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Individual Semantics Building for HFLTS Possibility Distribution With Applications in Domain-Specific Collaborative Decision Making

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ABSTRACT Collaborative decision making (CDM) with linguistic computational techniques has recently achieved significant advancements. Due to the widespread use of sophisticated linguistic constructions, such as generalized comparative linguistic expressions (GCLEs), additional information associated with subjective appraisals has been exploited with the aim of addressing accuracy improvements in multifarious CDM, given that partial information loss is almost inevitable while dealing with complex linguistic comprehension. This paper brings an innovative perspective into CDM from COmponent ANalysis with GCLEs (COANG) to formalize problems involved in making optimal choices, mainly in CDM problems with participants who are usually characterized by domain specificity. Consequently, the focus of this paper is on the domain-specific CDM (DSCDM) in which individual semantics should be built predominantly to model various implications of their decision appraisals with heterogeneity in the knowledgeable domain for the effort of computational reinforcements. The attitude orientation and strength are crucial decompositions to incorporate COANG into DSCDM to establish an elastic paradigm that puts forward individual perception comprehension ahead of exerting collective efforts. The DSCDM based on COANG model enables agents to turn complex challenges of sophisticated linguistic constructions into substantial opportunities by translating them into customized individual semantics (CIS), and CIS into useful insights for making better decisions and improving results. The potential advantages of the proposed COANG-based DSCDM framework are validated with a clinical psychological practice related to the severity assessment of symptoms of schizophrenia.

INDEX TERMS Hesitant fuzzy linguistic term set, computing with words, schizophrenia, positive and negative syndrome scale (PANSS), domain-specific collaborative decision making.

I. INTRODUCTION

With the proliferation of exploring the challenge of applying complex linguistic constructions in developing practice-oriented collaborative decision-making (CDM) techniques, there is an explosive growth of the amount of participant-contributed opinion perceptions in the manifestation of decision appraisals [4], [12], [22]. The past decades have witnessed the prominent role of qualitative CDM technologies played in shaping human activity in organizations and society [2], [63]. Traditional qualitative CDM models deal with multiple points of view in the characterization of several instrumental features including (but not limited to): individual opinion representation, qualitative opinion fusion, consensus building and adjustment, and an exploitation process to arrive at an acceptable final decision [6], [17], [19]–[22], [43], [73].

Domain-specific CDM (DSCDM), in comparison with the traditional CDM, is a more general and complex theory with multiple aspects of content that need to be specified for different real-world conditions. The DSCDM theories of development hold that we have many independent and specialized knowledgeable participants who exhibit domain-specific expertise rather than share one cohesive expertise knowledge database [24]. Individuals with domain specificity are entitled to their own opinions in CDM, and they occupy a prominent place in collectively choosing from the alternatives in front of them, which is an integrated process calling for intensive discussions to make full use of their expertise and experience during domain-specific appraisal modeling.

Building on the tacit intention of DSCDM, the idea that all kinds of CDM processes in any situation can be accounted for by one limited general set of computational treatments has been replaced by a view on CDM that acknowledges the importance of the domain-specific expertise, as well as the nature of the heterogeneous individual meanings of opinions conveyed [10], [40]. Practical examples of DSCDM applications are available across a wide-ranging spectrum, which in the dermatology domain, for example, a medical specialty requires physicians to have image inspection experience [24]. Automating or at least aiding such efforts requires understanding physicians' reasoning processes and their use of domain knowledge.

Nowadays the goal of CDM is no longer to create general theories of CDM in the context of large amounts of input data and representation format diversification but to focus on how individuals construct and deliver their opinions associated with information that cannot be unreservedly observed in practice [10]. Despite its advancements hitherto across several fronts, the paradigm change in recognizing the opinion structure has become a barrier to the progress of participants' knowledge if the existing DSCDM methods were applied. This undesired feature necessitates advanced DSCDM methods able to deal with potential challenges stemming from their real-world applications. Consequential developments regarding the current DSCDM are manifold, which to be fostered in this study constitutes our primary motivation. The general framework for DSCDM proposed in this paper is introduced in Figure 1, in which multiple relevant facets are further detailed in the following subtopics:

- **Individual opinion representation**. Fine-tuned linguistic constructions capable of giving enough freedom for participants with domain specificity in CDM need to be elaborated as a means to articulate opinionated expressions that are easy to decipher [30], [54]. This study considers comparative linguistic expressions (CLEs) initially coined by Rodríguez *et al.* [56] from context-free grammar (CFG) to provide a higher level of flexibility and comprehensiveness to preferences elicited by hesitant fuzzy linguistic term set (HFLTS). Comparing possible linguistic terms in a fair and informative manner is often not straightforward in the presence of time pressure and requirement of prompt action. With the extension of CFG that usually known as extended CFG (ECFG), the CLEs were further advanced in Rodríguez *et al.* [57] to contain more open-ended expressions. The set of CLEs is conventionally used in its generalized form, i.e., GCLE[1] Thanks to the rule of approximate equivalent linguistic transformation (AppELT) by Rodríguez *et al.* [57], the smooth computational treatment of GCLEs can be guaranteed as the AppELT creates a simple and efficient route translating GCLEs into HFLTSs, and also the enhancement of collective intelligence decision-making processes.
- **Domain-specific appraisal modeling**. Domain-specific appraisal modeling is de facto a process of individual semantics building for decision appraisals [24], [60]. Handling qualitative individual opinion representation in the form of GCLEs approaches have limitations in some instances [10], [63]. Ideas and methods brought into customizing semantics for individuals, which is an indispensable part of domain-specific appraisal modeling, are extremely valuable in this endeavour as they are meant to deepen rational linguistic understanding [9], [10], [40]. Semantic interpretation following the uniform distribution-based approach by Wu and Xu [64] subject to Laplace decision criterion (LapDECR) is liable to provide misleading information [10], [41]. The AppELT-based transformation from

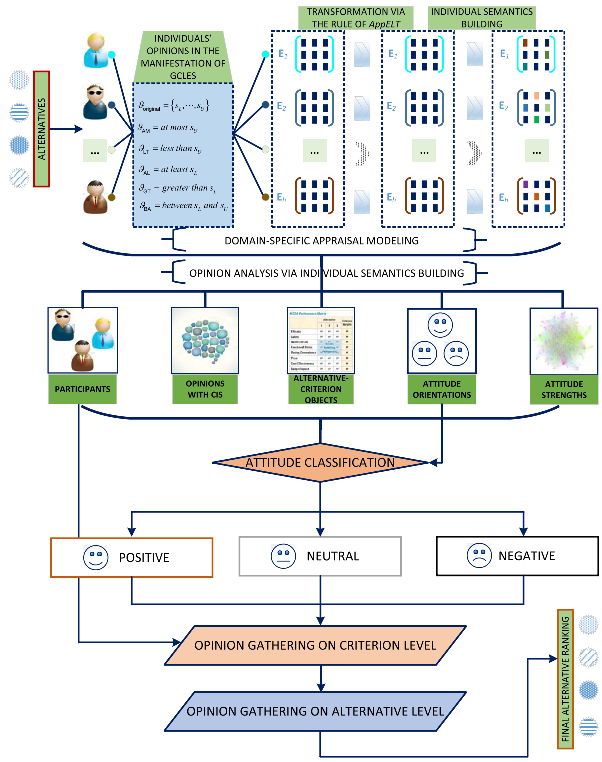

FIGURE 1. General framework for DSCDM.

[1]The expressions generated by the ECFG may be either single linguistic terms or CLEs. Both types define the expression domain of the ECFG, and they are collectively termed as generalized CLEs (GCLEs) [9], [10].

either homogeneous GCLEs[2] or heterogeneous GCLEs[3] to HFLTSs have the chances of receiving identical treatment because of the semantic equivalence. Semantic analysis for individuals is, then vital to their language-independent meanings.

- **Component analysis with GCLE**. The implementation of domain-specific appraisal modeling allows breaking down the problem of CDM into an integral structure consisting of several subcomponents, which are opinions with customized individual semantics (CIS), alternative-criterion object, participant, attitude orientation and attitude strength, respectively (See Section IV.A). COmponent ANalysis with GCLE (COANG) refers to the specification of the CDM structure bringing manifold perspectives to systematically identify, extract, quantify, and study affective states and subjective information. Generally speaking, COANG is a streamlined approach to declaring an aspect-oriented structure that stores multidimensional information generated from the process of text analysis for the sake that one-sided computational treatment concerning merely about the inputs themselves is evidently not enough. In the era of Big Data, the conventional means to make decisions in CDM from subjective data with mediocre quality on a large scale become insensitive to the inherent complexity of cognitive styles associated to involved participants from culturally and linguistically diverse backgrounds. Now, more than ever, it's vitally important to ensure data quality to support final decisions with prominent accuracy enhancements [45]. CIS supports determining attitude orientation and attitude strength that are two crucial decompositions to COANG, through the *Attitude Character* function proposed by Yager [68]. Novel CDM structure based on individual semantics building has to be defined for participants who are congregated in the selection of a satisfactory alternative.
- **The COANG-based DSCDM framework**. The main tasks of COANG focus on examining the interplay or predicting attitude polarities between positive and negative links. Given the multifaceted information embedded in the COANG model, expanding and diversifying the theoretical advancements as well as potential applications become feasible and valuable, and among them, the one development to be investigated in this study is for participants to undertake to make optimal choices subject to the satisfaction of various criteria. Under the COANG-based DSCDM framework, gaining a full insight into participants' behavior becomes a reality with its ability to complete tasks such as classification of subjective appraisals with different attitude orientations and group attitude detection with attitude strength. Due to the rise of social network platforms and advances in mobile/cloud computing [47], [48], the COANG-based DSCDM framework has the potential in processing comparative online reviews. Relations of similarities or differences between two targets are expressed via comparative opinions with emotional or rational sentiments, and the essential components of GCLEs can be elicited from them [26], [29]. However, we concentrate on subjective appraisals instead of comparative online reviews. Last but not least, the CIS can model dependably the attitude orientation and attitude strength embedded in GCLEs; however, it is nigh impossible to create a feasible solution in one unified paradigm for adapting CIS to comparative online reviews because they contain a considerable amount of variants.

In summary, this study aims at making an innovative contribution to the field of DSCDM by introducing the COANG model and concurrent aggregation techniques for the merger of decision appraisals and their associated attitude characters to address the key challenges of attitude recognition and exploitation in the context of GCLEs. Therefore it is investigated the role of attitude-related information in gaining more useful insights into DSCDM based on domain-specific appraisal modeling, and based on this to develop a novel CDM framework for structuring participant-contributed opinions to facilitate accuracy improvements in DSCDM.

More specifically, the objectives of this study are to:

▹ Investigate and develop an effective domain-specific appraisal modeling based on CIS building on heterogeneous subordinate possibilistic information (SPI) for capturing and visualising participants' complex cognition process.

▹ Design several optimization-based algorithms with the maximization of correlation between the pregenerated SPI and the modified SPI.

▹ Develop a COANG-based DSCDM model by decomposing the traditional CDM framework into several components from a systematic perspective.

▹ Implement this decision model through in clinical psychological diagnosis experience for demonstrating its manipulation, analysis, and evaluation.

[2]Considered here is that three experts have been requested to participate in an evaluation panel for material supplier selection. The experts are primarily asked to evaluate the business reputation of one of the candidate material suppliers. The opinions given are based on $S = \{s_{-4} : \textit{extremely poor}, s_{-3} : \textit{very poor}, s_{-2} : \textit{poor}, s_{-1} : \textit{slightly poor}, s_0 : \textit{fair}, s_1 : \textit{slightly good}, s_2 : \textit{good}, s_3 : \textit{very good}, s_4 : \textit{extremely good}\}$, which is a balanced linguistic term set (LTS). The three experts hold unexceptionally the same opinion, i.e., $\{\textit{between } s_1 \textit{ and } s_3\}$, toward the candidate material supplier in respect of its business reputation unanimously. Following the rule of AppELT derives the same HFLTS, i.e., $\{s_1, s_2, s_3\}$, and subsequently, using the uniform distribution-based approach generates its corresponding possibility distribution $(0, 0, 0, 0, 0, 1/3, 1/3, 1/3, 0)$. The individual opinions receive equal treatment while computing with them.

[3]Continued from footnote 2, we consider another case where experts offer their views in the manifestation of different GCLEs. Opinions offered by the three experts are *greater than* s_1, *at least* s_2, and *between* s_2 *and* s_4, respectively. Following the rule of AppELT in this case as well obtains the same conversion of these GCLEs, i.e., $\{s_2, s_3, s_4\}$. Computing with them loses their initial comparative implications as it finally leads into the processing of the same possibility distribution $(0, 0, 0, 0, 0, 0, 1/3, 1/3, 1/3)$. Evidently, the computational treatment conditioned on the uniform distribution-based approach to generating HFLTS possibility distribution is contentious.

- ▹ Conduct evaluation of the developed methodology in practical application background using available data sets collected that are privacy concern-eliminated.

The remainder of this paper is structured as follows. Section II introduces a basic description of the HFLTS and OWA concepts. Section III focuses on the domain-specific appraisal modeling in which heterogeneous SPI is used to generate attitudinal HFLTS possibility distribution through the construction of several nonlinear optimization models. In Section IV, the novel DSCDM framework with the COANG model is proposed in which the COANG model and the derived uncertain linguistic information are detailed. Section V conducts a case study for illustration of the proposed COANG-based DSCDM framework. Finally, Section VI concludes this paper.

II. PRELIMINARIES

This section reviews several basic definitions and notions that are crucial to the subsequent development of this study.

Let n be any nonzero natural integer, $\mathbb{N}$ be the set of strictly positive integers and set $[n] := \{1, 2, \cdots, n\}$ and $\lfloor n \rfloor := \{-n, \cdots, -1, 0, 1, \cdots, n\}$. Bold symbols are used to denote n-tuples. For instance, $(x_1, \cdots, x_n)$ is often written as $\boldsymbol{x}$. In the current paper, a possibility distribution $(p_{-\tau}, \cdots, p_i, \cdots, p_\tau)$ is written as P. The following terminologies are introduced to facilitate relevant descriptions. The cardinality of any set K is denoted by $\#K$. The subscript set of a subscript-symmetric additive LTS

$$S = \{s_t | t = -\tau, \cdots, -1, 0, 1, \cdots, \tau\}$$

is denoted by $\lfloor \tau \rfloor$. That is, $\lfloor \tau \rfloor = \{-\tau, \cdots, -1, 0, 1, \cdots, \tau\}$ and $S = \{s_t | t \in \lfloor \tau \rfloor\}$.

A. HFLTS POSSIBILITY DISTRIBUTION

The linguistic variable is a renowned concept coined by Zadeh [72] as to foster a structural modeling capacity to express semantical implications that cannot be established by merely a single term to manifest words or sentences in a natural or artificial language. Linguistic variables are suitable for characterizing phenomena that are too complex or too ill-defined to be amenable to conventional quantitative descriptions in an approximate manner. For the past decades, the linguistic variable has attracted broad attention since its inception from researchers and scholars who were, and are, actively advancing the theory of natural language processing in the use of fuzzy linguistic approaches.

The use of linguistic variables implies processes of computing with words (CW) that assists agents in gaining more insights into complex human activities, which in a way accomplishes the desired aim of distilling useful information from broad and non-specific thoughts [33]. The legitimate selection of linguistic descriptors of the linguistic terms and their semantics can commendably smooth the way for processing linguistic variables. One of the existing approaches to such selection is the determination of linguistic descriptors in which all the terms are distributed on a scale given a predefined order structure suggested by Delgado *et al.* [14].

Let $S = \{s_0, s_1, \cdots, s_g\}$ be a LTS with odd granularity $g + 1$, where the term s_j represents a possible value for a linguistic variable. Generally, the linguistic term set is required to satisfy the following characteristics:

(1) The set is ordered: $s_i \geq s_j$ if and only if $i \geq j$. Hence, a minimization operator and a maximization operator exist.

(2) There exists a negation operator: $neg(s_j) = s_{g-j}$.

Eliciting a single linguistic term may not be enough to characterize complex human cognitive mechanisms. People are frequently required of to make decisions given limited time or resources or faced with difficulties that are not easy to overcome, which add up to create the appraisal environment under which the cognitive process of experts endowed with hesitance needs to be elaborated. The proposal of HFLTS proposed by Rodríguez *et al.* [56], which is a generalization of hesitant fuzzy sets [61] on a qualitative setting, has proven to be an efficient surrogate to traditional information elicitation methodologies [39], [63]. The formal definition of HFLTS is given below.

Definition 2 [56]: Let $S = \{s_0, s_1, \cdots, s_g\}$ be a LTS. An HFLTS H_S on S is an ordered finite subset of consecutive linguistic terms in S.

Substituting subscript-symmetric additive LTS $S = \{s_t | t \in \lfloor \tau \rfloor\}$ for the traditional LTS $S = \{s_0, s_1, \cdots, s_g\}$ with odd granularity $g + 1$ in Definition 2 has been diffusely adopted. The $S = \{s_t | t \in \lfloor \tau \rfloor\}$ is a LTS in which the linguistic term s_0 is used to represent the median term and the remaining in S are placed uniformly and systematically around it. Serving as an alternative to the traditional LTS, it successfully incorporates the attitudinal dimension with a bipolar scale rather than a unipolar scale [35]. The current paper follows this convention without loss of generality. The operations defined on S can be found in [55]. The distance measure for any two linguistic terms in S are defined as $d(s_i, s_j) = (i - j)/2\tau$ for $s_i, s_j \in S$ [67].

Since the research on Zadeh's fuzzy linguistic approach and the CW paradigm becomes more in-depth and detailed [69], [72], computing with HFLTSs has been commonly seen as a subarea of linguistic computational intelligence systems [37], [42], [44], [59]. Thanks in particular to the proposal of CLEs in parallel with HFLTS, computing with HFLTSs has expanded the research of linguistic computational intelligence systems significantly because it has introduced many challenging research problems that had not been studied before. The concept of AppELT is crucial to this study as it is used as a transformation logic manipulation that creates a bridge between GCLEs and HFLTSs.

Definition 3 [57]: Let $\mathcal{E}$ be a function that transforms the linguistic expressions $ll \in S_{ll}$ obtained by the context-free grammar G_H into HFLTSs. S is the LTS used by G_H, and S_{ll} is the expression domain generated by G_H: $\mathcal{E} : S_{ll} \rightarrow H_S$. The linguistic expressions generated by G_H by using the *production rules will be transformed into HFLTS by means* *of the following transformations:*

$$\begin{cases} \mathcal{E}(s_i) = \{s_i | s_i \in S\}, \\ \mathcal{E}(\text{at most } s_i) = \{s_j | s_j \in S \text{ and } s_j \leq s_i\}, \\ \mathcal{E}(\text{lower than } s_i) = \{s_j | s_j \in S \text{ and } s_j < s_i\}, \\ \mathcal{E}(\text{at least } s_i) = \{s_j | s_j \in S \text{ and } s_j \geq s_i\}, \\ \mathcal{E}(\text{greater than } s_i) = \{s_j | s_j \in S \text{ and } s_j > s_i\}, \\ \mathcal{E}(\text{betweens}_i \text{ and } s_j) = \{s_k | s_k \in S \text{ and } s_i \leq s_k \leq s_j\}. \end{cases}$$

The implication of the first equation $\mathcal{E}(s_i) = \{s_i | s_i \in S\}$ was expanded in Chen *et al.* [9] as a signal that indicates the transformation of an HFLTS obtains itself in the use of AppELT; that is, $\mathcal{E}(H_S) = \{H_S\}$. The symbolic translation from GCLEs generated from ECFG to HFLTSs enables the implementation of linguistic computational techniques. However, the existing linguistic computational techniques need accuracy enhancement tools as computing with HFLTSs inevitably produces loss of information, which registers as imperative to the development of semantical associations for HFLTSs [9], [10]. The primitive semantical association, i.e., fuzzy envelope, with linguistic descriptors adopted for HFLTSs was developed by Liu and Rodríguez [36]. Following with fuzzy envelope, a collection of proposals including trapezoidal fuzzy set (TraFS) [7], discrete fuzzy numbers (DFN) [58], possibility distribution [64], proportional hesitant fuzzy linguistic fuzzy term set (PHFLTS) [66], linguistic intuitionistic term sets [71] and probabilistic linguistic term set (PLTS) [53] were successively proposed to associate machine manipulative formats with HFLTSs serving as quantitative equivalences. The possibilistic semantical description is adopted in this study because of the following reasons: first, the HFLTS possibility distribution in its mathematical representation is inimitably put in a succinct style, which strengthens the understanding of researchers showing curiosity to the theory of HFLTS. Second, the computational treatment of HFLTSs in the use of possibility distribution operates on the associated possibilities to each linguistic term, by means of which the computational simplicity and interpretability of the results can be retained. Finally, with the exception of some particular cases, the possibilistic, proportional, and probabilistic semantics for HFLTSs are essentially mathematically consistent. Therefore, this study considers the integration of possibility distribution into HFLTS, which was initially proposed by Wu and Xu [64] as follows.

Definition 4: Let $S = \{s_t | t \in \lfloor\tau\rfloor\}$. Let $H_S(\vartheta) = \{s_L, s_{L+1}, \cdots, s_U\}$ be an HFLTS given by a participant in DSCDM. The possibility distribution for H_S on S is represented by $P^{(2\tau+1)} = (p_{-\tau}, \cdots, p_l, \cdots, p_\tau)$, where p_l is given by

$$p_l = \begin{cases} 0, & l = -\tau, -\tau+1, \cdots, L-1, \\ 1/(U-L+1), & l = L, L+1, \cdots, U, \\ 0, & l = U+1, \cdots, \tau. \end{cases}$$

and p_l denotes the possibility degree under which the linguistic term s_l can be considered as the assessment value when evaluating alternatives or criteria such that $\sum_{l\in\lfloor\tau\rfloor} p_l = 1$ and $0 \leq p_l \leq 1, l \in \lfloor\tau\rfloor$.

The set of all HFLTS possibility distributions on S is denoted by $\mathcal{HP}(S)$. Indicated in Definition 4 is that HFLTS possibility distribution serves as a complex linguistic construction that is inconvenient for computational manipulation. Therefore, the numerically equivalent value (NEV) for an HFLTS possibility distribution is defined in [64] by $\mathsf{NEV}(P^{(2\tau+1)}) = \sum_{l\in\lfloor\tau\rfloor} \Delta_S^{-1}(s_l) p_l$ acting as its computational surrogate. To improve its interpretability, the linguistically equivalent value (LEV) $\mathsf{LEV}(P^{(2\tau+1)}) = \Delta_S(\mathsf{NEV}(P^{(2\tau+1)})) = \Delta_S(\sum_{l\in\lfloor\tau\rfloor} \Delta_S^{-1}(s_l) p_l)$ that translates the NEV into a linguistic 2-tuple is diffusely adopted.

The assignment of possibilities for each linguistic term s_l in an HFLTS H_S is subject to the LapDECR. Recent literature has shown that computing with HFLTS possibility distribution, in this case, degenerates merely to computing with HFLTS [9]. The reason behind this is that the existing computational treatment of HFLTSs is based on the equal likelihood of each linguistic term in an HFLTS. Such a rigorous restriction triggers debate over its justification in real-world applications taking into consideration its sacrifice of the possible inclusion of additional information despite that it alleviates the computation complexity. As a result, the possibility equivalence hypothesis has been removed in an assemblage of studies in succession [9], [10], [70].

B. ORDERED WEIGHTED AVERAGING OPERATOR

Individual semantics building for GCLEs requires the generation of HFLTS possibility distributions with certain Attitude Characters. The existing approaches to generating attitudinal HFLTS possibility distribution were developed via the correspondence established between HFLTS possibility distribution and the ordered weighted averaging (OWA) operator. The rationale that the relationship between linguistic terms in an HFLTS and their possibilities can be modeled between the aggregated arguments and their associated weights has been specified in [9] and [10], in which their arguments are based on the fact that HFLTS possibilities satisfy the basic axioms of complete weighting assignment, i.e., $p_l \in [0, 1]$ and $\sum_{l\in\lfloor\tau\rfloor} p_l = 1$. The resemblance serves as the theoretical basis for the adaption of methods of OWA weight determination in generating attitudinal HFLTS possibility distribution. Below a brief introduction of the OWA-related knowledge is provided.

The OWA operator along with its Attitudinal Character were originally proposed by Yager [68]. An OWA operator of dimension n is a mapping $\mathsf{F}_{w^{(n)}} : \mathcal{R}^n \to \mathcal{R}$ which has an associated weighting vector $w^{(n)} = (w_1, w_2, \cdots, w_n)$ satisfying $\sum_{i=1}^n w_i = 1, 0 \leq w_j \leq 1$ for $j \in \{1, 2, \cdots, n\}$ such that

$$\mathsf{F}_{w^{(n)}}(a_1, a_2, \cdots, a_n) = \sum_{i\in[n]} w_i b_i,$$

where b_j is the j-th largest value out of $a = (a_1, a_2, \cdots, a_n)$ (*i.e.*, $b_1 \geq b_2 \geq \cdots \geq b_n$).

Definition 5 [68]: The measure of Attitudinal Character associated with an OWA operator $\mathsf{F}_{w^{(n)}}$ of dimension n is defined as

$$\mathsf{orness}\left(w^{(n)}\right)=\sum_{i\in[n]}\frac{n-i}{n-1}w_i,$$

and the measure of andness associated with the OWA operator $\mathsf{F}_{w^{(n)}}$ is the complement of its orness, which means

$$\mathsf{andness}\left(w^{(n)}\right)=1-\mathsf{orness}\,(w)=\sum_{i\in[n]}\frac{i-1}{n-1}w_i.$$

With $w^*=(1,0,\cdots,0)$, $w_*=(0,\cdots,0,1)$ and $w_{\mathbf{A}}=(1/n,1/n,\cdots,1/n)$ given for an OWA operator we obtain the Max, Min and average mean AM, which, in the context of decision making, correspond to the fully optimistic decision, fully pessimistic decision and Laplace decision criteria, respectively [28]. The dispersion measures for an OWA operator F_w is defined as $\mathsf{Disp}\left(w^{(n)}\right)=-w_i\ln(w_i)$ [68].

Proposition 1 [28]: For any OWA weighting vectors $w^{(n)}=(w_1,w_2,\cdots,w_n)$ with $\mathsf{orness}\left(w^{(n)}\right)=\alpha$, and its reversed form: $w'^{(n)}=\left(w'_1,w'_2,\cdots,w'_n\right)=(w_n,w_{n-1},\cdots,w_1)$, we have $\mathsf{orness}\left(w^{(n)}\right)=\mathsf{andness}(w'^{(n)})$.

III. DOMAIN-SPECIFIC APPRAISAL MODELING

Domain-specific appraisal modeling is a core process to DSCDM that consists of the gathering of appraisal information and the construction of domain specificity [10]. The two consecutive tasks can be accomplished with the generation of HFLTS possibility distributions reflecting diverse individual cognitive styles. The inherent heterogeneity regarding cognitive styles of participants in DSCDM needs models capable of recognizing the cognitive differences among participants. Individual knowledge mining plays an integral role in achieving this purpose [38], [51]. Multiple drivers characterize evaluated cognitive systems for individuals with a sufficient amount of self-administered individual appraisals with linguistic construction based on LTS defined prior [38]. Individuals may be called upon to provide inputs for all stages of the modeling and management process, and specifically to inform the interpretation of results and the characterization of uncertainty [51]. In this sense, individual knowledge can be used as a reliable source of information accompanied by objective information to expect performance reinforcement of given decision support systems.

In terms of participants in DSCDM, their understanding of different objects are heterogeneously delineated by that in which way and to what extent, they can provide SPI sufficiently. As a consequence, HFLTS possibility distributions can be viewed as a composition of the principal linguistic construction (PLC) (in the form of HFLTS) for individuals and SPI (in the form of possibility distribution). In practice, the PLC is provided directly by participants in DSCDM, but the SPI can be of different formalizations in accordance with the distinctions among individual knowledgable domains. SPI is de facto a partial certainty over the possibility distribution expressed by each participant. In order to break the limitation posed by the possibility equivalence hypothesis, this section deals with the utilization of SPI in the generation of attitudinal HFLTS possibility distribution through the construction of several nonlinear optimization models.

A. GENERATION APPROACH WITH LAPDECR-BASED SPI

Translating GCLEs to PLC is a salient information process that can be automatically completed in accordance with the AppELT [56], [57]. However, whether or not SPI is available depends to a great extent on how much cognitive efforts have been devoted by individuals to articulate their preference. The SPI is set to be free of any additional restrictions under the condition that individuals serve no further knowledge to supplement the PLC. In this sense, the SPI is merely subject to the fundamental ordered possibility simplex (FOPS):

$$\mathcal{FOPS}=\left\{P^{(2\tau+1)}\in I^{(2\tau+1)}:\sum\nolimits_{j\in\lfloor\tau\rfloor}p_j=1,j\in\lfloor\tau\rfloor\right\}.$$

Given an HFLTS possibility distribution, the LapDECR is used as a primary restriction to be placed on the initial SPI, turning the FOPS into the LapDECR-based FOPS (LFOPS):

$$\mathcal{LFOPS}=\left\{P^{(2\tau+1)}\in I^{(2\tau+1)}:\sum\nolimits_{j\in\lfloor\tau\rfloor}p_j=1,\right.\\ \left.p_j=1\big/(2\tau+1),j\in\lfloor\tau\rfloor\right\}.$$

LFOPS uses all the information in PLC by assigning equal possibilities to the possible linguistic terms. It is predominantly used to deal with hesitancy endowed with participants when SPI is unavailable due to certain reasons. The attitude orientation and opinion strength are two instrumental features to be crawled from the LFOPS. Because of the correspondence between SPI and OWA weighting vectors, semantic analysis serving this task is feasible in the use of Attitudinal Character function. Following Definition 5 gives the Attitudinal Character value of the initial HFLTS possibility distribution with LFOPS as

$$\mathbb{AC}\left(P^{(2\tau+1)}\right)=\frac{1}{2\tau+1}\sum_{i\in\lfloor\tau\rfloor}\frac{\tau-i}{2\tau}\\ =\frac{1}{2\tau+1}\times\frac{(2\tau+1)2\tau}{4\tau}=\frac{1}{2},$$

which is independent of τ with the explicit implication that individuals evaluate a given object neutrally. Dispersion value for LFOPS reaches its maximum as PLC has been entirely utilized without favoritism shown towards any linguistic term. Incorporating Attitudinal Character into FOPS paves the way for the extraction of sentiment-related features. To model the arbitrary opinion strength of individuals, the intersection of Attitudinal Character with FOPS (ACFOPS) should be initially defined:

$$\mathcal{ACFOPS}\\ =\left\{P^{(2\tau+1)}\in I^{(2\tau+1)}:\sum\nolimits_{j\in\lfloor\tau\rfloor}\frac{\tau-j}{2\tau}p_j=\mathbb{AC}\,(P)\,,\right.\\ \left.\sum\nolimits_{j\in\lfloor\tau\rfloor}p_j=1,j\in\lfloor\tau\rfloor\right\}.$$

The ACFOPS includes the LFOPS as a special case, which highlights the capability of ACFOPS in identifying attitude orientation and modeling the opinion strength. In practice, the ACFOPS needs to adapt themselves to diversifying practical needs oriented to the transformation of PLC and its associated SPI into attitude-related features.

Computing an approximation of the Attitudinal Characters of participants' opinions in DSCDM can be performed by the similarity measure-based generation algorithm for attitudinal HFLTS possibility distributions [10]. The algorithm designed for the generation of initial HFLTS possibility distribution for ϑ_{AM} is adapted from [10] and briefed as follows.

The remaining algorithms for the rest of GCLEs can be analogously designed in reference to Algorithm 1.

Algorithm 1 Generating Initial HFLTS Possibility Distribution for ϑ_{AM} (Adapted From [10])

Input: A LTS $S = \{s_\alpha | \alpha = -\tau, \cdots, 0, \cdots, \tau\}$, $\vartheta_{AM} =$ *at most* s_U
Initialization: U, N, and τ
1: **for** $\vartheta_{AM} =$ *at most* s_U **do**
2: $\quad H_S(\vartheta_{\text{AM}}) \leftarrow \{s_{-\tau}, s_{-\tau+1}, \cdots, s_U\}$
3: $\quad$ **for** $l = -\tau, -\tau+1, \cdots, U$ and $n = 1, 2, \cdots, N$ **do**
4: $\quad\quad d(s_l, s_U) \leftarrow |s_l - s_U| / 2\tau$
5: $\quad\quad \rho^n(s_l, s_U) \leftarrow (1 - d^n(s_l, s_U))^{1/n}$
6: $\quad\quad p_l^n \leftarrow \rho(s_l, s_U) \big/ \sum_{l=-\tau}^{U} \rho(s_l, s_U)$
7: $\quad$ **end for**
8: $\quad$ **for** $l = U+1, U+2, \cdots, \tau$ and $n = 1, 2, \cdots, N$ **do**
9: $\quad\quad p_l^n \leftarrow 0$
10: $\quad$ **end for**
11: $\quad$ **for** $n = 1, 2, \cdots, N$ **do**
12: $\quad\quad P^n \leftarrow (p_{-\tau}^n, \cdots, p_l^n, \cdots, p_\tau^n)$
13: $\quad$ **end for**
14: **end for**
Output: The initial HFLTS possibility distribution P^n for ϑ_{AM}

DSCDM participants' beliefs and behaviors towards some appraisal objects are not usually stable because of social influences. Social psychology suggests that, despite the attitude change of individuals is subjected to diversifying internal and external stimuli, the individual's motivation to maintain cognitive consistency when cognitive dissonance occurs-when two attitudes or attitude and behavior conflict-is instinctual [46]. The attitudinal HFLTS possibility distribution should move itself away from the initial HFLTS possibility distribution with the cognitive consistency maximized. HFLTS possibility distributions are vectors in essence, and the consistency between arbitrary two of them can be measured by Pearson's correlation coefficient (PCC) [52]. PCC is a common tool for collaborative filtering, where it serves as an efficient measurement of user item similarity [62].

The PCC-based optimization model embedded with LFOPS for P and ACFOPS for P' for generating attitudinal HFLTS possibility distribution can be constructed as follows:

Model 1(M1):

$$\begin{cases} \text{Max} \dfrac{1}{2}\left(1 + \dfrac{\sum_{i\in\lfloor\tau\rfloor}(p_i - \bar{P})(p'_i - \bar{P}')}{\sqrt{\sum_{i\in\lfloor\tau\rfloor}(p_i - \bar{P})^2}\sqrt{\sum_{i\in\lfloor\tau\rfloor}(p'_i - \bar{P}')^2}}\right) \\ s.t.\ \begin{cases} \sum_{i\in\lfloor\tau\rfloor} \dfrac{\tau - i}{2\tau} p'_i = \mathbb{AC}(P') \\ \sum_{i\in\lfloor\tau\rfloor} p'_i = 1 \\ p'_i \in [0, 1] \quad for\ i \in \lfloor\tau\rfloor \\ p_i = 1/(2\tau+1) \quad for\ i \in \lfloor\tau\rfloor \end{cases} \end{cases}$$

In M1, the nonlinear objective function is a slight modification to the original PCC that maps the universe of discourse regarding PCC from $[-1, 1]$ to $[0, 1]$. The constraints of M1 are derived from the ACFOPS for P' and the LFOPS for P. In particular, the final constraint that $p_i = 1/(2\tau+1)$ for $i \in \lfloor\tau\rfloor$ can be dropped if the initial HFLTS possibility distribution was given. That is, M1 becomes simply the PCC-based optimization model embedded with ACFOPS for P':

Model 2(M2):

$$\begin{cases} \text{Max} \dfrac{1}{2}\left(1 + \dfrac{\sum_{i\in\lfloor\tau\rfloor}(p_i - \bar{P})(p'_i - \bar{P}')}{\sqrt{\sum_{i\in\lfloor\tau\rfloor}(p_i - \bar{P})^2}\sqrt{\sum_{i\in\lfloor\tau\rfloor}(p'_i - \bar{P}')^2}}\right) \\ s.t.\ \begin{cases} \sum_{i\in\lfloor\tau\rfloor} \dfrac{\tau - i}{2\tau} p'_i = \mathbb{AC}(P') \\ \sum_{i\in\lfloor\tau\rfloor} p'_i = 1 \\ p'_i \in [0, 1] \quad for\ i \in \lfloor\tau\rfloor \end{cases} \end{cases}$$

This is particularly the case when the initial HFLTS possibility distribution for GCLEs can be generated through the use of several existing methods such as the probability density function-based approach in [9] and the linguistic terms similarity measure-based approach in [10]. The initial HFLTS possibility distribution generated following these approaches leads to a more general case which can be described as in Algorithm 2. In addition, the algorithms can be analogously designed referring to Algorithm 2 as for the remaining GCLEs and therefore are omitted for space-saving.

B. GENERATION APPROACH WITH SPI OF PARTIAL AWARENESS

In terms of different GCLEs, their corresponding HFLTSs obtained from the implementation of AppELT have chances of being tied to a possibility distribution with specific requirements from individuals as a reflection of their domain specificity. Various arrangements of SPI lead to different generation approaches for attitudinal HFLTS possibility distribution. The SPI is provided by participants in DSCDM with heterogeneous structures characterized by different orderings of complete or partial linguistic terms in PLC. Main reasons for individual knowledge mining involving SPI of partial awareness include but not limit to: 1) the subordinate knowledge support might need to be served under time pressure,

Algorithm 2 Generating Attitudinal HFLTS Possibility Distributions Based on *Optimization Model (M2)*

Input: The linguistic term set, $S = \{s_t | t \in \lfloor\tau\rfloor\}$; CLE $\vartheta_{AM} = at\ most\ s_U$ given by an individual; the targeted Attitudinal Character of the HFLTS possibility distribution P' to be generated, $\mathbb{AC}(P')$.

Initialization: g

for $U \in \lfloor\tau\rfloor$ **do**

$H_S(\vartheta_{AM}) = \{s_{-\tau}, \cdots, s_U\} \leftarrow$ Transform $\vartheta_{AM} = at\ most\ s_U$ into HFLTS using the AppELT

$P = (p_{-\tau}, \cdots, p_l, \cdots, p_U) \leftarrow$ Generate the initial HFLTS possibility distribution

$\bar{P} = \sum_{l=-\tau}^{U} p_l \Big/ (U + \tau + 1) \leftarrow$ Calculate the average HFLTS possibility with respect to P

for P **do**

$P_{Rev} = \left(p_1^{Rev}, \cdots, p_k^{Rev}, \cdots, p_{U+\tau+1}^{Rev}\right) \leftarrow$ Reverse P with linguistic terms in the HFLTS rearranged in decreasing order with $p_k^{Rev} = p_{U+1-l}$

$\mathbb{AC}(P_{Rev}) = \sum_{k\in[U+\tau+1]} \frac{U+\tau+1-k}{U+\tau} p_k^{Rev} \leftarrow$ Calculate the Attitudinal Character of P_{Rev}

$\mathbb{AC}(P) \leftarrow \mathbb{AC}(P_{Rev})$

end for

Apply the following *Optimization Model (M2)*:

$$\begin{cases} \text{Max}\frac{1}{2}\left(1 + \frac{\sum_{k\in[U+\tau+1]}\left(p_i^{Rev} - \bar{P}\right)\left(p'^{Rev}_i - \bar{P}'_{Rev}\right)}{\sqrt{\sum_{k\in[U+\tau+1]}\left(p_i^{Rev} - \bar{P}\right)^2}\sqrt{\sum_{k\in[U+\tau+1]}\left(p'^{Rev}_i - \bar{P}'_{Rev}\right)^2}}\right) \\ s.t. \begin{cases} \sum_{k\in[U+\tau+1]} \frac{U+\tau+1-i}{U+\tau} p'^{Rev}_i = \mathbb{AC}\left(P'_{Rev}\right) \\ \sum_{k\in[U+\tau+1]} p'^{Rev}_i = 1 \\ p'^{Rev}_i \in [0,1]\ for\ i \in [U+\tau+1] \end{cases} \end{cases}$$

$P'_{Rev} \leftarrow \left(p'^{Rev}_1, \cdots, p'^{Rev}_k, \cdots, p'^{Rev}_{U+\tau+1}\right)$

$P' = \left(p'_{-\tau}, \cdots, p'_l, \cdots, p'_U\right) \leftarrow$ Reverse P'_{Rev} with linguistic terms in the HFLTS rearranged in increasing order with $p'_l = p'^{Rev}_{U+1-k}$

$Q = \mathbb{AC}(P') - \mathbb{AC}(P) \leftarrow$ Calculate the degree of attitude change in respect of the individual

end for

Output: All generated attitudinal HFLTS possibility distributions P', and as well the degree of attitude change Q for $U \in \lfloor\tau\rfloor$

2) the participants might not feel confident in providing accurate figures for the distinctions between linguistic terms compared in pairs, 3) the participants are averse to revealing their preferences over certain linguistic terms in public, and 4) in some cases, there are multiple transformed linguistic terms are available in a SPI providing problem at the outset that comparing and evaluating all of them in terms of preference differentiation in detail is not practical as participants are keen on the identification of the more promising linguistic evaluations. The remaining parts of this section are devoted to the specifications of several established ordering-embedded types of SPI that are expected to be frequently used in domain-specific appraisal modeling.

• *SPIWO-based generation approach*

SPI with weak ordering (SPIWO) implies that an individual has a specific preference orientation over the linguistic terms transformed from GCLEs. The linguistic terms in a transformed HFLTS are associated with a ranking dominating them that can be hailed as a constraint for conducting semantic analysis. A weak ordering of HFLTS possibilities can be given as follows:

$$\mathcal{WO} = \Big\{P'^{(2\tau+1)} \in I^{(2\tau+1)} : \sum_{j\in\lfloor\tau\rfloor} \frac{\tau - j}{2\tau} p_j = \mathbb{AC}(P),$$
$$p'_j \geq p'_k \geq 0, j \in \lfloor\tau\rfloor, k \in \lfloor\tau\rfloor \setminus j, \sum_{j\in\lfloor\tau\rfloor} p'_j = 1\Big\}.$$

The ranking of possibilities associated with linguistic terms in a transformed HFLTS does not strictly relate to an ascending ordering of their subscripts. Neither does it subject all linguistic terms in PLC to the SPIWO. As a result, the formulated SPIWO $\mathcal{WO}$ is the most substantial constraint it can be in the sense of weak ordering. The optimization model based on the SPIWO is formulated as follows:

Model 3(M3):

$$\begin{cases} \text{Max}\frac{1}{2}\left(1 + \frac{\sum_{i\in\lfloor\tau\rfloor}\left(p_i - \bar{P}\right)\left(p'_i - \bar{P}'\right)}{\sqrt{\sum_{i\in\lfloor\tau\rfloor}\left(p_i - \bar{P}\right)^2}\sqrt{\sum_{i\in\lfloor\tau\rfloor}\left(p'_i - \bar{P}'\right)^2}}\right) \\ s.t. \begin{cases} \sum_{i\in\lfloor\tau\rfloor} \frac{\tau - i}{2\tau} p'_i = \mathbb{AC}(P') \\ p'_j \geq p'_k \quad for\ j \in \lfloor\tau\rfloor, k \neq j \\ \sum_{i\in\lfloor\tau\rfloor} p'_i = 1 \\ p'_i \in [0,1] \quad for\ i \in \lfloor\tau\rfloor \end{cases} \end{cases}$$

• *SPISO-based generation approach*

SPI with a strict ordering (SPISO) considers distinct discrimination factors between HFLTS possibilities. The individual has a discernible preference orientation with exact possibility differentiation between the linguistic terms transformed from GCLEs. A strict ordering of HFLTS possibilities can be mathematically formulated as a set of several constraints by

$$\mathcal{SO}=\left\{P'^{(2\tau+1)}\in I^{(2\tau+1)}:\sum\nolimits_{j\in\lfloor\tau\rfloor}\frac{\tau-j}{2\tau}p_j=\mathbb{AC}\left(P\right),\right.$$
$$p'_j-p'_{j+k}\geq\varepsilon_j,p'_{\tau+1}=0,j\in\lfloor\tau\rfloor,$$
$$\left.k\in[\tau-j],\sum\nolimits_{j\in\lfloor\tau\rfloor}p'_j=1\right\}.$$

Individuals can primarily discern the preference orientation in terms of a given PLC, and subsequently, the possibility differentiation can be as well measured such that individual's subjective and partial certainty is reflected over certain comparable linguistic terms. In contrast to the SPIWO, the SPISO $\mathcal{SO}$ is the weakest constraint it can be as the differentiation between two HFLTS possibilities forms the bare minimum to extract a strict ordering. The optimization model based on the SPISO is formulated as follows:

Model 4(M4):

$$\begin{cases}\text{Max }\dfrac{1}{2}\left(1+\dfrac{\sum_{i\in\lfloor\tau\rfloor}\left(p_i-\bar{P}\right)\left(p'_i-\bar{P}'\right)}{\sqrt{\sum_{i\in\lfloor\tau\rfloor}\left(p_i-\bar{P}\right)^2}\sqrt{\sum_{i\in\lfloor\tau\rfloor}\left(p'_i-\bar{P}'\right)^2}}\right)\\ s.t.\begin{cases}\sum_{i\in\lfloor\tau\rfloor}\dfrac{\tau-i}{2\tau}p'_i=\mathbb{AC}\left(P'\right)\\ p'_j-p'_{j+k}\geq\varepsilon_j\quad for\ j\in\lfloor\tau\rfloor\ and\ k\in[\tau-j]\\ \sum_{i\in\lfloor\tau\rfloor}p'_i=1\\ p'_i\in[0,1]\quad for\ i\in\lfloor\tau\rfloor\end{cases}\end{cases}$$

• *SPIOM-based generation approach*

Pairwise comparison of HFLTS possibilities is in general a process of comparing entities of PLC in pairs to judge which linguistic term is preferred. The ratio that showing one linguistic term has a greater amount of some quantitative property over another can be crafted in the form of $p'_i/p'_j=\alpha_{ij}$. One of the logical justification behind using this formation is that when it is used in the analytic hierarchy process, the parameter α_{ij} corresponds to the well-established verbal description which reflects exactly the preference. In this sense, SPI with a ordering with multiples (SPIOM) for HFLTS possibilities can be considered:

$$\mathcal{OM}=\left\{P'^{(2\tau+1)}\in I^{(2\tau+1)}:\sum\nolimits_{j\in\lfloor\tau\rfloor}\frac{\tau-j}{2\tau}p_j=\mathbb{AC}\left(P\right),\right.$$
$$p'_j\geq\alpha_{j(j+k)}p'_{j+k},j\in\lfloor\tau\rfloor\setminus\tau,k\in[\tau-j],$$
$$\left.\sum\nolimits_{j\in\lfloor\tau\rfloor}p'_j=1\right\}.$$

The method of pairwise comparison has been diffusely used in the scientific study of preferences, attitudes, voting systems and social choice, etc. The SPIOM follows strictly the law of comparative judgment. Participants in DSCDM comparing linguistic terms in pairs estimate ratios for each paired comparison that express their authentic feelings. Selected entities from the PLC are not subjected to a consecutive requirement. Even more, the constraints imposed on j and k can be enhanced in consideration of the actual willingness of participants' cooperation to mature the complex linguistic construction. The optimization model based on the SPIOM can be built as follows:

Model 5(M5):

$$\begin{cases}\text{Max}\dfrac{1}{2}\left(1+\dfrac{\sum_{i\in\lfloor\tau\rfloor}\left(p_i-\bar{P}\right)\left(p'_i-\bar{P}'\right)}{\sqrt{\sum_{i\in\lfloor\tau\rfloor}\left(p_i-\bar{P}\right)^2}\sqrt{\sum_{i\in\lfloor\tau\rfloor}\left(p'_i-\bar{P}'\right)^2}}\right)\\ s.t.\begin{cases}\sum_{i\in\lfloor\tau\rfloor}\dfrac{\tau-i}{2\tau}p'_i=\mathbb{AC}\left(P'\right)\\ p'_j\geq\alpha_{j(j+k)}p'_{j+k}\quad for\ j\in\lfloor\tau\rfloor\setminus\tau\ and\ k\in[\tau-j]\\ \sum_{i\in\lfloor\tau\rfloor}p'_i=1\\ p'_i\in[0,1]\quad for\ i\in\lfloor\tau\rfloor\end{cases}\end{cases}$$

• *SPIOD-based generation approach*

Given a PLC that transformed from a GCLE, pairwise comparisons among the linguistic terms in it may derive the following judgment from participants in DSCDM that linguistic term s_i is of significant importance over linguistic term s_j, and linguistic term s_k is weakly crucial than linguistic term s_l. The SPI, in this case, can be modeled as an ordering of differences denoting a bi-level strength of HFLTS possibility. The SPI with an ordering of differences (SPIOD) is formulated as follows:

$$\mathcal{OD}=\left\{P'^{(2\tau+1)}\in I^{(2\tau+1)}:\sum\nolimits_{j\in\lfloor\tau\rfloor}\frac{\tau-j}{2\tau}p_j=\mathbb{AC}\left(P\right),\right.$$
$$p'_i-p'_j\geq p'_k-p'_l,p'_{\tau+1}=0,i,j,k,l\in\lfloor\tau\rfloor,$$
$$\left.\sum\nolimits_{j\in\lfloor\tau\rfloor}p'_j=1\right\}.$$

The capability of individuals to distinguish further the differences between paired comparisons among linguistic terms in a transformed PLC from one to another is highly dependent upon their domain specificity. In general, an additional requirement for $\mathcal{OD}$ is $j=k$ in consideration of the psychological characteristic that making consecutive comparisons is more natural and as well much more accessible for human beings to process than randomized comparisons. However, individual knowledge mining is a process of discovering SPI being regularly unpredictable due to diversifying cognitive efforts that participants in DSCDM collaborate to ramp up. As such, the additional requirement unnecessarily serves as a preexist condition for forming SPIOD and the PCC-based optimization model embedded with ACFOPS. Another fact worthy of mention is that the SPIOD and SPISO are not mutually exclusive from a mathematical perspective. The SPIOD becomes the SPISO in general if we take the right-handed paired comparison between p'_k and p'_l as the discrimination factors, and inversely, the SPISO can be viewed as SPIOD given that its discrimination factors are provided implicitly by individuals with rough estimates from comparing linguistic terms in pairs.

The optimization model based on the SPIOD is constructed as follows:

Model 6(M6):

$$\begin{cases} \text{Max} \frac{1}{2}\left(1+\frac{\sum_{i\in\lfloor\tau\rfloor}\left(p_i-\bar{P}\right)\left(p'_i-\bar{P}'\right)}{\sqrt{\sum_{i\in\lfloor\tau\rfloor}\left(p_i-\bar{P}\right)^2}\sqrt{\sum_{i\in\lfloor\tau\rfloor}\left(p'_i-\bar{P}'\right)^2}}\right) \\ s.t.\begin{cases} \sum_{i\in\lfloor\tau\rfloor}\frac{\tau-i}{2\tau}p'_i=\mathbb{AC}\left(P'\right) \\ p'_i-p'_j\geq p'_k-p'_l \, for\ i,j,k,l\in\lfloor\tau\rfloor \\ p'_{\tau+1}=0 \\ \sum_{i\in\lfloor\tau\rfloor}p'_i=1 \\ p'_i\in[0,1] \, for\ i\in\lfloor\tau\rfloor \end{cases} \end{cases}$$

• *SPIIB-based generation approach*

Interval possibilistic judgment is another representation structure frequently adopted by participants in DSCDM to deliver SPI. The upper and lower bounds of HFLTS possibilities attached to certain linguistic terms in PLC are not both entailed to be offered by participants. This is because open-ended judgments are allowed as the closure will be accomplished by the natural constraint that these possibilistic judgments belong to the unit interval, which is indicated with parentheses. The SPI with an interval bounds (SPIIB) is formulated as follows:

$$\mathcal{IB}=\Big\{P'^{(2\tau+1)}\in I^{(2\tau+1)}:\sum_{j\in\lfloor\tau\rfloor}\frac{\tau-j}{2\tau}p_j=\mathbb{AC}\left(P\right), \kappa_j+\varepsilon_j\geq p'_j\geq\kappa_j, \kappa_j,\varepsilon_j\in[0,1], j\in\lfloor\tau\rfloor, \sum_{j\in\lfloor\tau\rfloor}p'_j=1\Big\}.$$

Several picked linguistic terms could be the potential authentic opinion of the participants, in the sense that the SPIIB could act as an auxiliary tool for participants to reassess their original PLCs. The cardinality of PLC has considerable chances of being reduced in accordance with the possible opinion adjustment. The fact that using all these SPI with partial awareness facilitates the cohesiveness-improving of PLCs applies to all subcategories of SPI defined above, and mathematically, the SPIIB may overlap with any of them. However, the prominent distinction among them are their information sources of SPI determined by participants' subjective cognition that differs from one to another. The gathering of appraisal information and the construction of domain specificity can be accomplished by seeking solutions to these SPI-based optimization models, which are frequently adopted strategies for domain-specific appraisal modeling that will be covered along with the optimization model based on the SPIIB constructed below.

Model 7(M7):

$$\begin{cases} \text{Max}\ \frac{1}{2}\left(1+\frac{\sum_{i\in\lfloor\tau\rfloor}\left(p_i-\bar{P}\right)\left(p'_i-\bar{P}'\right)}{\sqrt{\sum_{i\in\lfloor\tau\rfloor}\left(p_i-\bar{P}\right)^2}\sqrt{\sum_{i\in\lfloor\tau\rfloor}\left(p'_i-\bar{P}'\right)^2}}\right) \\ s.t.\begin{cases} \sum_{i\in\lfloor\tau\rfloor}\frac{\tau-i}{2\tau}p'_i=\mathbb{AC}\left(P'\right) \\ \kappa_j+\varepsilon_j\geq p'_j\geq\kappa_j \quad for\ \kappa_j,\varepsilon_j\in[0,1]\ and\ j\in\lfloor\tau\rfloor \\ \sum_{i\in\lfloor\tau\rfloor}p'_i=1 \\ p'_i\in[0,1] \quad for\ i\in\lfloor\tau\rfloor \end{cases} \end{cases}$$

IV. NOVEL DSCDM FRAMEWORK WITH THE COANG MODEL

The aim of COANG based on individual semantics obtained from domain-specific appraisal modeling is to define automatic tools able to extract subjective information from decision appraisals in DSCDM, such as attitude orientation and attitude strength, so as to create structured and actionable knowledge to be used by either a decision support system or a decision maker. The individual semantics adds the Attitude Character as a new dimension to intensify our understanding of GCLEs with their connotative rather than denotative meaning [10]. Functionally, building individual semantics for GCLEs allows distinguishing the precisely same transformed temporal HFLTSs during the implementation of AppELT to guarantee the accuracy of any GCLE-based computational techniques for DSCDM. More importantly, GCLE is unequivocally built to discover the connotative implications that avoid any confusion. Based on the individual semantics established in the previous section, we propose the COANG model tailored to DSCDM with GCLEs.

A. THE COANG MODEL

A COANG model can be formulated as a septuple

$$\mathcal{COANG}=\left\langle \mathsf{CIS},\mathsf{AC},\mathsf{P},\mathsf{AO},\mathsf{AS},\mathsf{T},\mathsf{O}_{\mathcal{COANG}}\right\rangle,$$

where CIS, AC, P, AO and AS stand for finite sets of opinions with customized individual semantics (CIS), alternative-criterion objects, participants who express their opinions over AC, attitude orientation of their opinions, and attitude strength of their opinions respectively. The set $\mathsf{AC}=\{positive, negative, neutral\}$ defines the attitude orientations embedded in opinions, and the attitude strength $\mathsf{AS}\in[0,1]$ represents the degree of feeling perceived by a participant. The set of time points T indicates when the set of opinions CIS is held. Finally, the opinion formulation function $\mathsf{O}_{\mathcal{COANG}}$ conveys a valid opinion tuple, that is, who holds a particular opinion for a specific alternative-criterion object at a particular point of time. In the current paper we are not intended to deal with dynamic DSCDM with evolving opinion networks; therefore, the T dimension will not be explored for the rest of this paper. Consequently, the COANG model degenerates to the following sextuple

$$\mathcal{COANG}=\left\langle \mathsf{CIS},\mathsf{AC},\mathsf{P},\mathsf{AO},\mathsf{AS},\mathsf{O}_{\mathcal{COANG}}\right\rangle.$$

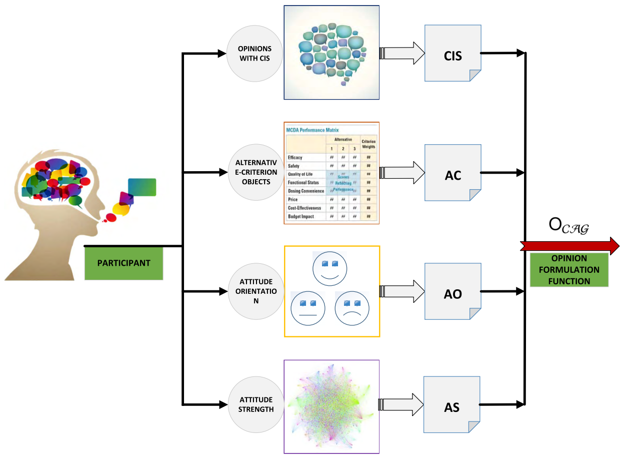

FIGURE 2. **The general idea of the proposed COANG model.**

The components of the COANG model are analyzed in detail as follows (see graphically in Figure 2).

- **Opinions with CIS**, CIS: The opinions in the context of DSCDM are specially considered in the manifestation of GCLEs. In the case of more general settings such as social networks, opinions refer to the subjective feelings of participants concerning some opinion targets (e.g., products, services, events and political figures).
- **Alternative-criterion object**, AC: The alternative-criterion objects are toward which the opinions are expressed. Mostly, a DSCDM consists of multiple alternatives and as well several conflicting criteria. However, not all alternative-criterion objects are associated with specific opinions, even hesitant ones. The missing opinion inputs can be caused by many reasons such as the unfamiliarity of participants with specific objects and the unwillingness of participants to share their views (i.e., non-cooperative behaviors). To be more precise, alternatives can be viewed as the target entities, and criteria are the target aspects of each entity on which the opinion has been given. In this study, alternatives and criteria are considered inseparably as an object pair because they are toward which participants would refer to simultaneously.
- **Participant**, P: The participant in a group may specifically mean social network users, agents, decision makers or experts depending on their specialized contexts. Without loss of generality, we use "participant" in general to refer to subjects involving in the discussion of DSCDM.
- **Attitude orientation**, AO: The provision of CIS that associated with GCLEs facilitate the attitude orientation detection of each participant evaluating alternative-criterion objects. Attitude Character function $\mathrm{AC(P)} \in [0, 1]$ serves as an essential standard with, given a specific participant, $\mathrm{AC(P)} > 0.5$ implying the participant being optimistic, $\mathrm{AC(P)} < 0.5$ reflecting the participant being pessimistic, and otherwise representing the neutral attitude of the participant.
- **Attitude strength**, AS: The attitude strength $\mathrm{S} \in [0, 1]$ in this case is functionally equivalent to the Attitude Character function $\mathrm{AC(P)} \in [0, 1]$ with different associated implications. Basically, the attitude strength on the positive attitude orientation is projected from the $\mathrm{AC(P)} \in (0.5, 1]$ whereas that on the negative attitude orientation is projected from the $\mathrm{AC(P)} \in [0, 0.5)$.
- **Opinion formulation function**, $O_{\mathcal{COANG}}$: The opinion formulation function brings together the former components of $\mathcal{COANG}$ to deliver a valid opinion tuple. It unequivocally includes all facets of information that we expect to cover in order to facilitate the constriction of novel DSCDM framework. By contrast, the traditional CDM models base themselves on every aspect of the $\mathcal{COANG}$ excluding attitude orientation, attitude strength as well as the opinion formulation function.

The COANG model provides a framework to transform unstructured individual semantic analysis to structured data. The sextuple is basically a database schema, based on which the extracted opinions can be put into a database table. Then a rich set of qualitative, quantitative, and trend analyses of opinions can be performed using a whole suite of database management systems and online analytical processing tools.

B. DERIVED UNCERTAIN LINGUISTIC INFORMATION

The idea of basic uncertain information (BUI) was initially coined by Jin *et al.* [27] and appeared primarily in Mesiar *et al.* [45]. Serving the purpose as a fundamental "Basic Unit" in uncertain theory, it is fine-tunely designed in a bid to explain a plethora of uncertain information representations such as LDA, PHFLTS, PLTS, and so on. BUI is used to derive a new classification of uncertain information representation called "Derived Uncertain Information (DUI)" [45]. DUI obtains their meaning from the original definition of the BUI. Therefore, BUI is the source of all knowledge for each and every generalizations of it in every aspect. Special attention is paid to derived uncertain linguistic element (DULE) in this section. DULEs are adapted from BUI through the integration of a participant' complex linguistic construction (as manifested by PLC) for an evaluated object and its indicating attitude strength. The formal definition of DULE is given as follows.

Definition 6: Let $S = \{s_t | t \in \lfloor\tau\rfloor\}$ be a LTS and $\bar{S}$ the 2-tuple set associated with S defined as $\bar{S} = S \times [-0.5, 0.5)$. Let $H_S(\vartheta) = \{s_L, s_{L+1}, \cdots, s_U\}$ be an HFLTS given by a participant in DSCDM citing his/her decision appraisal toward an alternative-criterion object, and the possibility distribution for H_S on S is calculated as $P^{(2\tau+1)} = (p_{-\tau}, \cdots, p_l, \cdots, p_\tau)$. The binary pair $\left(P^{(2\tau+1)}, \mathrm{AS}\right)$ is called a DULE, where AS indicates the opinion strength calculated from the Attitudinal Character $\mathrm{AC(P)}$ that the participant shows implicitly in the decision appraisal.

The set of all DULEs on $\bar{S}$ is denoted by $\mathcal{U}(\bar{S})$. In addition to the opinions with CIS and the attitude strength, the remaining components of the COANG model are not factored in the construction of DULE since they have been explicitly included in the logical statements compiling Definition 6. The mathematical form of the proposed DULE appears to be akin to that for the proportional linguistic pairs defined in [8] if, and only if, the first element is substituted with LEV and its symbolic translation takes the value of zero. Even more, the DULE resembles mathematically the possibilistic 2-tuple linguistic pairs (P2TLPs) [11] if, and only if, the first element is simply substituted with the LEV. However, the second elements in DULE, proportional linguistic pairs and P2TLPs differ in their underlying implications. That is, the proportional linguistic pairs are essentially constructed as a reflection of group homogeneity with the inclusion of objective proportional information and the P2TLP presents the level of confidence for a given 2-tuple linguistic input, whereas DULE models individually twofold aspects of the assessment representation structure including HFLTS possibility distribution and its attitude strength.

The development of DULE-based linguistic computational techniques generally necessitates the accurate comparison among them. Motivated by [11], DULEs are compared lexicographically using the parallel comparison of the LEV and the attitude strength. This means that to compare equal, two DULEs must compare equal in both positions. If not equal, the BULEs posses differing elements. Similar to the P2TLP, a DULE as well does not necessarily have $P^{(2\tau+1)}$ as its first element and AS as its second element. The reverse order of a DULE can be regarded as a DULE. But the initial form served in Definition 5 is retained as structural consistency is necessitated for computing with DULEs. The product of the LEV and its associated attitude strength, i.e., $\mathsf{NEV}\left(P^{(2\tau+1)}\right) \times \mathsf{AS}$, is called the numerical equivalent transformation of a DULE U_i and denoted by $\mathsf{NET}(U_i)$. With all these preexist conditions, the comparison laws for DULEs can be commendably adapted from that for P2TLPs offered in [11].

Definition 7: Let $U_i = \left(P_i^{(2\tau+1)}, \mathsf{AS}_i\right)$ and $U_j = \left(P_j^{(2\tau+1)}, \mathsf{AS}_j\right)$ be two arbitrary DULEs on $\mathcal{U}\left(\bar{S}\right)$. The comparison laws between them are defined as follows:

1) if

$$\left(\mathsf{NET}(U_i) > \mathsf{NET}(U_j)\right) \vee \Big(\left(\mathsf{NET}(U_i) = \mathsf{NET}(U_j)\right) \wedge \left(\mathsf{NEV}\left(P_i^{(2\tau+1)}\right) > \mathsf{NEV}\left(P_j^{(2\tau+1)}\right)\right)\Big),$$

then $U_i \succ U_j$;

2) if

$$\left(\mathsf{NET}(U_i) < \mathsf{NET}(U_j)\right) \vee \Big(\left(\mathsf{NET}(U_i) = \mathsf{NET}(U_j)\right) \wedge \left(\mathsf{NEV}\left(P_i^{(2\tau+1)}\right) < \mathsf{NEV}\left(P_j^{(2\tau+1)}\right)\right)\Big),$$

then $U_i \prec U_j$;

3) if

$$\left(\mathsf{NEV}\left(P_i^{(2\tau+1)}\right) = \mathsf{NEV}\left(P_j^{(2\tau+1)}\right)\right) \wedge \left(\mathsf{AS}_i = \mathsf{AS}_j\right),$$

then $U_i \sim U_j$.

The condition 3) is guaranteed by that Δ_S is a one to one correspondence. In this sense, we have $\mathsf{LEV}\left(P_i^{(2\tau+1)}\right) = \mathsf{LEV}\left(P_j^{(2\tau+1)}\right)$ from $\mathsf{NEV}\left(P_i^{(2\tau+1)}\right) = \mathsf{NEV}\left(P_j^{(2\tau+1)}\right)$, which in combination with $\mathsf{AS}_i = \mathsf{AS}_j$ guarantees the equivalence of U_i and U_j. A set of DULEs $\{U_i | \, i \in [n]\}$ on $\mathcal{U}\left(\bar{S}\right)$ is said to be ordered if they are arranged in descending order as per Definition 6. Thanks to the relations given in Definition 7, uncertain cases when both U_i precedes U_j and U_j precedes U_i have been eliminated by antisymmetry. Meanwhile, any pair of DULEs on $\mathcal{U}\left(\bar{S}\right)$ are comparable under the relation in the sense that the property of "totality" is satisfied. The proposed comparison laws are implicitly implied with transitivity and therefore form a strict total ordering of DULEs over $\mathcal{U}\left(\bar{S}\right)$.

Definition 8: Let $U_i = \left(P_i^{(2\tau+1)}, \mathsf{AS}_i\right)$ $(i \in [n])$ be a set of DULEs on $\mathcal{U}\left(\bar{S}\right)$. The aggregation function for a set of $P_i^{(2\tau+1)}$ $(i \in [n])$ is feasible for construction with the mapping $\mathsf{Agg}_{\mathcal{HP}} : \mathcal{HP}(S)^n \to \mathcal{HP}\left(S\right)$. The aggregation function defined for n DULEs on $\mathcal{U}\left(\bar{S}\right)$ is a mapping $\mathsf{Agg}_{\mathcal{U}} : \mathcal{U}(\bar{S})^n \to \mathcal{U}\left(\bar{S}\right)$ called a DULE-aggregation function that can be expressed as $\mathsf{Agg}_{\mathcal{U}}\left(\left(P_1^{(2\tau+1)}, \mathsf{AS}_1\right), \cdots, \left(P_n^{(2\tau+1)}, \mathsf{AS}_n\right)\right) = \left(P_{\mathsf{Agg}}^{(2\tau+1)}, \mathsf{AS}_{\mathsf{Agg}}\right)$ such that

DULE 1: for any $(U_1, \cdots, U_n) \in \mathcal{U}\left(\bar{S}\right)^n$ where $i \in [n]$,

$$\mathsf{Agg}_{\mathcal{U}}\left(\left(P_1^{(2\tau+1)}, \mathsf{AS}_1\right), \cdots, \left(P_n^{(2\tau+1)}, \mathsf{AS}_n\right)\right) = \left(\mathsf{Agg}_{\mathcal{HP}}\left(P_1^{(2\tau+1)}, \cdots, P_1^{(2\tau+1)}\right), \mathsf{AS}_{\mathsf{Agg}}\right)$$

for $\mathsf{AS}_{\mathsf{Agg}} \in [0, 1]$, which is on par with

$$\mathsf{Agg}_{\mathcal{HP}}\left(P_1^{(2\tau+1)}, \cdots, P_n^{(2\tau+1)}\right) = \mathbb{P}_{\mathcal{HP}}\left(\mathsf{Agg}_{\mathcal{U}}\left(\left(P_1^{(2\tau+1)}, \mathsf{AS}_1\right), \cdots, \left(P_n^{(2\tau+1)}, \mathsf{AS}_n\right)\right)\right),$$

being independent of AS_i where $\mathbb{P}_{\mathcal{HP}} : \mathcal{U}\left(\bar{S}\right) \to \mathcal{HP}(S)$.

DULE 2: for any $\left(P_1^{(2\tau+1)}, \cdots, P_n^{(2\tau+1)}\right) \in \mathcal{HP}(S)^n$ where $i \in [n]$, the mapping $\mathsf{Agg}_{\mathcal{AS}} : [0, 1]^n \to [0, 1]$ is given by

$$\mathsf{Agg}_{\mathcal{AS}}\left(\mathsf{AS}_1, \cdots, \mathsf{AS}_n\right) = \mathbb{P}_{\mathcal{AS}}\left(\mathsf{Agg}_{\mathcal{U}}\left(\left(P_1^{(2\tau+1)}, \mathsf{AS}_1\right), \cdots, \left(P_n^{(2\tau+1)}, \mathsf{AS}_n\right)\right)\right),$$

where $\mathbb{P}_{\mathcal{AS}} : \mathcal{U}\left(\bar{S}\right) \to [0, 1]$.

DULE 3:

$$\mathsf{Agg}_{\mathcal{HP}}\left(P_1^{(2\tau+1)}, \cdots, P_n^{(2\tau+1)}\right) = \mathsf{Agg}_{\mathcal{HP}}\left(P_1'^{(2\tau+1)}, \cdots, P_n'^{(2\tau+1)}\right)$$

such that

$$\mathsf{NEV}\left(P_i^{(2\tau+1)}\right) = \mathsf{NEV}\left(P_i'^{(2\tau+1)}\right)$$

for all $i \in [n]$.

Theoretical foundations for the definition of DULE-based aggregation functions can be found in Beliakov *et al.* [5] and Mesiar *et al.* [45].

C. THE COANG-BASED DSCDM FRAMEWORK

The COANG model brings an innovative perspective to scrutinize the internal information structure of DSCDM in the use of component analysis. With the aspect-oriented inclusion of attitude orientation and attitude strength, semantic analysis is commendably feasible and productive through the implementation of COANG model. A plethora of useful conclusions can be obtained by accomplishing the tasks of semantic classification and opinion significance determination. In the section, we build a novel DSCDM framework on the basis of the COANG model. Thanks to the CIS, semantic components of COANG model can thus determine how participants feel about a specific alternative by presenting their attitude orientation and attitude strength. The management of the COANG-based DSCDM process follows the necessary steps detailed below to carry out the discussions.

- **Basic components of COANG-based DSCDM**: The COANG-based CDM algorithms are characterized by several instrumental features including the alternative-criterion description, the enumeration of participants, and the appraisals of each alternative-criterion object in the manifestation of GCLEs. To facilitate the description of the proposed framework of COANG-based DSCDM, the alternatives and criteria considered in this study are denoted by $\mathbb{A} = \{a_1, a_2, \cdots, a_M\}$ and $\mathbb{C} = \{c_1, c_2, \cdots, c_N\}$. In addition, the participants in DSCDM are denoted by $\mathbb{P} = \{p_1, p_2, \cdots, p_H\}$ and the decision appraisals toward each alternative-criterion pair (a_i, c_j) from participants are manifested by $\mathcal{G}_{ij}^k$, which means that the k-th participant holds the opinion $\mathcal{G}_{ij}^k$ toward the alternative-criterion pair (a_i, c_j).
- **Domain-specific appraisal modeling**: The individual semantics associated with every single appraisal can be obtained via modeling their PLC and the attached SPI based on the selection of apt PCC-based optimization models. In practice, the SPI is provided by the participants in the form of mixture orderings of HFLTS possibilities. The quality of the submitted information is considerably subject to the cognitive modes that the participants essentially follow. The cognitive psychology recognizes two principal cognitive models. The unconscious or instinctive cognitive model suggests that a participant has little cognitive load and performs best when the prompt action is required, and the conscious or deliberative cognitive model that involves rational apparatus and has a higher cognitive amount taking into consideration the time for deliberation is available, and extensive analysis and contextual knowledge is requisite [41]. The attitude orientation and attitude strength discovered from individual semantics building for GCLEs are easy to detect as they register as a personal reflection of participants' cognitive styles.
- **The COANG-based DSCDM**: The COANG-based DSCDM framework is proposed to accomplish several tasks to satisfy heterogeneous demands in practical applications. It is essentially developed to answer the following instrumental questions:

  a) *How to discover potential defective features in candidate alternatives to guide alternative designers to improve certain aspects, upon the determination of the best alternative*?

  b) *How to detect the attitude change and recognize the influence paradigm in terms of the specific feature*?

  c) *How to select the best alternative taking simultaneously into account the decision appraisal values as well as several attitude-related aspects*?

— **Opinion gathering on criterion level**.

The pooling of participants' opinions toward each alternative-criterion pair in accordance with the following:

$$\begin{aligned}\mathfrak{C}_{ij} &= \mathsf{Agg}_{\mathcal{U}}\left(\mathcal{G}_{ij}^1, \cdots, \mathcal{G}_{ij}^H\right) = \sum_{k\in[H]}\left\langle P_{ij}^k, \mathsf{AS}_{ij}^k\right\rangle \\ &= \left\langle\left(\sum_{k\in[H]} p_{-\tau}^{ijk}, \cdots, \sum_{k\in[H]} p_l^{ijk}, \cdots, \sum_{k\in[H]} p_\tau^{ijk}\right),\right. \\ &\qquad \left.\frac{\sum_{k\in[H]} as^{ijk}\sum_{l\in\lfloor\tau\rfloor} p_l^{ijk}s_l^{ijk}}{H - \sum_{k\in[H]} as^{ijk} + \sum_{k\in[H]} as^{ijk}\sum_{l\in\lfloor\tau\rfloor} p_l^{ijk}s_l^{ijk}}\right\rangle. \end{aligned} \tag{1}$$

which is a mapping $\mathcal{U}(\bar{S})^H \to \mathcal{U}(\bar{S})$ that satisfying the three properties offered in Definition 8 that could be supported by the proof descriptions given below.

DULE 1: For any $\left(\mathcal{G}_{ij}^1, \cdots, \mathcal{G}_{ij}^H\right) \in \mathcal{U}(\bar{S})^H$ where $i \in [M], j \in [N]$,

$$\mathsf{Agg}_{\mathcal{HP}}\left(P_{ij}^1, \cdots, P_{ij}^H\right) = \mathbb{P}_{\mathcal{HP}}\left(\mathcal{G}_{ij}^1, \cdots, \mathcal{G}_{ij}^H\right),$$

being independent of AS_{ij}^k.

DULE 2: For any $\left(P_{ij}^1, \cdots, P_{ij}^H\right) \in \mathcal{HP}(S)^H$ where $i \in [M], j \in [N]$,

$$\mathsf{Agg}_{\mathcal{AS}}\left(\mathsf{AS}_{ij}^1, \cdots, \mathsf{AS}_{ij}^H\right) = \mathbb{P}_{\mathcal{AS}}\left(\mathsf{Agg}_{\mathcal{U}}\left(\mathcal{G}_{ij}^1, \cdots, \mathcal{G}_{ij}^H\right)\right).$$

DULE 3: It is clear that $\mathsf{Agg}_{\mathcal{HP}}\left(P_{ij}^1, \cdots, P_{ij}^H\right) = \mathsf{Agg}_{\mathcal{HP}}\left(P'^1_{ij}, \cdots, P'^H_{ij}\right)$ such that $\mathsf{NEV}\left(P_{ij}^k\right) = \mathsf{NEV}\left(P'^k_{ij}\right)$ for all $k \in [H]$.

Inspired by [15], position searching in terms of a specific criterion C_j follows strictly the dominating cardinality function $\mathcal{P}_g^i = \left\{C_j \middle| \mathfrak{C}_{ij} \succ \mathfrak{C}_{ig}, j \in [N]\right\}$, which denotes the set of the criteria whose decision appraisal values are greater than that of the criterion $\mathfrak{C}_{ig}$. Given that $\#\left(\mathcal{P}_g^i\right) = 0$, we are entirely guaranteed for sure that $\mathfrak{C}_{ig} = \mathsf{Max}\left\{\mathfrak{C}_{i1}, \cdots, \mathfrak{C}_{ij}, \cdots, \mathfrak{C}_{iN}\right\}$, and in the case of $\#\left(\mathcal{P}_g^i\right) = N - 1$, we directly obtain

$\mathfrak{C}_{ig} = \mathrm{Min}\left\{\mathfrak{C}_{i1}, \cdots, \mathfrak{C}_{ij}, \cdots, \mathfrak{C}_{iN}\right\}$. The position ranking function for criteria $C_j (j \in [N])$ can be defined as

$$\mathcal{R}_g^i = \#\left(\left\{C_j \middle| \mathfrak{C}_{ij} \succ \mathfrak{C}_{ig}, j \in [N]\right\}\right) + 1,$$

from which we can subsequently obtain the following results:
a) if $C_j \succ C_h \Leftrightarrow \mathcal{R}_j^i < \mathcal{R}_h^i$ for $i \in [M]$ and $j, h \in [N]$, then we have $C_j \succ C_h \Leftrightarrow \mathfrak{C}_{ij} < \mathfrak{C}_{ih}$;
b) if $C_j \preceq C_h \Leftrightarrow \mathcal{R}_j^i \geq \mathcal{R}_h^i$ for $i \in [M]$ and $j, h \in [N]$, then we have $C_j \preceq C_h \Leftrightarrow \mathfrak{C}_{ij} \leq \mathfrak{C}_{ih}$.
The prioritization of criteria considered on participant level is meant to provide additional information to assist decision-makers in differentiating the significance of different criteria in respect of each alternative. Because of the inclusion of attitude strength, it has more practical implications in real-world applications. Take new product development for example, as indicated by the QFD (Quality Function Deployment) model, the product attributes should be translated into engineering features when product development decisions are made. More than that, the prioritization of the product attributes can tell product designers which features significantly affect the customer utility and how much the customer utility would be improved and how their attitudes would potentially change if the specific attribute were developed. In this sense, the COANG-based criterion analysis can easily supplement the product design decision process, or cooperate with other requirement measurement methods.

— **Opinion gathering on alternative level**.
The task of opinion gathering on alternative level is essentially to construct multi-criteria decision rule. The pooling of participants' opinions toward each alternative follows

$$\begin{aligned}\mathfrak{A}_i &= \sum_{k\in[H]}\sum_{j\in[N]}\left\langle P_{ij}^k, \mathrm{OS}_{ij}^k\right\rangle \\ &= \left\langle\left(\sum_{k\in[H]}\sum_{j\in[N]} p_{-\tau}^{ijk}, \cdots, \sum_{k\in[H]}\sum_{j\in[N]} p_l^{ijk}, \cdots, \sum_{k\in[H]}\sum_{j\in[N]} p_\tau^{ijk}\right),\right. \\ &\left.\frac{\sum\limits_{k\in[H]}\sum\limits_{j\in[N]} os^{ijk}\sum\limits_{l\in\lfloor\tau\rfloor} p_l^{ijk} s_l^{ijk}}{N+H-\sum\limits_{k\in[H]}\sum\limits_{j\in[N]} os^{ijk}+\sum\limits_{k\in[H]}\sum\limits_{j\in[N]} os^{ijk}\sum\limits_{l\in\lfloor\tau\rfloor} p_l^{ijk} s_l^{ijk}}\right\rangle. \end{aligned} \tag{2}$$

The dominating cardinality function

$$\mathcal{D}_e = \{A_i | \mathfrak{A}_i > \mathfrak{A}_e, i \in [M]\},$$

which denotes the set of the alternatives whose collective appraisal values are greater than that of $\mathfrak{A}_g$.
In particular, assigning $\#(\mathcal{D}_e)$ with zero obtains $\mathfrak{A}_g = \mathrm{Max}\{\mathfrak{A}_1, \cdots, \mathfrak{A}_i, \cdots, \mathfrak{A}_M\}$, and letting $\#(\mathcal{D}_e)$ with $M-1$ infers $\mathfrak{A}_g = \mathrm{Min}\left\{\mathfrak{A}_{i1}, \cdots, \mathfrak{A}_{ij}, \cdots, \mathfrak{A}_{iN}\right\}$. The position ranking function for alternatives $A_i (i \in [M])$ can be defined as

$$\mathcal{R}_e = \#\left(\{A_i | \mathfrak{A}_i > \mathfrak{A}_e, i \in [M]\}\right) + 1,$$

from which the following rules can be generated:
a) if $A_i \succ A_f \Leftrightarrow \mathcal{R}_i < \mathcal{R}_f$ for $i \in [M]$, then we have $A_i \succ A_f \Leftrightarrow \mathfrak{A}_i < \mathfrak{A}_f$;
b) if $A_i \preceq A_f \Leftrightarrow \mathcal{R}_i \geq \mathcal{R}_f$ for $i \in [M]$, then we have $A_i \preceq A_f \Leftrightarrow \mathfrak{A}_i \leq \mathfrak{A}_f$.

V. REAL-LIFE HEALTHCARE CASE STUDY

Schizophrenia is one of the most important public health problems in the world. A survey by the World Health Organization ranks schizophrenia among the top ten illnesses that contribute to the global burden of disease. Because of its early age of onset and its subsequent tendency to persists chronically, often at significant levels of severity, it produces great suffering for patients and also for their family members. It is also a relatively common illness. Although estimates of rates in the general population vary, it appears to affect from 0.5% to 1% of people worldwide [1]. Some of its symptoms, such as delusions and hallucinations, produce great subjective psychological pain. Other facets of the illness produce impaired dysfunction as well, such as cognitive function degradation. Furthermore, it is an illness that affects the essence of a person's identity, the most complex function that the brain mediates. It affects the ability to think clearly, to experience emotions, to read social situations and to have normal interpersonal relationships, and to interpret past experiences and plan for the future.

The concept of schizophrenia as a disorder entity as described by Kraepelin and Bleuler has been badly compromised by evidence that schizophrenia syndrome may comprise a number of specific entities [65]. The diagnostic criteria DSM-5 used currently can be considered provisional constructs with some face validity. However, for clinical purposes and based on epidemiological data, many researches support the clinical usefulness of psychopathological dimensions [65]. Therefore, there are some studies showing a continuous distribution of psychosis-like symptoms and even of dimensions of positive, negative, and general psychopathology [31]. Actually, Positive and Negative Syndrome Scale (PANSS) is one of psychotic assessments which are based on the dimensions of symptoms and widely used in clinical evaluation. PANSS (See Table 1) is a standard well-defined instrument for positive-negative symptom ratings that can be applied in relatively brief time and can be used repeatedly for longitudinal severity assessment [49]. It is developed on the basis of a seven-granularity LTS:

$$S = \left\{\begin{array}{l} s_{-3} = \mathit{Minimal}\ (\mathit{Min}), s_{-2} = \mathrm{Mild(Mil)}, \\ s_{-1} = \mathrm{Mild\text{-}Moderate(MM)}, s_0 = \mathrm{Moderate(Mod)}, \\ s_1 = \mathrm{Moderate\text{-}Severe(MS)}, s_2 = \mathrm{Severe(S)}, \\ s_3 = \mathrm{Extreme(E)} \end{array}\right\}.$$

TABLE 1. PANSS ratings on the three patients offered by visiting staff one.

Positive scale (P)								
PS_1	Delusions	Minimal	Mild	Mild-Moderate	Moderate	Moderate-Severe	Severe	Extreme
PS_2	Conceptual disorganization	Minimal	Mild	Mild-Moderate	Moderate	Moderate-Severe	Severe	Extreme
PS_3	Hallucinatory behavior	Minimal	Mild	Mild-Moderate	Moderate	Moderate-Severe	Severe	Extreme
PS_4	Excitement	Minimal	Mild	Mild-Moderate	Moderate	Moderate-Severe	Severe	Extreme
PS_5	Grandiosity	Minimal	Mild	Mild-Moderate	Moderate	Moderate-Severe	Severe	Extreme
PS_6	Suspiciousness/persecution	Minimal	Mild	Mild-Moderate	Moderate	Moderate-Severe	Severe	Extreme
PS_7	Hostility	Minimal	Mild	Mild-Moderate	Moderate	Moderate-Severe	Severe	Extreme
Negative scale (N)								
NS_1	Blunted affect	Minimal	Mild	Mild-Moderate	Moderate	Moderate-Severe	Severe	Extreme
NS_2	Emotional withdrawal	Minimal	Mild	Mild-Moderate	Moderate	Moderate-Severe	Severe	Extreme
NS_3	Poor rapport	Minimal	Mild	Mild-Moderate	Moderate	Moderate-Severe	Severe	Extreme
NS_4	Passive/apathetic social withdrawal	Minimal	Mild	Mild-Moderate	Moderate	Moderate-Severe	Severe	Extreme
NS_5	Difficulty in abstract thinking	Minimal	Mild	Mild-Moderate	Moderate	Moderate-Severe	Severe	Extreme
NS_6	Lack of spontaneity & flow of conversation	Minimal	Mild	Mild-Moderate	Moderate	Moderate-Severe	Severe	Extreme
NS_7	Stereotyped thinking	Minimal	Mild	Mild-Moderate	Moderate	Moderate-Severe	Severe	Extreme
General psychopathology scale (G)								
GPS_1	Somatic concern	Minimal	Mild	Mild-Moderate	Moderate	Moderate-Severe	Severe	Extreme
GPS_2	Anxiety	Minimal	Mild	Mild-Moderate	Moderate	Moderate-Severe	Severe	Extreme
GPS_3	Guilt feelings	Minimal	Mild	Mild-Moderate	Moderate	Moderate-Severe	Severe	Extreme
GPS_4	Tension	Minimal	Mild	Mild-Moderate	Moderate	Moderate-Severe	Severe	Extreme
GPS_5	Mannerisms & posturing	Minimal	Mild	Mild-Moderate	Moderate	Moderate-Severe	Severe	Extreme
GPS_6	Depression	Minimal	Mild	Mild-Moderate	Moderate	Moderate-Severe	Severe	Extreme
GPS_7	Motor retardation	Minimal	Mild	Mild-Moderate	Moderate	Moderate-Severe	Severe	Extreme
GPS_8	Uncooperativeness	Minimal	Mild	Mild-Moderate	Moderate	Moderate-Severe	Severe	Extreme
GPS_9	Unusual thought content	Minimal	Mild	Mild-Moderate	Moderate	Moderate-Severe	Severe	Extreme
GPS_{10}	Disorientation	Minimal	Mild	Mild-Moderate	Moderate	Moderate-Severe	Severe	Extreme
GPS_{11}	Poor attention	Minimal	Mild	Mild-Moderate	Moderate	Moderate-Severe	Severe	Extreme
GPS_{12}	Lack of judgment & insight	Minimal	Mild	Mild-Moderate	Moderate	Moderate-Severe	Severe	Extreme
GPS_{13}	Disturbance of volition	Minimal	Mild	Mild-Moderate	Moderate	Moderate-Severe	Severe	Extreme
GPS_{14}	Poor impulse control	Minimal	Mild	Mild-Moderate	Moderate	Moderate-Severe	Severe	Extreme
GPS_{15}	Preoccupation	Minimal	Mild	Mild-Moderate	Moderate	Moderate-Severe	Severe	Extreme
GPS_{16}	Active social avoidance	Minimal	Mild	Mild-Moderate	Moderate	Moderate-Severe	Severe	Extreme

It is common to put forward and discuss a clinical case in a conference which can be beneficial to produce a sophisticated treatment plan. Initially, residents will pick up patients who present complex symptoms in inpatient wards or outpatient departments. Afterwards, two visiting staffs participate in the case conference and are in charge of the disposition of the case. Apart from that, clinical psychologists who perform professional clinical assessments and therapy and social workers who provide family information and social support for patients also take part in the discussion. During the case

TABLE 2. Clinical symptoms of three cases [18].

Patient one (21 years old, female)
1. Convinced that her classmates were making fun of her and noticed that they would snort and sneeze whenever she entered the classroom; 2. A boy she was dating broke off the relationships with her, she believed that she had been "replaced" by a look-alike; 3. She called the police and asked for help to solve the "kidnapping"; 4. Her academic performance in school declined dramatically.
Patient two (34 years old, female)
1. Become increasingly euphoric, irritable and over talkative; 2. Suspect her husband having affairs; 3. Suddenly develop the unshakable belief that a physician in love with her and fell passionately in love with him, but said nothing and become increasingly distressed each time she saw him.
Patient three (54 years old, male)
1. Become more suspicious and withdrawn; 2. Think he take part in a large experiment to discover the secret of his "superior intelligence"; 3. "Millions of dollars" are involved in keeping him under surveillance; 4. Hearing voices during working and eventually fired; 5. Has been hospitalized 12 times; 6. Usually stop taking outpatient medication after leaving the hospital; 7. Suicide ideation.

conference, confirming the severity of the patient is a vital step which will influence the whole treatment plan.

Here, there are three cases (P_1, P_2, P_3) (see Table 2) come from Department of psychiatry, Kaohsiung Medical University Chung-Ho Memorial Hospital in Taiwan. They described bizarre symptoms and behaviors list in Table 2. All the symptoms have lasted more than 6 months. Three psychiatrists including two visiting staffs (E_1, E_2) and one resident (E_3), who are all male by the way, will use PANSS to evaluate the severity of positive and negative symptoms. Therefore, based on the ratings, they can be figure out the suitable priority and further treatment protocols. The assessment outcomes can be observed from Tables 3-5. It should be noted that, due to the fact that taking advantage of the self-administrative assessment results from real clinical cases will substantially violate their privacy and legal rights, our research constructs all the severity evaluations largely based on the clinical experience, research articles and case studies [18].

The basic components of the COANG-based DSCDM in the context of severity check for the three patients from the three psychiatrists have been shaped by the aspects of the entire descriptions brought up in the application background. Provided Tables 3-5 from the two visiting staffs and the resident we can kick-start the translation process from the current GCLE evaluations into HFLTS possibility distributions, and then from HFLTS possibility distributions into DULE assessment representations that reflect attitude strength. The transformed data set is of massive scale, and therefore, we will present it in the supplementary files. The similarity measure-based generation approach was adopted in a bid to translate the GCLE evaluations into HFLTS possibility distributions, and the results are provided as Tables 9-11 in the supplementary file of this paper, which kick off the construction of domain-specific appraisal modeling. It should be pointed out that the similarity measure-based generation approach was deemed diffusely as a feasible option as it solves the cold-start problem while generating initial HFLTS possibility distributions modeling the individual semantics. The consecutive information transition process converting HFLTS possibility distributions into DULE assessment representations can be accomplished in the use of the OWA-based approach to generating attitudinal HFLTS possibility distributions. The attitudinal character function introduced into the OWA theory models simultaneously both the attitude orientation and attitude strength. The results with DULE assessment representations are provided as Tables 12-14 in the supplementary file.

The follow-up operation of the proposed COANG-based DSCDM requires the three psychiatrists to be invited to reexamine their PANSS assessments along with the transformed PLC. The heterogeneous SPI given by the three psychiatrists serves as a useful indicator of their domain specificity in the diagnosis of schizophrenia as well as enhances significantly the quality and reliability of the subjective decision appraisals.

The visiting staff one had a second thought of several PANSS assessments and offered several additional information for their SPI generated by the similarity measure-based generation approach. Regarding the assessments on symptom-patient pairs (PS_4, P_3), (NS_7, P_3), (GPS_9, P_3), the visiting staff one confirmed that the previous PLCs are those he showed reservations towards before but would like to contribute to its accuracy enhancement with more specifications. The modified SPI as per the request from visiting staff one for (PS_4, P_3) is supposed to be constrained by that the possibilities for s_{-2} and s_{-1} exhibit a difference of 0.1, and the possibility for s_{-1} is 1.2 times that for s_0. The modified SPI for (NS_7, P_3) is constrained by that the possibility difference between s_{-3} and s_{-2} is greater than that between s_{-2} and s_{-1} and the possibilities for them decrease across the increasing subscripts for these linguistic terms. In addition, the modified SPI for (GPS_9, P_3) restricts itself to the constraint that the possibility difference between s_{-1} and s_1 is between 0.05 and 0.1, excluding 0.05 and 0.1. It has been pointed

TABLE 3. Severity check for the three patients conducted by the visiting staff one.

Visiting Staff One		E1		
Positive scale (P)		P1	P2	P3
PS_1	Delusions	Between MS and S	Between Mil and MM	Greater than S
PS_2	Conceptual disorganization	MM	Min	Between S and E
PS_3	Hallucinatory behavior	Lower than Mil	Lower than Mod	At least Mod
PS_4	Excitement	Between MM and Mod	Min	Greater than MM
PS_5	Grandiosity	MM	Between Min and Mil	Mod
PS_6	Suspiciousness/persecution	Between S and E	Min	Between MS and S
PS_7	Hostility	At most MM	Min	MM
Negative scale (N)				
NS_1	Blunted affect	Between Min and Mil	MM	Mil
NS_2	Emotional withdrawal	At most Mil	Mod	Lower than MM
NS_3	Poor rapport	Between Mil and MM	Min	Between MM and Mod
NS_4	Passive/apathetic social withdrawal	Between Min and Mil	At most MM	MM
NS_5	Difficulty in abstract thinking	Min	Between MM and Mod	At least MS
NS_6	Lack of spontaneity & flow of conversation	Between Min and Mil	Mil	Mil
NS_7	Stereotyped thinking	At most MM	Min	At most Mod
General psychopathology scale (G)				
GPS_1	Somatic concern	At most MM	Min	At most Mil
GPS_2	Anxiety	Between Mod and MS	Lower than Mil	Mil
GPS_3	Guilt feelings	Min	MM	Mil
GPS_4	Tension	At most MM	Lower than Mil	Min
GPS_5	Mannerisms & posturing	Min	MM	Between Mil and MM
GPS_6	Depression	Between Min and Mil	Lower than Mod	Mil
GPS_7	Motor retardation	Min	Between Mil and MM	Min
GPS_8	Uncooperativeness	Mil	Mil	At least MM
GPS_9	Unusual thought content	MM	Min	At most MS
GPS_{10}	Disorientation	Between Min and Mil	Mil	Mil
GPS_{11}	Poor attention	Min	At most Mil	Between MM and Mod
GPS_{12}	Lack of judgment & insight	Min	Min	MS
GPS_{13}	Disturbance of volition	Between Min and Mil	Between Min and Mil	Min
GPS_{14}	Poor impulse control	At most MM	Min	Lower than Mil
GPS_{15}	Preoccupation	MM	Between Mil and MM	MM
GPS_{16}	Active social avoidance	Between Min and Mil	Mil	Between Min and Mil

out in our previous research that the disparate psychological states are predicated through their responsive expressions with measurable Attitudinal Character values. Presumptions are as well made here for the attitudinal characteristics of visiting staff one, which is that he leans toward optimism pessimism with $\text{orness}(E_1) = 0.9$ when providing update to (PS_4, P_3) and leans in the direction of pessimism with $\text{orness}(Exp_1) = 0.1$ when providing the rest. The following

SPI-based optimization models with the goal of maximizing the cognitive consistency among individuals are built in a bid to facilitate the domain specific appraisal modeling.

The SPI optimization model built for (PS_4, P_3) as per E_1's request is as follows:

$$\begin{cases} \text{Max}\frac{1}{2}\left(1-\frac{\sum_{i=1}^{7}\left(p_i-\bar{P}_{43}\right)\left(p'_i-\bar{P}'_{43}\right)}{\sqrt{\sum_{i=1}^{7}\left(p_i-\bar{P}_{43}\right)^2}\sqrt{\sum_{i=1}^{7}\left(p'_i-\bar{P}'_{43}\right)^2}}\right) \\ s.t.\begin{cases}\frac{1}{6}p'_2+\frac{1}{3}p'_3+\frac{1}{2}p'_4=0.7\\ p'_2-p'_3\geq 0.1\\ p'_3=1.2p'_4\\ \sum_{i=1}^{7}p'_i=1\\ p'_i\in[0.05,1]\quad for\ i=1,2,3,4\end{cases}\end{cases}$$

The SPI optimization model built for (NS_7, P_3) as per E_1's request is as follows:

$$\begin{cases} \text{Max}\frac{1}{2}\left(1-\frac{\sum_{i=1}^{7}\left(p_i-\bar{P}_{73}\right)\left(p'_i-\bar{P}'_{73}\right)}{\sqrt{\sum_{i=1}^{7}\left(p_i-\bar{P}_{73}\right)^2}\sqrt{\sum_{i=1}^{7}\left(p'_i-\bar{P}'_{73}\right)^2}}\right) \\ s.t.\begin{cases}\frac{1}{6}p'_2+\frac{1}{3}p'_3+\frac{1}{2}p'_4=0.3\\ p'_1-p'_2>p'_2-p'_3\geq 0\\ \sum_{i=1}^{7}p'_i=1\\ p'_i\in[0.05,1]\quad for\ i=1,2,3,4\end{cases}\end{cases}$$

The SPI optimization model built for (GPS_9, P_3) as per E_1's request is as follows:

$$\begin{cases} \text{Max}\frac{1}{2}\left(1-\frac{\sum_{i=1}^{7}\left(p_i-\bar{P}_{93}\right)\left(p'_i-\bar{P}'_{93}\right)}{\sqrt{\sum_{i=1}^{7}\left(p_i-\bar{P}_{93}\right)^2}\sqrt{\sum_{i=1}^{7}\left(p'_i-\bar{P}'_{93}\right)^2}}\right) \\ s.t.\begin{cases}\frac{1}{6}p'_2+\frac{1}{3}p'_3+\frac{1}{2}p'_4+\frac{2}{3}p'_5=0.3\\ 0.05<p'_3-p'_5<0.1\\ \sum_{i=1}^{7}p'_i=1\\ p'_i\in[0.05,1]\quad for\ i=1,2,\cdots,5\end{cases}\end{cases}$$

It should be pointed out that $\bar{P}_{43}$, $\bar{P}_{73}$, and $\bar{P}_{93}$ are possibility distributions for the PANSS assessments in the manifestation of HFLTS transformed from GCLEs. In addition, the visiting staff two E_2 confirmed that he urged no further modification to his original PANSS assessments. The resident E_3, however, made the request to cement his previous assessments on symptom-patient pairs including (PS_5, P_3) and (NS_2, P_2). The modified SPI for (PS_5, P_3) was placed a restriction from E_3 that the possibility difference between s_0 is between 0.1 and 0.3, excluding 0.1 and 0.3. Two constraints were set for the modified SPI for (NS_2, P_2), which are that the possibility for s_{-3} is smaller than that for s_1 and the possibility difference between s_{-1} and s_0, showing that superior of s_{-1}, is greater than or equal to 0.03. Presumptions are as well made here for the attitudinal characteristics of the resident, which is that he leans toward pessimism with $\text{orness}(E_3) = 0.4$ with regards to these two modifications. Likewise, the following SPI-based optimization models can be built.

The SPI optimization model built for (PS_5, P_3) as per E_3's request is as follows:

$$\begin{cases} \text{Max}\frac{1}{2}\left(1-\frac{\sum_{i=1}^{7}\left(p_i-\bar{P}_{53}\right)\left(p'_i-\bar{P}'_{53}\right)}{\sqrt{\sum_{i=1}^{7}\left(p_i-\bar{P}_{53}\right)^2}\sqrt{\sum_{i=1}^{7}\left(p'_i-\bar{P}'_{53}\right)^2}}\right) \\ s.t.\begin{cases}\frac{1}{6}p'_2+\frac{1}{3}p'_3+\frac{1}{2}p'_4=0.3\\ 0.1<p'_4<0.3\\ \sum_{i=1}^{7}p'_i=1\\ p'_i\in[0.05,1]\quad for\ i=1,2,3,4\end{cases}\end{cases}$$

The SPI optimization model built for (NS_2, P_2) as per E_3's request is as follows:

$$\begin{cases} \text{Max}\frac{1}{2}\left(1-\frac{\sum_{i=1}^{4}\left(p_i-\bar{P}_{22}\right)\left(p'_i-\bar{P}'_{22}\right)}{\sqrt{\sum_{i=1}^{4}\left(p_i-\bar{P}_{22}\right)^2}\sqrt{\sum_{i=1}^{4}\left(p'_i-\bar{P}'_{22}\right)^2}}\right) \\ s.t.\begin{cases}\frac{1}{6}p'_2+\frac{1}{3}p'_3+\frac{1}{2}p'_4=0.3\\ p'_3\leq p'_4\\ \sum_{i=1}^{4}p'_i=1\\ p'_i\in[0.05,1]\quad for\ i=1,2,3,4\end{cases}\end{cases}$$

The DULE assessment representations with all these updates made are provided as Tables 15-17 in the supplementary file. Eventually, the implementation of the proposed COANG-based DSCDM can be set forth as two successive opinion-gathering processes at different levels. The whole landscape of the computational treatment on the basis of our approach will be pictured in our ensuing descriptions in an attempt to integrate the collective wisdom of the three psychiatrists into the severity evaluation framework. The opinion gathering process initiates on the symptom level and produces the collective assessments for each patient from the three psychiatrists, which are given in Table 6. It ends on the patient level and suggests that the overall severity check results for patients P_1, P_2, and P_3 are

$$\mathfrak{A}_1=\left\langle\begin{pmatrix}0.4164, 0.2136, 0.2478, 0.0352,\\ 0.0304, 0.0269, 0.0297\end{pmatrix}, 0.4838\right\rangle,$$
$$\mathfrak{A}_2=\left\langle\begin{pmatrix}0.4764, 0.2755, 0.1576, 0.0917,\\ 0.0000, 0.0000, 0.0000\end{pmatrix}, 0.2151\right\rangle,$$
$$\mathfrak{A}_3=\left\langle\begin{pmatrix}0.2133, 0.2513, 0.2118, 0.1027,\\ 0.0917, 0.0630, 0.0662\end{pmatrix}, 0.7906\right\rangle.$$

The information equivalences of $\mathfrak{A}_1$, $\mathfrak{A}_2$, and $\mathfrak{A}_3$ manifested by linguistic 2-tuples associated with attitude strength are $\langle(s_{-2}, 0.25), 0.4838\rangle$, $\langle(s_{-2}, -0.13), 0.2151\rangle$, and $\langle(s_{-1}, 0.06), 0.7906\rangle$. The diagnostic conclusion reached in the use of the proposed COANG-based decision support model suggests that patient three got a higher severity degree than patient one and, more than that, they are both at a

TABLE 4. Severity check for the three patients conducted by the visiting staff two.

Visiting Staff Two	E2		
	P1	P2	P3
Positive scale (P)			
PS_1 Delusions	MS	At most Mil	At least S
PS_2 Conceptual disorganization	Lower than MM	Between Min and Mil	Between S and E
PS_3 Hallucinatory behavior	At most Mil	Between MM and Mod	Mod
PS_4 Excitement	Between MM and MS	Lower than Mil	MM
PS_5 Grandiosity	MM	Min	Between Mod and MS
PS_6 Suspiciousness/persecution	Greater than S	Lower than Mil	Between MS and S
PS_7 Hostility	MM	Min	At most MM
Negative scale (N)			
NS_1 Blunted affect	Min	Between Min and MM	At most Mil
NS_2 Emotional withdrawal	Between Mil and MM	Mod	Lower than MM
NS_3 Poor rapport	Between Mil and MM	At most Mil	MM
NS_4 Passive/apathetic social withdrawal	Min	Between Mil and MM	Lower than MM
NS_5 Difficulty in abstract thinking	Between Min and Mil	Mod	Between MS and S
NS_6 Lack of spontaneity & flow of conversation	Min	At most Mil	Mil
NS_7 Stereotyped thinking	Between Mil and Mod	Min	Mod
General psychopathology scale (G)			
GPS_1 Somatic concern	Between Min and MM	Min	Between Mil and MM
GPS_2 Anxiety	Between MM and Mod	Mil	At most Mil
GPS_3 Guilt feelings	Min	Lower than MM	Between Min and Mil
GPS_4 Tension	At most MM	Mil	At most Mil
GPS_5 Mannerisms & posturing	Min	Lower than MM	Mil
GPS_6 Depression	At most Mil	Mod	Between Mil and MM
GPS_7 Motor retardation	Min	Mil	Min
GPS_8 Uncooperativeness	At most Mil	Between Mil and MM	Between MM and Mod
GPS_9 Unusual thought content	At most MM	Min	MS
GPS_{10} Disorientation	Min	Min	Min
GPS_{11} Poor attention	Min	Min	MM
GPS_{12} Lack of judgment & insight	Between Min and Mil	Lower than Mil	Greater than MS
GPS_{13} Disturbance of volition	Min	Min	Lower than Mil
GPS_{14} Poor impulse control	Between MM and Mod	At most Mil	Mil
GPS_{15} Preoccupation	MM	Mil	Lower than MM
GPS_{16} Active social avoidance	MM	Between Mil and MM	Min

more concerned status than patient two. The overall severity check results assist the group of psychiatrists attending the case conference in figuring out the suitable priorities among the patients and furthering treatment protocols for each of them. The case conference created the protocols following certain clinical guidelines on the treatment of Schizophrenia, which can be seen from Figure 3 and Table 7. As for patient three, the degree of severity was rated algorithmically by

TABLE 5. Severity check for the three patients conducted by the Resident.

Resident		E3		
Positive scale (P)		P1	P2	P3
PS_1	Delusions	Greater than MS	Mil	E
PS_2	Conceptual disorganization	MM	Min	Between MS and S
PS_3	Hallucinatory behavior	Between Mil and MM	Mod	Between Mod and MS
PS_4	Excitement	Greater than Mod	Min	MM
PS_5	Grandiosity	MM	Between Min and Mil	At most Mod
PS_6	Suspiciousness/persecution	At least S	Min	Between MS and S
PS_7	Hostility	Between Mil and MM	Min	MM
Negative scale (N)				
NS_1	Blunted affect	Min	MM	Between Mil and Mod
NS_2	Emotional withdrawal	Between Mil and MM	At most Mod	MM
NS_3	Poor rapport	Between Mil and MM	Min	Between Mil and MM
NS_4	Passive/apathetic social withdrawal	Min	MM	MM
NS_5	Difficulty in abstract thinking	Min	Between MM and Mod	Greater than MS
NS_6	Lack of spontaneity & flow of conversation	Between Min and Mil	Between MM and Mod	Mil
NS_7	Stereotyped thinking	Between MM and Mod	Min	Mod
General psychopathology scale (G)				
GPS_1	Somatic concern	Min	Between Min and Mil	MM
GPS_2	Anxiety	Mod	Mil	Mil
GPS_3	Guilt feelings	Min	Between MM and Mod	Between Min and Mil
GPS_4	Tension	Between Mil and MM	At most Mil	Mil
GPS_5	Mannerisms & posturing	Min	MM	At most Mil
GPS_6	Depression	Min	Between MM and Mod	At most Mod
GPS_7	Motor retardation	Min	At most Mil	Min
GPS_8	Uncooperativeness	Mil	Between Mil and MM	MM
GPS_9	Unusual thought content	Between MM and Mod	Min	MS
GPS_{10}	Disorientation	Between Min and Mil	Min	Min
GPS_{11}	Poor attention	Min	Min	Between MM and Mod
GPS_{12}	Lack of judgment & insight	Min	Min	MS
GPS_{13}	Disturbance of volition	Min	Lower than Mil	Min
GPS_{14}	Poor impulse control	MM	Min	Between Min and Mil
GPS_{15}	Preoccupation	Between Min and Mil	Mil	MM
GPS_{16}	Active social avoidance	MM	Mil	Min

the collective wisdom of the psychiatrist group as Mild-Moderate, according to which they offered the treatment advice of *moderate dosage psychopharmacology, mid-term psychological therapy, and mid-term social work follow-up*.

The patients one and two shared a relatively low severity status, Mild, and therefore, were offered the treatment advice of *low dosage psychopharmacology, short-term psychological therapy, and short-term social work follow-up*.

TABLE 6. Opinion gathering on symptom level.

Psychiatrists	Patients	Collective assessments on symptom level
E_1	P_1	$\langle(0.4096, 0.2404, 0.2333, 0.0333, 0.0333, 0.0333, 0.0167), 0.4021\rangle$
	P_2	$\langle(0.4752, 0.2682, 0.2067, 0.0500, 0.0000, 0.0000, 0.0000), 0.2157\rangle$
	P_3	$\langle(0.1974, 0.2730, 0.1756, 0.1078, 0.0919, 0.0679, 0.0864), 0.6733\rangle$
E_2	P_1	$\langle(0.4228, 0.2172, 0.2600, 0.0222, 0.0444, 0.0000, 0.0333), 0.3972\rangle$
	P_2	$\langle(0.4854, 0.3369, 0.0811, 0.1000, 0.0000, 0.0000, 0.0000), 0.2187\rangle$
	P_3	$\langle(0.2498, 0.2702, 0.1633, 0.1167, 0.0833, 0.0697, 0.0470), 0.6169\rangle$
E_3	P_1	$\langle(0.4167, 0.1833, 0.2500, 0.0500, 0.0133, 0.0475, 0.0392), 0.4823\rangle$
	P_2	$\langle(0.4686, 0.2214, 0.1850, 0.1250, 0.0000, 0.0000, 0.0000), 0.2813\rangle$
	P_3	$\langle(0.1927, 0.2107, 0.2964, 0.0836, 0.1000, 0.0515, 0.0652), 0.6490\rangle$

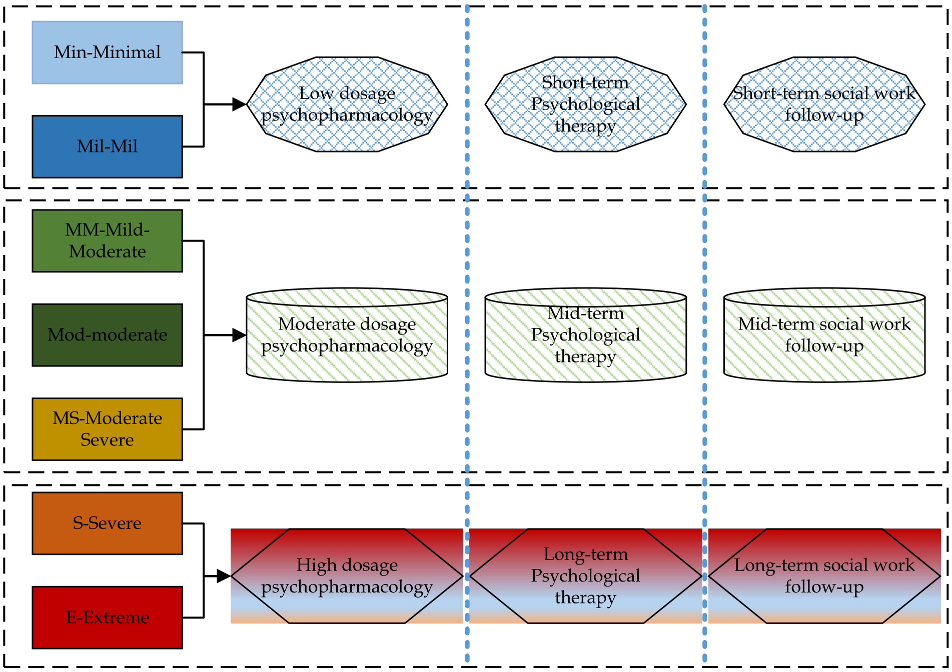

FIGURE 3. Primary treatment advices based on symptoms severity.

VI. DISCUSSION

The CDM has been deemed as a process of gathering a set of participants in a bid to extract their individual knowledge towards certain objects being evaluated. There is no theoretical ground that it could exclude the attitudinal dimension of opinion delivering without losing a certain amount of information that matters. In an effort to get rid of the preconceived idea created following the traditional view solidly built in CDM practice citing existing studies, this paper has revisited opinion structures embedded in the DSCDM from the perspective of component analysis acknowledging the domain-specific expertise that uses individual semantics as a connotative manifestation. The purpose of this paper is to offer the chance of enhancing domain-specific appraisal modeling by proposing multiple PCC-based optimization models serving for the determination of SPI that governs the semantic implications of distinct PLCs.

The proposal of COANG model proceeds to provide automatic tools able to extract subjective information from decision appraisals in DSCDM, such as attitude orientation and attitude strength, to create structured and actionable knowledge aiming at introducing more specifications of decision appraisals. The DULE then bridges the gap between the domain-specific appraisals and the decomposing structure of the COANG model. With the comparison rules and aggregation paradigm developed for DULE, the novel COANG-based DSCDM framework was constructed in order to harness the explosion of digital data with semantical distinctions and computational power with advanced algorithms to enable collaborative and natural interactions between people and machines that extend the abilities of decision support systems to sense, learn, and understand participants individually and holistically. The research infuses individual opinion representation, domain-specific appraisal modeling, and the COANG model with the ability to evaluate, reason, analyze, and perform with humanlike decision-making skill and agility.

The illustrative application of the proposed methodological development to severity check and treating priority identification in the context of psychological disease diagnosis has verified its feasibility and effectiveness. Clinical psychological

TABLE 7. Therapies for schizophrenia [13].

Psychopharmacology (Major drugs used in treating schizophrenia)

Drug Category	Generic Name	Trade Name
Phenothiazine	Chlorpromazine Fluphenazine decanocate	Thorazine Prolixin
Butyrophenone	Haloperidol	Haldol
Thioxanthene	Thiothixene	Navane
Tricyclic dibenzodiazepine	Clozapine	Clozaril
Thienbenzodiazepine	Olanzapine	Zyprexa
Benzisoxazole	Risperidone	Risperdal

Psychological treatments

- Psychodynamic therapies

Free association is a method that simply tell patients to speak freely about whatever thoughts crossed their mind, and it becomes a cornerstone of Freud's famous treatment. However, Harry Stack Sullivan pioneered the use of psychoanalysis with schizophrenic patient and developed a psychoanalytic treatment.

- Cognitive-Behavioral therapy

1. systematic desensitization
   A technique for eliminating fears that has three key elements.
   The first is relaxation training using progressive muscle relaxation. The second is constructing a hierarchy of fears ranging from very mild to very intense. The third part is the learning process, maintaining relaxation while confronting fears [50].
2. Aversion therapy
   This therapy is to create, not eliminate an unpleasant response. This technique is used primarily in treating substance use disorder, such as alcoholism and cigarette [50]. For example: one form of aversion therapy pairs the sight, smell and taste of alcohol with severe nausea produced artificially by a drug.
3. Contingency management
   Contingency management directly changes rewards and punishments for identified behaviors. It involves changing the relationship between a behavior and its consequences.
4. Social-skills training
   This treatment is to teach patients new ways of behaving that are both desirable and likely to be rewarded in everyday life. Two commonly taught skills are assertiveness training and social problem solving [50]. In teaching assertiveness, therapists frequently use role playing, an acting technique that allows clients to rehearse new social skills. Social problem solving is a multistep process that has been used to teach children and adults ways to go about solving a variety of life's problems. The first step involves defining the problem in detail, breaking a complex difficulty into smaller and more manageable pieces. Coming up with many alternative solutions is the second step. The third step involves carefully evaluating these options. Finally, the best solution is chosen and applied.
5. Cognitive techniques
   1)Attribution retraining involves changing attributions, often by asking clients to abandon intuitive strategies. Instead, clients are instructed in more scientific methods, such as objectively testing hypotheses about themselves and others. 2)Self-instruction training is another cognitive technique that is often used with children. In Meichenbaum's self-instruction training, the adult first models an appropriate behavior while saying the self-instruction aloud. Next, the child is asked to repeat the action and to say the self-instruction until children internalize the self-instruction and learn internal controls over their behavior [32].
   3)Beck's cognitive therapy challenges cognitive errors, often by having clients analyze their thoughts more carefully [3].
6. Rational-Emotive therapy
   Albert Ellis's rational-emotive therapy is also designed to challenge cognitive distortions. According to Ellis [16], emotional disorders are caused by irrational beliefs. The rational-emotive therapist searches for a client's irrational beliefs, point out the impossibility of fulfilling them, and uses any and every opportunity to persuade the client to adopt more realistic beliefs [16].
7. Third-wave CBT
   Third-wave CBT focus on broad, abstract principles such as acceptance, mindfulness, values, and relationships [25]. For example, dialectical behavior therapy includes an emphasis on mindfulness, increased awareness of your feelings, thoughts, and motivations [34]. Acceptance and commitment therapy encourage accepting oneself, not just on making changes [25].

Social work

- Family therapy

Family therapy might include two, three, or more family members in a treatment designed to improve communication, negotiate conflicts and perhaps change relationships and roles [50].

- Case management

Case managers can help patients cope with the mental health system and they are able to get patients into contact with providers of whatever services the patients required.

TABLE 8. Opinion gathering on symptom level.

Psychiatrists	Patients	Collective assessments on symptom level
E_1	P_1	$\langle (0.4096, 0.2404, 0.2333, 0.0333, 0.0333, 0.0333, 0.0167), 0.4021\rangle$
	P_2	$\langle (0.4752, 0.2682, 0.2067, 0.0500, 0.0000, 0.0000, 0.0000), 0.2157\rangle$
	P_3	$\langle (0.1892, 0.2821, 0.1759, 0.1167, 0.0985, 0.0643, 0.0733), 0.6613\rangle$
E_2	P_1	$\langle (0.4228, 0.2172, 0.2600, 0.0222, 0.0444, 0.0000, 0.0333), 0.3972\rangle$
	P_2	$\langle (0.4854, 0.3369, 0.0811, 0.1000, 0.0000, 0.0000, 0.0000), 0.2187\rangle$
	P_3	$\langle (0.2498, 0.2702, 0.1633, 0.1167, 0.0833, 0.0697, 0.0470), 0.6169\rangle$
E_3	P_1	$\langle (0.4167, 0.1833, 0.2500, 0.0500, 0.0133, 0.0475, 0.0392), 0.4823\rangle$
	P_2	$\langle (0.4692, 0.2271, 0.1926, 0.1111, 0.0000, 0.0000, 0.0000), 0.2715\rangle$
	P_3	$\langle (0.1929, 0.2108, 0.2963, 0.0833, 0.1000, 0.0515, 0.0652), 0.6489\rangle$

practice suggests that linguistic evaluations have been diffusely used to measure the symptom servility degrees, which forms the theoretical ground for the formation of the diagnostic criteria DSM-5 [1]. Extracting useful diagnostic clues from the estimation of explicit servility status for each patient in a bid to facilitate accurate clinical diagnostic experiences requires psychiatrists' domain knowledge acquired through years of systematic study and clinical training. The COANG-based DSCDM framework provides the potential of automating or at least aiding such efforts as it contributes an innovative paradigm to the in-depth understanding of psychiatrists' reasoning processes and their use of domain knowledge. The comparison of COANG-based DSCDM framework with other methodologies and approaches based on DULE are not available at the current stage as this is, as far as we know, the first paper attempting at developing DULE-based linguistic computational techniques. However, making comparisons for DULE-based models by including whether or not the SPI provided by the three psychiatrists remains feasible. With the SPI excluded the opinion gathering process initiating on the symptom level produces the collective assessments for each patient from the three psychiatrists, which are given in Table 8. It as well ends on the patient level and suggests that the overall severity check results for patients P_1, P_2, and P_3 are

$$\left\langle \begin{pmatrix} 0.4164, 0.2136, 0.2478, 0.0352, \\ 0.0304, 0.0269, 0.0297 \end{pmatrix}, 0.4838 \right\rangle,$$
$$\left\langle \begin{pmatrix} 0.4766, 0.2774, 0.1601, 0.0870, \\ 0.0000, 0.0000, 0.0000 \end{pmatrix}, 0.2103 \right\rangle,$$
$$\left\langle \begin{pmatrix} 0.2106, 0.2544, 0.2119, 0.1056, \\ 0.0940, 0.0618, 0.0618 \end{pmatrix}, 0.7867 \right\rangle.$$

The information equivalences of $\mathfrak{A}_1$, $\mathfrak{A}_2$, and $\mathfrak{A}_3$ manifested by linguistic 2-tuples associated with attitude strength are $\langle (s_{-2}, 0.25), 0.4838\rangle$, $\langle (s_{-2}, -0.15), 0.2151\rangle$, and $\langle (s_{-1}, 0.05), 0.7906\rangle$. The diagnostic conclusion reached in this case is simply the same to the previous one where the patient three got a higher severity degree than patient one, and they are both at a more concerned status than patient two. The slight modification to certain SPI in accordance with their heterogeneous requirements produces accuracy-oriented changes in the aggregation of DULE assessments. More insightful observations can be anticipated in our future work as more DULE-based linguistic computational techniques are expected to be developed in the use of established decision-making theoretical frameworks.

VII. CONCLUSIONS

DSCDM nowadays has gained increasing attention from the academic world as the emergence of decision experts involved in complicated CDM comes along with diversifying backgrounds reflecting their domain-specific expertise. The existing DSCDM methods fail to process participants' knowledge by shifting the existing paradigm into the recognition of the opinion structure. This undesired feature necessitates advanced DSCDM methods able to deal with potential challenges in practical applications. Suggested in this paper is a COANG-based DSCDM framework consisting of the domain-specific appraisal modeling and the COANG model tailored to DSCMD with GCLEs.

Our contributions in this research paper show that:

1) An effective domain-specific appraisal modeling based on CIS building on heterogeneous SPI facilitates the capture and visualization of the cognition process of human beings.
2) The SPI-based optimization models developed facilitate maximizing the cognitive consistency among individuals in the domain-specific appraisal modeling.
3) The COANG-based DSCDM model introduced in this paper can classify subjective appraisals with different attitude orientations and group attitude detection with distinct attitude strength.
4) The application of the COANG-based DSCDM framework into real-world problems provides important improvements in supporting decisions through knowledge-intensive computer-based solutions that ultimately support and improve the performance.

In the future, the research and tools that result from this study and their advancements are expected to be woven into existing and new software embedded with clinical decision support systems.

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