *moderator agent*, and this sends to each *expert agent* its corresponding suggestion list.

e) *Analyst agent:* This agent assumes the task of storing all the information concerning each consensus process in a database. In this way, this information may be recovered and analyzed in the future.

B. System ontology

A key topic in any MAS communication is the definition of a suitable ontology to deal with a particular problem. Ontologies are used to capture knowledge about some domain of interest. An ontology describes the concepts in the domain as well as the relationships among them [13]. COMAS follows the idea proposed by Kacprzyk and Zadrozny [8] in which two different ontologies must be defined: (i) a domain ontology that represents in an abstract way the general concepts of the consensus problem and (ii) a application ontology that describes the specific details of each consensus problem (e.g. features of each alternative).

In our proposal, we follow a similar idea but adapting it to

our particular context. So, the specific ontologies designed for COMAS are graphically depicted in Fig. 3 and described below:

1) COMAS domain ontology (see Fig. 3(a)): it defines general concepts that COMAS uses to carry out a consensus reaching process. These concepts are closely related to both elements and concepts described in Section II. The high level of abstraction of the domain ontology allows that it can be used to represent the environment of different consensus problems because the particular details of each problem are obviated. Therefore concepts such as *PreferenceRelation* or *Expert*, represent abstract concepts of the problem domain. About twenty concepts represented as classes and subclasses[6] have been defined, although this number will possibly increase as other aggregation operators and information domains are considered.

2) COMAS application ontology (see Fig. 3(b)): it defines concepts and actions of the specific problem considered, i.e., application ontology depends on each problem. Therefore, specific tasks of our consensus model such as *rateConsensus* to measure the consensus degree or *MakeRecommendation* for suggesting the changes to an agent expert are defined.

Separating the knowledge in both ontologies, it can be managed in a different way general aspects of the problem by using the domain ontology, and specific aspects related to a particular problem by using the application ontology, achieving flexibility to undertake different consensus problems with minimal adaptations and efforts.

C. Technical issues

Regarding the implementation, both the system and the ontologies have been implemented with the JADE develop-

[6]Class diagrams have been generated by using the terminology of the Protégé software, http://protege.stanford.edu

ment platform. The ontologies in JADE define three main elements:

a) *Concepts*, expressions that identify entities (abstracts or concretes) with a complex structure.

b) *Predicates* (or facts), expressions that say something about the status of some problem topic and can be true or false.

c) *Agent actions*, special concepts that indicate actions that can be performed by some agents.

These elements are implemented by using Java objects which are defined in both *jade.content* and *jade.ontology* packages. The information among agents is exchanged using the standard FIPA-ACL (Agent Communication Language).

IV. EXAMPLE OF THE SYSTEM PERFORMANCE

Right now COMAS is an initial prototype but it can simulate examples of consensus reaching problems. In this section a general description of the system performance is presented. Afterwards, a summary of the results of a particular consensus reaching process carried out by COMAS is shown.

a) COMAS performance. The following steps describe the system performance:

Step 1: *Moderator agent* fixes the problem features, such as maximum number of consensus rounds, number of alternatives and the consensus threshold.

Step 2: *Moderator agent* invites the rest of agents (expert agents, consensus evaluator agent, etc.) to participate in the consensus process by using the communication protocol *Propose*[7].

Step 3: If a minimum number of *expert agents* accept the invitation, the first consensus round starts.

Step 4: *Moderator agent* asks for experts' preferences by means of the *Request* communication protocol. *Expert agents* accept this petition and send their preferences to *moderator agent*.

Step 5: *Moderator agent* requests *consensus evaluator agent* to compute the consensus degree. This returns it the current consensus degree.

Step 6: *Moderator agent* checks the level of agreement and if is not enough, requests *change detector agent* the preference changes.

Step 7: *Change detector agent* sends to *moderator agent* the changes and this sends them to each *expert agent*. The first round finishes.

Step 8: In the second round, *moderator agent* requests the experts' updated preferences and the Step 5 and following ones are repeated until either the consensus threshold is reached or the maximum number of consensus rounds is exceed.

[7]Communications protocols *Propose* and *Request* belong to the FIPA Interaction Protocol Library.


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MULTI-AGENT BASED CONSENSUS SUPPORT SYSTEM

MODERATOR AGENT: Spanish wine selection problem description
MODERATOR AGENT: Fixing problem parameters
- MAXIMUM NUMBER OF ROUNDS: 10
- NUMBER OF ALTERNATIVES: 4
- CONSENSUS THRESHOLD: 0.75

MODERATOR AGENT: Proposing the participation in a consensus problem...
Accept-proposal received from 8 expert agents, consensus evaluator agent and change detector agent

ROUND 1 BEGINS
MODERATOR AGENT: Requesting experts preferences...
EXPERT AGENTS: Sending expert preferences to moderator agent...
MODERATOR AGENT: Experts' preferences received. Passing them to Consensus Evaluator Agent...
Ok
CONSENSUS EVALUATOR AGENT: Computing consensus degrees...
Ok
Alternative-level consensus degree: CA1:0,655  CA2:0,604  CA3:0,659  CA4:0,617
CR: 0,634
No consensus reached. Current degree is less than consensus threshold

CHANGE DETECTOR AGENT: Generating change recommendations ...
Assessments (pairs of alternatives) that must be changed by some experts: (2,1) (2,3) (4,1) (4,2) (4,3)
CHANGE DETECTOR AGENT: Sending change recommendations to Moderator Agent...
MODERATOR AGENT: Sending each change recommendation to the corresponding expert agent...
Ok
TOTAL ROUND CHANGES=40
ACCUMULATED CHANGES NUMBER: 40

ROUND 2 BEGINS
EXPERT AGENTS: Updating preferences, when necessary...
MODERATOR AGENT: Experts' preferences received. Passing them to Consensus Evaluator Agent...
Ok
CONSENSUS EVALUATOR AGENT: Computing consensus degrees...
Ok
Alternative-level consensus degree: CA1:0,655  CA2:0,676  CA3:0,659  CA4:0,728
CR: 0,679
No consensus reached. Current degree is less than consensus threshold

CHANGE DETECTOR AGENT: Generating change recommendations ...
Assessments (pairs of alternatives) that must be changed by some experts: (1,2) (2,4) (3,2)
CHANGE DETECTOR AGENT: Sending change recommendations to Moderator Agent...
MODERATOR AGENT: Sending each change recommendation to the corresponding expert agent...
Ok
TOTAL ROUND CHANGES=24
ACCUMULATED CHANGES NUMBER: 64

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ROUND 5 BEGINS
EXPERT AGENTS: Updating preferences, when necessary...
MODERATOR AGENT: Experts' preferences received. Passing them to Consensus Evaluator Agent...
Ok
CONSENSUS EVALUATOR AGENT: Computing consensus degrees...
Ok
Alternative-level consensus degree: CA1:0,769  CA2:0,775  CA3:0,764  CA4:0,822
CR: 0,782
Consensus reached: consensus degree is more or equal than consensus threshold, the consensus process finishes
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Figure 4.   COMAS system log

b) Consensus reaching process example in COMAS. To illustrate a real consensus process in COMAS, it has been defined a problem regarding the quality of four Spanish wines according to the opinions of eight experts. The process carried out is depicted in Fig. 4 that shows a screenshot of the system log that saves every single activity made by each agent during the consensus process. Obviously the shown log has been summarized to clarify the main system tasks. Once the problem parameters (maximum number of consensus rounds = 10, number of alternatives = 4 and the consensus threshold = 0.75) are fixed, the *moderator agent* sends an invitation to take part in the consensus process to other agents. All agents accept such an invitation. *Expert agents* send their preferences to *moderator agent* and this requests *consensus evaluator agent* the consensus degree. The consensus degree $cr = 0.634$ is not enough and *moderator agent* requests *change detector agent* the set of recommendations. This sends the list of pair of alternatives that should be changed $((2,1),(2,3),...)$ to *moderator agent*. These changes are then sent to each expert (in our example, expert 6 and expert 8 should change their preferences about the pair $(2,3)$), and the first round finishes. The second rounds begins by requiring the updating the preferences and all the process is repeated again. As we can seen in

Fig. 4 in the second round and following ones the level of agreement is improved until in the fifth round the consensus threshold is reached and the consensus process is over.

V. Conclusion

Consensus reaching processes have been deeply investigated and many different models proposed. Although some ideas about consensus support systems appears in the literature, there are no real implementations of such types of systems. In this contribution a novel system based on a MAS, called COMAS, has been presented to support and automate consensus reaching processes. The system architecture as well as preliminary ontologies to represent the problem knowledge and to allow the communication among agents are introduced. This initial version of COMAS obtains very interesting results and provides and excellent platform as basis for future developments with different consensus scenarios.

Acknowledgment

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