A Consensus-based Best-Worst Method for Multi-criteria Group Decision-Making

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Abstract. The resolution of Multi-criteria Decision-Making (MCDM) problems driven by human knowledge involves collecting their opinions, which usually implies the emergence of inconsistencies. The Best-Worst Method (BWM) was proposed to reduce such inconsistencies and, consequently, obtain more reliable solutions for MCDM problems. Classically, the BWM finds the optimal weights for a set of criteria from the preferences of only one stakeholder, but lately it has been extended to deal with multi-criteria group decision-making (MCGDM) problems. However, when several Decision-Makers (DMs) take part in a decision process, disagreements may appear among them. If these conflicts are neglected, experts may feel unsatisfied with the solution chosen by the group or even question the decision process. Therefore, this contribution proposes an extension of the BWM to smooth disagreements and obtain consensual solutions in MCGDM problems. To do so, an optimization model is introduced which derives a collectively agreed solution for the criteria weights. Additionally, such an optimization model is based on linear programming, which provides accurate results and the ability to deal with hundreds or thousands of DMs.

Keywords: Best-Worst method · Multi-criteria Group decision-making · Consensus

1 Introduction

The use of human knowledge in some decision-making problems is essential in contexts in which the available information is neither objective nor quantitative, but uncertain and qualitative [18,25]. In this regard, group decision-making (GDM) problems aim at improving decision processes by taking into account the points of view of multiple experts to make a decision. However, when multiple DMs are involved in the resolution of a decision-making problem, the emergence of conflicts among them is unavoidable [7]. Therefore, it is usual to include a consensus mechanism in the GDM process to soften these discrepancies and provide a collectively agreed solution for the group decision situation [13]. According to

Martínez and Montero [16], the most popular methodology to manage the idea of consensus may be the notion of Soft Consensus, proposed by Kacprzyk [11], which is based on the concept of fuzzy majority.

On the other hand, when a human stakeholder is asked to give his/her opinion using pairwise comparison matrices [26], the DM usually does not provide such comparisons consistently, but by incurring some contradictions. Due to the fact that such inconsistencies would negatively affect the decision process, the Best-Worst Method (BWM) [20] was initially proposed as an algorithm to solve some behavioral errors in similar Multi-criteria Decision-Making (MCDM) methods and, consequently, reduce the number of pairwise comparisons and inconsistencies. Concretely, the BWM aims at deriving the weights of the criteria in a MCDM problem by just considering the pairwise comparisons between the best and the worst criteria with all the others. To do so, BWM considers an optimization model whose solution provides weights for the criteria that are similar to the original preferences elicited from the DM.

Even though there are some proposals which aim at adapting the BWM to GDM [15, 17], they neglect the notion of Soft Consensus [11] and the use of consensus measures [19] to manage the disagreements among DMs. Therefore, this contribution aims to simultaneously deal with both the inconsistencies in the preferences given by human DMs and the conflicts that appear among the participants in GDM problems. To do so, here it is proposed a reformulation of classic BWM which allows managing several DMs to provide agreed collective weights derived from pairwise comparisons involving their opinions about the best and the worst criteria. Furthermore, the proposed Consensus-based BWM (C-BWM) is stated in terms of pairwise comparisons given in a 0-1 scale, which allow linearizing the optimization model to obtain more accurate results than the ones provided by nonlinear solvers (see Fig 1).

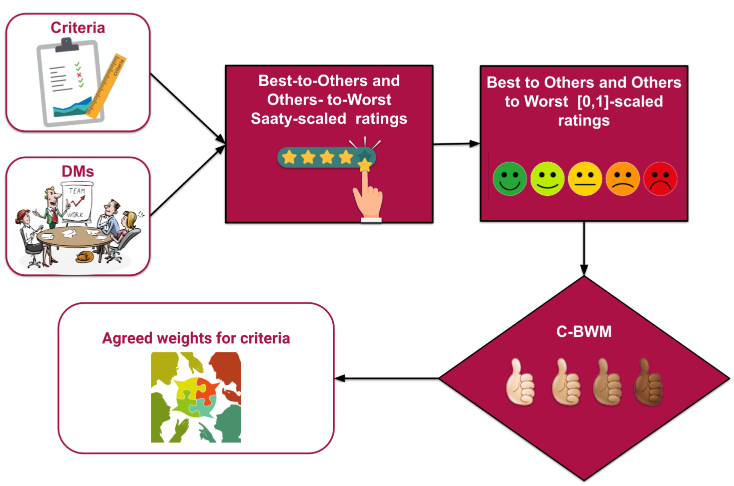

Fig. 1: Scheme of C-BWM model

In addition, this linearization improves the computational performance with respect to model resolution, and consequently qualifies the proposal to quickly manage GDM problems involving hundreds or thousands of DMs [23].

The remainder of the contribution is organized as follows: Section 2 reviews basic concepts related to GDM, consensus, and BWM. Section 3 introduces the proposal, a C-BWM for MCGDM. In Section 4, the performance of the proposal is shown through the resolution of a MCGDM problem. To conclude, Section 5 draws some conclusions together with future research lines.

2 Background

This section is devoted to revising some basic concepts related to the proposal.

2.1 Group Decision-Making and Consensus

Different situations in real-world contexts, such as business, work, social or personal life, require making a choice among different options [1, 9]. Even though sometimes the available information regarding such decisions is objective, when the accessible data is vague or uncertain, the complexity of the decision process increases, and it is necessary to take into account the knowledge of a group of human experts to consider multiple views [13, 18]. In particular, when a group of experts $E = \{E_1, E_2, \ldots, E_m\}$ is asked to evaluate possible alternatives $A = \{A_1, A_2, \ldots A_r\}$ according to different criteria $C = \{C_1, C_2, \ldots, C_n\}$, the decision problem is a MCGDM problem [10, 12].

Despite the participation of several DMs in the decision process presents several advantages, it also gives rise to a relevant phenomenon: the emergence of disagreements between them [13]. If disagreements are not properly managed before selecting a solution for the decision problem, such a solution may not satisfy some experts, which could question the trustworthiness of the process [7]. To overcome this drawback, Consensus Reaching Processes (CRPs) are performed before choosing the best alternative to smooth out the possible conflicts among experts' opinions and, in this way, achieve an agreed solution for the decision problem [8, 18]. In a CRP, experts talk to each other, exchange views, and, if they consider, change their initial opinions to increase the level of agreement within the group. This process is usually driven by a moderator, who is in charge of identifying conflicts and suggesting changes to experts about their opinions [14]. Several consensus approaches have been proposed in the specialized literature [19]. Some of them include feedback mechanisms, in which experts are asked if they want to change their preferences [22]. On the other hand, other proposals provide an automatic process in which experts are not questioned about modifying their preferences, but they are automatically modified [2] (see Fig. 2).

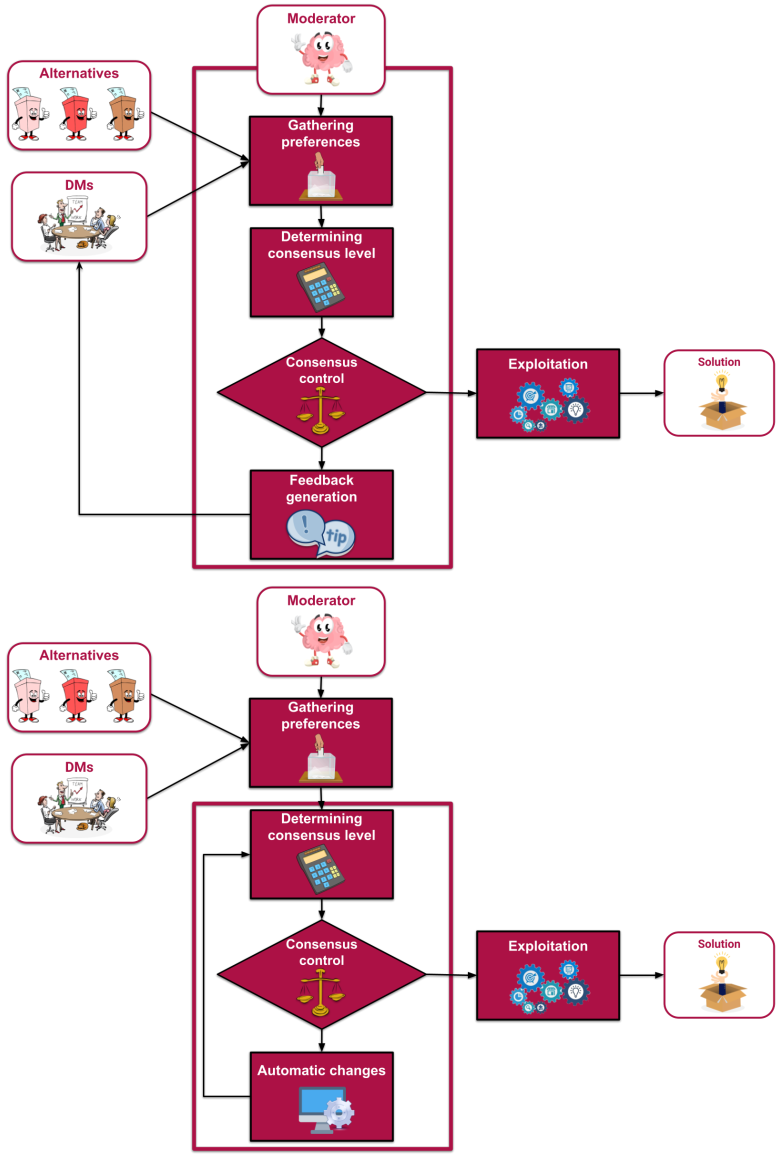

Fig. 2: Schemes of CRPs with and without feedback mechanisms

2.2 Best-Worst Method

The classic BWM [6, 20, 21] aims to determine the priority of the criteria $C = \{C_1, C_2, \ldots, C_n\}$ in a certain MCDM problem by reducing inconsistencies in the elicitation process and obtaining more consistent solutions.

To do so, the DM should provide, according to his point of view, the best and the worst criteria, which are denoted C_B and C_W, respectively. Furthermore, such DM must point out the comparison of C_B and the remaining criteria to obtain the Best-to-Others (BO) vector,

$$BO = \{a_{B1}, a_{B2}, \ldots, a_{Bn}\},$$

where $a_{Bi} \in [1, 9] \cap \mathbb{N}$ denotes the degree of preference of C_B over the criterion C_i. In the same way, the DM also compares all criteria with C_W to obtain the Others-to-Worst (OW) vector,

$$OW = \{a_{1W}, a_{2W}, \ldots, a_{nW}\},$$

where $a_{iW} \in [1, 9] \cap \mathbb{N}$ denotes the preference degree of the criterion C_i over C_W.

These values are then used as the input of an optimization model to obtain the weights for the criteria $\{w_1^*, w_2^*, \ldots, w_n^*\}$:

$$\begin{aligned} &\min_{w} \quad \max_{i=1,2,\ldots,n} \left\{ |a_{Bi} - \frac{w_B}{w_i}|, |a_{iW} - \frac{w_i}{w_W}| \right\} \\ &\text{s.t.} \quad \begin{cases} \sum_{i=1}^{n} w_i = 1, \\ w_i \geq 0, \quad \forall \;\; i = 1, 2, ..., n, \end{cases} \end{aligned} \tag{BWM}$$

Due to the full consistency of a multiplicative pairwise comparison matrix in 1-9 Saaty's scale [24] is given by $a_{ij} = a_{ik}a_{kj} \; \forall \; i, j, k \in \{1, 2, ..., n\}$, the weights obtained as output from the BWM allow constructing a fully consistent multiplicative pairwise comparison matrix ($\hat{a}_{ij} := \frac{w_i}{w_j}$) which is close to the original preferences BO and OW given by the DM.

3 Consensus-based Best-Worst Method

Let us consider a MCGDM problem in which a group of $m \in \mathbb{N}$ experts $E = \{E_1, E_2, ..., E_m\}$ wants to reach a collective agreed solution about the importance of $n \in \mathbb{N}$ criteria. To do so, each expert provides his/her opinions using Best-Worst preferences, i.e., each expert E_k points out which are the best (B^k) and the worst (W^k) criteria according to his/her point of view and also provides two pairwise comparison vectors: for the best criteria C_{B^k}

$$BO^k = (a_{B^k1}, a_{B^k2}, ..., a_{B^kn})$$

and for the worst criteria C_{W^k}

$$OW^k = (a_{1W^k}, a_{2,W^k}, ..., a_{n,W^k})$$

These preferences are given by experts using a 1-9 Saaty scale [24]. To simplify the notation and linearize the optimization model, this information is first remapped into a linear scale by using the function $f : [\frac{1}{9}, 9] \to [0, 1]$ defined as $f(x) = \frac{1}{2}(1 + \log_9(x)) \; \forall \; x \in [\frac{1}{9}, 9]$, and then it is stored as follows:

- A matrix $P \in \mathcal{M}_{m,n}$ containing the preferences BO^k, where $P_{ki} = f(a_{B^k i})$ $\forall\ 1 \leq k \leq m, 1 \leq i \leq n$,
- A matrix $Q \in \mathcal{M}_{m,n}$ containing the preferences OW^k, where $Q_{ki} = f(a_{iW^k})$ $\forall\ 1 \leq k \leq m, 1 \leq i \leq n$,
- A vector $B = (B^1, B^2, ..., B^m) \in \mathbb{R}^m$, $B^k \in \{1, 2, ..., n\}$, containing the best criterion for each expert,
- A vector $W = (W^1, W^2, ..., W^m) \in \mathbb{R}^m$, $W^k \in \{1, 2, ..., n\}$, containing the worst criterion for each expert.

In the same way, the solution for the collective BWM problem is stored in a matrix with the corresponding individual weights $(w_{ki}) \in \mathcal{M}_{m\times n}([0,1])$, where w_{ki} represents the individual weight of the expert E_k for the criterion C_i. Respectively, the collective weights are provided in a vector $(g_1, g_2, ..., g_n) \in \mathbb{R}^n$, where $g_i = \frac{1}{m}\sum_{k=1}^{m} w_{ki}$ $i \in \{1, 2, ..., n\}$ satisfy the consensus constraint $|w_{ki} - g_i| \leq \varepsilon$ $\forall\ i = 1, 2, ..., n, k = 1, 2, ..., m.$ and $\varepsilon \in]0, 1]$.

When using ratings on a $0-1$ linear scale, the distance between the weights that the method aims to obtain and the original preferences elicited from the experts can be defined by $\xi : \mathcal{M}_{m,n} \times \mathcal{M}_{m,n} \times \mathbb{R}^n \times \mathbb{R}^n \times \mathcal{M}_{m,n} \to \mathbb{R}^+$

$$\xi(P, Q, B, W, w) = \sum_{k=1}^{m}\sum_{i=1}^{n}(|P_{ki} - 0.5 - w_{kB^k} + w_{ki}| + |Q_{ki} - 0.5 - w_{ki} + w_{kW^k}|)$$

Remark 1 *Note that the function ξ stands for an additive distance in a 0-1 scale, easier to linearize than the original multiplicative distance for a 1-9 Saaty's scale.*

Therefore, for fixed values P, Q, B, W, the C-BWM is defined as

$$\begin{aligned}
&\min_{w,g}\ \xi(P, Q, B, W, w) \\
&\text{s.t.} \begin{cases} \sum_{i=1}^{n} w_{ki} = 1,\ \forall\ k = 1, 2, ..., m \\ w_{ki} \geq 0,\ \ \forall\ \ i = 1, 2, ..., n, k = 1, 2, ..., m, \\ g_i = \frac{1}{m}\sum_{k=1}^{m} w_{ki}\ \forall\ i = 1, 2, ..., n, \\ |w_{ki} - g_i| \leq \varepsilon\ \ \forall\ \ i = 1, 2, ..., n, k = 1, 2, ..., m. \end{cases}
\end{aligned} \tag{C-BWM}$$

The output of this model consists of individual and collective weights which minimize the distance function ξ for the given preferences and satisfy a consensus condition. In addition, the presented model allows considering both preferences elicited in a $1-9$ Saaty's scale [24] and preferences in the range $[0-1]$ (see Fig. 1). Furthermore, the proposed model substitutes the original objective function in BWM [20] for another one based on linear combinations and absolute values, which facilitates the linearization of the model to deal with hundreds or even thousands of DMs and provide more precise results in the numeric resolution [23].

Remark 2 *Note that the optimization model C-BWM provides a consensual solution in which the DMs' preferences are modified as little as possible, since it minimizes the distance between the weights that the method obtains and the original preferences given by the DMs.*

Remark 3 *Note that this model considers the function f to remap the 1-9 multiplicative scale into a 0-1 additive scale. Consequently, quotient-based expressions in traditional BWM are now expressed in terms of sums and differences. For this reason, the aggregation of the weights has been conducted using an arithmetic mean instead of the geometric mean.*

4 Illustrative Example

In this section, the proposed model is applied to select the agreed criteria weights in a MCGDM problem.

The soft drink company BAT-Cola has started the recruitment process for a marketing manager. The recruitment team, formed by five experts $E = \{E_1, E_2, E_3, E_4, E_5\}$, considers that four key aspects should be taken into account in the selection process: C=\{C_1: Proficiency in languages, C_2: Work experience, C_3: Leadership capacity, C_4: Age\}. However, experts have different opinions about the importance of these aspects in selecting the best candidate, and the company would prefer an agreed importance of these criteria. For this reason, before the selection process, the company asks the team to provide their preferences about the importance of the skills and, from them, obtain collective consensual weights for each aspect. These preferences are compiled in Table 1.

Table 1: BO and OW preferences in 1-9 Saaty's scale.

BO	C_1	C_2	C_3	C_4	Best
E_1	1	2	6	3	1
E_2	8	1	6	3	2
E_3	1	3	6	3	1
E_4	4	1	8	8	2
E_5	4	1	4	5	2

OW	C_1	C_2	C_3	C_4	Worst
E_1	6	3	1	2	3
E_2	1	8	2	6	1
E_3	6	5	1	3	3
E_4	2	8	1	5	3
E_5	4	5	4	1	4

Experts have provided their preferences using the 1-9 Saaty scale, which are transformed into a 0-1 scale using the function $f : [\frac{1}{9}, 9] \to [0, 1]$ defined as $f(x) = \frac{1}{2}(1 + \log_9(x))\ \forall\ x \in [\frac{1}{9}, 9]$ (see Table 2) to apply the C-BWM lately and derive the consensual weights.

Table 2: BO and OW preferences in 0-1 scale.

BO	C_1	C_2	C_3	C_4	Best
E_1	0.5	0.66	0.91	0.75	1
E_2	0.97	0.5	0.91	0.75	2
E_3	0.5	0.75	0.91	0.75	1
E_4	0.82	0.5	0.97	0.97	2
E_5	0.82	0.5	0.82	0.87	2

OW	C_1	C_2	C_3	C_4	Worst
E_1	0.91	0.75	0.5	0.66	3
E_2	0.5	0.97	0.66	0.91	1
E_3	0.91	0.87	0.5	0.75	3
E_4	0.66	0.97	0.5	0.87	3
E_5	0.82	0.87	0.82	0.5	4

From the experts' preferences, the C-BWM allows obtaining consensual collective weights ($\varepsilon = 0.05$). The approximated results of the optimization model are shown in Table 3. To obtain the results, we have used the solver Clp for the Julia 1.6 programming language [3] on the cloud service Google Colaboratory [4] (2.20GHz Intel(R) Xeon(R) CPU and 13 GB RAM).

Table 3: Results.

Weights	C_1	C_2	C_3	C_4
E_1	0.42	0.4	0.01	0.17
E_2	0.32	0.5	0.09	0.09
E_3	0.42	0.4	0.01	0.17
E_4	0.32	0.5	0.03	0.15
E_5	0.37	0.45	0.1	0.08
Collective	0.37	0.45	0.05	0.13

Therefore, according to the optimization model, the most relevant criterion is C_2: Work Experience (0.45), which makes sense taking into account that the majority of experts in the group think that this aspect is the most important. In the same sense, C_3: Leadership capacity (0.05) is selected as the worst rated criterion by the majority of the experts, and this is reflected in the consensual collective weight. It should be highlighted that, even though some experts have had to modify their original preferences (for example, E_2), these modifications are the minimum required to satisfy the consensus condition.

To graphically show the significance of including consensus in the classical BWM approach, the individual weights obtained by applying the classical BWM to the BO and OW preferences ($\varepsilon = 1$ in the C-BWM) have been compared with the modified agreed opinions obtained by using the C-BWM with $\varepsilon = 0.05$. Fig. 3 shows the multidimensional scaling (MDS) representation [5] of both non-agreed and agreed preferences and their respective collective opinion. As expected, the figure indicates that the preferences corresponding to the consensual approach are much closer to the group opinion.

To sum up, the introduced optimization model is able to detect and smooth disagreements in the experts' preferences in a GDM problem and provide an agreed solution to derive the criteria weights when managing multiple DMs.

5 Conclusions

This contribution has introduced an extension of BWM for MCGDM, which allows obtaining agreed weights for the given criteria according to the preferences of several experts.

To do so, the original optimization-based BWM proposal [20] has been modified to take into account the preferences of several experts and provide a collectively agreed solution to weight the considered criteria. In addition, this proposal has been presented in terms of linear variables, objective function, and constraints, which allow providing more precise results than when using nonlinear optimization. Furthermore, this linear approach also implies an improvement

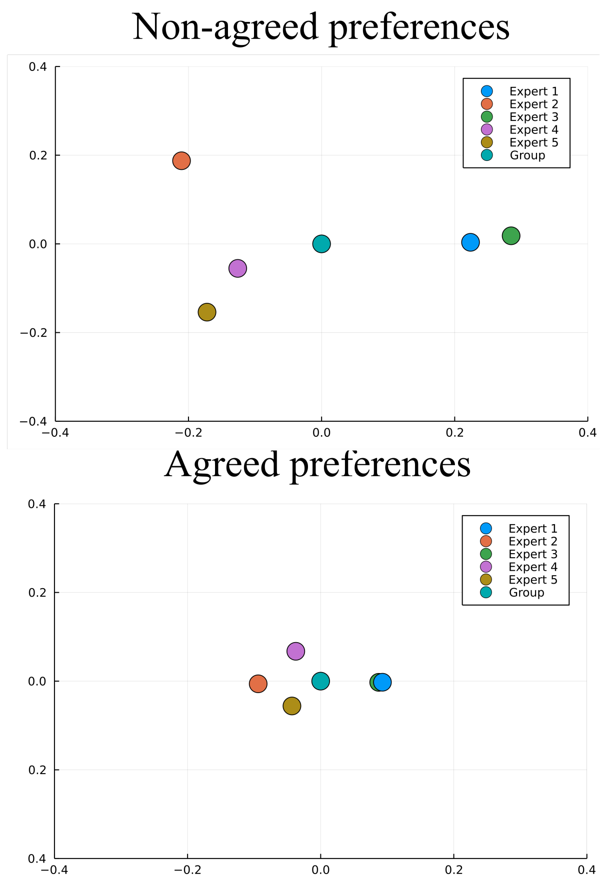

Fig. 3: Graphic MDS [5] visualization showing the relative distances between the DMs represented in two dimensions.

of computational efficiency, which guarantees the good performance of the model when dealing with decision problems which consider hundreds or thousands of experts.

Future studies should focus on exploring the relationship between this proposal and the recently proposed Comprehensive Minimum Cost Consensus (CMCC) models [13] as well as introducing new BWM-CMCC models which could be based on multi-objective optimization or bilevel optimization.

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