Hesitant Fuzzy Linguistic Term Sets for Decision Making

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1 Summary

Decision problems are usually defined in contexts where the information is vague and imprecise. Different approaches have been proposed to deal with such an uncertainty. One of them is the fuzzy linguistic approach that has provided successful results on decision making problems defined under uncertainty.

In decision situations, it might happen that an expert hesitates among several values to provide their assessments over an alternative. These hesitant situations may appear in both quantitative and qualitative contexts. In quantitative settings, Torra introduced the concept of Hesitant Fuzzy Sets (HFS), to manage those hesitant situations where several values are possible for the definition of a membership function of a fuzzy set. Similarly to these situations, in qualitative settings, it may occur that experts hesitate among several values to assess a linguistic variable. Therefore, to model multiple linguistic terms keeping the basis of the fuzzy linguistic approach and extending the idea of HFS to linguistic contexts, we propose the concept of Hesitant Fuzzy Linguistic Term Sets (HFLTS).

Definition 1 *Let S be a linguistic term set, $S = \{s_0, \ldots, s_g\}$, an HFLTS, H_S, is an ordered finite subset of consecutive linguistic terms of S.*

$H_S = \{s_i, s_{i+1}, \ldots, s_j\}$, *such that,* $s_k \in S, k \in \{i, \ldots, j\}$

Different basic operations are defined to compute with HFLTS, such as, *upper bound* and *lower bound*, that obtain the minimum and maximum of a HFLTS; *complement*, *intersection* and *union* of two HFLTS. In addition, the properties of commutativity, associative and distributive have been studied for the intersection and union operations.

In the computational processes of the decision making problems are necessary more complex operations. Thus, to facilitate the computations with HFLTS, we define the concept of envelope of a HFLTS.

Definition 2 *The envelope of a HFLTS, $env(H_S)$, is a linguistic interval whose limits are obtained by means of upper bound (*max*) and lower bound (*min*), hence:*

$$env(H_S) = [H_{S^-}, H_{S^+}], \quad H_{S^-} <= H_{S^+}$$

In order to establish an order between two HFLTS, it has been defined the comparison operation of HFLTS which is based on the concept of envelope of a HFLTS. To do so, the acceptability function proposed by Sengupta has been extended to linguistic intervals.

The concept of HFLTS allows modeling multiple linguistic terms, nevertheless, in decision making problems when an expert hesitates among different linguistic terms, he/she does not use multiple linguistic terms, but rather linguistic expressions similar to the expressions used by human beings in decision problems. Therefore, to improve the flexibility of the elicitation of the linguistic information, it is proposed the use of context-free grammars, because they formalize the generation of linguistic expressions. We have defined a basic context-free grammar G_H, that generates simple but rich linguistic expressions that can be easily represented by means of HFLTS.

Definition 3 *Let G_H be a context-free grammar and $S = \{s_0, \ldots, s_g\}$ a linguistic term set. The elements of $G_H = (V_N, V_T, I, P)$ are defined as follows:*

$V_N = \{\langle primary\ term\rangle\ ,\ \langle composite\ term\rangle\ ,\ \langle unary\ relation\rangle\ ,$
$\langle binary\ relation\rangle\ ,\langle conjunction\rangle\},$
$V_T = \{greater\ than,\ lower\ than,\ between,\ and,\ s_0, \ldots, s_g\},$
$I \in V_N.$
$P = \{I ::= \langle primary\ term\rangle|\langle composite\ term\rangle$
$\langle composite\ term\rangle ::= \langle unary\ relation\rangle\langle primary\ term\rangle|\ \langle binary\ relation\rangle \langle primary\ term\rangle\langle conjunction\rangle\langle primary\ term\rangle$
$\langle primary\ term\rangle ::= s_0|s_1|\ldots|s_g$
$\langle unary\ relation\rangle ::= greater\ than|lower\ than$
$\langle binary\ relation\rangle ::= between$
$\langle conjunction\rangle ::= and\}$

To carry out computational processes with the comparative linguistic expressions generated by the context-free grammar G_H, a transformation function E_{G_H} that converts the comparative linguistic expressions into HFLTS is defined.

Definition 4 *Let E_{G_H} be a function that transforms comparative linguistic expressions, ll, obtained by G_H, into HFLTS, H_S, where S is the linguistic term set used by G_H, and S_{ll} is the set of comparative linguistic expressions generated by G_H.*

$$E_{G_H} : S_{ll} \longrightarrow H_S$$

In addition, we propose a multicriteria linguistic decision making model in which experts can express their assessments by means of comparative linguistic expressions close to the human beings cognitive model or by single linguistic terms. This decision model manages such linguistic expressions by its representation using HFLTS. To combine these linguistic expressions, we have defined two symbolic aggregation operators, *min_upper* and *max_lower*, that provide a linguistic interval for each alternative. Afterwards, an exploitation process based on the application of the non-dominance choice degree to obtain the solution set of alternatives is used.

Finally, a decision making problem is solved to show the usefulness and effectiveness of the proposed model.