Group Decision Making with Comparative Linguistic Terms

Rosa M. Rodríguez[1], Luis Martínez[1], and Francisco Herrera[2]

[1] University of Jaén, Computer Science Department
23071, Jaén, Spain
{rmrodrig,martin}@ujaen.es
[2] University of Granada, Computer Science and A.I. Department
18071, Granada, Spain
herrera@decsai.ugr.es

Abstract. In group decision making (GDM) framework, we focus on decision problems defined under uncertainty where decision makers can hesitate among several values to elicit their preferences. In such cases, the use of hesitant fuzzy linguistic term sets (HFLTS) can facilitate the elicitation of decision makers preferences. In this contribution, our aim is to propose a linguistic GDM model that allows to decision makers use single linguistic terms or comparative linguistic terms to express their preferences and obtain the solution set of alternatives of the GDM problem.

Keywords: Group decision making, hesitant fuzzy linguistic term sets, comparative linguistic terms, context-free grammar.

1 Introduction

Decision making is a usual process for human beings and companies in different areas such as, engineering [10], planning [20], etc. In decision making problems with multiple experts, each expert expresses his/her preferences depending on the nature of the alternatives and on his/her own knowledge over them. Usually, this knowledge is vague and imprecise. In such cases, the fuzzy logic [8] and fuzzy linguistic approach [18] provide suitable tools to deal with this type of uncertainty. The use of linguistic information implies to carry out processes of computing with words (CWW) [11,19]. There are different linguistic computing models to accomplish such processes [5,9,15]. However, such approaches are limited to model qualitative settings where decision makers hesitate among different values, because they are thinking of several linguistic terms to provide their preferences.

Torra introduced the concept of hesitant fuzzy sets [14] to manage situations in quantitative settings, when decision makers hesitate among different values to determine the membership of an element into a set. In qualitative settings it may occur a similar situation, decision makers hesitate among different linguistic

S. Greco et al. (Eds.): IPMU 2012, Part I, CCIS 297, pp. 181–190, 2012.

terms. Rodríguez et al. proposed the concept of HFLTS [12] to facilitate the elicitation of such linguistic information by comparative linguistic terms.

The aim of this contribution is to develop a linguistic GDM model capable to manage hesitant information by means of comparative linguistic terms represented by HFLTS. These comparative terms facilitate the elicitation of linguistic information to decision makers in hesitant situations. The proposed GDM model will manage this type of information by using linguistic intervals.

This paper is structured as follows: Section 2, introduces a basic scheme of a GDM problem and makes a brief review about fuzzy linguistic approach. Section 3, revises the elicitation of comparative linguistic terms represented by HFLTS. Section 4, presents a linguistic GDM model that deals with comparative linguistic terms. Section 5 shows an illustrative example of a GDM problem, and finally, Section 6 points out some concluding remarks.

2 Preliminaries

This section introduces a basic scheme for a GDM problem and reviews the fuzzy linguistic approach basis of the HFLTS.

2.1 Scheme of a Group Decision Making Problem

A GDM problem is defined as a decision situation where a finite set of experts, $E = \{e_1, \ldots, e_m\}$ $(m >= 2)$, express their preferences over a finite set of alternatives, $X = \{x_1, \ldots, x_n\}, (n >= 2)$ to obtain a solution set of alternatives for the decision problem [7]. Usually, each expert, e_k, provides her/his preferences on X by means of a preference relation P^k, $\mu_{P^k} : X \times X \longrightarrow D$,

$$P^k = \begin{pmatrix} p^k_{11} & \cdots & p^k_{1n} \\ \vdots & \ddots & \vdots \\ p^k_{n1} & \cdots & p^k_{nn} \end{pmatrix}$$

where each assessment, $\mu_{P^k}(x_i, x_j) = p^k_{ij}$, represents the degree of preference of the alternative x_i over x_j according to expert e_k.

Usually, GDM problems have been solved performing a selection process where experts obtain the best alternative from their preferences [13]. The selection process consists of two phases (see Fig.1).

- *Aggregation phase*: the experts preferences are aggregated to obtain a collective preference matrix that reflects the preferences provided by all experts.
- *Exploitation phase*: it selects the best alternative/s to solve the decision problem by ranking the collective preferences obtained in the previous phase by using a choice function [3].

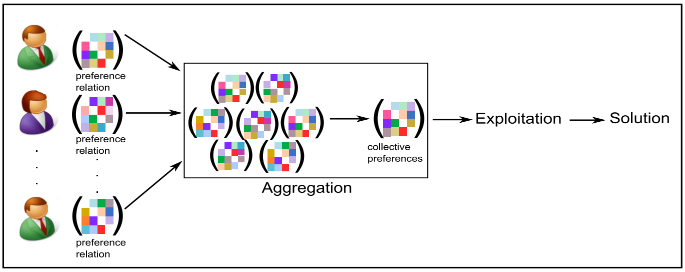

Fig. 1. General schema of a group decision making problem

2.2 Fuzzy Linguistic Approach

The fuzzy linguistic approach [18] represents qualitative settings by means of linguistic variables. The concept of linguistic variable was introduced by Zadeh [18] as "a variable whose values are not numbers but words or sentences in a natural or artificial language". To model linguistically the information is necessary to choose the appropriate linguistic descriptors for the linguistic term set and their semantics. To do so, there are different possibilities [16]. We will use one of them that consists of applying directly the term set by considering all the terms distributed on a scale that has an order defined [16]. In these cases, it is required that in the linguistic term set there are the following operators:

1. Negation: $\text{Neg}(s_i) = s_j$ with $j = g$-i (g+1 is the granularity of the term set).
2. Maximization: $\text{Max}(s_i, s_j) = s_i$ if $s_i \geq s_j$.
3. Minimization: $\text{Min}(s_i, s_j) = s_i$ if $s_i \leq s_j$.

The semantics of the terms is represented by fuzzy numbers defined in the interval [0,1], described by membership functions [1].

We aforementioned that the use of linguistic information implies processes of CWW. To perform these computations in the fuzzy linguistic approach appeared two classical computational models:

– Semantic model that computes with linguistic terms by means of operations associated to their membership functions based on the Extension Principle [2].
– Symbolic model that uses the ordered structure of the linguistic terms to operate [16].

Symbolic models have been widely used in decision making because of their simplicity and understandability. In this contribution, we will use a symbolic model in the proposal for the GDM model.

3 Elicitation of Comparative Linguistic Terms

Our interest is focused on GDM problems under uncertainty where decision makers may hesitate among different values to assess qualitative settings. To manage such a situation, we propose the use of comparative linguistic terms represented by HFLTS. In [12] Rodríguez et al. defined the following context-free grammar to generate expressions with comparative linguistic terms.

Definition 1. *[12] Let G_H be a context-free grammar and $S = \{s_0, \dots, s_g\}$ a linguistic term set. The elements of $G_H = (V_N, V_T, I, P)$ are defined as follows:*

$V_N = \{\langle primary\, term\rangle, \langle composite\, term\rangle, \langle unary\, relation\rangle, \langle binary\, relation\rangle, \langle conjunction\rangle\}$

$V_T = \{lower\ than, greater\ than, between, and, s_0, s_1, \dots, s_g\}$

$I \in V_N$

The production rules are defined in an extended Backus Naur Form so that the brackets enclose optional elements and the symbol | indicates alternative elements [1]. For the context-free grammar, G_H, the production rules are the following:

$P = \{I ::= \langle primary\ term\rangle | \langle composite\ term\rangle$

$\langle composite\ term\rangle ::= \langle unary\ relation\rangle\langle primary\ term\rangle | \langle binary\ relation\rangle \langle primary\ term\rangle | \langle conjunction\rangle\langle primary\ term\rangle$

$\langle primary\ term\rangle ::= s_0|s_1|\dots|s_g$

$\langle unary\ relation\rangle ::= lower\ than|greater\ than$

$\langle binary\ relation\rangle ::= between$

$\langle conjunction\rangle ::= and\}$

These linguistic expressions are represented by HFLTS.

Definition 2. *[12] An HFLTS, H_S, is an ordered finite subset of consecutive linguistic terms of S, where $S = \{s_0, \dots, s_g\}$ is a linguistic term set.*

For example, let $S = \{nothing, very\_low, low, medium, high, very\_high, perfect\}$ be a linguistic term set and X an alternative, an HFLTS might be:

$$H_S(X) = \{high, very\_high, perfect\}$$

To obtain HFLTS from the comparative linguistic terms generated by the context-free grammar G_H, was defined the transformation function E_{G_H}.

Definition 3. *[12] Let E_{G_H} be a function that transforms linguistic expressions, ll, obtained by G_H, into HFLTS, H_S, where S is the linguistic term set used by G_H.*

$$E_{G_H} : S_{ll} \longrightarrow H_S \tag{1}$$

In decision making is often to carry out comparisons between values. The comparison between two HFLTS is complex, because an HFLTS is a set of linguistic terms. Therefore, to compare two HFLTS was introduced the concept of envelope of an HFLTS.

Definition 4. *[12] The envelope of a HFLTS, $env(H_S)$, is a linguistic interval whose limits are obtained by means of upper bound (*max*) and lower bound (*min*):*

$$env(H_S) = [H_{S^-}, H_{S^+}], \quad H_{S^-} \leq H_{S^+} \tag{2}$$

where

$H_{S^+} = max(s_i) = s_j,\ s_i \in H_S\ and\ s_i \leq s_j\ \forall i\ and$
$H_{S^-} = min(s_i) = s_j,\ s_i \in H_S\ and\ s_i \geq s_j\ \forall i$

Following the previous example, $H_S(X) = \{high, very\_high, perfect\}$, its envelope is:

$$env(H_S) = [high, perfect]$$

Once obtained the envelopes of HFLTS, the comparison is conducted by interval values. Different approaches can be applied to carry out such comparison [12]. More operations with HFLTS and properties can be found in [12].

4 Linguistic Group Decision Making Model Dealing with Comparative Linguistic Terms

The aim of this contribution is to propose a linguistic GDM model that copes with hesitant situations in qualitative settings in which decision makers provide linguistic information by means of single linguistic terms or comparative linguistic terms. This model based on the classical symbolic model uses the indexes of the linguistic term set to operate across the decision making process. It extends the decision resolution scheme shown in Fig. 1 adding a phase to manage linguistic information by means of HFLTS. It consists mainly of three phases (see Fig. 2):

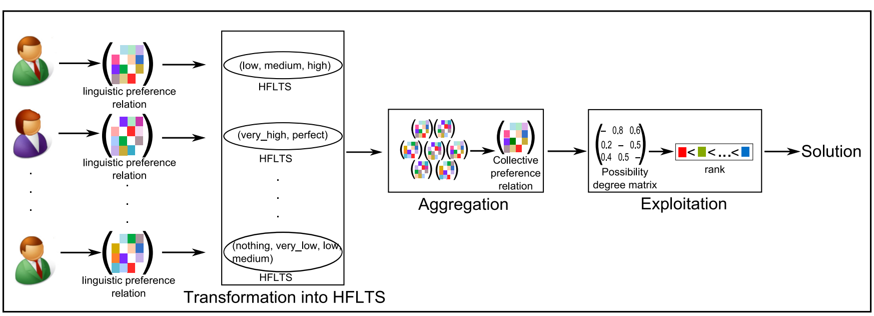

Fig. 2. Scheme of the linguistic group decision making model

1. Transformation of the comparative linguistic terms preference relations into HFLTS
   Experts provide their preference relation, P^k, by using single linguistic terms or comparative linguistic terms, $\mu_{P^k} : X \times X \longrightarrow S_{ll}$,

$$P^k = \begin{pmatrix} p_{11}^k & \dots & p_{1n}^k \\ \vdots & \ddots & \vdots \\ p_{n1}^k & \dots & p_{nn}^k \end{pmatrix}$$

where each assessment $p_{ij}^k \in S_{ll}$, represents the preference degree of the alternative x_i over x_j according to expert e_k, expressed in the information domain S_{ll}. To solve the GDM problem the comparative linguistic terms are transformed into HFLTS by means of the transformation function E_{G_H}. Afterwards, it is computed an envelope for each HFLTS that obtains a linguistic interval that will be used to aggregate the preferences provided by experts, $env(H_S(p_{ij}^k)) = [p_{ij}^{k-}, p_{ij}^{k+}]$,

$$P^k = \begin{pmatrix} [p_{11}^{k-}, p_{11}^{k+}] & \dots & [p_{1n}^{k-}, p_{1n}^{k+}] \\ \vdots & \ddots & \vdots \\ [p_{n1}^{k-}, p_{n1}^{k+}] & \dots & [p_{nn}^{k-}, p_{nn}^{k+}] \end{pmatrix}$$

2. Aggregation of the preference relations represented by linguistic intervals
   The linguistic intervals are aggregated to obtain a collective preference relation P_C. We use the LOWA aggregation operator [4] to aggregate the right limits, p_{ij}^{k+}, and the left limits, p_{ij}^{k-} of the intervals.

$$P_C = \begin{pmatrix} [p_{11}^{-}, p_{11}^{+}] & \dots & [p_{1n}^{-}, p_{1n}^{+}] \\ \vdots & \ddots & \vdots \\ [p_{n1}^{-}, p_{n1}^{+}] & \dots & [p_{nn}^{-}, p_{nn}^{+}] \end{pmatrix}$$

   where $i, j \in \{1, \dots, n\}$ and n is the number of alternatives.
3. Exploitation phase
   Once the linguistic intervals have been aggregated, the set of alternatives is ordered to select the best one/s. To do so, we use the approach proposed by Jiang [6] that deals with interval preference relations and obtains a ranking of alternatives based on numerical possibility degrees according to the following steps:
   (a) Firstly, it is calculated the mean preference relation $\bar{P}_C = (\bar{p}_{ij})_{n \times n}$, and the error matrix $\delta = (\delta_{ij})_{n \times n}$, that represents the mean distance of the limits of the intervals of P_C,

$$\bar{p}_{ij} = \frac{1}{2}(p_{ij}^- + p_{ij}^+) \tag{3}$$

$$\delta_{ij} = \frac{1}{2}(p_{ij}^+ - p_{ij}^-) \tag{4}$$

   where $i, j \in \{1, 2, \dots, n\}$

   Remark 1. We note that to deal with linguistic intervals symbolically, these functions are adapted, so $\bar{p}_{ij} = \frac{1}{2}(ind(p_{ij}^-) + ind(p_{ij}^+))$, $\delta_{ij} = \frac{1}{2}(ind(p_{ij}^+) - ind(p_{ij}^-))$; $ind(s_i) = i$.

   (b) Afterwards, it is used the error propagation principle [17] to obtain the priority vector $\bar{w} = (\bar{w}_1, \dots, \bar{w}_n)$ of the mean preference relation, $\bar{P}_C$.

$$\bar{w}_i = \frac{(\sum_{j=1}^{n} \bar{p}_{ij} + \frac{n}{2} - 1)}{n(n-1)} \quad i = 1, 2, \ldots, n \tag{5}$$

It is calculated an error vector $\Lambda w = (\Lambda w_1, \ldots, \Lambda w_n)$ of $\bar{w}$ due to the imprecise values of $\bar{p}_{ij}$, by using the following function.

$$\Lambda w_i = \frac{1}{n(n-1)} \sqrt{\sum_{j=1}^{n} \delta_{ij}^2}, \quad i = 1, 2, \ldots, n \tag{6}$$

And thus it is got the priority vector $w = (w_1, \ldots, w_n)^T$ of the collective matrix, P_C, where $w_i = [\bar{w}_i - \Lambda w_i, \bar{w}_i + \Lambda w_i], \quad i = 1, \ldots, n$.

(c) To rank these interval weights $w_i (i = 1, \ldots, n)$, each w_i is compared with all w_i by using the possibility degree function, and it is then built a possibility degree matrix $PD = (pd_{ij})_{n \times n}$.

$$pd_{ij} = p(\bar{w}_i \geq \bar{w}_j) = \frac{min(2(\Lambda w_i + \Lambda w_j), max(\bar{w}_i + \Lambda w_i - (\bar{w}_j - \Lambda w_j), 0))}{2(\Lambda w_i + \Lambda w_j)} \tag{7}$$

A non-dominance choice degree is applied to the possibility degrees to obtain the solution set of alternatives. To do so, the possibility degrees of the alternatives pd_{ij}, are summed by rows, and they are ranked in a descending order.

$$pd_i = \sum_{j=1}^{n} pd_{ij} \quad i = 1, \ldots, n \tag{8}$$

Finally, the alternatives are ordered according to pd_i and then the best alternative is selected.

5 Illustrative Example

Here, we present a GDM problem solved by the proposed GDM model.

Let a GDM problem be defined in qualitative settings where a set of experts, $E = \{e_1, e_2, e_3\}$, provide their preferences over a set of alternatives, $X = \{x_1, x_2, x_3, x_4\}$. Experts provide their preferences by using the comparative linguistic terms generated by the context-free grammar G_H, (see Def. 1). Such linguistic expressions are represented by HFLTS. The linguistic term set used for the context-free grammar is $S = \{nothing(n), very\_low(vl), low(l), medium(m), high(h), very\_high(vh), perfect(p)\}$ and the preference relations provided by the experts are the following ones:

$$P^1 = \begin{pmatrix} - & \textit{less than vl} & vh & \textit{more than h} \\ \textit{more than vl} & - & \textit{between h and vh} & \textit{less than m} \\ l & \textit{less than h} & - & \textit{more than vh} \\ \textit{less than vh} & \textit{more than h} & \textit{less than m} & - \end{pmatrix}$$

$$P^2 = \begin{pmatrix} - & less\ than\ m & more\ than\ h & between\ vl\ and\ l \\ more\ than\ h & - & h & vl \\ less\ than\ vh & l & - & more\ than\ vh \\ less\ than\ l & vh & between\ vh\ and\ p & - \end{pmatrix}$$

$$P^3 = \begin{pmatrix} - & more\ than\ h & between\ vl\ and\ l & h \\ less\ than\ vh & - & more\ than\ m & more\ than\ h \\ less\ than\ l & less\ than\ vh & - & vh \\ l & less\ than\ vh & vl & - \end{pmatrix}$$

According to the Fig. 2, the GDM process consists of:

1. *Transformation of the comparative linguistic terms preference relations into HFLTS*
   The linguistic preference relations provided by the experts are transformed into HFLTS by means of the transformation function E_{G_H}:

$$P^1 = \begin{pmatrix} - & \{n, vl\} & \{vh\} & \{h, vh, p\} \\ \{vl, l, m, h, vh, p\} & - & \{h, vh\} & \{n, vl, l, m\} \\ \{l\} & \{n, vl, l, m, h\} & - & \{vh, p\} \\ \{n, vl, l, m, h, vh\} & \{h, vh, p\} & \{n, vl, l, m\} & - \end{pmatrix}$$

$$P^2 = \begin{pmatrix} - & \{n, vl, l, m\} & \{h, vh, p\} & \{vl, l\} \\ \{h, vh, p\} & - & \{h\} & \{vl\} \\ \{n, vl, l, m, h, vh\} & \{l\} & - & \{vh, p\} \\ \{n, vl, l\} & \{vh\} & \{vh, p\} & - \end{pmatrix}$$

$$P^3 = \begin{pmatrix} - & \{h, vh, p\} & \{vl, l\} & \{h\} \\ \{n, vl, l, m, h, vh\} & - & \{m, h, vh, p\} & \{h, vh, p\} \\ \{n, vl, l\} & \{n, vl, l, m, h, vh\} & - & \{vh\} \\ \{l\} & \{n, vl, l, m, h, vh\} & \{vl\} & - \end{pmatrix}$$

   The envelopes obtained for each HFLTS are the following ones:

$$P^1 = \begin{pmatrix} - & [n, vl] & [vh, vh] & [h, p] \\ \text{[vl,p]} & - & [h, vh] & [n, m] \\ \text{[l,l]} & [n, h] & - & [vh, p] \\ \text{[n,vh]} & [h, p] & [n, m] & - \end{pmatrix} P^2 = \begin{pmatrix} - & [n, m] & [h, p] & [vl, l] \\ \text{[h,p]} & - & [h, h] & [vl, vl] \\ \text{[n,vh]} & [l, l] & - & [vh, p] \\ \text{[n,l]} & [vh, vh] & [vh, p] & - \end{pmatrix}$$

$$P^3 = \begin{pmatrix} - & [h, p] & [vl, l] & [h, h] \\ \text{[n,vh]} & - & [m, p] & [h, p] \\ \text{[n,l]} & [n, vh] & - & [vh, vh] \\ \text{[l,l]} & [n, vh] & [vl, vl] & - \end{pmatrix}$$

2. *Aggregation of the preference relations represented by linguistic intervals*
   The linguistic intervals are aggregated by using the LOWA operator to obtain the collective preferences matrix,

$$P_C = \begin{pmatrix} - & [vl, m] & [h, vh] & [m, vh] \\ \text{[l,p]} & - & [h, p] & [m, h] \\ \text{[vl,h]} & [vl, h] & - & [vh, p] \\ \text{[vl,m]} & [h, p] & [l, m] & - \end{pmatrix}$$

3. *Exploitation phase*
Once obtained the collective preferences from experts, it is used the approach proposed by Jiang [6] to obtain the solution set of alternatives.
   a) Mean preference relation $\bar{P}_C$, and error-matrix δ, of the collective preference relation P_C:

$$\bar{P}_C = \begin{pmatrix} - & 2 & 4.5 & 4 \\ 4 & - & 5 & 3.5 \\ 2.5 & 2.5 & - & 5.5 \\ 2 & 5 & 2.5 & - \end{pmatrix} \delta = \begin{pmatrix} - & 1 & 0.5 & 1 \\ 2 & - & 1 & 0.5 \\ 1.5 & 1.5 & - & 0.5 \\ 1 & 1 & 0.5 & - \end{pmatrix}$$

   b) Priority vector $\bar{w}$, and error vector Λw:

$$\bar{w} = (0.958, 1.125, 0.958, 0.875)$$
$$\Lambda w = (0.125, 0.191, 0.182, 0.125)$$

   c) Possibility degree matrix PD:

$$PD = \begin{pmatrix} - & 0.236 & 0.5 & 0.667 \\ 0.764 & - & 0.723 & 0.895 \\ 0.5 & 0.276 & - & 0.635 \\ 0.333 & 0.104 & 0.364 & - \end{pmatrix}$$

   d) Finally a dominance choice degree is applied over the possibility degree of the alternatives

$$pd_1 = 1.403 \quad pd_2 = 2.382 \quad pd_3 = 1.411 \quad pd_4 = 0.801$$

   and then the ranking of the alternatives is:

$$x_2 > x_3 > x_1 > x_4,$$

   being the best alternative of the GDM problem, $\mathbf{x_2}$.

6 Conclusions

GDM is a key area in many different fields such that decision makers may face situations in which they hesitate among several linguistic terms to provide their preferences. In this contribution, we have presented a linguistic GDM model capable to deal with HFLTS, that facilitates the elicitation of hesitant information to decision makers.

Acknowledgments. This work is partially supported by the Research Project TIN-2009-08286 and FEDER funds.

References

1. Bordogna, G., Pasi, G.: A fuzzy linguistic approach generalizing boolean information retrieval: A model and its evaluation. Journal of the American Society for Information Science 44, 70–82 (1993)

2. Degani, R., Bortolan, G.: The problem of linguistic approximation in clinical decision making. Int. Journal of Approximate Reasoning 2, 143–162 (1988)
3. Herrera, F., Herrera-Viedma, E.: Choice functions and mechanisms for linguistic preference relations. European J. of Operational Research 120(1), 144–161 (2000)
4. Herrera, F., Herrera-Viedma, E., Verdegay, J.L.: A sequential selection process in group decision making with a linguistic assessment approach. Information Sciences 85(4), 223–239 (1995)
5. Herrera, F., Martínez, L.: A 2-tuple fuzzy linguistic representation model for computing with words. IEEE Transactions on Fuzzy Systems 8(6), 746–752 (2000)
6. Jiang, Y.: An approach to group decision making based on interval fuzzy preference relations. J. of Systems Science and Systems Engineering 16(1), 113–120 (2007)
7. Kacprzyk, J.: Group decision making with a fuzzy linguistic majority. Fuzzy Sets and Systems 18, 105–118 (1986)
8. Klir, G.J., Yuan, B.: Fuzzy sets and fuzzy logic: Theory and Applications. Prentice-Hall PTR (1995)
9. Lawry, J.: A methodology for computing with words. International Journal of Approximate Reasoning 28, 51–89 (2001)
10. Liu, J., Martínez, L., Wang, H., Rodríguez, R.M., Novozhilov, V.: Computing with words in risk assessment. International Journal of Computational Intelligence Systems 3(4), 396–419 (2010)
11. Martínez, L., Ruan, D., Herrera, F.: Computing with words in decision support systems: An overview on models and applications. International Journal of Computational Intelligence Systems 3(4), 382–395 (2010)
12. Rodríguez, R.M., Martínez, L., Herrera, F.: Hesitant fuzzy linguistic term sets for decision making. IEEE Transactions on Fuzzy Systems 20(1), 109–119 (2012)
13. Roubens, M.: Fuzzy sets and decision analysis. Fuzzy Sets and Systems 90, 199–206 (1997)
14. Torra, V.: Hesitant fuzzy sets. Int. J. of Intelligent Systems 25(6), 529–539 (2010)
15. Türkşen, I.B.: Type 2 representation and reasoning for CWW. Fuzzy Sets and Systems 127, 17–36 (2002)
16. Yager, R.R.: An approach to ordinal decision making. International Journal of Approximate Reasoning 12(3-4), 237–261 (1995)
17. Yoon, K.: The propagation of errors in multiple-attribute decision analysis: a practical approach. Journal of the Operational Research Society 40, 681–686 (1989)
18. Zadeh, L.: The concept of a linguistic variable and its applications to approximate reasoning. Information Sciences, Part I, II, III (8,9), 199–249, 301–357, 43–80 (1975)
19. Zadeh, L.: Fuzzy logic = computing with words. IEEE Transactions on Fuzzy Systems 94(2), 103–111 (1996)
20. Zhang, T., Zhang, G., Ma, J., Lu, J.: Power distribution system planning evaluation by a fuzzy multi-criteria group decision support system. International Journal of Computational Intelligence Systems 3(4), 474–485 (2010)