The Interval Rough Number of the Extended ARAS Method for Solving Multi-Criteria Group Decision Making

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***Abstract*—In Multi-criteria-group decision making (MCGDM) problems dealing with qualitative information, the experts express their judgments about alternatives according to their degree of knowledge. It is difficult that experts provide precisely their preference about alternatives according to criteria. The interval rough number (IRN) achieved a high attraction to handle this ambiguity in many MCGDM problems. The main objective of this contribution is to create the IRN of the extended linguistic hierarchical ARAS method (IRN-ELH-ARAS) taking into account decision makers uncertainty. A MCGDM case study is given to illustrate the efficiency of the proposed approach. This example is related to the choice of the best option to invest a sum of money for an investment company.**

Keywords— multi-criteria group decision making, extended linguistic hierarchy model, ARAS-ELH, interval rough number

I. INTRODUCTION

Multicriteria decision-making (MCDM) methods put strategies to select the best alternative according to a set of criteria for solving a decision problem. The MCDM methods have been extensively applied to different fields in real-world scenarios. Recently, MCDM methods are extended for groups of decision makers which are called multi-criteria group decision-making methods (MCGDM).

In the MCGDM problem dealing with qualitative criteria and uncertain information, the use of linguistic values is suitable for the experts in order to give their judgements. In a decision situations with multiple experts, each one has his/her own degree of knowledge for giving their alternative assessments according to criteria. So, the use of the fuzzy environment is essential to take into consideration the uncertainty of decision makers.

Many extensions outranking methods have been proposed to manage uncertainty in MCGDM. In this paper, we shed the light on ARAS method. The ARAS method has been developed within different information environments to improve its applicability. We can mention triangular fuzzy set [1], hesitant fuzzy linguistic term set [2], interval- valued intuitionistic fuzzy set [3], rough set [4], z- number [5], and probability multivalued neutrosophic set [6].

The rest of the paper is divided into six parts: in section 2, a literature review is presented on MCGDM problem. In section 3, some essential preliminaries of the interval rough number, the extended linguistic hierarchical model and the interval-valued 2-Tuple ordered weighted average operator are given. Then, the proposed IRN-ARAS-ELH algorithm is presented in section 4. In section 5, a case study is given for an investment company. Finally, a conclusion and future research are presented in section 6.

II. LITERATURE REVIEW

The ARAS method is proposed by [7] to select the best alternative according to a set of criteria through the utility degree indicator which can eliminate the influence of different measurement units and the different optimization directions in MCDM. In such cases, one decision-maker commonly cannot deal with complex problems efficiently. Therefore, GDM is suitable to make their evaluation with different knowledge backgrounds which makes the decision process more efficient.

Many studies extend the ARAS method to GDM environment. A fuzzy group ARAS method (fuzzy GARAS) was developed by [8] for selecting the best waste dump site in Ayerma phosphate mine located in Yasouj, Iran. The advantage of fuzzy GARAS technique is its ability to deal with the inherent uncertainty implicated in the process of modeling a real-life. A MCGDM problem was presented by [9] for selecting candidates during the process of the recruitment based on the combination of the SWARA method and the ARAS method. The ARAS fuzzification method is applicated for ranking alternatives, whereas the SWARA method is applicated for the determination of weighting factors. An extended the ARAS method to fuzzy number is proposed by [10] for designing a suitable performance measurement system in the supply chain management of small-medium-sized enterprises. Reference [11] proposed a MCGDM model by combining the ARAS method and the Best Worst Method (BWM) under the hesitant fuzzy linguistic environment. An application of this model is presented to solve the decision-making problem of selecting the suitable digital supply chain finance (DSCF) supplier. An extension of the ARAS method to MCGDM model is also

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proposed by [12] to handle the probability multi-valued neutrosophic sets based on the obtained weight information of decision makers and criteria. Reference [13] presented the integration between the ARAS method and the interval-valued triangular fuzzy number for the development of a firm export performance measurement model.

In this contribution, we extend the ARAS-ELH method to the interval-rough number environment (IRN-ARAS-ELH).

III. PRELIMUNARIES

A. Interval rough numbers

In this section, some preliminaries on the rough numbers with interval values (IRN) are given which are needed for the proposed approach.

Definition 1 [14]. Let two sets include the upper class of the object $R'_{\mathrm{u}} = (R'_{\mathrm{u1}}, R'_{\mathrm{u2}}, \ldots, R'_{\mathrm{uk}})$ and the lower class of the objects $R'_{\mathrm{l}} = (R'_{\mathrm{l1}}, R'_{\mathrm{l2}}, \ldots, R'_{\mathrm{lk}})$. In both classes of objects (upper or lower) $I'_{ui} \epsilon R$ ($1 \leq i \leq k$) and $I'_{li} \epsilon R$ $(1 \leq i \leq j)$, the lower approximation of I'_{ui} and I'_{li} are defined as:

$$\underline{Apr}\ (I'_{li}) = U\ \{Y \epsilon\ U/(R'_{l}(Y) \leq I'_{li}\} \tag{1}$$

$$\underline{Apr}\ (I'_{ui}) = U\ \{Y \epsilon\ U/(R'_{u}(Y) \leq I'_{ui}\} \tag{2}$$

The upper approximation of I'_{ui} and I'_{li} are defined as :

$$\overline{Apr}(I'_{li}) = U\ \{Y \epsilon\ U/\ (R'_{l}(Y) \geq I'_{li}\} \tag{3}$$

$$\overline{Apr}(I'_{ui}) = U\ \{Y \epsilon\ U/\ (R'_{u}(Y) \geq I'_{ui}\} \tag{4}$$

Definition 2 [14]. Let (($\underline{Lim}\ (I'_{li})$, $\underline{Lim}\ (I'_{ui})$) and ($\overline{Lim}(I'_{li}), \overline{Lim}(I'_{ui})$) be the lower and upper limit of objects respectively, the lower limits are defined by:

$$\underline{Lim}\ (I'_{li}) = \frac{1}{M_L} \sum (R'_{l}(Y) \mid Y \epsilon\ \underline{Apr}\ (I'_{li}) \tag{5}$$

$$\underline{Lim}\ (I'_{li}) = \frac{1}{M'_L} \sum (R'_{l}(Y) \mid Y \epsilon\ \underline{Apr}\ (I'_{ui}) \tag{6}$$

The upper limits are defined using these two formulas :

$$\overline{Lim}(I'_{li}), = \frac{1}{M_U} \sum (R'_{l}(Y) \mid Y \epsilon\ \overline{Apr}(I'_{li}) \tag{7}$$

$$\overline{Lim}(I'_{ui}), = \frac{1}{M'_U} \sum (R'_{l}(Y) \mid Y \epsilon\ \overline{Apr}(I'_{ui}) \tag{8}$$

Definition 3 [14]. The rough number of the vague class I'_{li}, I'_{ui} can be represented as follows:

$$RN(I'_{li}) = [\ \overline{Lim}(I'_{li}), \underline{Lim}\ \ (I'_{li})] \tag{9}$$

$$RN(I'_{ui}) = [\overline{Lim}(I'_{ui}), \underline{Lim}\ \ (I'_{ui})] \tag{10}$$

Definition 4 [14]. Let $IRN(I'_{i})$ be the interval rough number which is based on its upper and lower limits of objects classes.

$$IRN(I'_{i}) = [RN(I'_{li}), RN(I'_{ui})] \tag{11}$$

B. Extended linguistic hierarchical model

Generally, when decision-makers encounter decision problems with different linguistics scales, they expect a flexible framework in which any term set can be used. An extended linguistic hierarchy (ELH) model has been proposed by Martinez et al. [15]. The ELH model provides accurate information as well as an easy understanding of the final results by the experts. To build the LH differently, the authors proposed a new way of the unification process. The ELH is based on two extended rules to handle any scale in the multi-granular linguistic framework.

The first one consists in defining a finite number of levels. Each level represents in its turn the multi-granular linguistic framework that the decision-makers need to express their judgment. It is not compulsory to keep the previous modal points with each other.

The second one consists in adding a new level which aims at keeping all the previous modal points of all the precedent levels within this new one.

As a matter of fact, the multigranular linguistic information has to be transformed into a one linguistic field, called S_T. Since the transformation within the levels of the LH are bijective, we can choose any level of linguistic term set.

Definition 5 [15]. While the transformation function from a linguistic label in level t to a label in level t^*, the linguistic hierarchy basic rules have to be respected. The latter is presented as:

$$TF_{t^*}^{t}: l(t, n(t)) \longrightarrow l(t^*, n(t^*))$$

$$TF_{t^*}^{t}\left(s_i^{n(t)}, \alpha^{n(t)}\right) = \Delta_{n(t^*)}\left(\frac{\Delta^{-1}{}_{n(t)}\left(s_i^{n(t)}, \alpha^{n(t)}\right).\ n(t^*)-1}{n(t)-1}\right) \tag{12}$$

Definition 6 [15]. The extended transformation function between linguistic terms in different levels in ELH is presented as:

$$ETF_{t}^{t^*}\left(ETF_{t^*}^{t}\left(s_i^{n(t)}, \alpha_i^{n(t)}\right)\right) = \left(s_i^{n(t)}, \alpha_i^{n(t)}\right) \tag{13}$$

C. The interval-valued 2-Tuple ordered weighted average (IVL2TOWA) operator

Definition 7 [16]. Let $X = \{[(S_1, \alpha_1), (S'_1, \alpha'_1)][(S_2, \alpha_2), (S'_2, \alpha'_2)], [(S_1, \alpha_1), (S'_1, \alpha'_1)][(S_2, \alpha_2), (S'_2, \alpha'_2)]\}$ be a set of interval-valued 2-tuples and $W = (w_1, w_2, \ldots, w_n)^T$ be their associated weights, with $w_i \in [0.1]$ and $\sum_1^n w_i = 1$. The Interval-Valued 2-Tuple Weighted Average (IVL2TWA) operator is defined as:

$$IVL2TWA\ ([(S_1, \alpha_1), (S'_1, \alpha'_1)],\ [(S_2, \alpha_2), (S'_2, \alpha'_2)], \ldots, [(S_n, \alpha_n), [(S'_n, \alpha'_n)]) = \Delta[\textstyle\sum_{i=1}^{n} w_i\, \Delta^{-1}(S_i, \alpha_i), \sum_{i=1}^{n} w_i\, \Delta^{-1}(S'_i, \alpha'_i)] \tag{13}$$

IV. THE PROPOSED IRN-ARAS-ELH ALGHORITHM

This paper presents a novel IRN-ARAS-ELH approach to handle with multi granular linguistic scale information in group decision-making. The interval rough numbers have been used to resolve the problem of ambiguity in multi-experts decision-making and to tackle imprecision, vagueness, and uncertainty in data analysis. The following section describes the steps of a novel IRN-ARAS-ELH algorithm.

For $h = t$

Step 1: Form a multi-criteria model i.e determine the set of criteria $C = \{y_1, \ldots, y_j\}$ and the set of alternatives $A = \{x_1, \ldots, x_i\}$. Furthermore, a group of experts $E = \{DM_1, \ldots, DM_4\}$ will be selected to evaluate the alternatives according to criteria.

Step 2: Determine a finite number of levels $l(t, n(t))$ of the hierarchy tree, where each level t is a linguistic term set, $S^{n(t)} = \left\{S_0^{n(t)}, \ldots, S_{n(t)-1}^{n(t)}\right\}$, with different granularity $n(t)$ to the rest of levels of the hierarchy. Afterwards, insert a new level to build an ELH, $l(t^*, n(t^*))$ with $t^* = m + 1$, and with granularity $n(t^*)$ [15].

Step 3: Form a rough matrix i.e determine the rough extended linguistic preference evaluation over the set of alternatives A under the criterion C. The assessment of experts are stand on multigranular linguistic term sets of any level of the hierarchy that she/he is chosen $l(t, n(t))$.

For $h = t^*$

Step 4: Unify the multi-granular linguistic information evaluated in multiple scales in any term set of the ELH using the transformation function (12).

Step5: Transform the obtained extended linguistic hierarchy (ELH) decision matrix into interval rough number extended linguistic hierarchy (IRN-ELH) decision matrix by applying the equations (1) – (11).

Step 6: Form the group IRN-ELH decision matrix using the interval-valued 2-Tuple ordered weighted average (IVL2TWA) operator (equation 14) [16].

Step 7: Calculate the based interval rough normalized 2-Tuple linguistic hierarchy decision matrix based on ELH model using ARAS method [7].

Step 8: Compute the weighted of the based normalized IRN-2-tuple linguistic decision matrix for all criteria [7].

Step 9: Calculate the values of optimality function for the i^{th} alternative and the utility alternative degree [7].

That it would be possible to get the partial-preorder of alternatives from ranking in decreasing order t7he value utility degree K_i.

For $h = t^* - i$

Step 10: Transform the group IRN-ELH decision matrix (see step 6) obtained in level t^* into any level of the original linguistic term set $l\big(t, n(t)\big)$ [15].

Go to step 6.

Go to step 7.

Step 11: Compute the new multigranular linguistic values of optimality function (S'_i) and the new multigranular linguistic utility degree (K'_i) for the i^{th} alternative [7].

Based on the IRN-ARAS-ELH proposal approach, determine the complete pre order at any level of the ELH, $l(t, n(t))$ (i.e., the final ranking of the alternatives).

V. An Illustrative Example

In this section, we will apply the suggested decision approach to an investment company problem. Let $A = \{x_1, x_2, x_3, x_4\}$ be four investment possibilities where x_i stands for arms company, food company, computer company and car industry respectively, and the universe $C = \{y_1, y_2, y_3, y_4\}$ be four criteria where y_i stands for pollution, potential customer and stability of the market, company's financial profitability and ability of uncertainty anticipation respectively. The criterion $y_2, y_3,$ and y_4 are benefit attributes while y_1 is cost attribute. Four experts $E = \{DM_1, \ldots, DM_4\}$ from four consultancy departments $D = \{z_1, z_1, z_1, z_1\}$ are chosen by the computer company to give their preferences throughout a set of criteria. Each department is handled by an expert where z_1 is the risk analysis department. The latter is directed by DM_1. z_2 is the growth analysis department which is managed by DM_2. z_3 is the social-political analysis department. The latter is directed by DM_3. z_4 is the environmental effect analysis department which is managed by DM_4. More specifically, DM_1 gives his preference in l(3,9), DM_2 gives his preference in l(1,5), DM_3 gives his preference in l(2,7) and DM_4 gives his preference in l(3,9).

For $h = t$

DM evaluates the alternatives according to criteria at any level $l(t, n(t))$ of the linguistic hierarchy. The interval linguistic preference values of each expert are shown in TABLE I (initial decision matrix $\Re_K$). In this example, the next step consist in uniting the non-homogeneous information from a linguistic label in level t with $l\big(t, n(t)\big)$ to label in level t^* with $l(4,25)$. So, the RN-2-Tuple. So, the RN-2-Tuple homogeneous linguistic decision matrix of each expert are obtained.

TABLE I. Heterogeneous rough number Input data of each expert

E	**A**	**C**			
		Y_1	Y_2	Y_3	Y_4
DM_1	x_1	$[S_5^9, S_6^9]$	$[S_2^9, S_2^9]$	$[S_6^9, S_7^9]$	$[S_4^9, S_5^9]$
	x_2	$[S_6^9, S_7^9]$	$[S_3^9, S_4^9]$	$[S_7^9, S_8^9]$	$[S_8^9, S_8^9]$
	x_3	$[S_8^9, S_8^9]$	$[S_4^9, S_5^9]$	$[S_5^9, S_6^9]$	$[S_3^9, S_4^9]$
	x_4	$[S_2^9, S_3^9]$	$[S_6^9, S_6^9]$	$[S_2^9, S_3^9]$	$[S_5^9, S_6^9]$
	W_j	0,24	0,21	0,18	0,24
DM_2	x_1	$[S_1^7, S_2^7]$	$[S_2^7, S_2^7]$	$[S_2^7, S_3^7]$	$[S_2^7, S_3^7]$
	x_2	$[S_6^7, S_6^7]$	$[S_4^7, S_5^7]$	$[S_2^7, S_2^7]$	$[S_6^7, S_6^7]$
	x_3	$[S_4^7, S_5^7]$	$[S_4^7, S_4^7]$	$[S_4^7, S_5^7]$	$[S_4^7, S_5^7]$
	x_4	$[S_6^7, S_6^7]$	$[S_2^7, S_3^7]$	$[S_2^7, S_3^7]$	$[S_4^7, S_5^7]$
	W_j	0,2	0,24	0,24	0,24
DM_3	x_1	$[S_3^5, S_4^5]$	$[S_3^5, S_4^5]$	$[S_2^5, S_1^5]$	$[S_1^5, S_1^5]$
	x_2	$[S_2^5, S_3^5]$	$[S_1^5, S_1^5]$	$[S_2^5, S_2^5]$	$[S_2^5, S_3^5]$
	x_3	$[S_1^5, S_1^5]$	$[S_2^5, S_2^5]$	$[S_4^5, S_3^5]$	$[S_4^5, S_4^5]$
	x_4	$[S_2^5, S_3^5]$	$[S_2^5, S_3^5]$	$[S_2^5, S_3^5]$	$[S_3^5, S_4^5]$
	W_j	0,24	0,18	0,24	0,24
DM_4	x_1	$[S_5^9, S_6^9]$	$[S_2^9, S_3^9]$	$[S_7^9, S_8^9]$	$[S_8^9, S_8^9]$
	x_2	$[S_8^9, S_8^9]$	$[S_5^9, S_5^9]$	$[S_6^9, S_7^9]$	$[S_4^9, S_5^9]$
	x_3	$[S_1^9, S_2^9]$	$[S_5^9, S_6^9]$	$[S_2^9, S_3^9]$	$[S_5^9, S_5^9]$
	x_4	$[S_7^9, S_8^9]$	$[S_4^9, S_4^9]$	$[S_5^9, S_6^9]$	$[S_3^9, S_4^9]$
	W_j	0,24	0,18	0,24	0,24

In this stage, the transformation of RN-2-Tuple homogeneous linguistic decision matrix to IRN-2-Tuple homogeneous linguistic decision matrix of each experts is obtained by applying equations (1) – (11) (TABLE II). In this step, the combined IRN-2-Tuple-ARAS decision matrix of all experts is obtained by applying IVL2TWA operator (14) (TABLE III).

TABLE II. HETEROGENEOUS ROUGH NUMBER INPUT DATA OF EACH EXPERT

E	A	Y_1	Y_{j-1}	Y_4
DM_1	x_1	$([(S_{11}^{25},0.333)\,;(S_{16}^{25},0)],[(S_{18}^{25},-0.334)\,;(S_{20}^{25},0)])$		$([(S_9^{25},-0.333)\,;(S_{18}^{25},0)],[(S_{11}^{25},0)\,;(S_{20}^{25},-0.5)])$
	x_2	$([(S_{15}^{25},0)\,;(S_{22}^{25},0)],[(S_{20}^{25},-0.5)\,;(S_{23}^{25},0)])$		$([(S_{18}^{25},0)\,;(S_{24}^{25},0)],[(S_{20}^{25},0.25)\,;(S_{24}^{25},0)])$
	x_3	$([(S_{12}^{25},0.25)\,;(S_{24}^{25},0)],[(S_{14}^{25},0)\,;(S_{24}^{25},0)])$		$([(S_9^{25},0)\,;(S_{16}^{25},0)],[(S_{12}^{25},0)\,;(S_{17}^{25},0)])$
	x_4	$([(S_6^{25},0)\,;(S_{16}^{25},-0.25)],[(S_9^{25},0)\,;(S_{19}^{25},-0.25)])$		$([(S_{12}^{25},0)\,;(S_{16}^{25},0.33)],[(S_{15}^{25},0)\,;(S_{21}^{25},0.33)])$
	W_j	0,24		0,24
DM_2	x_1	$([(S_4^{25},0)\,;(S_{13}^{25},0)],[(S_8^{25},0)\,;(S_{17}^{25},0)])$		$([(S_7^{25},0)\,;(S_{15}^{25},-0.333)],[(S_9^{25},0)\,;(S_{17}^{25},0)])$
	x_2	$([(S_{20}^{25},-0.5)\,;(S_{24}^{25},0)],[(S_{22}^{25},-0.25)\,;(S_{24}^{25},0)])$		$([(S_{18}^{25},0)\,;(S_{24}^{25},0)],[(S_{20}^{25},0.25)\,;(S_{24}^{25},0)])$
	x_3	$([(S_8^{25},0.33)\,;(S_{20}^{25},0)],[(S_{11}^{25},-0.333)\,;(S_{22}^{25},0)])$		$([(S_{13}^{25},0.33)\,;(S_{20}^{25},0)],[(S_{15}^{25},-0.33)\,;(S_{22}^{25},0)])$
	x_4	$([(S_{16}^{25},-0.25)\,;(S_{24}^{25},0)],[(S_{19}^{25},-0.25)\,;(S_{24}^{25},0)])$		$([(S_{13}^{25},0.33)\,;(S_{17}^{25},0)],[(S_{17}^{25},-0.33)\,;(S_{22}^{25},0)])$
	W_j	0,2		0,24
DM_3	x_1	$([(S_{13}^{25},0)\,;(S_{18}^{25},0)],[(S_{17}^{25},0)\,;(S_{24}^{25},0)])$	…………	$([(S_6^{25},0)\,;(S_{13}^{25},-0.5)],[(S_6^{25},0)\,;(S_{14}^{25},0)])$
	x_2	$([(S_{12}^{25},0)\,;(S_{20}^{25},-0.5)],[(S_{18}^{25},0)\,;(S_{22}^{25},-0.25)])$		$([(S_{12}^{25},0)\,;(S_{18}^{25},0)],[(S_{17}^{25},-0.5)\,;(S_{22}^{25},0)])$
	x_3	$([(S_5^{25},-0.5)\,;(S_{15}^{25},0.333)],[(S_6^{25},0)\,;(S_{14}^{25},0)])$		$([(S_{16}^{25},0)\,;(S_{24}^{25},0)],[(S_{17}^{25},0)\,;(S_{24}^{25},0)])$
	x_4	$([(S_9^{25},0)\,;(S_{19}^{25},0)],[(S_{24}^{25},-0.5)\,;(S_{22}^{25},0)])$		$([(S_{15}^{25},-0.5)\,;(S_{18}^{25},0)],[(S_{19}^{25},-0.5)\,;(S_{24}^{25},0)])$
	W_j	0,24		0,24
DM_4	x_1	$([(S_{11}^{25},0.333)\,;(S_{16}^{25},0)],[(S_{15}^{25},-0.334)\,;(S_{20}^{25},0)])$		$([(S_{13}^{25},-0.5)\,;(S_{24}^{25},0)],[(S_{14}^{25},0.25)\,;(S_{24}^{25},0)])$
	x_2	$([(S_{20}^{25},-0.5)\,;(S_{24}^{25},0)],[(S_{22}^{25},-0.5)\,;(S_{24}^{25},0)])$		$([(S_{12}^{25},0)\,;(S_{18}^{25},0)],[(S_{15}^{25},0)\,;(S_{22}^{25},0)])$
	x_3	$([(S_3^{25},0)\,;(S_{12}^{25},0.25)],[(S_6^{25},0)\,;(S_{14}^{25},0)])$		$([(S_{12}^{25},0)\,;(S_{18}^{25},0.33)],[(S_{12}^{25},0)\,;(S_{17}^{25},0)])$
	x_4	$([(S_{13}^{25},0)\,;(S_{23}^{25},-0.5)],[(S_{19}^{25},-0.25)\,;(S_{24}^{25},0)])$		$([(S_9^{25},0)\,;(S_{15}^{25},-0.5)],[(S_{12}^{25},0)\,;(S_{19}^{25},-0.5)])$
	W_j	0,24	W_{j-1}	0,24

TABLE III. AGGREGATION OF THE UNIFIED IRN-2-TUPLE-ARAS INPUT DATA OF ALL EXPERT

A	Y_1	Y_2	Y_3	Y_4
x_1	$([(S_9^{25},0.358),(S_{15}^{25},-0.4)],[(S_{12}^{25},-0.003),(S_{19}^{25},-0.24)])$	$([(S_8^{25},-0.462),(S_{13}^{25},0.235)],[(S_9^{25},0.176),(S_{18}^{25},0.106)])$	$([(S_9^{25},0.153),(S_{14}^{25},-0.07)],[(S_9^{25},-0.473),(S_{16}^{25},0.31)])$	$([(S_8^{25},0.198),(S_{17}^{25},-0.402)],[(S_{10}^{25},-0.34),(S_{18}^{25},-0.06)])$
x_2	$([(S_{15}^{25},0.06),(S_{21}^{25},-0.48)],[(S_{19}^{25},-0.43),(S_{21}^{25},-0.3)])$	$([(S_{10}^{25},0.225),(S_{16}^{25},-0.4)],[(S_{11}^{25},0.21),(S_{17}^{25},0.13)])$	$([(S_9^{25},-0.286),(S_{14}^{25},-0.22)],[(S_9^{25},0.194),(S_{16}^{25},-0.285)])$	$([(S_{14}^{25},0.358),(S_{20}^{25},0.16)],[(S_{17}^{25},0.28),(S_{22}^{25},0.08)])$
x_3	$([(S_6^{25},0.406),(S_{16}^{25},0.379)],[(S_8^{25},0.372),(S_{17}^{25},-0.12)])$	$([(S_{14}^{25},-0.1),(S_{16}^{25},0.228)],[(S_{15}^{25},0.254),(S_{18}^{25},0.369)])$	$([(S_6^{25},0.191),(S_{10}^{25},-0.243)],[(S_7^{25},-0.408),(S_{11}^{25},0.145)])$	$([(S_{12}^{25},0.079),(S_{19}^{25},-0.2)],[(S_{13}^{25},0.358),(S_{19}^{25},0.2)])$
x_4	$([(S_{10}^{25},-0.13),(S_{19}^{25},0.46)],[(S_{14}^{25},-0.35),(S_{20}^{25},0.34)])$	$([(S_{11}^{25},0.367),(S_{16}^{25},0.26)],[(S_{14}^{25},0.43),(S_{18}^{25},0.3)])$	$([(S_8^{25},0.362),(S_{15}^{25},-0.178)],[(S_{11}^{25},-0.46),(S_{14}^{25},0.131)])$	$([(S_{12}^{25},-0.281),(S_{16}^{25},-0.2)],[(S_{15}^{25},-0.082),(S_{20}^{25},0.438)])$
W_j	0,23	0,21	0,215	0,24

After the calculation of the normalized IRN-2-Tuple linguistic value, the normalized IRN-2-Tuple linguistic decision matrix is obtained. Then, we build the IRN-ELH-ARAS weighted normalized decision-making matrix $\hat{R}_{ij}$ in which we compute the values of the optimality function (S_i), and the utility degree (K_i) to obtain a ranking of all the alternatives (TABLE IV). The priority order of the investment company can be presented as:

$$A_3 \succ A_4 \succ A_2 \succ A_1$$

TABLE IV. SOLUTION RESULTS

	S_i^R	S_i	K_i	Rank
OV	(Δ[0,13707645; 0,24450306]; Δ[0,17085678; 0,373832346])	0,23156716	1	
x_1	(Δ[0,10522227; 0,19496656]; Δ[0,12051822; 0,314229515])	0,18373414	0,79343783	4
x_2	(Δ[0,12032799; 0,18895036]; Δ[0,13983103; 0,289135255])	0,18456116	0,79700921	3
x_3	(Δ[0,11628693; 0,2247082]; Δ[0,14411791; 0,335750915])	0,20521599	0,88620506	1
x_4	(Δ[0,11678116; 0,19615561]; Δ[0,15414889; 0,30655936])	0,19341126	0,83522749	2

For $h = t^* - 1$

For facilating the comprehension to the different DMs, we transform the IRN- 2-Tuple collective value of the decision makers (TABLE III) into level 3 of the original linguistic term set l(3,9) because in our case the most of experts have expressed their preference in it (TABLE V). The transformation is done by applying transformation function (12). So, the new based normalized extended linguistic IRN 2-Tuple-ARAS value is obtained. Therefore, the optimality function (S'_i), the utility degree (K'_i) and a final ranking is obtained (TABLE VI).

The priority order of the investment company is presented as:

$$A_3 \succ A_4 \succ A_2 \succ A_1$$

TABLE V. TRANSFORMATION OF THE UNIFIED IRN- 2-TUPLE INPUT DATA OF DMS FROM $l(4,25)$ INTO $l(3,9)$

A	Y_1	Y_2	Y_3	Y_4
x_1	$([(S_3^9, 0.119), (S_5^9, -0.133)], [(S_5^9, -0.001), (S_6^9, 0.253)])$	$([(S_3^9, -0.487), (S_4^9, 0.412)], [(S_3^9, -0.058), (S_6^9, 0.035)])$	$([(S_3^9, -0.071), (S_5^9, -0.356)], [(S_3^9, -0.158), (S_5^9, 0.436)])$	$([(S_3^9, -0.267), (S_6^9, -0.467)], [(S_3^9, 0.22), (S_6^9, -0.02)])$
x_2	$([(S_5^9, 0.02), (S_7^9, -0.16)], [(S_6^9, 0.25), (S_7^9, 0.1)])$	$([(S_3^9, 0.258), (S_5^9, 0.199)], [(S_4^9, -0.4), (S_6^9, -0.29)])$	$([(S_3^9, -0.095), (S_5^9, -0.406)], [(S_3^9, 0.064), (S_5^9, 0.238)])$	$([(S_5^9, -0.2), (S_7^9, -0.28)], [(S_6^9, -0.24), (S_7^9, 0.36)])$
x_3	$([(S_2^9, 0.135), (S_5^9, 0.46)], [(S_3^9, -0.2), (S_6^9, -0.373)])$	$([(S_5^9, -0.366), (S_5^9, 0.409)], [(S_5^9, 0.085), (S_6^9, 0.123)])$	$([(S_2^9, 0.064), (S_3^9, 0.252)], [(S_2^9, 0.198), (S_4^9, -0.434)])$	$([(S_4^9, 0.026), (S_6^9, 0.266)], [(S_4^9, 0.453), (S_6^9, 0.4)])$
x_4	$([(S_3^9, 0.29), (S_6^9, 0.18)], [(S_5^9, -0.45), (S_7^9, -0.22)])$	$([(S_4^9, -0.21), (S_5^9, 0.246)], [(S_5^9, -0.19), (S_6^9, -0.1)])$	$([(S_3^9, -0.213), (S_5^9, -0.059)], [(S_4^9, -0.487), (S_5^9, -0.29)])$	$([(S_4^9, 0.906), (S_5^9, 0.266)], [(S_5^9, -0.027), (S_7^9, 0.187)])$
W_j	0,23	0,21	0,215	0,24

TABLE VI. SOLUTION RESULTS

	S'_i	K'_I	Rank
OV	0,23195168	1	
x_1	0,18475414	0,79651996	4
x_2	0,18603366	0,80203625	3
x_3	0,20463923	0,88224939	1
x_4	0,19389604	0,83593287	2

The company should choose computer company for its investment.

As can be noticed, we obtain the same ranking when we use the linguistic terms sets of the third (l(3,9)) and the fourth levels (l(4,25)) of the ELH. The complete pre order of alternative ranking makes it easier for decision-makers to understand the results.

VI. CONCLUSION

This paper present a new model for decision-making. The main objective of IRN-ARAS-ELH approach is to determine the best alternative among the various alternatives taking into account uncertainty problem. Imprecision in group decision making is taken into account when applying interval rough numbers in combination with the extended linguistic hierarchy ARAS method. Consequently , the final objectives results are obtained. In future research, other extensions of ordinary rough number will be developed.

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