# A Consensus Model for Group Decision Making with Hesitant Fuzzy Linguistic Information

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***Abstract*—Group Decision Making (GDM) is a usual process in companies and administration in which complex decision problems are solved taking into account different points of view from different experts involved in the decision situation. Notwithstanding, in principle group decisions should be better accepted than decisions made by a single decision maker because they try to include several viewpoints, sometimes the decision processes do not consider the agreement in the solution, therefore such solutions can fail in their goal. To overcome such a problem, a consensus reaching process is added to GDM processes to obtain solutions with a high degree of agreement. The complexity of GDM problems are often due to the uncertainty related to the imprecision and vagueness of the meaning of the decision situation that is modelled by linguistic descriptors. Different linguistic *consensus* models have successfully dealt with these GDM problems. However, recently it has been pointed out that in GDM problems dealing with linguistic information may be necessary to offer a higher flexibility to experts for eliciting their preferences to manage mainly their hesitancy about linguistic assessments when a single linguistic term does not adjust enough to their knowledge/preference. This contribution provides a novel *consensus* model for GDM problems dealing with Hesitant Fuzzy Linguistic Term Sets (HFLTS) that have been proposed to deal with hesitancy in linguistic GDM problems.**

***Keywords*-group decision making; hesitant fuzzy linguistic term sets; consensus reaching process.**

## I. Introduction

Decision making is a typical task for human beings in their daily life that each one may accomplish by her/himself, but when decision problems are complex it is common that a group provides more knowledge than a single decision maker. Therefore, we talk about Group Decision Making (GDM) problems in which a group of decision makers try to obtain a solution for a decision making problem that consists of choosing the best alternative from a set of possible alternatives, by eliciting their preferences [2], [19]. It should be remarked that GDM problems aim at achieving an agreed solution accepted by all decision makers involved in the decision situation. Generally, GDM problems have been solved by applying selection based approaches [4] that do not guarantee to reach such a collective agreed solution. Hence, Consensus Reaching Processes (CRPs) become a necessary task in GDM (see Fig. 1), in which an iterative discussion process supervised by a human figure so-called *moderator* tries to guide by providing advice to experts in order to obtain agreed solutions by all decision makers participating in the decision problem [17].

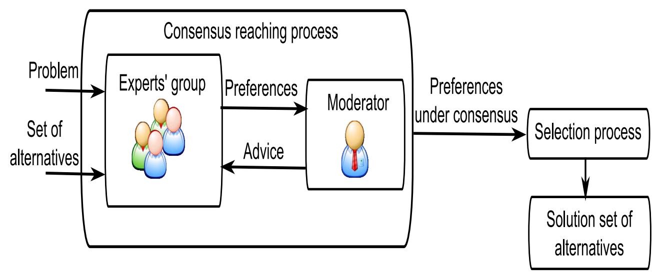


Figure 1. Consensus and GDM scheme

In GDM problems experts elicit their preferences according to the nature of the alternatives and to their own knowledge [9]. Usually, in real world GDM problems is fairly common that the definition framework involves uncertainty. We focused in this contribution on those decision situations in which the uncertainty is qualitative in nature and the use of linguistic information is adequate to model and elicit preferences about it [8]. Usually, GDM solving processes require that decision makers elicit assessments to express their preferences about the alternatives and most of linguistic GDM models provide linguistic modelling that only use single linguistic terms [10]. However, in decision situations with high degree of uncertainty, it may be quite hard for decision makers to provide just a single linguistic value because of the lack of knowledge, time pressure or complexity. In such cases, it might be more adequate the elicitation of multiples values or complex linguistic expressions. There are different approaches on linguistic decision making that use linguistic expressions instead of single terms [5], [18]. However, those proposals are not close to human beings cognitive processes. Rodríguez et al., based on the idea of Hesitant Fuzzy Sets [15], have recently introduced the concept of Hesitant Fuzzy Linguistic Term Set (HFLTS) [13] to facilitate the modelling of hesitant situations in linguistic contexts by elicitation of comparative linguistic expressions.

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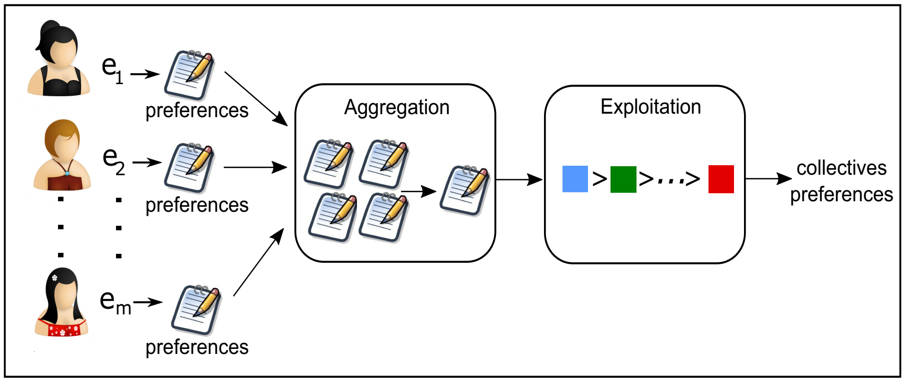


Figure 2. Selection process for a GDM problem

And it was also proposed in [14] a GDM model to deal with comparative linguistic expressions that provide a way to solve GDM under this uncertainty but however does not guarantee the achievement of agreed solutions for the GDM problem. Therefore, in this contribution it is proposed a novel *consensus* model dealing with comparative linguistic expressions modelled by HFLTS.

This paper is structured as follows: Section 2 revises some preliminary concepts about GDM and linguistic modelling with HFLTS. Section 3 presents the consensus model dealing with comparative linguistic expressions modelled by HFLTS. Section 4 concludes this contribution.

## II. Linguistic Modelling and Group Decision Making

This paper aims at introducing a consensus model capable of dealing with comparative linguistic expressions as preference assessments in hesitant decision situations. Before presenting this model, this section briefly reviews some basic and necessary concepts about GDM and comparative linguistic expressions based on HFLTS to understand our proposal.

### A. Group Decision Making

The complexity and impact of decision making in real world lead to the necessity of taking into account different points of view in such decision problems. Therefore, in many situations is assumed that a group of experts can provide richer knowledge and obtain better solutions and hence they take part in the decision making process.

A GDM problem is defined as a decision situation in which two or more experts, $E = \{e_1, \ldots, e_m\}$ $(m \geq 2)$, express their preferences over a finite set of alternatives, $X = \{x_1, \ldots, x_n\}$, $(n \geq 2)$ to obtain a solution set of alternatives for the decision problem [2], [12]. Experts, $e_i$, provide their preferences on $X$ by using a preference relation $P_i$, $\mu_{P_i} : X \times X \longrightarrow D$,

$$P_i = \begin{pmatrix} p_i^{11} & \ldots & p_i^{1n} \\ \vdots & \ddots & \vdots \\ p_i^{n1} & \ldots & p_i^{nn} \end{pmatrix},$$

being each assessment, $\mu_{P_i}(x_l, x_k) = p_i^{lk}$, the preference of the alternative $x_l$ over $x_k$ according to expert $e_i$. And $D$ the expression domain utilized to model the information elicited by experts. In this contribution, it is considered that $D$ will be a linguistic term set, $S = \{s_0, \ldots, s_g\}$, and the information can be expressed either by linguistic terms or by comparative linguistic expressions built from the term set, $S$.

Usually, GDM problems have been solved by means of a selection process where experts obtain the best alternative or subset of alternatives from their preferences [16]. This process consists of two phases (see Fig. 2):

- *Aggregation* phase fuses the experts' preferences by using aggregation operators to obtain a collective preference matrix of all experts involved in the decision problem.
- *Exploitation* selects the best set of alternative(s) to solve the decision problem from the previous collective preferences by using a choice function.

### B. Elicitation of Comparative Linguistic Expressions: Hesitant Linguistic Information

The use of linguistic information have provided successful results modelling vagueness and imprecision in decision making problems [7], [20]. But, most of the linguistic approaches model the assessments by using just one linguistic term [6], [7]. Sometimes, because of the lack of information or knowledge about the problem, the use of one linguistic term is not enough to elicit their assessments, because decision makers might hesitate among different linguistic terms. Therefore, it would be convenient to provide more complex linguistic expressions than single linguistic terms that allow to reflect such hesitation. Different approaches [5], [18] were proposed to improve the elicitation of complex linguistic expressions. Nevertheless, such linguistic expressions were far from the cognitive process used by human beings to provide their opinions. Therefore, in this contribution, it is considered another recent approach based on the concept of HFLTS [13] that models this type of hesitation and facilitates the generation of comparative linguistic expressions close to the human beings' cognitive model by using context-free grammars.

For GDM problems it is considered the following basic context-free grammar $G_H$, that builds comparative linguistic expressions suitable for eliciting assessments in GDM.

*Definition 1:* [3] Let $G_H$ be a context-free grammar and $S = \{s_0, \ldots, s_g\}$ be a linguistic term set. The elements of $G_H = (V_N, V_T, I, P)$ are defined as follows:

$$\begin{array}{l} V_N = \{\langle primary\ term\rangle\ ,\ \langle composite\ term\rangle\ ,\ \langle unary\ relation\rangle \\ \langle binary\ relation\rangle\ ,\ \langle conjunction\rangle\}, \\ V_T = \{at\ most,\ at\ least,\ between, and,\ s_0, \ldots, s_g\}, \\ I \in V_N, \\ P = \{I ::= \langle primary\ term\rangle | \langle composite\ term\rangle \\ \quad \langle composite\ term\rangle ::= \langle unary\ relation\rangle\langle primary\ term\rangle | \\ \quad \langle binary\ relation\rangle\langle primary\ term\rangle\langle conjunction\rangle \\ \langle primary\ term\rangle \\ \quad \langle primary\ term\rangle ::= s_0|s_1|\ldots|s_g \\ \quad \langle unary\ relation\rangle ::= at\ most|at\ least \\ \quad \langle binary\ relation\rangle ::= between \\ \quad \langle conjunction\rangle ::= and\}. \end{array}$$

The set of expressions $ll$, generated by the context-free grammar $G_H$, defines the expression domain $ll \in S_{ll}$ that are then transformed into HFLTS by means of a transformation function, $E_{G_H}$, to operate with them.

*Definition 2:* [13] Let $S = \{s_0, \ldots, s_g\}$ be a linguistic term set, a HFLTS $H_S$, is defined as an ordered finite subset of consecutive linguistic terms of $S$:

$$H_S = \{s_i, s_{i+1}, \ldots, s_j\} \quad such\ that \;\; s_k \in S, \;\; k \in \{i, \ldots, j\} \tag{1}$$

The function, $E_{G_H}$, was defined to obtain HFLTS from comparative linguistic expressions.

*Definition 3:* [13] Let $E_{G_H}$ be a function that transforms comparative linguistic expressions $ll \in S_{ll}$, obtained from a context-free grammar $G_H$, into HFLTS $H_S$, where $S$ is the linguistic term set used by $G_H$, and $S_{ll}$ is the set of linguistic expressions generated by $G_H$.

$$E_{G_H} : S_{ll} \longrightarrow H_S \tag{2}$$

$E_{G_H}$ performance depends on the comparative linguistic expressions generated by the context-free grammar $G_H$. The transformations for the context-free grammar $G_H$, introduced in Def. 1 are as follows:

- $E_{G_H}(s_i) = \{s_i | s_i \in S\}$
- $E_{G_H}$(at most $s_i$) $= \{s_j | s_j \in S \ and\ s_j \leq s_i\}$
- $E_{G_H}$(at least $s_i$) $= \{s_j | s_j \in S \ and\ s_j \geq s_i\}$
- $E_{G_H}$(between $s_i$ and $s_j$) $= \{s_k | s_k \in S \ and\ s_i \leq s_k \leq s_j\}$

Therefore a preference relation $P_i$, $\mu_{P_i}$ $X \times X \longrightarrow S_{ll}$, with the linguistic term set, $S = \{very\_low, low, medium, high, very\_high\}$, may be as follows:

$$P_i = \begin{pmatrix} - & bt\ low\ and\ medium & high \\ at\ least\ high & - & at\ most\ low \\ low & bt\ medium\ and\ high & - \end{pmatrix}$$

Note that *bt* stands for between.

In order to facilitate the operational processes with HFLTS, a fuzzy envelope for HFLTS was proposed in [3] which represents the linguistic expressions by means of a fuzzy membership function obtained by the aggregation of the linguistic terms that compound the HFLTS.

*Definition 4:* [3] Let $H_S = \{s_i, s_{i+1}, \ldots, s_j\}$ be a HFLTS, so that $s_k {\in} S = \{s_0, \ldots, s_g\}$, $k {\in} \{i, \ldots, j\}$.

$$env_F\left(H_S\right) = T(a, b, c, d), \tag{3}$$

being $T(\cdot)$ a trapezoidal or triangular fuzzy membership function.

## III. A Consensus Model for GDM with Comparative Linguistic Expressions

So far we have seen that despite the convenience of using comparative linguistic expressions in some GDM problems. The models, to deal with this information in GDM [14], carry out a selection process that obtains the solution of the problem without considering the agreement on the solution. Therefore, these solutions may provoke that one or more experts in the GDM problem do not feel identified with the decision made and they do not accept it, because they consider that their individual concerns have not been considered sufficiently to reach the solution made. Here, it is presented a consensus model for GDM problems that deals with comparative linguistic expressions in order to obtain agreed solutions for the GDM when experts elicit linguistic terms or comparative linguistic expressions.

Classically, consensus models proposed in the literature consist of an iterative process that guides experts across different discussion rounds to make closer their opinions removing disagreements and looking for agreed solutions. Usually such classical consensus models make use of a human figure so-called moderator, who is responsible for coordinating the overall CRP [17]. The proposed approach will facilitate the automation of his/her tasks if necessary, by implementing such a model into a *Consensus Support System* based on intelligent techniques [11].

A scheme of our proposal for a consensus model in GDM problems with HFLTS is depicted in Fig. 3, and its phases are described in further detail below:

1) *Determining Group Decision Problem*: The first phase consists of determining the GDM problem defining the alternatives, experts and domains of expression for experts' preferences.

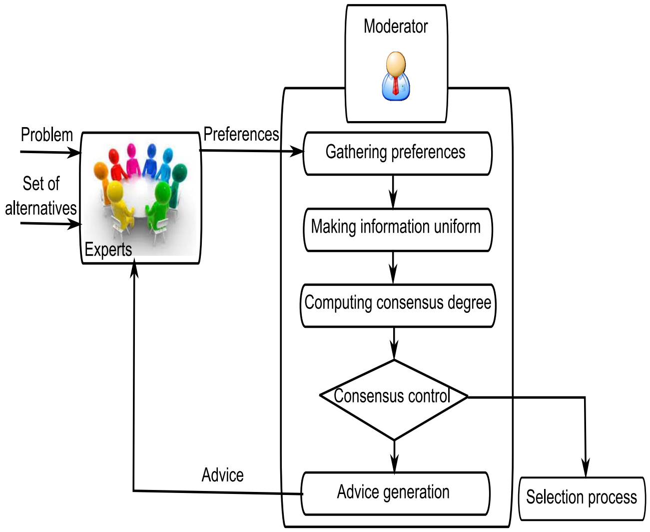


Figure 3. Consensus model scheme

2) *Gathering Preferences*: Given that the GDM problem is defined in a framework dealing with linguistic terms and comparative linguistic expressions, each $e_i$ provides his/her preferences on $X$ by means of a preference relation $P_i$, consisting of a $n \times n$ matrix of assessments $p_i^{lk} \in D = \{S_{ll}\}$.

3) *Making Information Uniform*: Preferences expressed by decision makers are conducted into a unified expression domain to operate among the linguistic terms and the comparative linguistic expressions. Such an unification process adapts the approach proposed in [1], that unifies heterogeneous information into fuzzy sets in a common linguistic term set, $S_T$, such that in this case the fuzzy membership functions representing the linguistic terms and the comparative linguistic expressions are unified into fuzzy sets:

*Definition 5:* [6] Let $s_i = T(a,b,c,d)$ and $S_T = \{s_0^T, ..., s_g^T\}$ be either the semantics of a linguistic term in $S_{ll}$ or the fuzzy representation of a comparative linguistic expression (see Def. 4) and the common linguistic term set respectively. The unification transformation function, $\tau_{sS_T}$ is then defined as:

$$\tau_{sS_T} : s_i \longrightarrow F(S_T)$$

$$\tau_{sS_T}(s_i) = \sum_{k=0}^{g_T} s_k^T / \gamma_k^j \quad (4)$$

$$\gamma_k^j = \max_y \min\{\mu_{s_i}(y), \mu_{s_k^T}(y)\},$$

being $F(S_T)$ the set of fuzzy sets defined in $S_T$, $\mu_{s_i}(y)$ and $\mu_{s_k^T}(y)$ the membership functions of the fuzzy sets associated to the terms $s_i$ and $s_k^T$, respectively.

Assuming that each unified assessment is represented by $p_i^{lk} = (\gamma_{i0}^{lk}, \dots, \gamma_{ig}^{lk})$, each decision maker's unified preference relation is a fuzzy set represented as follows:

$$P_i = \begin{pmatrix} - & \dots & (\gamma_{i0}^{1n}, \dots, \gamma_{ig}^{1n}) \\ \vdots & \ddots & \vdots \\ (\gamma_{i0}^{n1}, \dots, \gamma_{ig}^{n1}) & \dots & - \end{pmatrix}$$

4) *Computing Consensus Degree*: It computes the degree of agrement or consensus degree, $cr$, amongst decision makers [2], measured as a value in [0,1] such that the more value the better. Its computation is carried out by the following steps:

a) For each unified assessment (fuzzy set) $p_i^{lk} = (\gamma_{i0}^{lk}, \dots, \gamma_{ig}^{lk})$, a *central value* $cv_i^{lk} \in [0, g]$ is obtained, as follows:

$$cv_i^{lk} = \frac{\sum_{j=0}^{g} index(s_j) \cdot \gamma_{ij}^{lk}}{\sum_{j=0}^{g} \cdot \gamma_{ij}^{lk}}, \quad s_j \in S_T \quad (5)$$

being $index(s_j) = j \in \{0, \dots, g\}$.

b) For each pair of decision makers $e_i, e_t$, $(i < t)$, a *similarity matrix* $SM_{it} = (sm_{it}^{lk})_{n \times n}$ is computed. Each similarity value $sm_{it}^{lk} \in [0,1]$ represents the agreement level between $e_i$ and $e_t$ in their opinion on $(x_l, x_k)$, computed as:

$$sm_{it}^{lk} = 1 - \left| \frac{cv_i^{lk} - cv_t^{lk}}{g} \right| \quad (6)$$

c) A *consensus matrix* $CM = (cm^{lk})_{n \times n}$ is obtained by aggregating similarity values, by means of an aggregation operator, $\zeta$:

$$cm^{lk} = \zeta(SIM^{lk}), \quad (7)$$

being $SIM^{lk} = \{sm_{12}^{lk}, \dots, sm_{1m}^{lk}, \dots, sm_{(m-1)m}^{lk}\}$ the set of all pairs of decision makers' similarities in their opinion on $(x_l, x_k)$, with $|SIM^{lk}| = \binom{m}{2}$, and $cm^{lk}$ the degree of consensus achieved by the group in their opinion on the pair $(x_l, x_k)$.

d) Consensus degrees $ca^l$ on each alternative $x_l$, are computed as

$$ca^l = \frac{\sum_{k=1, k\neq l}^{n} cm^{lk}}{n-1} \quad (8)$$

e) Eventually, an overall consensus degree, $cr$, is obtained as follows:

$$cr = \frac{\sum_{l=1}^{n} ca^l}{n} \quad (9)$$

5) *Consensus Control*: Consensus degree $cr$ is compared with a consensus threshold $\mu \in [0,1]$, established a priori by the group. If $cr \geq \mu$, the CRP ends and the group moves on the selection process [14]; otherwise, the process requires further discussion. A parameter

$Maxrounds \in \mathbb{N}$ can be used to control the maximum number of discussion rounds.

6) *Advice Generation*: When consensus required is not achieved, $cr < \mu$, decision makers are advised to modify their preferences to make them closer to each other and increase the consensus degree in the following CRP round. As stated above, despite a human moderator has been traditionally responsible for advising and guiding decision makers during CRPs, the proposed model allows an automation of moderator's tasks [11], being the most of them performed in this phase of the CRP. This phase consists of the following steps (based on central values $cv_i^{lk}$):

   a) Compute a collective preference and proximity matrices: A collective preference $P_c = (p_c^{lk})_{n\times n}$, $p_c^{lk} \in [0, g]$, is computed for each pair of alternatives by aggregating preference relations:

   $$p_c^{lk} = \nu(cv_1^{lk}, \ldots, cv_m^{lk}) \quad (10)$$

   Afterwards, a proximity matrix $PP_i$ between each $e_i$ preference relation and $P_c$ is obtained:

   $$PP_i = \begin{pmatrix} - & \ldots & pp_i^{1n} \\ \vdots & \ddots & \vdots \\ pp_i^{n1} & \ldots & - \end{pmatrix}$$

   Proximity values $pp_i^{lk} \in [0, 1]$ are obtained for each pair $(x_l, x_k)$ as follows:

   $$pp_i^{lk} = 1 - \left| \frac{cv_i^{lk} - p_c^{lk}}{g} \right| \quad (11)$$

   Proximity values are used to identify the furthest preferences from the collective opinion, which should be modified by some decision makers.

   b) Identify preferences to be changed ($CC$): Pairs of alternatives $(x_l, x_k)$ whose consensus degrees $ca^l$ and $cp^{lk}$ are not enough, are identified:

   $$CC = \{(x_l, x_k) | ca^l < cr \wedge cp^{lk} < cr\} \quad (12)$$

   Afterwards, the model identifies decision makers who should change their opinions on each of these pairs, i.e. those experts, $e_i$, whose assessment $p_i^{lk}$ on $(x_l, x_k) \in CC$ whose $cv_i^{lk}$ is furthest to $p_c^{lk}$. To do so, an average proximity $\overline{pp}^{lk}$ is calculated, by using an aggregation operator $\lambda$:

   $$\overline{pp}^{lk} = \lambda(pp_1^{lk}, \ldots, pp_m^{lk}) \quad (13)$$

   As a result, decision makers, $e_i$, whose $pp_i^{lk} < \overline{pp}^{lk}$ are advised to modify their assessments $p_{ij}^{lk}$ on $(x_l, x_k)$.

   c) Establish change directions: Several direction rules are applied to suggest the direction of changes proposed to decision makers, in order to increase the level of agreement in the following rounds. An acceptability threshold $\varepsilon \geq 0$, which should take a positive value close to zero, is used to allow a margin of acceptability when $cv_i^{lk}$ and $p_c^{lk}$ are close to each other.

   - DIR.1: If $(cv_i^{lk} - p_c^{lk}) < -\varepsilon$, then $e_i$ should *increase* his/her assessment $p_i^{lk}$ on $(x_l, x_k)$.
   - DIR.2: If $(cv_i^{lk} - p_c^{lk}) > \varepsilon$, then $e_i$ should *decrease* his/her assessment $p_i^{lk}$ on $(x_l, x_k)$.
   - DIR.3: If $-\varepsilon \leq (cv_i^{lk} - p_c^{lk}) \leq \varepsilon$ then $e_i$ should not modify his/her assessment $p_i^{lk}$ on $(x_l, x_k)$.

## IV. Conclusions

In decision making is usual that a group of decision makers take part in the decision solving process. In Group Decision Making (GDM) problems decision makers often express their preferences by using single values. In qualitative settings, decision makers use single linguistic terms. But sometimes there are decision situations in which decision makers hesitate among different linguistic terms and they would prefer to use comparative linguistic expressions close to human beings cognitive model. Such hesitant situations can be modelled by means of Hesitant Fuzzy Linguistic Term Sets (HFLTS) that facilitate the elicitation of linguistic preferences by using comparative linguistic expressions. In this contribution, we have presented a consensus model that deals with HFLTS information in GDM to achieve agreed solutions in this type of GDM problems

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