A COMPARISON AMONG SYMBOLIC COMPUTATIONAL MODELS IN LINGUISTIC DECISION MAKING

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Decision Making (DM) problems are carried out in many areas of the real world in different situations. We focus on this contribution in DM problems under uncertainty where the information is vague and imprecise. In such a case, the use of linguistic information has provided successful results and implies processes of computing with words (CW). Different computational models have been proposed in the literature to deal with processes of CW. This contribution is aimed to compare different linguistic symbolic computational models applied to Linguistic Decision Making (LDM).

Keywords: decision making, linguistic information, 2-tuple.

1. Introduction

Decision making is a usual process for human beings and companies in different areas such as, planning, engineering, and so on. To solve this type of problem, classic decision theory provides determinist models and probabilistic models, but when in a DM problem appears non-probabilistic uncertainty, these models are not adequate. To manage uncertainty in such problems there exist different approaches and tools. One of them is the Fuzzy Linguistic Approach (FLA) [8]. That has provided successful results on DM problems under uncertainty. The use of linguistic information implies processes of CW. There exist two classical linguistic computational models to accomplish them: the model based on the extension principle (semantic model) [1] and the Symbolic model [7]. The latter presents results closer to the cognitive model of human beings. But, it presents drawbacks related to its lack of precision and computations. In order to improve these drawbacks different symbolic approaches based on the FLA have been defined to improve the classical one [2,4,6].

In this contribution we carry out a comparative analysis of the more widely used symbolic computational models as the 2-Tuple [2], the Virtual linguistic [6] and the Proportional 2-tuple [4]. The aim in this study is to investigate if the FLA is the basis of all of them as they claimed, and which one is more appropriated in each decision situation.

This paper is structured as follows: in Section 2, we introduce some preliminaries. In Section 3, we review different symbolic computational models. In Section 4, a comparative analysis among them in a LDM problem is carried out, and finally we point out some concluding remarks.

2. Preliminaries

Here we review briefly the FLA and its classical computational models. We also present a scheme of a LDM problem and a general solving process.

2.1. *Fuzzy Linguistic Approach*

We usually work on quantitative environments where the information is expressed by means of numerical values. However, many aspects of the real world cannot be assessed in this form, but rather in a qualitative way with vague and imprecise knowledge. In this case, a better approach could be to use linguistic assessments instead of numerical values. The FLA represents qualitative aspects by means of linguistic variables [8]. In order to model linguistically the information we have to choose the appropriate linguistic descriptors for the linguistic term set and their semantics. To do so, there exist different possibilities [7]. One of them consists of supplying directly the term set by considering all the terms distributed on a scale which has an order defined [7]. For example, a set of seven terms S could be:

$$S = \{s_0 : nothing\ (n), s_1 : very\ low\ (vl), s_2 : low\ (l), s_3 : medium\ (m), \\ s_4 : high\ (h), s_5 : very\ high\ (vh), s_6 : perfect\ (p)\}$$

In these cases, it is required that in the linguistic term set there exists the following operators:

(1) Negation: $\mathrm{Neg}(s_i) = s_j$ such that $j = g - i$ ($g + 1$ is the cardinality)
(2) Maximization: $\max(s_i, s_j) = s_i\ if\ s_i \geq s_j$
(3) Minimization: $\min(s_i, s_j) = s_i\ if\ s_i \leq s_j$

The semantics of the terms are represented by fuzzy numbers defined in the interval $[0, 1]$ described by membership functions. A way to characterize a fuzzy number is to use a representation based on parameters of its membership function [2].

2.2. *Classical Linguistic Computational Models*

We aforementioned that the use of linguistic variables implies processes of CW. To carry out these computations based on the FLA were developed two computational models:

- Semantic model [1]: This model computes with linguistic terms by means of operations associated to their membership functions based on the Extension Principle. So, the obtained results are fuzzy numbers that usually do not match with the initial linguistic terms.
- Symbolic Model [7]: This model uses the ordered structure of the linguistic terms set $S = \{s_0, \ldots, s_g\}$ where $s_i < s_j$ if $i < j$, to operate. The results are numeric values which will be approximated to a numeric value which indicates the index of the associated linguistic term.

Both of them produce loss of information due to the necessity of an approximation process and hence a lack of precision in the results.

2.3. *Linguistic Decision Making Problem*

A LDM problem consists of a finite set of alternatives, $X = \{x_1, ..., x_n\}$, in which a set of experts, $P = \{p_1, \ldots, p_m\}$, express their evaluations on a linguistic term set, $x_i^j \in S$, $i = 1, \ldots, n$ $j = 1, ..., m$ in order to select the best alternative of the problem.

A solution general schema of a DM problem consists of two phases [3]: (i) an aggregation phase that obtains collective valuations of each alternative and (ii) an exploitation phase that obtains the solution set of alternatives of the problem (see Fig. 1) .

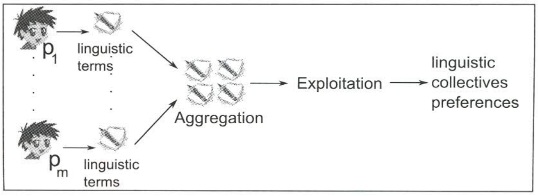

Fig. 1. Decision making schema

3. Recent Proposals

The need of dealing with linguistic information in LDM has led to new models to improve the processes of CW. Here we review in short the representation and the operational laws of the most wide-use symbolic models such as: the 2-tuple model [2], the virtual model [6] and the proportional 2-tuple model [4].

3.1. *2-tuple Linguistic Model*

This model was presented in [2] in order to improve the precision in processes of CW.

(a) *Representation model*: The linguistic information is represented by means of a pair of values (s_i, α), where s_i is a linguistic term and

α a numerical value which represents the symbolic translation [2]. This linguistic model defines a set of functions Δ and Δ^{-1} in order to carry out transformations between numeric values and 2-tuples [2].

(b) *Computational model*: It is based on the functions Δ and Δ^{-1}. In [2] were defined different operations as comparison, negation and aggregation operators.

3.2. *Virtual Linguistic Model*

This model was proposed by Xu in [6] in order to avoid the loss of information and increase the number of operators in processes of CW.

(a) *Representation model*: This model extends the discrete term set S to a continuous linguistic term set $\bar{S} = \{s_\alpha | s_l < s_\alpha \leq s_t, \alpha \in [1, t]\}$, where, if $s_\alpha \in S$, s_α is called *an original linguistic term*, otherwise, s_α is called *virtual linguistic term* which does not have assigned any semantics.

(b) *Computational model*: To carry out processes of CW, Xu presented several operations [5] that extends the previous ones of the 2-tuple.

3.3. *Proportional 2-tuple Linguistic Model*

This model presented in [4] extends and generalizes the 2-tuple model [2].

(a) *Representation model*: This model represents the linguistic information by means of a proportional 2-tuple, such as $(0.3A, 0.7B)$ that means a value of 30%A and 70%B. The authors remark that if B were used as the approximative grade then some performance information would be lost. This approach is based on the concept of *symbolic proportion* [4]. In the same way to [2], Wang and Hao defined the functions π and π^{-1} to facilitate the operations with this type of representation.

(b) *Computational model*: This model defined the following operators: comparison, negation and aggregation operators [4].

4. Comparative Analysis

Here we present and to solve a LDM problem with the models presented in Section 3 and make a comparative study among them.

4.1. *Decision Making Problem*

Let a company be that wants to renew the screens of their programming employees' computers, $P = \{p_1, p_2, p_3, p_4\}$. To do so, experts (employees) give their opinions on the set of alternatives, $X = \{x_1 : TFT\ 17", x_2 : TFT\ 19", x_3 : TFT\ 21", x_4 : TFT\ 23"\}$, assessed in the linguistic term, $S = \{s_0 : n, s_1 : vl, s_2 : l, s_3 : m, s_4 : h, s_5 : vh, s_6 : p\}$. Each employee provides a preference vector:

		alternatives			
	μ_{ij}	x_1	x_2	x_3	x_4
experts	p_1	l	h	h	l
	p_2	m	h	l	n
	p_3	h	vh	h	vl
	p_4	h	h	vh	l

where $\mu_{i,j}$ is the degree of preference that the expert, p_i, provides over the alternative, x_j.

Using the solving process presented in Fig. 1 and the different symbolic models reviewed in Section 3, we obtain the results showed in Table 1. The best alternative is always x_2, but if we observe the Table 1, we notice that there exist important differences that we analyze below.

Table 1. Solution of the LDM problem with different symbolic models.

	$\overline{x}_1^e$	$\overline{x}_2^e$	$\overline{x}_3^e$	$\overline{x}_4^e$
2-Tuple	$(m, .25)$	**(vh,.25)**	$(h, -.25)$	$(vl, .25)$
Virt. Ling.	$(s_{3.25})$	$(\mathbf{s_{5.25}})$	$(s_{3.75})$	$(s_{1.25})$
Prop. 2-T	$(0.75m, 0.25h)$	**(0.75vh,0.25p)**	$(0.25m, 0.75h)$	$(0.75vl, 0.25l)$

4.2. *Comparative Survey*

The analysis of the previous results points out different features of the symbolic models regarding their representation, accuracy and comprehension. Besides, it shows that not all of them are based on the FLA despite their authors claim that origin.

- ***Fuzzy representation***: the virtual linguistic model is not based on the FLA, because its linguistic variables do not use either semantic or syntaxis, as it is defined in [8]. The proportional 2-tuple does not keep either a fuzzy representation, because it uses the proportions (no semantics) of two consecutive linguistic terms to represent the result. The 2-tuple model is the only one that keeps a fuzzy representation of the linguistic information.
- ***Accuracy***: the computational model of the virtual model is accuracy in any term set, because it does not use semantics. Besides, it can use values which are out of the universe of discourse. The others just can obtain values in the universe of discourse. The 2-tuple model is accurate when the labels are symmetrically distributed and the proportional 2-tuple when the terms have the same width in their support.
- ***Comprehension***: by comparing the previous results, we observe that the virtual model obtains values difficult to understand because are not

linguistic. However, the 2-tuple model and proportional 2-tuple offer qualitative results easy to understand, though the latter is a little bit more complex, because it uses four values to express just one valuation.

As we aforementioned, the 2-tuple model is the only model based on FLA, since keeps a syntaxis and fuzzy semantics. It is accurate and easy to understand. Therefore, it seems the most adequate symbolic model to deal with linguistic information in DM problems.

5. Concluding Remarks

In this contribution we have reviewed the 2-tuple, the virtual and the proportional 2-tuple symbolic models in order to carry out a comparative analysis among them in LDM. In such analysis, we have showed that the 2-tuple is the only model based on the FLA, because it keeps the syntaxis and fuzzy semantics in its results. While that the results obtained by the others do not present any semantics. Additionally, the 2-tuple is accurate and easy to understand.

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