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MANAGING HESITANT HETEROGENEOUS INFORMATION IN DECISION MAKING

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Abstract:

Decision making is a usual process for human beings in their daily life. The complexity of real world decision making problems imply the necessity of multiple points of view. Since experts may provide their assessments by using different domains according to their knowledge area and background. Different approaches have been introduced in the literature, nevertheless, none of them has considered the use of hesitant information. In this contribution, we propose an approach that manages hesitant heterogeneous information such as hesitant fuzzy sets and hesitant fuzzy linguistic term sets with other information as numerical, linguistic and interval-valued. A hesitant heterogeneous decision making model is also presented.

Keywords:

Heterogeneous information, hesitant fuzzy set, hesitant fuzzy linguistic term set, decision making

1. Introduction

Experts are increasingly involved in complex real decisions that require multiple points of view such as, evaluation, planning, etc. Therefore, each expert may express his/her assessments in different information domains, depending on expert's knowledge area. Usually, in quantitative contexts the experts provide their assessments by using numerical or interval-valued values, and in qualitative contexts they use linguistic terms. In such a case, the decision problem is defined in a heterogeneous framework and for managing such a framework, a suitable approach is required. In the literature can be found different approaches that deal with heterogeneous frameworks in decision problems [3, 4, 11].

Herrera et al. [3] proposed an approach that unifies the heterogeneous information into linguistic information to facilitate the computations and obtain understandable results. Li et al. [4] introduced an approach that does not unify the heterogeneous information but rather it computes the distances to the Ideal Solution and Negative Ideal Solution for each criterion defined in the decision problem. Zhang et al. [11] presented another approach that unifies the heterogeneous information into triangular fuzzy numbers and obtains an index ranking for each alternative by using a distance measure.

Previous approaches provide different ways of managing heterogeneous frameworks that take into account mainly numerical, linguistic and interval-valued values. However, none of them considers those decision situations with high degree of uncertainty where experts hesitate among several values to provide their assessments. To manage such situations, Torra [9] introduced the definition of Hesitant Fuzzy Sets (HFS) to fulfill the management of decision situations in quantitative contexts, where experts hesitate among different membership degrees to fix a membership function. Similarly, in qualitative contexts, Rodríguez et al. [8] proposed the concept of Hesitant Fuzzy Linguistic Terms Set (HFLTS) to manage those decision situations in which experts hesitate among several linguistic terms to assess a linguistic variable.

In this contribution, we propose an approach that extends the model presented by Herrera et al. [3] by adding the use of HFS and HFLTS. The proposed model unifies the heterogeneous information in a linguistic domain by means of the 2-tuple linguistic representation [2] that allows to accomplish the computing with words processes in a symbolic and precise way, obtaining linguistic results. To do so, we propose different transformation functions to manage these types of information.

The remainder of the paper is structured as follows: Section 2 reviews in short hesitant information in quantitative and qualitative settings. Section 3 presents a heterogeneous approach that integrates the use of hesitant information. Section 4 presents a multi-expert multi-criteria decision making model that uses the proposed approach. An illustrative example is also introduced in this section. Finally, Section 5 points out some conclusions.

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2. Dealing with hesitant information in quantitative and qualitative contexts

The need of managing situations where experts hesitate among several values has driven to the introduction of HFS and HFLTS to deal with such situations.

2.1. Hesitant fuzzy sets

In [9] Torra presented the concept of HFS to manage decision situations in quantitative contexts where experts hesitate among several membership values to define a membership function. A HFS is formally defined as follows.

Definition 1 *[9] Let X be a reference set, a HFS on X is a function h that returns a subset of values in [0,1]:*

$$h : X \to P([0,1])$$

A HFS can be also defined in terms of the union of their membership degrees to a set of fuzzy sets.

Definition 2 *[9] Let* $M = \{\mu_1, \ldots, \mu_n\}$ *be a set of n membership functions. The HFS* h_M*, is defined as:*

$$h_M : M \to P([0,1])$$

$$h_M(x) = \bigcup_{\mu \in M} \mu(x)$$

Some basic operations were defined for HFS.

Definition 3 *[9] Let* h *be a HFS, its lower and upper bounds are:*

$h^-(x) = min\ h(x)$
$h^+(x) = max\ h(x)$

In [9] was also proved that the envelope of a HFS is an intuitionistic fuzzy set by the following definition.

Definition 4 *[9] Let* h *be a HFS, its envelope* $A_{env(h)}$*, is*

$$A_{env(h)} = \{x, \mu_A(x), \nu_A(x)\}$$

where $A_{env(h)}$ *is an intuitionistic fuzzy set,* $\mu_{A(x)} = h^-(x)$ *and* $\nu_{A(X)} = 1 - h^+(x)$.

2.2. Hesitant fuzzy linguistic term sets

Similarly to the decision situations managed by means of HFS, in qualitative setting, it may occur that experts hesitate among several linguistic terms to assess a linguistic variable. To deal with such situations Rodríguez et al. [8] proposed the concept of HFLTS.

Definition 5 *[8] Let* $S = \{s_0, \ldots, s_g\}$ *be a linguistic term set, a HFLTS* H_S*, is defined as a ordered finite subset of consecutive linguistic terms of S:*

$$H_S = \{s_i, s_{i+1}, \ldots, s_j\} \quad such\ that\ \ s_k \in S,\ \ k \in \{i, \ldots, j\}$$

Two operators were defined to obtain the maximun and the minimun bounds of a HFLTS.

Definition 6 *[8] The upper bound* H_S^+*, and lower bound* H_S^-*, of the HFLTS* H_S*, are defined as:*

$H_{S^+} = max(s_i) = s_j,\ s_i \in H_S\ and\ s_i \leq s_j\ \forall i$
$H_{S^-} = min(s_i) = s_j,\ s_i \in H_S\ and\ s_i \geq s_j\ \forall i$

To facilitate the computing with words processes with HFLTS was introduced the concept of envelope of a HFLTS.

Definition 7 *[8] The envelope of a HFLTS* $env(H_S)$*, is a linguistic interval whose limits are obtained by means of upper bound (max) and lower bound (min):*

$$env(H_S) = [H_{S^-}, H_{S^+}],\ \ H_{S^-} <= H_{S^+}$$

Usually, experts do not use multiple linguistic terms to express their assessments, but rather linguistic expressions. A context-grammar G_H, was defined in [8] to generate expressions close to human beings expressions. The elements of $G_H = (V_N, V_T, I, P)$ are defined as follows:

$V_N = \{\langle primary\ term \rangle, \langle composite\ term \rangle, \langle unary\ relation \rangle, \langle binary\ relation \rangle, \langle conjunction \rangle\}$,
$V_T = \{greater\ than,\ lower\ than,\ between,\ and,\ s_0, \ldots, s_g\}$,
$I \in V_N$.
$P = \{I ::= \langle primary\ term \rangle | \langle composite\ term \rangle$
$\langle composite\ term \rangle ::= \langle unary\ relation \rangle \langle primary\ term \rangle | \langle binary\ relation \rangle \langle primary\ term \rangle \langle conjunction \rangle \langle primary\ term \rangle$
$\langle primary\ term \rangle ::= s_0 | s_1 | \ldots | s_g$
$\langle unary\ relation \rangle ::= greater\ than | lower\ than$
$\langle binary\ relation \rangle ::= between$
$\langle conjunction \rangle ::= and\}$

These linguistic expressions can be represented into HFLTS by means of the transformation function E_{G_H} (further detail see [8]).

3. Integrating of the hesitant information in the heterogeneous framework

So far, the approaches that manages heterogeneous frameworks take into account mainly numerical, linguistic and interval-valued values. However, in decision situations with high degree of uncertainty, it might occur that experts hesitate among several values to provide their assessments and prefer using more than one value. The use of HFS or HFLTS is suitable in these situations. Therefore, the aim of this contribution is to extend the heterogeneous framework by introducing the management of hesitant information. To do so, the heterogeneous approach introduced by Herrera et al. [3] is used as basis because it obtains linguistic results which allow carrying out the computing with words processes in a precise way.

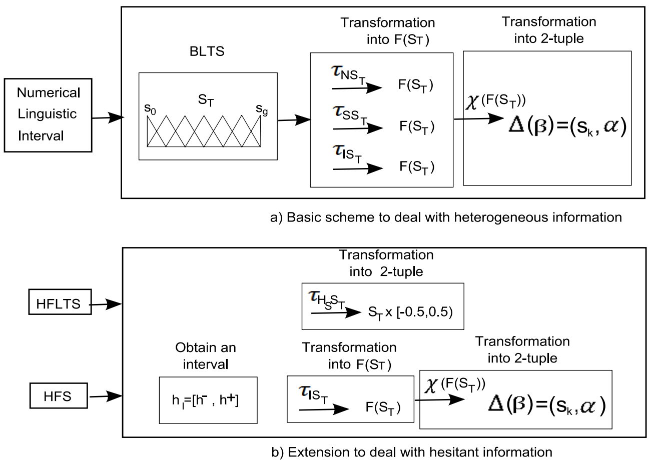

Figure 1. Scheme to manage hesitant heterogeneous information

The proposed approach in [3] unifies the information into a common linguistic domain, so-called Basic Linguistic Term Set (BLTS), $S_T = \{s_0, \ldots, s_g\}$, whose granularity is chosen according to the suggestions provided in [3]. Afterwards, each assessment is unified into a fuzzy set in S_T, $F(S_T)$, by using a transformation function according to the nature of the information. Figure 1 (a) shows the unification process of such an approach.

1. *Numerical domain*

Definition 8 *Let $\vartheta \in [0,1]$ be a numerical value and $S_T = \{s_0, \ldots, s_g\}$ a linguistic term set. The transformation function $\tau_{NS_T} : [0,1] \rightarrow F(S_T)$ defined by $\tau_{NS_T}(\vartheta) = \sum_{i=0}^{g} s_i/\gamma_i$ transforms a numerical value into a fuzzy set in S_T:*

$$\gamma_i = \mu_{s_i}(\vartheta) = \begin{cases} 0, & \vartheta < a \text{ or } \vartheta > c, \\ \frac{\vartheta - a}{b - a}, & a < \vartheta < b, \\ 1, & b \leq \vartheta \leq d, \\ \frac{c - \vartheta}{c - d}, & d < \vartheta < c, \end{cases} \quad (1)$$

being $F(S_T)$ the set of fuzzy sets on S_T, $\gamma_i = \mu_{s_i}(\vartheta) \in [0,1]$ the membership degree of ϑ to $s_i \in S_T$, and (a,b,d,c) a parametric membership function.

2. *Linguistic domain*

Definition 9 *Let $S = \{s_0, \ldots, s_h\}$ be a linguistic term set with $h < g$, the transformation function $\tau_{SS_T} : S \rightarrow F(S_T)$ defined by $\tau_{SS_T}(s_j) = \sum_{i=0}^{g} s_i/\gamma_i$ transforms a linguistic term into a fuzzy set in S_T:*

$$\gamma_i = \max_y \min\{\mu_{s_j}(y), \mu_{s_i}(y)\},\ i = \{0 \ldots, g\}$$

being $F(S_T)$ the set of fuzzy sets on S_T, μ_{s_j} and μ_{s_i} the membership functions of the fuzzy sets associated to the terms $s_j \in S$ and $s_i \in S_T$ respectively.

3. *Interval domain*

Definition 10 *Let $I = [\underline{a}, \overline{a}]$ be an interval in $[0,1]$, the transformation function $\tau_{IS_T} : I \rightarrow F(S_T)$ defined by $\tau_{IS_T}(I) = \sum_{i=0}^{g} s_i/\gamma_i$ transforms an interval I into a fuzzy set S_T:*

$$\gamma_i = \max_y \min\{\mu_I(y), \mu_{s_i}(y)\},\ i = \{0 \ldots, g\}$$

where $F(S_T)$ is the set of fuzzy sets on S_T, and μ_I and μ_{s_i} the membership functions of the fuzzy sets associated to the interval I and the terms $s_i \in S_T$, respectively.

Finally, the fuzzy sets obtained are transformed into linguistic 2-tuples [2] to facilitate the computing with words processes and produce understandable results. To do so, it is used the transformation function χ.

Definition 11 *[7] Let $F(S_T)$ be a fuzzy set in S_T, the function is defined as:*

$$\chi(F(S_T)) = \Delta\left(\frac{\sum_{j=0}^{g} j\gamma_j}{\sum_{j=0}^{g} \gamma_j}\right) = \Delta(\beta) = (s_l, \alpha)$$

where the fuzzy set $F(S_T)$ can be obtained from τ_{NS_T},τ_{SS_T} or τ_{IS_T}, respectively.

To integrate hesitant information in the approach shown in Figure 1 (a), it is necessary to define different transformation functions that unity the hesitant information in a linguistic domain (see Figure 1 (b)). These functions are defined as follows.

- *Transforming HFLTS into a linguistic domain*

  According to the definition of a HFLTS, it is compounded of several linguistic terms. Therefore, to transform a HFLTS into a linguistic domain, the linguistic terms of the HFTLS are aggregated by using the OWA operator [10] and the result is represented by means of a linguistic 2-tuple value [2].

  An important aspect of the OWA operator is the computation of the OWA weights. There are different approaches to compute the weights [1, 5, 6]. We will use the approach presented in [5], because it allows reflecting different importance among the linguistic terms that compound a HFLTS.

  Definition 12 *Let $H_{S_1} = \{s_i, \ldots, s_j\}$ be a HFLTS, the transformation function $\tau_{H_S S_T} : H_S \rightarrow S_T \times [-0.5, 0.5)$ is defined as follows:*

  $$\tau_{H_S S_T}(H_{S_1}) = \Delta(\sum_{k=i}^{j} w_k * s_k)$$

  where $s_k \in S$, $w_k \in [0,1]$, $k = \{i, \ldots, j\}$ and $\sum_{k=i}^{j} w_k =$ 1.

- *Transforming HFS into a linguistic domain*

  A HFS cannot be directly transformed into a 2-tuple value, therefore the unification phase is divided into three steps.

  1. *Obtain an interval*: Firstly, a numeric interval is built by using the lower and upper bounds defined for a HFS.

     Definition 13 *Let h_1 be a HFS, the interval of the h_1 is:*

     $$h_{I_1} = [h_1^-, h_1^+]$$

     being $h_1^- = min\ h_1$ and $h_1^+ = max\ h_1$.
  2. *Transform into fuzzy sets*: Once, the interval is obtained, we use the transformation function τ_{IS_T} : $I \rightarrow F(S_T)$ which transforms an interval h_I into a fuzzy set in S_T (see Def. 10).
  3. *Transform into 2-tuple*: Finally, the fuzzy set F_{S_T}, is converted into a 2-tuple value by using the transformation function $\chi : F(S_T) :\rightarrow S_T \times [-0.5, 0.5)$ introduced in Definition 11.

4. A multi-expert multi-criteria decision making model in a hesitant heterogeneous framework

In this section, we present a multi-expert multi-criteria decision making model in which experts can provide their assessments by means of different information domains. Afterwards, an illustrative example to show the usefulness and effectiveness of the proposed model is introduced.

4.1. Multi-expert multi-criteria decision making model

The proposed decision making model consists of 6 phases.

1. *Definition of information domains*: The proposed hesitant heterogeneous approach is able to manage different information domains, therefore the domains must be defined in this phase.
2. *Gathering of assessments*: Each expert $E = \{e_1, \ldots, e_l\}$ provides his/her assessments, over the criteria $C = \{c_1, \ldots, c_m\}$ defined for each alternative $X = \{x_1, \ldots, x_n\}$ by using different information domains (numerical, linguistic, interval-valued, HFS, HFLTS) according to his/her knowledge.
3. *Unification into a linguistic domain*: To carry out the computing with words processes in the *aggregation phase*, the assessments provided by experts are unified into a linguistic domain by the transformation functions introduced in Section 3.
4. *Selection of an aggregation operator*: To aggregate the linguistic information, it is necessary to choice an aggregation operator φ.
5. *Aggregation*: This phase is carried out in a two-step aggregation process.
   - *Computing collective assessments for each criteria*: A collective assessment v_{ij}, for each criterion c_j, for each alternative x_i, is obtained by using the aggregation operator selected in the previous phase.

     $$v_{ij} = \varphi(v_{ij}^k) \quad \forall k \in \{1, \ldots, l\}$$
   - *Computing collective assessment for each alternative*: A collective assessment v_i, for each alternative x_i, is computed by using an aggregation operator ϕ that may be the same as φ or not.

     $$v_i = \phi(v_{ij}) \quad \forall j \in \{1, \ldots, m\}$$

6. *Exploitation*: In this phase the collective assessments of the alternatives are compared by using the comparison operation of 2-tuples [2] to obtain a ranking of alternatives and select the best one.

4.2. Illustrative example

Let us suppose that a computer center of a university wants to change its information system to improve the work productivity. After preliminary screening, three alternatives $X = \{x_1, x_2, x_3\}$ have remained in the candidate list. A committee compound by 4 experts with different background $E = \{e_1, e_2, e_3, e_4\}$ must make a decision about which alternative is the best one considering four criteria C=$\{c_1$:Costs of hardware/software investment,c_2:Contribution to organization performance,c_3:Effort to transform from current system,c_4:Outsourcing software developer reliability$\}$. All experts are equally important and the weights of the criteria are $w = (0.4, 0.2, 0.2, 0.2)$.

Experts can provide their assessments by using different information domains (numerical, linguistic, interval-valued) according to their knowledge. Additionally, if the expert hesitate among different values he/she can use HFS or HFLTS.

1. *Definition of information domains*:
   - Numerical: [0,1]
   - Linguistic: $\{neither(n), very\_low(vl), low(l), medium(m), high(h), very\_high(vh), absolute(a)\}$
   - Interval-valued: I([0,1])
   - HFS: $P([0,1)]$
   - HFLTS: linguistic expressions generated by G_H
2. *Gathering of assessments*: The assessments provided by experts are shown in Tables 1, 2 and 3.

TABLE 1: ASSESSMENTS FOR x_1

	Alternative x_1			
v^k_{1j}	c_1	c_2	c_3	c_4
e_1	0.6	0.9	0.5	0.5
e_2	vh	h	l	h
e_3	{0.7,0.8}	{0.7,0.8,0.9}	{0.5,0.7}	{0.6,0.7}
e_4	gr than h	h	btw h and vh	m

TABLE 2: ASSESSMENTS FOR x_2

	Alternative x_2			
v^k_{2j}	c_1	c_2	c_3	c_4
e_1	0.4	0.8	0.3	0.4
e_2	l	vh	vl	vl
e_3	{0.4,0.5}	{0.6,0.7}	{0.7,0.8}	{0.5,0.6,0.7}
e_4	btw h and vh	h	vh	btw vl and l

TABLE 3: ASSESSMENTS FOR x_3

	Alternative x_3			
v^k_{3j}	c_1	c_2	c_3	c_4
e_1	0.9	0.8	0.9	0.8
e_2	vh	a	vh	a
e_3	{0.8,0.9}	{0.7,0.8,0.9}	{0.8,0.9}	{0.7,0.9}
e_4	gr than vh	btw h and vh	vh	vh

where btw stand for between and gr for greater.

3. *Unification into a linguistic domain*: The first step to unify the information is to select the linguistic domain S_T. In this case, we have chosen the linguistic term set used by the expert e_2 to provide their assessments, its semantics is the following one:

$$nothing = (0, 0, .17) \quad very\_low = (0, .17, .33)$$
$$low = (.17, .33, .5) \quad medium = (.33, .5, .67)$$
$$high = (.5, .67, .83) \quad very\_high = (.67, .83, 1)$$
$$absolute = (.83, 1, 1).$$

Afterwards, all the assessments are transformed into the selected domain by using the transformation functions introduced in section 3. Finally, the fuzzy sets are unified into linguistic 2-tuple values. Because of limited space, we only show the assessments transformed for the alternative x_1 (see Table 4).

TABLE 4: TRANSFORMATION INTO 2-TUPLE

	Alternative x_1			
	c_1	c_2	c_3	c_4
e_1	(h,-.4)	(vh,.4)	(m,0)	(m,0)
e_2	(vh,0)	(h,.0)	(l,0)	(h,0)
e_3	(vh,-.5)	(vh,-.18)	(h,-.36)	(h,-.12)
e_4	(vh,.11)	(h,0)	(vh,-.5)	(m,0)

where the assessment provided by the expert e_4 over the criterion c_1 is transformed as follows:

$$\tau_{H_s S_S}(h, vh, a) = \Delta(\tfrac{16}{36}a + \tfrac{8}{36}vh + \tfrac{2}{6}h) = (vh, .11)$$

The weights has been obtained by using the approach presented in [5].

4. *Selection of an aggregation operator*: Without loss of generality and due to each criterion has different importance, the aggregation operator is the weighted mean.
5. *Aggregation*: It is divided into two steps.
   - *Computing collective assessments for each criterion*: The assessments of experts are aggregated by using

the selected aggregation operator. Due to all experts are equally important, the weights are $w = (0.25, 0.25, 0.25, 0.25)$. Table 5 shows the results.

TABLE 5: COLLECTIVE VALUES FOR CRITERIA

	Criteria			
	c_1	c_2	c_3	c_4
x_1	(vh,-.45)	(vh,.44)	(m,.29)	(h,-.53)
x_2	(m,-.12)	(h,.42)	(m,.08)	(l,.14)
x_3	(vh,.34)	(vh,.03)	(vh,.13)	(vh,.16)

The value obtained for the criterion c_1 for the alternative x_1 is computed as follows:
$v_{11} = \Delta(0.25 * \Delta^{-1}(h, -.4) + 0.25 * \Delta^{-1}(vh, 0) + 0.25 * \Delta^{-1}(vh, -.5) + 0.25 * \Delta^{-1}(vh, .11)) = (vh, -.45)$

- *Computing collective assessments for each alternative*: In this step the criteria are aggregated by using the weighted mean aggregation operator. The weights are $w = (0.4, 0.2, 0.2, 0.2)$ and the results are shown in Table 6.

TABLE 6: COLLECTIVE VALUES FOR ALTERNATIVES

Alternatives		
x_1	x_2	x_3
(h,.08)	(m,.08)	(vh,.2)

6. *Exploitation*: In this phase, a ranking of alternatives is obtained by using the comparison operation of 2-tuple.

$$x_3 = (vh, .2) > x_1 = (h, .08) > x_2 = (m, .08)$$

Therefore, the best alternative is $x_3 = (vh, .2)$.

5. Conclusions

In this contribution, a heterogeneous approach that introduces the management of new information domains in hesitant situations such as, HFS in quantitative settings and HFLTS in qualitative ones has been proposed. A multi-expert multi-criteria decision making model where experts can provide their assessments by means of different information domains has been presented and applied for solving a decision making problem.

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