An optimization-based approach to estimate the range of consistency in hesitant fuzzy linguistic preference relations

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Abstract—**The study of consistency is a very important problem in decision making using preference relations. This paper focuses on measuring the consistency of hesitant fuzzy linguistic preference relations (HFLPRs). In this paper we propose the optimization-based approach to estimate the range of consistency degree in a HFLPR. The underlying idea of the proposed approach consists in measuring the pessimistic consistency index (PCI) of HFLPRs, and also the optimistic consistency index (OCI) of HFLPRs. The PCI of HFLPRs is determined by its linguistic preference relation with the worst consistency degree, and the OCI of HFLPRs is determined by its linguistic preference relation with the best consistency degree. Furthermore, numerical examples are provided to show the use of the proposed consistency measure.**

Keywords—hesitant; linguistic preference relations; optimization; consistency

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I. INTRODUCTION

In real-world decision-making activities, decision makers often provide their opinions linguistically. Solving a decision problem with linguistic information implies the need for computing with words (CW) [13, 19, 24, 33]. In particular, Herrera and Martínez [14, 20] proposed the 2-tuple linguistic representation model to deal with uniformly and symmetrically distributed linguistic term sets. The 2-tuple linguistic model has been successfully used in a wide range of applications (e.g., [3, 10, 16, 21, 22, 23, 30]). In recent years, different models based on linguistic 2-tuples, such as the proportional 2-tuple linguistic representation model [17, 31, 34], the model based on the linguistic hierarchy [5, 15], and numerical scale model [4, 6, 8, 12], have been developed to deal with term sets that are not uniformly and symmetrically distributed.

Generally, when using linguistic models in decision making problems, experts provide a single term as an expression of their knowledge (or preferences). However, in some situations because of lack of information, time pressure and so on; the experts doubt and cannot easily elicit a single term as an expression of their knowledge, and they may prefer to think of several terms at the same time to provide their preferences instead of a single linguistic term. To overcome the limitation, Rodríguez et al. [25] introduced the concept of a hesitant fuzzy linguistic term set (HFLTS) to serve as the basis of increasing the flexibility of the elicitation of linguistic information by means of linguistic expressions. Rodríguez et al. [26] further developed a group decision making model dealing with comparative linguistic expressions based on HFLTSs. Wei et al. [32]

defined operations on HFLTSs, and gave possibility degree formulas for comparing HFLTSs and also presented two new linguistic aggregation operators for HFLTSs. Liu and Rodríguez [18] proposed a new representation of HFLTSs by means of a fuzzy envelope to carry out the CW processes. Dong et al. [7] proposed an optimization-based consensus model to minimize adjusted simple terms in the consensus reaching process with hesitant linguistic assessments in group decision making. To present a clear view on the use of hesitant fuzzy sets in decision making, Rodríguez et al. [27, 28] presented a complete review on hesitant fuzzy sets and recent results on HFLTS.

It is well known that quantifying consistency is a crucial issue in decision-making with preference relations. The lack of consistency can lead to inconsistent conclusions. In the specialized literature, several consistency measurement methods of hesitant fuzzy linguistic preference relations (HFLPRs) have been proposed (see e.g., [35, 36]). For example, Zhu and Xu [35] studied the weak consistency of a hesitant fuzzy preference relation, but this consistency measure can be only used for the hesitant fuzzy preference relation with strict comparison information, and does not conform to the reality. In [36], Zhu and Xu studied the additive consistency of HFLPR, but there are some problems in the normalization method, such as the normalization method biases original information.

In this paper, we focus on the study of measuring the consistency of HFLPRs by using symbolic linguistic computations based on the linguistic 2-tuple model. We propose an optimization-based approach to estimate the range of consistency degree in a HFLPR. The underlying idea of this approach consists of measuring the pessimistic consistency index (PCI) of HFLPRs, and also the optimistic consistency index (OCI) of HFLPRs. The PCI of HFLPRs is determined by its linguistic preference relation with the worst consistency degree, and the OCI of HFLPRs is determined by its linguistic preference relation with the best consistency degree.

The rest of the paper is organized as follows. Section 2 introduces a basic description of the 2-tuple linguistic model, hesitant fuzzy linguistic term sets, context-free grammar, linguistic preference relations and hesitant fuzzy linguistic preference relations. Section 3 presents an optimization-based approach to estimate the range of consistency degree in a HFLPR. In Section 4, numerical examples are provided. Finally, concluding remarks are included in Section 5.

II. PRELIMINARIES

This section introduces the basic knowledge regarding the 2-tuple linguistic model, hesitant fuzzy linguistic term sets, context-free grammar, linguistic preference relations and hesitant fuzzy linguistic preference relations, which are necessary to understand our proposals.

A. The 2-tuple linguistic model

The basic notations and operational laws of linguistic variables were introduced in [33]. Let $S=\{s_j \mid j=0,\cdots,g\}$ be a linguistic term set with odd granularity $g+1$, where the term s_j represents a possible value for a linguistic variable. The linguistic term set is usually required to satisfy the following additional characteristics:

(1) The set is ordered: $s_i \le s_j$ if and only if $i \le j$;

(2) There is a negation operator: $Neg(s_j)=s_{g-j}$.

The 2-tuple linguistic representation model, presented in Herrera and Martínez [14], represents the linguistic information by a 2-tuple $(s_i,\alpha)\in \overline{S}=S\times[-0.5,0.5)$, where $s_i \in S$ and $\alpha\in[-0.5,0.5)$. Formally, let $S=\{s_0,s_1,\ldots,s_g\}$ be a linguistic term set and $\beta\in[0,g]$ be a value representing the result of a symbolic aggregation operation. The 2-

tuple that expresses the equivalent information to β is then obtained as:

$$\Delta:[0,g]\rightarrow\overline{S}, \tag{1}$$

being

$$\Delta(\beta)=(s_i,\alpha),\ with\begin{cases}s_i, i=round(\beta)\\ \alpha=\beta-i,\ \alpha\in[-0.5,0.5)\end{cases} \tag{2}$$

Function Δ, it is a one to one mapping whose inverse function $\Delta^{-1}:\overline{S}\rightarrow[0,g]$ is defined as $\Delta^{-1}(s_i,\alpha)=i+\alpha$. When $\alpha=0$ in (s_i,α) is then called simple term.

In [14] it was also defined a computational model for linguistic 2-tuples in which different operations were introduced:

(1) A 2-tuple comparison operator: let (s_k,α) and (s_l,r) be two 2-tuples. Then:

(a) if $k<l$, then (s_k,α) is smaller than (s_l,r).

(b) if $k=l$, then

(i) if $\alpha=r$, then (s_k,α), (s_l,r) represents the same information.

(ii) if $\alpha<r$, then (s_k,α) is smaller than (s_l,r).

(2) A 2-tuple negation operator:

$$Neg((s_i,\alpha))=\Delta(g-(\Delta^{-1}(s_i,\alpha)))$$

(3) Several 2-tuple linguistic aggregation operators have been developed (see [14, 20]).

B. Hesitant fuzzy linguistic term sets

Torra [29] introduced the hesitant fuzzy set. Similar to the situations that are described and managed by hesitant fuzzy sets in [29], decision makers may hesitate between several linguistic terms before assessing an alternative. Bearing this idea in mind, Rodríguez et al. [25] gave concepts regarding HFLTSs as follows:

Definition 1 [25]: Let $S=\{s_j\,|\,j=0,\cdots,g\}$ be a linguistic term set, where $g+1$ is odd. A hesitant fuzzy linguistic term set (HFLTS), M_S, is an ordered finite subset of consecutive linguistic terms of S.

Once the concept of the HFLTS has been introduced, some operation laws can be performed on HFLTSs.

Let $S=\{s_j\,|\,j=0,\cdots,g\}$ be a linguistic term set, where $g+1$ is odd. Let M_S, M_{S1} and M_{S2} be three HFLTSs of S.

Definition 2 [25]: The upper bound M_S^+ and lower bound M_S^- of the HFLTS M_S are defined as:

(1) $M_S^+=\max(s_i)=s_j$, where $s_i\in M_S$;

(2) $M_S^-=\min(s_i)=s_j$, where $s_i\in M_S$.

Definition 3 [25]: The envelope of the HFLTS M_S, denoted as $env(M_S)$, is a linguistic interval whose limits are obtained by means of lower bound M_S^- and upper bound M_S^+, i.e.,

$$env(M_S)=[M_S^-,M_S^+].$$

Definition 4: Let M_S be defined as before, then the negation operator of M_S is defined as follows,

$$Neg(M_S)=\{s\,|\,s=Neg(h),h\in M_S\}$$

C. Context-free grammar for elicting linguistic information based on HFLTS

A context-free grammar G_H provides a way to generate simple but rich linguistic expressions that can be easily represented by means of HFLTS [25].

Definition 5 [25]: Let G_H be a context-free grammar, and $S=\{s_j\,|\,j=0,\cdots,g\}$ be a linguistic term set. The elements of $G_H = (V_N, V_T, I, P)$ are defined as follows:

$$V_N = \{\langle primary\ term \rangle, \langle composite\ term \rangle,$$
$$\langle unary\ relation \rangle, \langle binary\ relation \rangle, \langle conjunctiond \rangle\}$$
$$V_T = \{lower\ than, greater\ than, between, and, s_0, s_1, ..., s_g\}$$
$$I \in V_N.$$

The production rules are defined in an extended Backus–Naur form so that the brackets enclose optional elements and the symbol "|" indicates alternative elements. For the context free grammar G_H, the production rules are as follows:

$$P = \{I ::= \langle primary\ term \rangle | \langle composite\ term \rangle$$
$$\langle composite\ term \rangle ::= \langle unary\ relation \rangle \langle primary\ term \rangle |$$
$$\langle binary\ relation \rangle \langle primary\ term \rangle \langle conjuction \rangle \langle primary\ term \rangle$$
$$\langle primary\ term \rangle ::= s_0 \mid s_1 \mid ... \mid s_g$$
$$\langle unary\ relation \rangle ::= lower\ than \mid greater\ than$$
$$\langle binary\ relation \rangle ::= between$$
$$\langle conjuction \rangle ::= and\}$$

The expressions produced by the context-free grammar G_H, may be either single valued linguistic terms $s_i \in S$, or comparative linguistic expressions. Both types of expressions define the expression domain generated by G_H and that is noted as S_{ll}. These comparative linguistic expressions generated by G_H, cannot be directly used for CWW, therefore in [25] a transformation function was proposed to transform them into HFLTS.

Definition 6 [25]: Let E_{G_H} be a function that transforms linguistic expressions, $ll \in S_{ll}$, obtained by using G_H, into HFLTS, M_S. S is the linguistic term set used by G_H and S_{ll} is the expressions domain generated by G_H:

$$E_{G_H} : S_{ll} \to M_S$$

The comparative linguistic expressions generated by G_H using the production rules are converted into HFLTS by means of the following transformations:

$$E_{G_H}(s_i) = \{s_i | s_i \in S\};$$
$$E_{G_H}(less\ than\, s_i) = \{s_j | s_j \in S\ and\ s_j \le s_i\};$$
$$E_{G_H}(greater\ than\, s_i) = \{s_j | s_j \in S\ and\ s_j > s_i\};$$
$$E_{G_H}(between\, s_i\ and\ s_j) = \{s_k | s_k \in S\ and\ s_i \le s_k \le s_j\}.$$

D. Linguistic preference relations and hesitant fuzzy linguistic preference relations

Let $X = \{X_1, X_2, ..., X_n\} (n \ge 2)$ be a finite set of alternatives. When a decision maker makes pairwise comparisons using the linguistic term set S, he/she can construct a linguistic preference relation $L = (l_{ij})_{n \times n}$, whose element l_{ij} estimates the preference degree of alternative X_i over X_j. Linguistic preference relations based on linguistic 2-tuples can be formally defined as Definition 6.

Definition *7* [1, 2]: The matrix $L = (l_{ij})_{n \times n}$, where $l_{ij} \in S$, is called a simple linguistic preference relation. The matrix $L = (l_{ij})_{n \times n}$, where $l_{ij} \in \overline{S}$, is called a 2-tuple linguistic preference relation. If $l_{ij} = Neg(l_{ji})$ for $i, j = 1, 2, ..., n$, then L is considered reciprocal.

Additive transitivity is often used to character the consistency of linguistic preference relations (see Definition 7).

Definition 8 [1, 9, 11]: Let $L = (l_{ij})_{n \times n}$ be a linguistic preference relation based on S. L is considered consistent if $\Delta^{-1}(l_{ij}) + \Delta^{-1}(l_{jk}) - \Delta^{-1}(l_{ik}) = \dfrac{g}{2}$ for $i, j, k = 1, 2, ..., n$.

Based on Definition 8, the consistency index (CI) of a linguistic preference relation L is defined as,

$$CI(L) = 1 - \frac{2}{3gn(n-1)(n-2)} \sum_{i,j,k=1}^{n} \left| \Delta^{-1}(l_{ij}) + \Delta^{-1}(l_{jk}) - \Delta^{-1}(l_{ik}) - \frac{g}{2} \right| \quad (3)$$

The larger the value of $CI(L)$ the more consistent L is. If $CI(L) = 1$, then L is a consistent linguistic preference relation.

Definition 9: Let H_S be a set of HFLTSs based on S. A hesitant fuzzy linguistic preference relation (HFLPR) based on S is presented by a matrix $H = (H_{ij})_{n\times n}$, where $H_{ij} \in H_S$ and $Neg(H_{ij}) = H_{ji}$.

Definition 10: Let H be a HFLPR. $L = (l_{ij})_{n\times n}$ is a linguistic preference relation of H if $l_{ij} \in H_{ij}$ and $l_{ij} = Neg(l_{ji})$.

We denote N_H as the set of the linguistic preference relation associated to H.

III. OPTIMIZATION-BASED CONSISTENCY MEASURE OF HFLPRs

Let $S = \{s_j \mid j = 0, \cdots, g\}$ be a linguistic term set. Let $H = (H_{ij})_{n\times n}$ be a HFLPR of S.

In the following, we propose the optimization-based approach to estimate the range of consistency degree in a HFLPR. The underlying idea of this consistency measure consists in measuring the pessimistic consistency index of H, denoted as $PCI(H)$, and also the optimistic consistency index of H, denoted as $OCI(H)$.

The value of $PCI(H)$ is determined by its linguistic preference relation with the worst consistency degree, i.e.,

$$PCI(H) = \min_{L\in N_H} CI(L) \quad (4)$$

The value of $OCI(H)$ is determined by its linguistic preference relation with the highest consistency degree, i.e.,

$$OCI(H) = \max_{L\in N_H} CI(L) \quad (5)$$

According to Eq. (3),

$$CI(L) = 1 - \frac{2}{3gn(n-1)(n-2)} \sum_{i,j,k=1}^{n} \left| \Delta^{-1}(l_{ij}) + \Delta^{-1}(l_{jk}) - \Delta^{-1}(l_{ik}) - \frac{g}{2} \right|$$

Based on Definition 9, $L \in N_H$ equals to

$$\begin{cases} l_{ij} \in H_{ij} \\ l_{ij} = Neg(l_{ji}) \end{cases} \quad (6)$$

Thus, Eq. (4) can be equivalently transformed into a model (7),

$$\min_{L\in N_H} 1 - \frac{2}{3gn(n-1)(n-2)} \sum_{i,j,k=1}^{n} \left| \Delta^{-1}(l_{ij}) + \Delta^{-1}(l_{jk}) - \Delta^{-1}(l_{ik}) - \frac{g}{2} \right|$$

$$s.t. \quad \begin{cases} l_{ij} \in H_{ij} \\ l_{ij} = Neg(l_{ji}) \end{cases} \quad (7)$$

Eq. (5) can be equivalently transformed into a model (8),

$$\max_{L\in N_H} 1 - \frac{2}{3gn(n-1)(n-2)} \sum_{i,j,k=1}^{n} \left| \Delta^{-1}(l_{ij}) + \Delta^{-1}(l_{jk}) - \Delta^{-1}(l_{ik}) - \frac{g}{2} \right|$$

$$s.t. \quad \begin{cases} l_{ij} \in H_{ij} \\ l_{ij} = Neg(l_{ji}) \end{cases} \quad (8)$$

Solving models (7) and (8), we obtain the range of consistency degree of H. Clearly, the consistency index of any linguistic preference relations L associated to H are in the interval $[PCI(H), OCI(H)]$, i.e., $CI(L) \in [PCI(H), OCI(H)]$ for any $L \in N_H$.

Note: In this paper, we do not discuss the method to obtain the optimal solutions to models (7) and (8). In the future, an extended version of this conference paper will be provided to discuss this problem in detail.

IV. ILLUSTRATIVE EXAMPLES

In this section, two numerical examples are provided to illustrate how to estimate the range of consistency degree in a HFLPR based on the use of models (7) and (8).

Let

$S = \{s_0 = neither, s_1 = very\,low, s_2 = low,$
$s_3 = slightly\,low, s_4 = medium, s_5 = slightly\,high,$
$s_6 = high, s_7 = very\,high, s_8 = absolute\}$.

be the linguistic pre-establish label set used in these examples.

A. Example 1

In this example, based on the context-free grammar introduced in Definition 5 and the transformation function E_{G_H} in Definition 6, a decision maker provides a HFLPR, H.

$$H = \begin{pmatrix} - & \{s_3, s_4\} & \{s_5, s_6\} & \{s_1, s_2\} & \{s_0, s_1\} \\ \{s_4, s_5\} & - & \{s_6, s_7\} & \{s_2, s_3\} & \{s_0, s_1, s_2\} \\ \{s_2, s_3\} & \{s_1, s_2\} & - & \{s_5, s_6\} & \{s_4, s_5, s_6\} \\ \{s_6, s_7\} & \{s_5, s_6\} & \{s_2, s_3\} & - & \{s_4, s_5, s_6\} \\ \{s_7, s_8\} & \{s_6, s_7, s_8\} & \{s_2, s_3, s_4\} & \{s_2, s_3, s_4\} & - \end{pmatrix}$$

To obtain the range of consistency of H, we construct the following linear programming models.

$$\min\,(1 - \frac{1}{12n(n-1)(n-2)} \sum_{i,j,k=1; i \neq j \neq k}^{5} \left| \Delta^{-1}(l_{ij}) + \Delta^{-1}(l_{jz}) - \Delta^{-1}(l_{iz}) - 4 \right|)$$

$s.t.$

$$\begin{cases} l_{12} \in \{3,4\}; l_{13} \in \{5,6\}; l_{14} \in \{1,2\}; l_{15} \in \{0,1\} \\ l_{23} \in \{6,7\}; l_{24} \in \{2,3\}; l_{25} \in \{0,1,2\}; l_{34} \in \{5,6\} \\ l_{35} \in \{4,5,6\};\ l_{45} \in \{4,5,6\} \\ l_{ij} = Neg(l_{ji}) \end{cases} \quad (9)$$

and

$$\max\,(1 - \frac{1}{12n(n-1)(n-2)} \sum_{i,j,k=1; i \neq j \neq k}^{5} \left| \Delta^{-1}(l_{ij}) + \Delta^{-1}(l_{jz}) - \Delta^{-1}(l_{iz}) - 4 \right|)$$

$s.t.$

$$\begin{cases} l_{12} \in \{3,4\}; l_{13} \in \{5,6\}; l_{14} \in \{1,2\}; l_{15} \in \{0,1\} \\ l_{23} \in \{6,7\}; l_{24} \in \{2,3\}; l_{25} \in \{0,1,2\}; l_{34} \in \{5,6\} \\ l_{35} \in \{4,5,6\}; l_{45} \in \{4,5,6\} \\ l_{ij} = Neg(l_{ji}) \end{cases} \quad (10)$$

Solving model (9) obtains the $\min_{L \in N_H} CI(L) = CI(L_1) = 0.6416$, where

$$L_1 = \begin{pmatrix} - & s_4 & s_6 & s_1 & s_0 \\ s_4 & - & s_7 & s_3 & s_0 \\ s_2 & s_1 & - & s_6 & s_6 \\ s_7 & s_5 & s_2 & - & s_6 \\ s_8 & s_8 & s_2 & s_2 & - \end{pmatrix}$$

So $PCI(H) = CI(L_1) = 0.6416$.

Solving model (10) obtains the $\max_{L \in N_H} CI(L) = CI(L_2) = 0.8416$, where

$$L_2 = \begin{pmatrix} - & s_3 & s_5 & s_1 & s_1 \\ s_5 & - & s_6 & s_2 & s_2 \\ s_3 & s_2 & - & s_5 & s_4 \\ s_7 & s_6 & s_3 & - & s_4 \\ s_7 & s_6 & s_4 & s_4 & - \end{pmatrix}$$

So $OCI(H) = CI(L_2) = 0.8416$.

In this example, L_1 reflects the worst consistency degree of H, and L_2 reflects the best consistency degree of H. Clearly, the $CI(L) \in [0.6416, 0.8416]$ for any $L \in N_H$.

B. Example 2

In this example, similar to Example 1, we give the following HFLPR H'.

$$H'=\begin{pmatrix} - & \{s_1,s_2\} & \{s_6,s_7\} & \{s_1,s_2\} & \{s_4,s_5,s_6\} \\ \{s_6,s_7\} & - & \{s_4,s_5,s_6\} & \{s_1,s_2\} & \{s_0,s_1,s_2\} \\ \{s_1,s_2\} & \{s_2,s_3,s_4\} & - & \{s_5,s_6,s_7\} & \{s_3,s_4\} \\ \{s_6,s_7\} & \{s_6,s_7\} & \{s_1,s_2,s_3\} & - & \{s_3,s_4\} \\ \{s_2,s_3,s_4\} & \{s_6,s_7,s_8\} & \{s_4,s_5\} & \{s_4,s_5\} & - \end{pmatrix}$$

To obtain the range of consistency of H', we construct the following two linear programming models.

$$\min\ (1-\frac{1}{12n(n-1)(n-2)}\sum_{i,j,k=1;i\neq j\neq k}^{5}\left|\Delta^{-1}(l_{ij})+\Delta^{-1}(l_{jz})-\Delta^{-1}(l_{iz})-4\right|)$$

$s.t.$

$$\begin{cases} l_{12}\in\{1,2\};l_{13}\in\{6,7\};l_{14}\in\{1,2\};l_{15}\in\{4,5,6\} \\ l_{23}\in\{4,5,6\};l_{24}\in\{1,2\};l_{25}\in\{0,1,2\} \\ l_{34}\in\{5,6,7\};l_{35}\in\{3,4\};l_{45}\in\{3,4\} \\ l_{ij}=Neg(l_{ji}) \end{cases} \quad (11)$$

and

$$\max\ (1-\frac{1}{12n(n-1)(n-2)}\sum_{i,j,k=1;i\neq j\neq k}^{5}\left|\Delta^{-1}(l_{ij})+\Delta^{-1}(l_{jz})-\Delta^{-1}(l_{iz})-4\right|)$$

$s.t.$

$$\begin{cases} l_{12}\in\{1,2\};l_{13}\in\{6,7\};l_{14}\in\{1,2\};l_{15}\in\{4,5,6\} \\ l_{23}\in\{4,5,6\};l_{24}\in\{1,2\};l_{25}\in\{0,1,2\} \\ l_{34}\in\{5,6,7\};l_{35}\in\{3,4\};l_{45}\in\{3,4\} \\ l_{ij}=Neg(l_{ji}) \end{cases} \quad (12)$$

Solving model (11) obtains the $\min_{L\in N_{H'}} CI(L)=CI(L_1')=0.5916$, where

$$L_1'=\begin{pmatrix} - & s_1 & s_7 & s_1 & s_6 \\ s_7 & - & s_6 & s_1 & s_0 \\ s_1 & s_2 & - & s_7 & s_4 \\ s_7 & s_7 & s_1 & - & s_4 \\ s_2 & s_8 & s_4 & s_4 & - \end{pmatrix}$$

So $PCI(H')=CI(L_1')=0.5916$.

Solving model (12) obtains the $\max_{L\in N_{H'}} CI(L)=CI(L_2')=0.8$, where

$$L_2'=\begin{pmatrix} - & s_2 & s_6 & s_2 & s_4 \\ s_6 & - & s_4 & s_2 & s_2 \\ s_2 & s_4 & - & s_5 & s_3 \\ s_6 & s_6 & s_3 & - & s_4 \\ s_4 & s_6 & s_5 & s_4 & - \end{pmatrix}$$

So $OCI(H')=CI(L_2')=0.8$.

In this example, L_1' reflects the worst consistency degree of H', and L_2' reflects the best consistency degree of H'. Clearly, the $CI(L)\in[0.5916,\ 0.8]$ for any $L\in N_{H'}$.

V. Conclusion

In this paper, we propose the optimization-based approach to estimate the range of consistency degree in a HFLPR. The underlying idea of the optimization-based approach is to measure the consistency degree of the HFLPR based on the best and worst consistency of its associated linguistic preference relations. The PCI of HFLPRs is determined by its linguistic preference relation with the worst consistency degree, and the OCI of HFLPRs is determined by its linguistic preference relation with the best consistency degree.

In the future we aim at optimizing the consistency of HFLPRs.

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