A FUZZY ENVELOPE FOR HESITANT FUZZY LINGUISTIC TERM SETS BASED ON CHOQUET INTEGRAL

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Recently, it has been presented the concept of Hesitant Fuzzy Linguistic Term Sets (HFLTS) to manage hesitant situations under qualitative settings in which experts hesitate among different linguistic terms to express their preferences or assessments. It was also defined the concept of envelope for an HFLTS to carry out the computational processes with them. However, this envelope does not keep the fuzzy representation because it is represented by a symbolic linguistic interval. Therefore, in this contribution we propose a new envelope for HFLTS based on Choquet integral that keeps the fuzzy representation in the computational processes with HFLTS.

1. Introduction

Decision problems defined in context with uncertainty are quite common in real world. Our interest in this contribution is focused on decision situations dealing with qualitative aspects in which the uncertainty is because of the vagueness of meanings of the qualitative values provided by the decision makers. The fuzzy linguistic approach [7] has obtained successful results in the modeling and management of this type of uncertainty. The use of linguistic information in

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decision making implies processes of computing with words (CWW) [3, 4] that can be accomplished by different linguistic computing models [3]. Nevertheless, such models present some limitations in qualitative settings when experts prefer using multiple linguistic terms to express their preferences or assessments. In order to manage these hesitant situations, Rodríguez et al. proposed the concept of hesitant fuzzy linguistic term set (HFLTS) [6] to facilitate the elicitation of linguistic information by using comparative linguistic terms. To carry out computational processes in decision making with HFLTS was introduced the concept of envelope of HFLTS that consists of a linguistic interval. This envelope manages in a symbolic way the HFLTS, therefore it does not keep the fuzzy representation. Hence, in this contribution we propose a new HFLTS envelope based on the Choquet integral [5] that computes an aggregated fuzzy number (fuzzy envelope) that represents the hesitant information of an HFLTS. This fuzzy envelope can be strongly used in fuzzy decision models, such as, fuzzy AHP [1], fuzzy TOPSIS [2], etc.

The remaining of the paper is structured as follows: Section 2 revises some basic concepts. Section 3 proposes a fuzzy HFLTS envelope by using the fuzzy Choquet integral and Section 4 points out with some conclusions.

2. Preliminaries

In this section, we review the elicitation of comparative linguistic terms represented by HFLTS and introduce some basic concepts about the fuzzy Choquet integral.

2.1. *Elicitation of Comparative Linguistic Terms*

Rodríguez et al. introduced the concept of HFLTS [6] to manage decision situations in qualitative contexts when experts hesitate among different linguistic terms to express their preferences or assessments.

Definition 1. *[6]* An HFLTS, H_S, is an ordered finite subset of consecutive linguistic terms of S, where $S=\{s_0,..,s_g\}$ is a linguistic term set.

Example 1. Let S be a linguistic term set such as, S={s_0:null satisf., s_1:very low satisf., s_2:low satisf., s_3:medium satisf., s_4:high satisf., s_5:very high satisf., s_6:perfect satisf.} and X a linguistic variable, therefore an HFLTS might be:

$$H_S(X)=\{\text{very low satisf., low satisf., medium satisf.}\}.$$

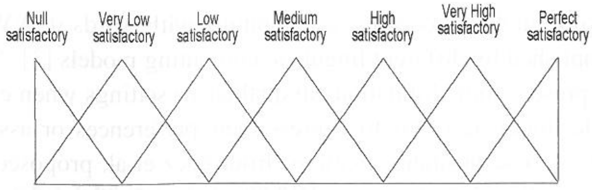

Figure 1: 7-linguistic labels term set

Nevertheless, experts usually do not express their preferences by using multiple linguistic terms, but rather expressions in natural language. Thus, Rodríguez et al. defined a context-free grammar G_H [6], that uses a linguistic term set S, based on fuzzy linguistic approach [7], as is shown in Figure 1, to generate expressions with comparative linguistic terms ll, such as, *greater than* s_i, *lower than* s_i, *between* s_i *and* s_j ($s_i, s_j \in S$).

To obtain an HFLTS from comparative linguistic terms ll, generated by the context-free grammar G_H, was defined the transformation function E_{G_H}.

Definition 2. *[6]* Let E_{G_H} be a function that transforms comparative linguistic terms, ll, obtained by G_H, into HFLTS, , where is the linguistic term set used by G_H.

$$E_{G_H} : S_{ll} \rightarrow H_S .$$

To facilitate the computations with HFLTS was introduced the concept of *envelope* of a HFLTS.

Definition 3. *[6]* The *envelope* of a HFLTS, $env(H_S)$, is a linguistic interval whose limits are obtained by means of upper bound (*max*) and lower bound (*min*):

$$env(H_S) = [H_{S^-}, H_{S^+}], \quad H_{S^-} \leq H_{S^+} \tag{1}$$

where the upper bound is defined as $H_{S^+} = max(s_i)$ and the lower bound is defined as $H_{S^-} = min(s_i)$, $s_i \in H_S$ and $i \in \{1,\ldots,g\}$.

Example 2. Following the previous example the envelope of H_S is:

$$env(H_S) = [\text{very low satisfactory, medium satisfactory}].$$

As we can observe in the Example 2, the envelope obtained is represented by a linguistic interval that for computations is managed in a symbolic way losing the fuzzy representation. Therefore, in this contribution we propose a fuzzy HFLTS envelope based on the fuzzy Choquet integral for computing with HFLTS. But first, some basic concepts about the Choquet integral are reviewed.

2.2. *Choquet Integral*

Definition 4. *[5]* Let $C=\{c_1,\ldots,c_n\}$ be a finite universe. A *fuzzy measure* or *capacity* is a set function $\nu : P(C)\rightarrow[0,1]$ which satisfies:

1. $\nu(\varnothing)=0$ and $\nu(C)=1$.
2. $A\subseteq B \Rightarrow \nu(A)\le \nu(B)$,

where $P(C)$ is the set of all subsets of C.

Definition 5. [5] Let ν be a fuzzy measure on C, the *Möbius* representation of ν is a set function $m_\nu : P(C)\rightarrow R$ given by

$$m_\nu(S)=\sum_{T\subseteq S}(-1)^{|S|-|T|}\ \nu(T)\,,\qquad \forall S\subseteq C. \tag{2}$$

Remark 1. Once known the *Möbius* representation is easy to recover the fuzzy measure from which was derived

$$\nu(T)=\sum\nolimits_{S\subseteq T} m_\nu(S),\quad \forall T\subseteq C\,.$$

This property allows to rewrite the Choquet integral by using only the Möbius representation via

$$F_\nu\left(y_1,\ldots,y_n\right)=\sum_{T\subseteq C} m_\nu(T)\bigwedge_{i\in T} y_i. \tag{3}$$

3. A Fuzzy Envelope for HFLTS

Here, we propose a fuzzy HFLTS envelope based on the fuzzy Choquet integral that increases the flexibility and richness of the computations with HFLTS due to the fact that the use of the fuzzy Choquet integral allows capture and keeps all the subjectivity of HFLTS and the interaction among the linguistic terms in it. To do so, we use fuzzy arithmetic based on extension principle of Zadeh [7] and a fuzzy extension of the Choquet integral defined in [5]:

Definition 6. Let ν be a fuzzy measure on C and m_ν the Möbius representation of ν, the fuzzy Choquet integral is defined by

$$F_\nu\left(y_1,\ldots,y_n\right)=\sum_{T\subseteq C}\left(m_\nu\left(T\right)\tilde{\cdot}\bigwedge_{i\in T} y_i\right) \tag{4}$$

where it is necessary to use the following operations.

Let y_1, y_2 two fuzzy numbers and μ_{y_1} and μ_{y_2} their membership functions, respectively.

- $y_1 \tilde{+} y_2$ is a fuzzy number with membership function

$$\mu_{y_1 \tilde{+} y_2}(x) = \sup_{a+b=x}\left(\min\{\mu_{y_1}(a) + \mu_{y_2}(b)\}\right)$$

- $\wedge(y_1, y_2)$ is a fuzzy number with membership function

$$\mu_{\wedge(y_1, y_2)}(x) = \sup_{a+b=x}\left(\min\{\mu_{y_1}(a), \mu_{y_2}(b)\}\right)$$

- $\tilde{x} \,.\, \tilde{y}$ denote the product of a crisp number x by a trapezoidal fuzzy number $\tilde{y}$. The last is represented in the usual way by $(a_{\tilde{y}}, b_{\tilde{y}}, c_{\tilde{y}}, d_{\tilde{y}})$. $\tilde{x} \,.\, \tilde{y}$ is a trapezoidal fuzzy number given by $(xa_{\tilde{y}}, xb_{\tilde{y}}, xc_{\tilde{y}}, xd_{\tilde{y}})$

Therefore, the definition of the fuzzy envelope for HFLTS is as follows,

Definition 7. Let V be a fuzzy measure on H_S and m_v the *Möbius* representation of v. The *fuzzy Choquet integral envelope* of a HFLTS, $env_v(H_S)$, is a fuzzy number given by:

$$env_v(H_S) = \sum_{T \subseteq H_S}\left(m_v(T) \tilde{\cdot} \wedge_{i \in T} s_i\right) \tag{5}$$

where s_i is the corresponding fuzzy number that represents the semantics of the linguistic term s_i, for all $i \in H_S$.

Example 3. Let see how is the new HFLTS envelope of the H_S for the example 1 with two particular fuzzy measures v_1 and v_2. The Figure 2 shows a new HFLTS envelope for v_1 with $v_1(s_1) = v_1(s_2) = v_1(s_3) = 0.1$, $v_1(s_1, s_3) = 0.3$, $v_1(s_1, s_2) = v_1(s_2, s_3) = 0.5$, and the Figure 3 shows a new HFLTS envelope for v_2 with $v_2(s_1) = 0.095$, $v_2(s_2) = 0.189$, $v_2(s_3) = 0.568$, $v_2(s_1, s_2) = 0.298$, $v_2(s_2, s_3) = 0.842$, $v_2(s_1, s_3) = 0.705$.

We can note that the use of this new envelope based on Choquet integral improves the previous one because it not only represents the limits of the HFLTS but also it considers all the linguistic terms included in it and their relationship.

4. Conclusions

This contribution has introduced a fuzzy envelope based on the fuzzy Choquet integral to facilitate the computing processes in those decision problems managing HFLTS. This new fuzzy envelope provides a fuzzy representation of the multiple linguistic terms belong to a HFLTS including their relationship.

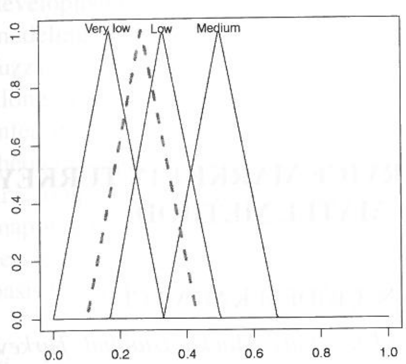

Figure 2: Fuzzy Choquet integral envelope by using the fuzzy measure V_1

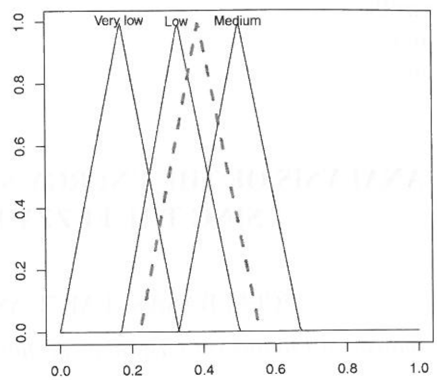

Figure 3: Fuzzy Choquet integral envelope by using the fuzzy measure V_2

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