

APPENDIX A
PREFERENCES USED IN THE ILLUSTRATIVE EXAMPLE

A. Initial HFLPRs

$$\begin{aligned}
O_1 &= \begin{pmatrix} E & W & W & Bt\ SW\ and\ E \\ B & E & SB & B \\ B & SW & E & Bt\ SB\ and\ B \\ Bt\ E\ and\ SB & W & Bt\ W\ and\ SW & E \end{pmatrix}, \\
O_2 &= \begin{pmatrix} E & SW & MW & W \\ SB & E & W & SW \\ MB & B & E & SB \\ B & SB & SW & E \end{pmatrix}, \\
O_3 &= \begin{pmatrix} E & SB & B & SW \\ SW & E & Bt\ E\ and\ SB & W \\ W & Bt\ SW\ and\ E & E & MW \\ SB & B & MB & E \end{pmatrix}. \\
O_4 &= \begin{pmatrix} E & SB & W & Bt\ W\ and\ SW \\ SW & E & Bt\ MW\ and\ W & E \\ B & Bt\ B\ and\ MB & E & B \\ Bt\ SB\ and\ B & E & W & E \end{pmatrix}, \\
O_5 &= \begin{pmatrix} E & SB & SW & B \\ SW & E & W & SB \\ SB & B & E & MB \\ W & SW & MW & E \end{pmatrix}
\end{aligned}$$

where *Bt* stands for *between*.

B. Initial EPRs

$$\begin{aligned}
O_1 &= \begin{pmatrix} (E,0)_0 & (W,0)_0 & (W,0)_0 & [(SW,0), (E,0)]_{00} \\ (B,0)_0 & (E,0)_0 & (SB,0)_0 & (B,0)_0 \\ (B,0)_0 & (SW,0)_0 & (E,0)_0 & [(SB,0), (B,0)]_{00} \\ [(E,0), (SB,0)]_{00} & (W,0)_0 & [(W,0), (SW,0)]_{00} & (E,0)_0 \end{pmatrix}, \\
O_2 &= \begin{pmatrix} (E,0)_0 & (SW,0)_0 & (s_0,0)_0 & (W,0)_0 \\ (SB,0)_0 & (E,0)_0 & (W,0)_0 & (SW,0)_0 \\ (MB,0)_0 & (B,0)_0 & (E,0)_0 & (SB,0)_0 \\ (B,0)_0 & (SB,0)_0 & (SW,0)_0 & (E,0)_0 \end{pmatrix}, \\
O_3 &= \begin{pmatrix} (E,0)_0 & (SB,0)_0 & (B,0)_0 & (SW,0)_0 \\ (SW,0)_0 & (E,0)_0 & [(E,0), (SB,0)]_{00} & (E,0)_0 \\ (E,0)_0 & [(SW,0), (E,0)]_{00} & (E,0)_0 & (MW,0)_0 \\ (SB,0)_0 & (B,0)_0 & (MB,0)_0 & (E,0)_0 \end{pmatrix}. \\
O_4 &= \begin{pmatrix} (E,0)_0 & (SB,0)_0 & (W,0)_0 & [(W,0), (SW,0)]_{00} \\ (SW,0)_0 & (E,0)_0 & [(MW,0), (W,0)]_{00} & (E,0)_0 \\ (B,0)_0 & [(B,0), (MB,0)]_{00} & (E,0)_0 & (B,0)_0 \\ [(SB,0), (B,0)]_{00} & (E,0)_0 & (W,0)_0 & (E,0)_0 \end{pmatrix}. \\
O_5 &= \begin{pmatrix} (E,0)_0 & (SB,0)_0 & (SW,0)_0 & (B,0)_0 \\ (SW,0)_0 & (E,0)_0 & (W,0)_0 & (SB,0)_0 \\ (SB,0)_0 & (B,0)_0 & (E,0)_0 & (MB,0)_0 \\ (W,0)_0 & (SW,0)_0 & (MW,0)_0 & (E,0)_0 \end{pmatrix}.
\end{aligned}$$

C. Initial TrFNs

$$\begin{aligned}
O_1 &= \begin{pmatrix} T(0.33, 0.5, 0.5, 0.67) & T(0.0, 0.17, 0.17, 0.33) & T(0.0, 0.17, 0.17, 0.33) & T(0.17, 0.33, 0.5, 0.67) \\ T(0.67, 0.83, 0.83, 1.0) & T(0.33, 0.5, 0.5, 0.67) & T(0.5, 0.67, 0.67, 0.83) & T(0.67, 0.83, 0.83, 1.0) \\ T(0.67, 0.83, 0.83, 1.0) & T(0.17, 0.33, 0.33, 0.5) & T(0.33, 0.5, 0.5, 0.67) & T(0.5, 0.67, 0.83, 1.0) \\ T(0.33, 0.5, 0.67, 0.83) & T(0.0, 0.17, 0.17, 0.33) & T(0.0, 0.17, 0.33, 0.5) & T(0.33, 0.5, 0.5, 0.67) \end{pmatrix}, \\
O_2 &= \begin{pmatrix} T(0.33, 0.5, 0.5, 0.67) & T(0.17, 0.33, 0.33, 0.5) & T(0.0, 0.0, 0.0, 0.17) & T(0.0, 0.17, 0.17, 0.33) \\ T(0.5, 0.67, 0.67, 0.83) & T(0.33, 0.5, 0.5, 0.67) & T(0.0, 0.17, 0.17, 0.33) & T(0.17, 0.33, 0.33, 0.5) \\ T(0.83, 1.0, 1.0, 1.0) & T(0.67, 0.83, 0.83, 1.0) & T(0.33, 0.5, 0.5, 0.67) & T(0.5, 0.67, 0.67, 0.83) \\ T(0.67, 0.83, 0.83, 1.0) & T(0.5, 0.67, 0.67, 0.83) & T(0.17, 0.33, 0.33, 0.5) & T(0.33, 0.5, 0.5, 0.67) \end{pmatrix}, \\
O_3 &= \begin{pmatrix} T(0.33, 0.5, 0.5, 0.67) & T(0.5, 0.67, 0.67, 0.83) & T(0.67, 0.83, 0.83, 1.0) & T(0.17, 0.33, 0.33, 0.5) \\ T(0.17, 0.33, 0.33, 0.5) & T(0.33, 0.5, 0.5, 0.67) & T(0.33, 0.5, 0.67, 0.83) & T(0.0, 0.17, 0.17, 0.33) \\ T(0.0, 0.17, 0.17, 0.33) & T(0.17, 0.33, 0.5, 0.67) & T(0.33, 0.5, 0.5, 0.67) & T(0.0, 0.0, 0.0, 0.17) \\ T(0.5, 0.67, 0.67, 0.83) & T(0.67, 0.83, 0.83, 1.0) & T(0.83, 1.0, 1.0, 1.0) & T(0.33, 0.5, 0.5, 0.67) \end{pmatrix}, \\
O_4 &= \begin{pmatrix} T(0.33, 0.5, 0.5, 0.67) & T(0.5, 0.67, 0.67, 0.83) & T(0.0, 0.17, 0.17, 0.33) & T(0.0, 0.17, 0.33, 0.5) \\ T(0.17, 0.33, 0.33, 0.5) & T(0.33, 0.5, 0.5, 0.67) & T(0.0, 0.0, 0.17, 0.33) & T(0.33, 0.5, 0.5, 0.67) \\ T(0.67, 0.83, 0.83, 1.0) & T(0.67, 0.83, 1.0, 1.0) & T(0.33, 0.5, 0.5, 0.67) & T(0.67, 0.83, 0.83, 1.0) \\ T(0.5, 0.67, 0.83, 1.0) & T(0.33, 0.5, 0.5, 0.67) & T(0.0, 0.17, 0.17, 0.33) & T(0.33, 0.5, 0.5, 0.67) \end{pmatrix}, \\
O_5 &= \begin{pmatrix} T(0.33, 0.5, 0.5, 0.67) & T(0.5, 0.67, 0.67, 0.83) & T(0.17, 0.33, 0.33, 0.5) & T(0.67, 0.83, 0.83, 1.0) \\ T(0.17, 0.33, 0.33, 0.5) & T(0.33, 0.5, 0.5, 0.67) & T(0.0, 0.17, 0.17, 0.33) & T(0.5, 0.67, 0.67, 0.83) \\ T(0.5, 0.67, 0.67, 0.83) & T(0.67, 0.83, 0.83, 1.0) & T(0.33, 0.5, 0.5, 0.67) & T(0.83, 1.0, 1.0, 1.0) \\ T(0.0, 0.17, 0.17, 0.33) & T(0.17, 0.33, 0.33, 0.5) & T(0.0, 0.0, 0.0, 0.17) & T(0.33, 0.5, 0.5, 0.67) \end{pmatrix}.
\end{aligned}$$

D. Resulting TrFNs

$$\begin{aligned}
T_1 &= \begin{pmatrix} T(0.33, 0.5, 0.5, 0.67) & T(0.17, 0.33, 0.33, 0.5) & T(0.0, 0.17, 0.17, 0.33) & T(0.17, 0.33, 0.5, 0.67) \\ T(0.5, 0.67, 0.67, 0.83) & T(0.33, 0.5, 0.5, 0.67) & T(0.22, 0.22, 0.67, 0.83) & T(0.47, 0.75, 0.75, 0.83) \\ T(0.67, 0.83, 0.83, 1.0) & T(0.17, 0.33, 0.78, 0.78) & T(0.33, 0.5, 0.5, 0.67) & T(0.5, 0.67, 0.83, 1.0) \\ T(0.33, 0.5, 0.67, 0.83) & T(0.17, 0.25, 0.25, 0.53) & T(0.0, 0.17, 0.33, 0.5) & T(0.33, 0.5, 0.5, 0.67) \end{pmatrix}, \\
T_2 &= \begin{pmatrix} T(0.33, 0.5, 0.5, 0.67) & T(0.17, 0.33, 0.33, 0.5) & T(0.0, 0.0, 0.0, 0.17) & T(0.0, 0.17, 0.17, 0.33) \\ T(0.5, 0.67, 0.67, 0.83) & T(0.33, 0.5, 0.5, 0.67) & T(0.0, 0.17, 0.17, 0.33) & T(0.17, 0.33, 0.33, 0.5) \\ T(0.83, 1.0, 1.0, 1.0) & T(0.67, 0.83, 0.83, 1.0) & T(0.33, 0.5, 0.5, 0.67) & T(0.5, 0.67, 0.67, 0.83) \\ T(0.67, 0.83, 0.83, 1.0) & T(0.5, 0.67, 0.67, 0.83) & T(0.17, 0.33, 0.33, 0.5) & T(0.33, 0.5, 0.5, 0.67) \end{pmatrix}, \\
T_3 &= \begin{pmatrix} T(0.33, 0.5, 0.5, 0.67) & T(0.5, 0.67, 0.67, 0.83) & T(0.17, 0.51, 0.51, 0.51) & T(0.17, 0.33, 0.33, 0.5) \\ T(0.17, 0.33, 0.33, 0.5) & T(0.33, 0.5, 0.5, 0.67) & T(0.22, 0.22, 0.67, 0.83) & T(0.04, 0.33, 0.33, 0.5) \\ T(0.49, 0.49, 0.49, 0.83) & T(0.17, 0.33, 0.78, 0.78) & T(0.33, 0.5, 0.5, 0.67) & T(0.5, 0.57, 0.57, 0.57) \\ T(0.5, 0.67, 0.67, 0.83) & T(0.5, 0.67, 0.67, 0.96) & T(0.43, 0.43, 0.43, 0.5) & T(0.33, 0.5, 0.5, 0.67) \end{pmatrix}, \\
T_4 &= \begin{pmatrix} T(0.33, 0.5, 0.5, 0.67) & T(0.5, 0.67, 0.67, 0.83) & T(0.0, 0.17, 0.17, 0.33) & T(0.0, 0.17, 0.33, 0.5) \\ T(0.17, 0.33, 0.33, 0.5) & T(0.33, 0.5, 0.5, 0.67) & T(0.0, 0.0, 0.17, 0.33) & T(0.33, 0.5, 0.5, 0.67) \\ T(0.67, 0.83, 0.83, 1.0) & T(0.67, 0.83, 1.0, 1.0) & T(0.33, 0.5, 0.5, 0.67) & T(0.67, 0.83, 0.83, 1.0) \\ T(0.5, 0.67, 0.83, 1.0) & T(0.33, 0.5, 0.5, 0.67) & T(0.0, 0.17, 0.17, 0.33) & T(0.33, 0.5, 0.5, 0.67) \end{pmatrix}, \\
T_5 &= \begin{pmatrix} T(0.33, 0.5, 0.5, 0.67) & T(0.5, 0.67, 0.67, 0.83) & T(0.17, 0.33, 0.33, 0.5) & T(0.5, 0.5, 0.5, 0.67) \\ T(0.17, 0.33, 0.33, 0.5) & T(0.33, 0.5, 0.5, 0.67) & T(0.0, 0.17, 0.17, 0.33) & T(0.5, 0.67, 0.67, 0.83) \\ T(0.5, 0.67, 0.67, 0.83) & T(0.67, 0.83, 0.83, 1.0) & T(0.33, 0.5, 0.5, 0.67) & T(0.83, 0.99, 0.99, 1.0) \\ T(0.33, 0.5, 0.5, 0.5) & T(0.17, 0.33, 0.33, 0.5) & T(0.0, 0.01, 0.01, 0.17) & T(0.33, 0.5, 0.5, 0.67) \end{pmatrix}, \\
\bar{T} &= \begin{pmatrix} T(0.33, 0.5, 0.5, 0.67) & T(0.37, 0.54, 0.54, 0.7) & T(0.07, 0.24, 0.24, 0.37) & T(0.17, 0.3, 0.37, 0.53) \\ T(0.3, 0.46, 0.46, 0.63) & T(0.33, 0.5, 0.5, 0.67) & T(0.09, 0.16, 0.37, 0.53) & T(0.3, 0.52, 0.52, 0.67) \\ T(0.63, 0.76, 0.76, 0.93) & T(0.47, 0.63, 0.84, 0.91) & T(0.33, 0.5, 0.5, 0.67) & T(0.6, 0.74, 0.78, 0.88) \\ T(0.47, 0.63, 0.7, 0.83) & T(0.33, 0.48, 0.48, 0.7) & T(0.12, 0.22, 0.26, 0.4) & T(0.33, 0.5, 0.5, 0.67) \end{pmatrix}.
\end{aligned}$$

E. Resulting EPRs

$$\begin{aligned}
T_1 &= \begin{pmatrix} (E, 0)_0 & (SW, 0)_0 & (W, 0.02)_0 & [(SW, -0.02), (E, 0)]_{(0,0)} \\ (SB, 0)_0 & (E, 0)_0 & [(W, 0.34), (SB, 0.02)]_{(0.17, -0.01)} & [(SB, 0.48), (SB, 0.48)]_{(-0.1, -0.1)} \\ (B, -0.02)_0 & [(SW, -0.02), (B, -0.34)]_{(0.01, -0.17)} & (E, 0)_0 & [(SB, 0.02), (B, -0.02)]_{(0,0)} \\ [(E, 0), (SB, 0.02)]_{(0, -0.01)} & [(SW, -0.48), (SW, -0.48)]_{(0.08, 0.1)} & [(W, 0.02), (SW, -0.02)]_{(0,0)} & (E, 0)_0 \end{pmatrix}, \\
T_2 &= \begin{pmatrix} (E, 0)_0 & (SW, 0)_0 & (MW, 0)_0 & (W, 0.02)_0 \\ (SB, 0)_0 & (E, 0)_0 & (W, 0.02)_0 & (E, -0.02)_0 \\ (MB, 0)_0 & (B, -0.02)_0 & (E, 0)_0 & (SB, 0.02)_0 \\ (B, -0.02)_0 & (SB, 0.02)_0 & (E, -0.02)_0 & (E, 0)_0 \end{pmatrix}, \\
T_3 &= \begin{pmatrix} (E, 0)_0 & (SB, 0.02)_0 & (E, 0.08)_0 & (E, -0.02)_0 \\ (E, -0.02)_0 & (E, 0)_0 & [(W, 0.34), (SB, 0.02)]_{(0.17, -0.01)} & [(E, -0.02), (E, -0.02)]_{(-0.1, 0)} \\ (E, -0.08)_0 & [(E, -0.02), (B, -0.34)]_{(0.01, -0.17)} & (E, 0)_0 & (E, 0.4)_0 \\ (SB, 0.02)_0 & [(SB, 0.02), (SB, 0.02)]_{(0, 0.12)} & (E, -0.4)_0 & (E, 0)_0 \end{pmatrix}, \\
T_4 &= \begin{pmatrix} (E, 0)_0 & (SB, 0.02)_0 & (W, 0.02)_0 & [(W, 0.02), (E, -0.02)]_{(0,0)} \\ (E, -0.02)_0 & (E, 0)_0 & [(MW, 0), (W, 0.02)]_0 & (E, 0)_0 \\ (B, -0.02)_0 & [(B, -0.02), (MB, 0)]_0 & (E, 0)_0 & (B, -0.02)_0 \\ [(SB, 0.02), (B, -0.02)]_{(0,0)} & (E, 0)_0 & (W, 0.02)_0 & (E, 0)_0 \end{pmatrix}, \\
T_5 &= \begin{pmatrix} (E, 0)_0 & (SB, 0.02)_0 & (E, -0.02)_0 & [(E, -0.01), (E, 0)]_{(0.2, 0)} \\ (E, -0.02)_0 & (E, 0)_0 & (W, 0.02)_0 & (SB, 0.02)_0 \\ (SB, 0.02)_0 & (B, -0.02)_0 & (E, 0)_0 & (MB, -0.09)_0 \\ [(E, 0), (E, 0.01)]_{(0, -0.17)} & (E, -0.02)_0 & (MW, 0.09)_0 & (E, 0)_0 \end{pmatrix}, \\
\bar{T} &= \begin{pmatrix} (E, 0)_0 & (E, 0.21)_0 & (W, 0.42)_0 & [(E, -0.2), (E, 0.2)]_{(0,0)} \\ (E, -0.21)_0 & (E, 0)_0 & [(W, -0.06), (E, 0.22)]_{(0.09, -0.01)} & (E, 0.09)_0 \\ (B, -0.42)_0 & [(SB, -0.22), (B, 0.06)]_{(0.01, -0.09)} & (E, 0)_0 & [(SB, 0.47), (B, -0.34)]_{(0, -0.1)} \\ [(SB, -0.2), (SB, 0.2)]_{(0, -0.03)} & (E, -0.09)_0 & [(W, 0.34), (E, -0.47)]_{(0.06, -0.02)} & (E, 0)_0 \end{pmatrix}.
\end{aligned}$$