

A Micro-Extended Belief Rule-Based System for Big Data Multiclass Classification Problems

Long-Hao Yang^{ID}, Jun Liu, *Member, IEEE*, Ying-Ming Wang, and Luis Martínez^{ID}, *Member, IEEE*

Abstract—Big data classification problems have drawn great attention from diverse fields, and many classifiers have been developed. Among those classifiers, the extended belief rule-based system (EBRBS) has shown its potential in both big data and multiclass situations, while the time complexity and computing efficiency are two challenging issues to be handled in EBRBS. As such, three improvements of EBRBS are proposed first in this paper to decrease the time complexity and computing efficiency of EBRBS for multiclass classification under the assumption of large amount of data, including the strategy to skip rule weight calculation, a simplified evidential reasoning algorithm, and the domain division-based rule reduction method. This turns out to be a micro version of the EBRBS, called Micro-EBRBS. Moreover, one of commonly used cluster computing, named *Apache Spark*, is then applied to implement the parallel rule generation and inference schemes of the Micro-EBRBS for big data multiclass classification problems. The comparative analyses of experimental studies demonstrate that the Micro-EBRBS not only can obtain a desired accuracy but also has the comparatively better time complexity and computing efficiency than some popular classifiers, especially for multiclass classification problems.

Index Terms—Apache spark, big data, extended belief rule-based system (EBRBS), multiclass.

I. INTRODUCTION

CLASSIFICATION problems are the common and fundamental ones involved in various real-world applications, such as intrusion detection [1], pattern recognition [19], image processing [2], and DNA sequence classification [5]. Classification becomes much more complex under big data

and multiclass situations. The former always implies a high requirement of the computing efficiency of classifiers, while the need of a cluster computing for implementing the classifiers is getting popular. The latter implies many overlaps among the data of different classes, which requires the classifiers to have a powerful ability to differentiate the class boundaries.

Among the many methodologies for multiclass classification problems, such as the support vector machine (SVM) [34], ensemble learning [21], and others, rule-based systems (RBSs) are a useful tool that has been a popular framework for designing classifiers over the past few decades. Basically, these RBSs can be divided into two categories depending on the construction methods of rule bases: RBSs based on the optimization model and iterative algorithm to determine the optimal values of parameters involved [33], [36], [39] or RBSs where rules are generated from sample data without the optimal model [8], [22], [28]. While handling big data multiclass classification problems, it is obvious that the RBSs without the optimal model are a better choice because of its high computing efficiency [26].

Some popular RBSs without the optimal model include fuzzy rule-based classification system (FRBCS) based on Chi *et al.* algorithm (Chi-FRBCS) [8] and the extended belief rule-based system (EBRBS) proposed by Liu *et al.* [22], both of them are originated from the work by Wang and Mendel [31]. It is worth noting that Chi-FRBCS has been applied to deal with big data classification problems recently (see details in Section II-B). Although there are some existing works about EBRBS [6], [11], [38], [39], it has yet to be developed as an efficient classifier to deal with big data classification problems. Hence, the goal of this paper is to propose a novel big data EBRBS classifier and show its performance, in terms of accuracy and efficiency, in big data multiclass classification problems.

EBRBS has shown its potential to address multiclass classification problems because of its rules with belief structure and the evidential reasoning (ER) algorithm used in the inference scheme to collectively handle multiclass information for classification [20], [23]–[25]; however, two challenges must be addressed to handle the big data situation.

- 1) The time complexity of the EBRBS must be reduced to ensure the high computing efficiency under big data problems. The proposed solution in this paper is to optimize the procedures of the EBRBS so that its time complexity can be reduced, which are mainly focused on three key procedures of the EBRBS: a) rule weight

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L.-H. Yang is with the Decision Sciences Institute, Fuzhou University, Fuzhou 350108, China, and also with the Department of Computer Science, University of Jaén, 23008 Jaén, Spain (e-mail: more026@hotmail.com).

J. Liu is with the School of Computing, Ulster University at Jordanstown Campus, Newtownabbey BT37 0QB, U.K. (e-mail: j.liu@ulster.ac.uk).

Y.-M. Wang is with the Decision Sciences Institute, Fuzhou University, Fuzhou 350108, China, and also with the Key Laboratory of Spatial Data Mining and Information Sharing of Ministry of Education, Fuzhou University, Fuzhou 350108, China (e-mail: msymwang@hotmail.com).

L. Martínez is with the Department of Computer Science, University of Jaén, 23008 Jaén, Spain (e-mail: martin@ujaen.es).

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calculation; b) the ER algorithm; and c) the size of rule base. The corresponding improvements are proposed, respectively, which forms a micro version of the EBRBS with much higher computing efficiency enables to deal with big data multiclass classification problems, called the Micro-EBRBS.

- 2) The cluster computing must be applied to implement the parallel computing to improve the computing efficiency.

Chi-FRBCS classifiers usually involve the cluster computing to deal with big data classification problems [10], [26], [29]. Likewise, it is necessary to propose a solution of implementing the EBRBS classifier by using the cluster computing to handle the big data problem. Apache Spark [4] is an open-source framework that supports the processing of large datasets in a distributed computing environment and provides primitives for in-memory cluster computing and APIs in Scala, Java, and Python. As such, Apache Spark-based implementation of the parallel rule generation and inference scheme is proposed to improve the computing efficiency of Micro-EBRBS in big data situation.

To verify the effectiveness and computing efficiency of the Micro-EBRBS, three experiments based on 14 classification datasets, in which four of these datasets have relatively large number of data, are carried out to test the performance of the Micro-EBRBS. Two main aspects, namely accuracy and computing time, are used to compare the Micro-EBRBS with the EBRBS, the conventional FRBCS and machine-learning classifiers, and the big data FRBCS classifiers.

The remainder of this paper is organized as follows. Section II briefly reviews the background and challenges of the EBRBS for classification. Section III introduces the Micro-EBRBS for big data multiclassification problems. Section IV discusses experiments to demonstrate the performance of the Micro-EBRBS, and this paper is concluded in Section V.

II. BACKGROUND AND CHALLENGES

In this section, the EBRBS for classification problems is reviewed first to provide the basic knowledge of this paper. Second, the related works of the Chi-FRBCS for big data problems are reviewed for the sake of comparison with the proposed Micro-EBRBS. Finally, the time complexity of the EBRBS is discussed to clarify the challenges to face for the novel classifier under big data and multiclass situation.

A. EBRBS for Classification Problems

An EBRBS consists of two components: 1) the extended belief rule base (EBRB) and 2) the inference scheme. The former can be regarded as a knowledge base to store a set of rules with uncertainty. The latter provides an inference engine to infer new results based on the interaction of test input and the EBRB.

Basically, an EBRB for classification problems includes M antecedent attributes and one consequent attribute, in which each antecedent attribute U_i ($i = 1, \dots, M$) has an attribute weight δ_i ($0 < \delta_i \leq 1$) and J_i reference values $A_{i,j}$ ($j = 1, \dots, J_i$) which are used as the discrete and representative evaluation grades for describing the i th antecedent

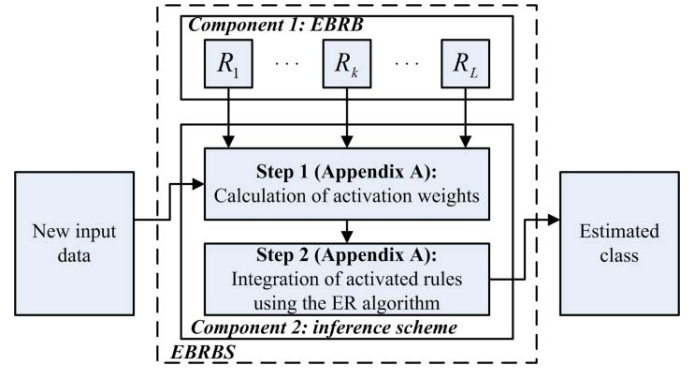


Fig. 1. Methodological framework of EBRBS for classification.

attribute; and the consequent attribute D has N classes D_n ($n = 1, \dots, N$). Thus, the k th ($k = 1, \dots, L$) extended belief rule (EBR) R_k in the EBRB can be written as

$$R_k : \text{IF } U_1 \text{ is } \{(A_{1,j}, \alpha_{1,j}^k); j = 1, \dots, J_1\} \wedge \dots \wedge U_M \text{ is } \{(A_{M,j}, \alpha_{M,j}^k); j = 1, \dots, J_M\}, \text{ THEN } D \text{ is } \{(D_n, \beta_{n,k}); n = 1, \dots, N\}, \text{ with } \theta_k \text{ and } \{\delta_1, \dots, \delta_M\} \quad (1)$$

where $\alpha_{i,j}^k$ ($0 \leq \alpha_{i,j}^k \leq 1$) and $\beta_{n,k}$ ($0 \leq \beta_{n,k} \leq 1$) are the belief degree to the reference value $A_{i,j}$ and the class D_n in the k th rule with $\sum_{j=1}^{J_i} \alpha_{i,j}^k \leq 1$ and $\sum_{n=1}^N \beta_{n,k} \leq 1$; $\{(A_{i,j}, \alpha_{i,j}^k); j = 1, \dots, J_i\}$ and $\{(D_n, \beta_{n,k}); n = 1, \dots, N\}$ are the belief structure embedding in the k th rule; θ_k ($0 < \theta_k \leq 1$) is the weight of the k th rule.

Remark 1: By assuming that: 1) $A_{i,j}$ ($i = 1, \dots, M$; $j = 1, \dots, J_i$) is a linguistic label modeled by a triangular membership function; 2) $\alpha_{i,j}^k = 1$ ($k = 1, \dots, L$) and $\alpha_{i,t}^k = 0$ ($t = 1, \dots, J_i$; $t \neq j$); 3) $\beta_{n,k} = 1$ and $\beta_{n,s} = 0$ ($s = 1, \dots, N$; $s \neq n$); and 4) $\delta_i = 1$ ($i = 1, \dots, M$), the k th EBR becomes a fuzzy rule in the Chi-FRBCS [29]

$$R_k : \text{IF } U_1 \text{ is } A_{1,j} \wedge \dots \wedge U_M \text{ is } A_{M,j} \text{ THEN } D \text{ is } D_n, \text{ with } \theta_k. \quad (2)$$

It is clear from the comparison of (1) and (2) that the fuzzy rule is a special case of the EBR. Moreover, the EBR is more flexible to express multiclass information under uncertainty and incompleteness, e.g., $\{(D_1, 0.6), (D_2, 0.4)\}$ means 60% sure that the class is D_1 , 40% sure that it is D_2 . $\sum_{j=1}^{J_i} \alpha_{i,j}^k = 0.9$ means $100\% - 90\% = 10\%$ ignorance in the i th antecedent attribute of the k th EBR.

Based on the above EBRB, the ER algorithm-based inference scheme is applied to integrate EBRs to produce estimated classes, i.e., the integrated result is belief distribution $\{(D_1, 0.4008), (D_2, 0.4275), (D_3, 0.1718)\}$ and finally produces the class D_2 as the output. A simple methodological framework of the EBRBS for classification is shown in Fig. 1 and the detailed step procedure can be referred to [22]. Additionally, the rule generation scheme is an indispensable part of the EBRBS and illustrated in Fig. 2.

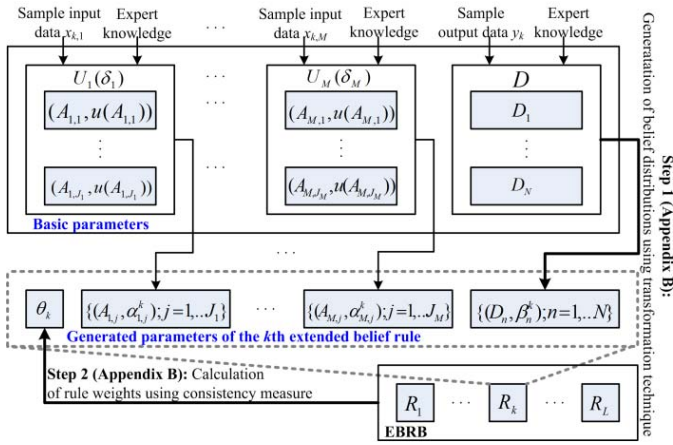


Fig. 2. Rule generation scheme of EBRBS.

As shown in Fig. 2, there are two kinds of parameters involved in the rule generation scheme of the EBRB. The first one is named as the basic parameters, including attribute weights, reference values, and utility values of antecedent attributes, and classes of the consequent attribute. All these basic parameters are always determined by using expert knowledge. The second one is the generated parameters, including rule weights and belief distributions of antecedent and consequent attributes. All these generated parameters have to be initialized according to the sample input-output data and the basic parameters, including two steps: 1) generation of belief distributions for antecedent and consequent attributes using transformation techniques and 2) calculation of rule weights using consistency measures. The detailed description of those steps can be found in Appendix B.

B. Chi-FRBCS in Big Data Classification Problems

FRBCSs are popular methods for classification problems with many versions developed so far, e.g., Chi-FRBCS [8], structural learning algorithm on vague environment (SLAVE) [15], fuzzy hybrid genetic-based machine learning algorithm (FH-GBML) [18], fuzzy unordered rule induction algorithm (FURIA) [16], and fuzzy association rule-based classification method for high-dimensional problems (FARC-HD) [3]. However, considering the limitations of standard fuzzy rule base learning approaches for large number of samples, the Chi-FRBCS was recognized by many researchers as the suitable FRBCS to handle big data classification problems [26], [29].

In the last few years, several big data classifiers based on the Chi-FRBCS have been proposed and made use of the Apache Hadoop to deploy the distributed system. For example, López *et al.* [26] proposed the first FRBCS capable of addressing big data and imbalance datasets, called *Chi-FRBCS-BigDataCS*, which utilized the Apache Hadoop to distribute the computational operations of Chi-FRBCS and also included cost-sensitive learning techniques to address imbalanced big data. After that, Rio *et al.* [29] developed a more general big data classifier based on the Chi-FRBCS and Apache Hadoop, called *Chi-FRBCS-BigData*, which includes

two versions: 1) Chi-FRBCS-BigData-Max (Chi-Max) and 2) Chi-FRBCS-BigData-Ave (Chi-Ave). Both show the ability to deal with big data problems providing competitive results and reasonable computing efficiencies. Fernández *et al.* [12] studied the relationship between the granularity and data scattering for Chi-FRBCS in big data classification problems and made use of Chi-FRBCS-BigData to accomplish their analysis. Later on, Fernández *et al.* [13] carried out many experimental studies regarding the use of Chi-FRBCS-BigData to analyze the differences in performance with respect to the lack of data for the learning stage, how rules are distributed among Maps, and their influences on the classification stage. Elkano *et al.* [10] proposed a global version of Chi-FRBCS-BigDataCS in order to address the problem of previous big data classifiers that it would become less accurate when more computing nodes are added into the cluster.

The above literatures review shows many potential applications of Chi-FRBCS in big data classification problems. However, they were mainly focused on two-class classification problems. Although the decomposition strategies [27], such as one-versus-one (OVO) and one-versus-all schemes, can be used to decompose a multiclass problem into multiple two-class problems, it is unavoidable to cause the increase of the time complexity. Considering that the EBRBS has an effective rule representation scheme better than the one in Chi-FRBCS owing to embedding the belief structure into both antecedent and consequent attributes, the present work aims to propose a novel classifier based on the EBRBS for big data problems in comparison with Chi-FRBCS in terms of accuracy and efficiency.

C. Challenges of EBRBS for Big Data Problems

Due to the importance of computing efficiency in big data problems, in the context of the EBRBS, the time complexity of the rule generation and inference scheme are analyzed in this section. For the discussion purposes, Figs. 3 and 4 provide the pseudocode and the time complexity of those two schemes involved in the EBRBS, respectively.

From Fig. 3, the time complexity of generating belief distributions shown in the first line to the eighth line is $O(T \times (\sum_{i=1, \dots, M} J_i + N))$ and the time complexity of calculating rule weights shown in the 9th line to the 14th line is $O(T^2 \times (\sum_{i=1, \dots, M} J_i + N))$, where T is the number of sample data, M is the number of antecedent attributes, J_i is the number of reference values in the i th antecedent attribute, and N is the number of classes. Clearly, for the rule generation scheme of the EBRBS, the calculation of rule weights requires the most computing time and sometimes it would be unacceptable while there are large amounts of sample data. For example, the dataset Poker has 1 025 010 samples (i.e., $T = 1\,025\,010$), ten attributes (i.e., $M = 10$), and ten classes (i.e., $N = 10$). If the number of reference values is assumed to be 3 for each attribute (i.e., $J_i = 3$; $i = 1, \dots, M$) and the computing time of each operation is 10^{-6} s, then the total computing time of generating rule weights is 11673.8 h.

From Fig. 4, the time complexity of the inference scheme is $O(S \times L \times (\sum_{i=1, \dots, M} J_i + N))$, where S is the number of test

Pseudocode of the rule generation scheme involved in the EBRBS		
01 Initialize $EBRB = \{\}$;		
02 For each training data x_i in $\{x_1, \dots, x_T\}$		
03 For each input data x_{ij} in $x_i = \{x_{i1}, \dots, x_{iM}\}$		
04 Generate $S(x_{ij}) = \{(A_{ij}, \alpha'_{ij}); j = 1, \dots, J_i\}$ shown in Eq. (16)	$O(\sum_{i=1}^M J_i)$	$O(T \cdot (\sum_{i=1}^M J_i + N))$
05 End for		
06 Generate $S(y_i) = \{(D_n, \beta'_n); n = 1, \dots, N\}$ shown in Eq. (19)	$O(N)$	
07 Update $EBRB = EBRB \cup \{S(x_{ij}), \dots, S(x_{iM}), S(y_i)\} >$;		
08 End for		
09 For each extended belief rule R_k in $EBRB$		
10 Calculate $SRA(l, k)$ ($l = 1, \dots, T; l \neq k$) shown in Eq. (B2)	$O(T \cdot \sum_{i=1}^M J_i)$	$O(T^2 \cdot (\sum_{i=1}^M J_i + N))$
11 Calculate $SRC(l, k)$ ($l = 1, \dots, T; l \neq k$) shown in Eq. (B3)	$O(T \cdot N)$	
12 Calculate $Incons(R_k)$ shown in Eq. (B5)	$O(T)$	
13 Calculate θ_k shown in Eq. (B6)		
14 End for		

Fig. 3. Pseudocode and time complexity of the rule generation scheme.

Pseudocode of the inference scheme involved in the EBRBS		
01 For each testing data x_i in $\{x_1, \dots, x_S\}$		
02 For each input data x_{ij} in $x_i = \{x_{i1}, \dots, x_{iM}\}$		
03 Calculate $S(x_{ij}) = \{(A_{ij}, \alpha'_{ij}); j = 1, \dots, J_i\}$ shown in Eq. (21)	$O(\sum_{i=1}^M J_i)$	$O(L \cdot \sum_{i=1}^M J_i)$
04 End for		
05 For each extended belief rule R_k in $\{R_1, \dots, R_L\}$		
06 For each antecedent attribute U_j in $\{U_1, \dots, U_M\}$		
07 Calculate $S^k(x_{ij}, U_j)$ shown in Eq. (22)	$O(L \cdot \sum_{i=1}^M J_i)$	$O(S \cdot L \cdot (\sum_{i=1}^M J_i + N))$
08 End for		
09 Calculate w_k shown in Eq. (23)		
10 End for		
11 Calculate β_n ($n = 1, \dots, N$) shown in Eq. (A1)	$O(L \cdot N)$	$O(N)$
12 Estimate $f(x)$ shown in Eq. (A3)		
13 End for		

Fig. 4. Pseudocode and time complexity of the inference scheme.

data, L is the number of EBRs in the EBRB. Considering that one EBR is directly transformed from one sample data [22], so $T = L$. Therefore, the time complexity of the inference scheme can be expressed as $O(S \times T \times (\sum_{i=1}^M J_i + N))$. Obviously, the computing time would also be unacceptable while there is large amount of sample data involved in the rule generation scheme, i.e., while the tenfold cross validation is utilized to test the dataset Poker, the number of sample data and test data is therefore 922 509 and 102 501, respectively. Finally, the computing time of the inference scheme is 1050.6 h.

The above discussions clearly show that although the EBRBS is the RBS without the optimal model, the time-consuming process found in the calculation of rule weights and the inference scheme would be serious challenges while the EBRBS is applied to address the classification problem with a large amount of data. Therefore, this paper aims to address these challenges.

III. NOVEL EBRBS FOR BIG DATA MULTICLASS CLASSIFICATION PROBLEMS

According to the challenges pointed out in Section II-C, the possible approaches are investigated to reduce the time complexity of the rule generation and the inference schemes,

followed by a new rule reduction method to downsize the EBRB. Based on these achievements, a novel EBRBS and its Apache Spark-based implementation are developed to deal with big data multiclass classification problems.

A. Analysis of Rule Weight Calculation and the ER Algorithm Involved in EBRBS

In order to reduce the time complexity of the EBRBS, the properties of the rule weight calculation and the ER algorithm are investigated as follows.

Theorem 1: The rule weight of each EBR will approximate to 1 while using large number of sample data to generate EBRs.

Proof: Suppose there are L EBRs. Based on the calculation of rule weights in (36) in Appendix B, we can get the first order partial derivative of the rule weight θ_k ($k = 1, \dots, L$) with respect to the inconsistency degree $Incons(R_k)$ as follows:

$$\begin{aligned} \frac{\partial \theta_k}{\partial Incons(R_k)} &= -\frac{\sum_{l=1}^L Incons(R_l) - Incons(R_k)}{\left[Incons(R_k) + \sum_{l=1, l \neq k}^L Incons(R_l) \right]^2} \\ &= -\frac{\sum_{l=1, l \neq k}^L Incons(R_l)}{\left[Incons(R_k) + \sum_{l=1, l \neq k}^L Incons(R_l) \right]^2} < 0. \end{aligned} \quad (3)$$

It follows that θ_k decreases while its $Incons(R_k)$ increases. In other words, when $Incons(R_k)$ is equal to 1, R_k has a minimum rule weight. This makes sense that if this rule causes the contradiction, then it will be useless.

Apparently, when the number of rules is increasing, the inconsistency degree of each rule is also increasing. It follows that when L is approaching to ∞ , the inconsistency degree of each rule is approaching to 1. Without loss of generality, we consider the rule weight of R_k as follows:

$$\theta_k = \lim_{L \rightarrow +\infty} \left[1 - \frac{Incons(R_k)}{\sum_{l=1}^L Incons(R_l)} \right] = 1 - \lim_{L \rightarrow +\infty} \frac{1}{L} = 1. \quad (4)$$

Now that L is equal to the number of sample data, this concludes the proof. ■

Example 1: Suppose there are L EBRs and two different EBRs, namely the k th rule and the i th rule ($k, i \in \{1, \dots, L\}; i \neq k$) with their inconsistency degree $Incons(R_k)$ and $Incons(R_i)$, respectively. The relationship between rule weights and inconsistency degrees can be shown in Fig. 5: while $Incons(R_i)$ is fixed, θ_k increases with the decrease of $Incons(R_k)$. Moreover, while $Incons(R_k) = 1$, the k th rule has a minimum rule weight.

Second, assume that $Incons(R_k) = 1$ ($k \in \{1, \dots, L\}$) and the inconsistency degree of other rules are set by using random values. Then the relationship between θ_k and L (from 1 to 1000) is shown in Fig. 6, in which each line (20 lines in total) denotes an independent experiment of using random values to determine θ_i ($i = 1, \dots, L; i \neq k$). From Fig. 6, $Incons(R_k) = 1$ leads to the minimum rule weight for the k th rule. However, with the increase of L , it turned out that θ_k approximates to 1.

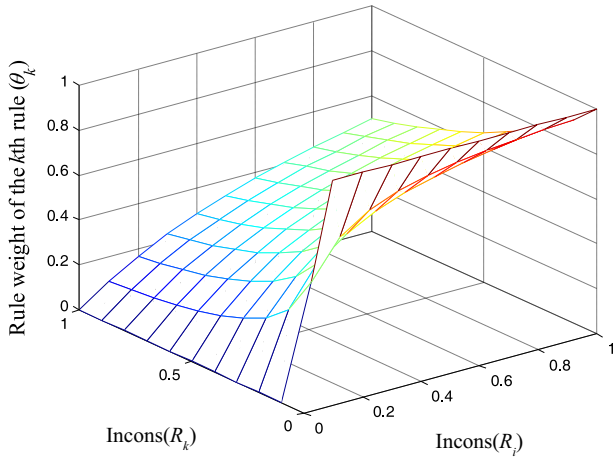
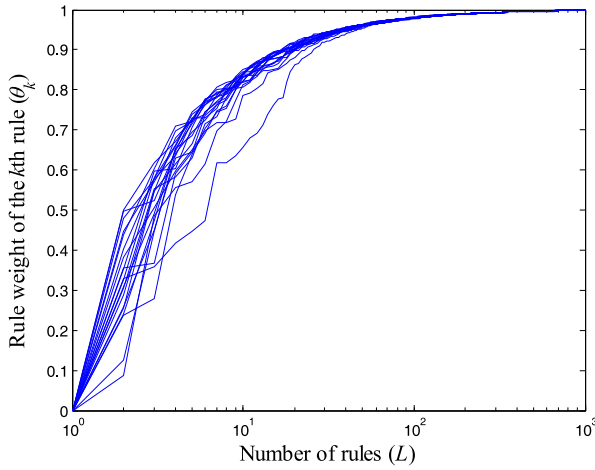


Fig. 5. Relationship between the rule weights and their inconsistency degrees.

Fig. 6. Relationship between the rule weights of R_k and the number of rules.

Remark 2: From Theorem 1, it is unnecessary to calculate each rule weight because its value will approximate to 1 while a large number of data are used to generate EBRs. As a result, the time complexity of the rule generation scheme is reduced from $O(T^2 \times (\sum_{i=1, \dots, M} J_i + N))$ to $O(T \times (\sum_{i=1, \dots, M} J_i + N))$.

Theorem 2: For classification problems, the following ER algorithm for classification (denoted as ER-C) is the same as the analytical ER algorithm shown in (28) in Appendix A:

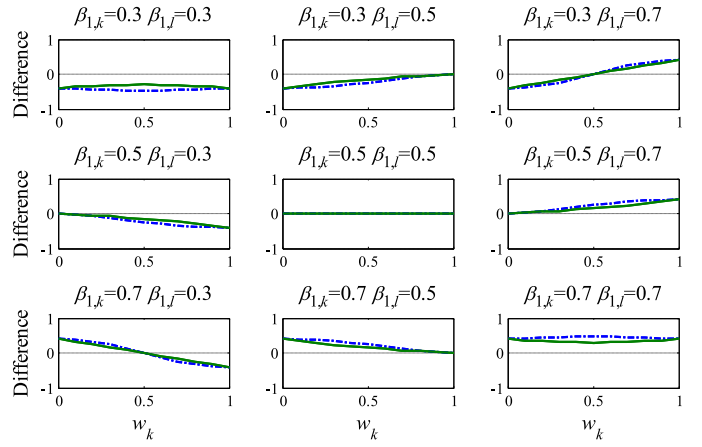
$$\beta_n^{\text{ER-C}} = \prod_{k=1}^L \left(w_k \beta_{n,k} + 1 - w_k \sum_{i=1}^N \beta_{i,k} \right). \quad (5)$$

Proof: Suppose a classification problem has N classes and the n th ($n = 1, \dots, N$) class is denoted as D_n . Assume the estimated class of the EBRBS is the n th class for the test input data \mathbf{x} . Hence, according to (30) in Appendix A, we have

$$\beta_n > \beta_t; \quad t = 1, \dots, N; t \neq n. \quad (6)$$

Based on the analytical ER algorithm in (28), we assume

$$\chi^1 = \prod_{k=1}^L \left(1 - w_k \sum_{i=1}^N \beta_{i,k} \right) \quad (7)$$

Fig. 7. Comparison of the ER-C and ER algorithms (the full line denotes $d^{\text{ER-C}}$ and the dotted line denotes d^{ER}).

$$\chi^2 = \prod_{k=1}^L (1 - w_k). \quad (8)$$

Hence, we can obtain

$$\begin{aligned} \beta_n &= \frac{\beta_n^{\text{ER-C}} - \chi^1}{\sum_{i=1}^N \beta_i^{\text{ER-C}} - (N-1)\chi^1 - \chi^2} \\ &> \frac{\beta_t^{\text{ER-C}} - \chi^1}{\sum_{i=1}^N \beta_i^{\text{ER-C}} - (N-1)\chi^1 - \chi^2} = \beta_t \\ &\Leftrightarrow \beta_n^{\text{ER-C}} > \beta_t^{\text{ER-C}}; \quad t = 1, \dots, N; t \neq n. \end{aligned} \quad (9)$$

It follows that the ER-C algorithm is the same as the analytical ER algorithm in EBRBS for classification. ■

Example 2: Suppose there are two EBRs and their belief distributions of the consequent attribute are shown as follows:

$$R_k : \{(D_n, \beta_{n,k}); n = 1, 2\} \text{ with } \sum_{n=1}^2 \beta_{n,k} = 1 \quad (10)$$

$$R_l : \{(D_n, \beta_{n,l}); n = 1, 2\} \text{ with } \sum_{n=1}^2 \beta_{n,l} = 1. \quad (11)$$

The activation weight of the rule R_k and R_l are assumed to be w_k and w_l , respectively, and $w_k + w_l = 1$.

According to the analytical ER algorithm in (28) and the ER-C algorithm in (5), the difference of the integrated belief degrees for the D_1 and D_2 can be expressed as follows:

$$d^{\text{ER}} = \beta_1 - \beta_2 \quad (12)$$

$$d^{\text{ER-C}} = \beta_1^{\text{ER-C}} - \beta_2^{\text{ER-C}}. \quad (13)$$

To illustrate the possible values of d^{ER} and $d^{\text{ER-C}}$, we consider nine illustrative cases under the assumptions that $\beta_{1,l} > \beta_{1,k}$, $\beta_{1,l} = \beta_{1,k}$, and $\beta_{1,l} < \beta_{1,k}$ while the value of w_k lies between 0 and 1. Without loss of generality, the value of both $\beta_{1,l}$ and $\beta_{1,k}$ is assumed as 0.3, 0.5, and 0.7, respectively. Hence, the curves of d^{ER} and $d^{\text{ER-C}}$ are shown in Fig. 7. Fig. 7 shows that d^{ER} and $d^{\text{ER-C}}$ have same negative and positive symbols for nine combinations based on different belief degrees and activation weights. Hence, the ER-C algorithm can produce the same estimated class as the ER algorithm.

Remark 3: From Theorem 2, the ER-C algorithm can be used to replace the ER algorithm in the inference scheme of the EBRBS while facing classification problems. Additionally, the existing studies of using the ER algorithm as inference engine for classification problems, such as [6] and [7], can also use the ER-C algorithm to replace the inference engine.

Remark 4: The advantages of the ER-C algorithm over the ER algorithm can be summarized as follows.

- 1) The ER-C algorithm has a much clean and simple formula than the ER algorithm because it derives from the core part of the ER algorithm.
- 2) The ER-C algorithm is more efficient than the ER algorithm according to their time complexity, in which the ER-C algorithm is $O(L \times N)$ and the ER algorithm is $O(L \times N^2)$.
- 3) In term of independence, the calculation of the integrated belief degree for all classes is independent of each other in the ER-C algorithm so that it is possible to have more solutions of parallelization for the EBRBS.

B. Domain Division-Based Rule Reduction Method for EBRBS

In order to further reduce the time complexity of the EBRBS, in this section, a domain division-based rule reduction method is proposed for the rule generation scheme. First, the main idea, which follows the similar way to the fuzzy partition [17] and the Wang–Mendel model [31], of the proposed rule reduction method are given based on the following definitions.

Definition 1 (Division Point): The division point is the intersection between transform functions used to calculate the belief degree to which the input data belongs to the reference value of antecedent attributes. For convenience, $P(A_{i,j}, A_{i,j+1})$ ($i = 1, \dots, M; j = 1, \dots, J_i - 1$) is defined to express the division point between the $A_{i,j}$ and $A_{i,j+1}$ in the i th antecedent attribute.

Definition 2 (Division Domain): The division domain is the local input space constructed by the two adjacent division points of each antecedent attribute. For convenience, $D(A_{1,j_1}, \dots, A_{M,j_M})$ ($j_i = 1, \dots, J_i; i = 1, \dots, M$) is defined to express the division domain constructed by the division point regarding the reference values $A_{1,j_1}, \dots, A_{M,j_M}$.

Example 3: Suppose that an EBRB includes one antecedent attribute U_1 with three reference values $A_{1,i}$ and their utility values $u(A_{1,i})$ ($i = 1, 2, 3$). Without loss of generality, the order of those utility values is $u(A_{1,1}) < u(A_{1,2}) < u(A_{1,3})$. x_1 is assumed to be the input variable of the attribute U_1 and the utility function used to generate belief degrees of the reference value $A_{1,i}$ ($i = 1, 2, 3$), is assumed to be piecewise linear [35], namely $\alpha_{1,i}(x_1)$, shown in Fig. 8. From Fig. 8, there are two division points, namely $P(A_{1,1}, A_{1,2})$ and $P(A_{1,2}, A_{1,3})$, and three division domains, namely $D(A_{1,1})$, $D(A_{1,2})$, and $D(A_{1,3})$.

Remark 5: Based on Definitions 1 and 2 together with Example 3, when an input data falls into a division domain, the relatively maximal belief degree can be generated from the input data for all reference values belonging to the division domain. In other words, each division domain can be

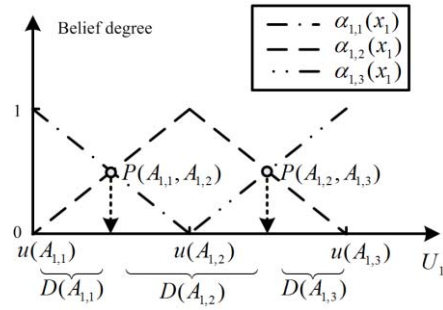


Fig. 8. Example of division point and division domain.

regarded as the clustering center of the input data which most likely belong to the reference value of division domain in the form of belief distribution.

Definition 3 (Rule Clustering Strategy): The rule clustering strategy is the map relationship between EBRs and division domains based on the relatively maximal belief degree in each antecedent attribute so that it can be defined as

$$R_k \rightarrow D(A_{1,j_1}, \dots, A_{M,j_M}) \quad (14)$$

where $j_i = \arg \max_{j=1, \dots, J_i} \{\alpha_{i,j}^k\}$ ($k = 1, \dots, L; i = 1, \dots, M$); $\alpha_{i,j}^k$ denotes the belief degree of the reference value $A_{i,j}$ in the k th EBR ($j = 1, \dots, J_i; i = 1, \dots, M$), M is the number of antecedent attributes, L is the number of EBRs, and J_i is the number of reference values used for the i th antecedent attribute.

Definition 4 (Rule Reduction Strategy): The rule reduction strategy is the combination strategy for the EBRs which are assigned to the same division domain so that it can be defined as

$$\bar{\alpha}_{i,j}^l = \frac{\sum_{k=1}^{L_l} \alpha_{i,j}^k}{L_l}, \bar{\beta}_{n,l} = \frac{\sum_{k=1}^{L_l} \beta_{n,k}}{L_l}; \text{ while } L_l > 0 \quad (15)$$

where L_l is the number of the rules gathered at the l th ($l = 1, \dots, \prod_{i=1}^M J_i$) division domain, and $\beta_{n,k}$ ($n = 1, \dots, N; k = 1, \dots, L$) denotes the belief degrees of the reference value $A_{i,j}$ and D_n in the k th rule, respectively.

Remark 6: As shown in (15), different denominators are used to calculate the belief degree of antecedent and consequent attributes. The main reason is that 1) the belief degree of antecedent attribute reflects the space relationship between different division domains in a sense, and therefore, the belief degree is calculated by using L_l , and 2) the belief degree of consequent attribute reflects how many rules have the same class; thus, the belief degree is calculated by using L .

Example 4: Suppose that an EBRB includes two antecedent attributes U_1 and U_2 with three reference values $A_{i,j}$ and their utility values $u(A_{i,j})$ ($i = 1, 2; j = 1, 2, 3$). Without loss of generality, the order of those utility values is $u(A_{1,1}) < u(A_{1,2}) < u(A_{1,3})$ in the i th antecedent attribute. In addition, the EBRB has three EBRs R_1 , R_2 , and R_3 , respectively, as shown in Fig. 9.

From Fig. 9, four division points and nine division domains can be constructed based on Definitions 1 and 2. After that, according to Definition 3, all three EBRs are assigned to the division domain $D(A_{1,2}, A_{2,2})$. Afterward,

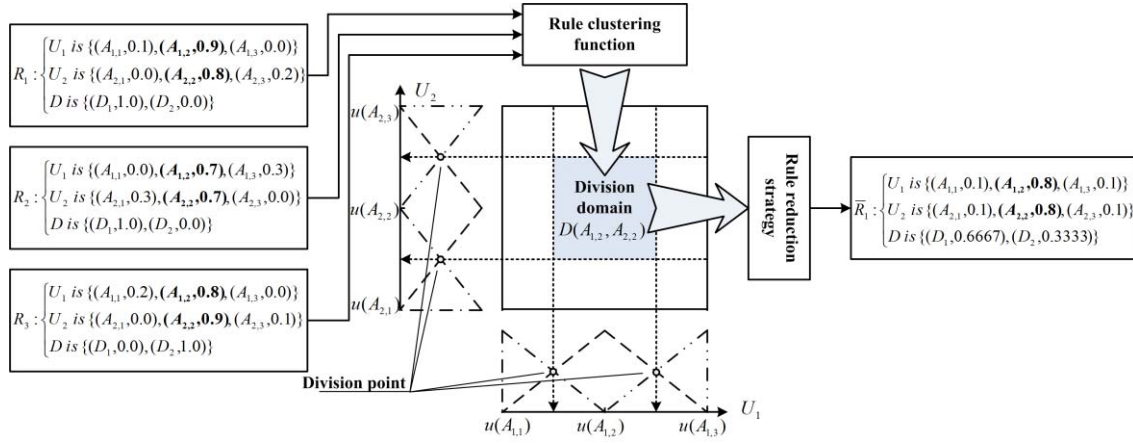


Fig. 9. Examples of rule clustering strategy and rule reduction strategy.

based on Definition 4, the EBRs gathered in $D(A_{1,2}, A_{2,2})$ are all combined as a new rule, denoted as \bar{R}_1 , which is obtained via (15), e.g., $\bar{\alpha}_{1,1}^1 = (0.1 + 0 + 0.2)/3 = 0.1$, $\bar{\alpha}_{1,2}^1 = (0.9 + 0.7 + 0.8)/3 = 0.8$, and $\bar{\alpha}_{1,3}^1 = (0 + 0.3 + 0)/3 = 0.1$ in the antecedent attribute U_1 , and $\bar{\beta}_{1,1} = (1 + 1 + 0)/3 = 0.6667$ and $\bar{\beta}_{2,1} = (0 + 0 + 1)/3 = 0.3333$ in the attribute D .

Remark 7: Based on Definitions 3 and 4 together with Example 4, when one more EBR is assigned to the same division domain, a new EBR would be generated, in which the information of the new EBR is considered complete because it is generated by using the belief distribution of the antecedent and consequent attributes of all original rules.

Based on the above Definitions 1–4 and Examples 3 and 4, the steps of the domain division-based rule reduction method are described as follows.

Step 1: To generate division points for each antecedent attribute by using the transform functions. Suppose that there are M antecedent attributes with J_i reference values for the i th ($i = 1, \dots, M$) antecedent attribute. Based on Definition 1, $J_i - 1$ division points, namely $\{P(A_{i,j}, A_{i,j+1}); j = 1, \dots, J_i - 1\}$, are generated for the i th antecedent attribute.

Step 2: To generate division domains for the EBRBS by using the division points. Based on Definition 2, $\prod_{i=1}^M J_i$ division domains, namely $\{D(A_{1,j_1}, \dots, A_{M,j_M}); j_i = 1, \dots, J_i; i = 1, \dots, M\}$, are generated for the EBRBS, in which the division domain is the clustering center according to Remark 5.

Step 3: To assign all EBRs to the division domains based on the rule clustering strategy. Suppose that there are T EBRs transformed from T sample data. Based on Definition 3, these T EBRs are all assigned to the $\prod_{i=1}^M J_i$ division domains.

Step 4: To generate new EBRs from the EBRs gathered in each division domain based on the rule reduction strategy. Suppose that there are \bar{L} division domains which include at least one EBR, based on Definition 4, all EBRs in each division domain are used to generate \bar{L} new EBR, respectively.

Remark 8: After utilizing the domain division-based rule reduction method to downsize the EBRB, \bar{L} new EBRs shown in step 4 can construct a reduced EBRB, in which the number of rules regarding the reduced EBRB should be no more than both the number of sample data T and the number of division domains $\prod_{i=1}^M J_i$.

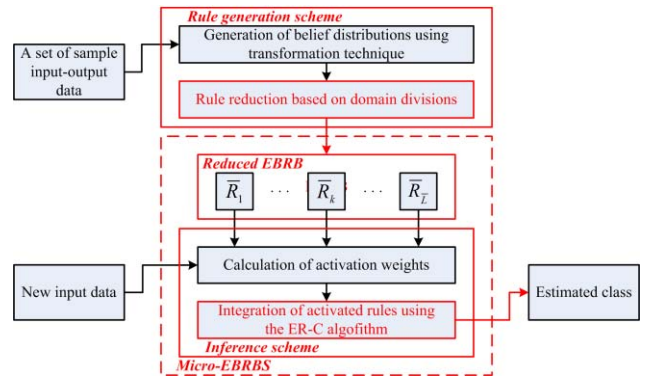


Fig. 10. Illustration of Micro-EBRBS.

C. Micro-EBRBS: EBRBS With Rule Reduction and ER-C Algorithm But Without Rule Weight Calculation

Based on the above analysis and new improvements, a novel EBRBS, called Micro-EBRBS which has a simplified rule generation and inference schemes and a downsized EBRB comparing to EBRBS, is developed and its methodological framework is shown in Fig. 10.

It is clear from Fig. 10 that, comparing to the EBRBS shown in Figs. 1 and 2, the Micro-EBRBS includes the process of rule reduction but excludes the process of rule weight calculation. Additionally, the ER-C algorithm is used to replace the ER algorithm in the process of activated rule integration. More specifically, the rule generation and the inference scheme of the Micro-EBRBS are described as follows.

For the rule generation scheme of the Micro-EBRBS, it consists of the following two steps.

Step 1: Generation of belief distributions using the transformation technique.

After determining the basic parameters based on expert knowledge, including attribute weights, utility values of the reference values used for antecedent and consequent attributes, the belief distributions of the antecedent and consequent attributes can be generated.

Suppose that $\{u(A_{i,j}); j = 1, \dots, J_i\}$ is a set of given utility values used for the i th ($i = 1, \dots, M$) antecedent attribute, and $x_{k,i}$ is the k th ($k = 1, \dots, L$) sample input data of

the i th antecedent attribute. Thus, the belief distribution of the i th antecedent attribute generated using the utility-based equivalence transformation technique [35] is

$$S(x_{k,i}) = \left\{ (A_{i,j}, \alpha_{i,j}^k); j = 1, \dots, J_i \right\} \quad (16)$$

where

$$\begin{aligned} \alpha_{i,j}^k &= \frac{u(A_{i,j+1}) - x_{k,i}}{u(A_{i,j+1}) - u(A_{i,j})} \\ \alpha_{i,j+1}^k &= 1 - \alpha_{i,j}^k, \text{ if } u(A_{i,j}) \leq x_{k,i} \leq u(A_{i,j+1}) \\ \alpha_{i,t}^k &= 0, \text{ for } t = 1, \dots, J_i \text{ and } t \neq j, j+1 \end{aligned} \quad (17)$$

$$\alpha_{i,t}^k = 0, \text{ for } t = 1, \dots, J_i \text{ and } t \neq j, j+1 \quad (18)$$

where $\alpha_{i,j}^k$ is the belief degree of $A_{i,j}$ in the k th EBR obtained from the sample input data $x_{k,i}$.

Next, when the k th sample output data is y_k and the given utility values attached to the consequent attribute D are $\{u(D_n); n = 1, \dots, N\}$, the belief distribution of consequent attribute is

$$S(y_k) = \left\{ (D_n, \beta_n^k); n = 1, \dots, N \right\}. \quad (19)$$

Finally, all belief distributions shown in (16) and (19) together with attribute weights determined by experts are used to construct an EBRB.

Step 2: Rule reduction based on domain divisions.

For the EBRB generated by step 1, the domain division-based rule reduction method shown in Section III-B is then used to reduce the number of rules. Suppose there are M antecedent attributes with J_i reference values $A_{i,j}$ ($j = 1, \dots, J_i$) and one consequent attribute D with N classes D_n ($n = 1, \dots, N$), the k th ($k = 1, \dots, \bar{L}$) rule of the reduced EBRB can be written as

$$\begin{aligned} \bar{R}_k : \text{ IF } U_1 \text{ is } \{ (A_{1,j}, \bar{\alpha}_{1,j}^k); j = 1, \dots, J_1 \} \wedge \dots \wedge U_M \text{ is } \\ \{ (A_{M,j}, \bar{\alpha}_{M,j}^k); j = 1, \dots, J_M \}, \text{ THEN } D \text{ is } \\ \{ (D_n, \bar{\beta}_{n,k}); n = 1, \dots, N \} \\ \text{ with } \theta_k = 1 \text{ and } \{ \delta_1, \dots, \delta_M \} \end{aligned} \quad (20)$$

where $\bar{\alpha}_{i,j}^k$ and $\bar{\beta}_{n,k}$ are the integrated belief degree of the $A_{i,j}$ and the class D_n using the rule reduction strategy.

Remark 9: As shown in the proposed rule reduction method, the most complex step is to generate $\bar{\alpha}_{i,j}^k$ ($k = 1, \dots, \bar{L}$; $i = 1, \dots, M$; $j = 1, \dots, J_i$) and $\bar{\beta}_{n,k}$ ($n = 1, \dots, N$) by using L rules of EBRB. Furthermore, Section II-C shows that the time complexity of generating belief distributions is $O(L \times (\sum_{i=1, \dots, M} J_i + N))$. Hence, the time complexity for the rule generation scheme of the Micro-EBRBS is $O(L \times \bar{L} \times (\sum_{i=1, \dots, M} J_i + N))$.

For the inference scheme of the Micro-EBRBS, it consists of the following two steps.

Step 1: Calculation of activation weights using the distance measure.

While a test input data is provided for the Micro-EBRBS, the activation weights can be calculated for each EBR of the reduced EBRB. Suppose that $\mathbf{x} = (x_1, \dots, x_M)$ is a test input data, each input x_i ($i = 1, \dots, M$) will be first transformed into a belief distribution of the reference values of the i th antecedent attribute using (17) and (18)

$$S(x_i) = \left\{ (A_{i,j}, \alpha_{i,j}); j = 1, \dots, J_i \right\}. \quad (21)$$

Next, the individual matching degree of the i th antecedent attribute in the k th rule, denoted as $S^k(x_i, U_i)$, is calculated by using the Euclidean distance

$$\begin{aligned} S^k(x_i, U_i) &= 1 - d^k(x_i, U_i) \\ &= 1 - \min \left\{ 1, \sqrt{\sum_{j=1}^{J_i} (\alpha_{i,j} - \alpha_{i,j}^k)^2} \right\} \end{aligned} \quad (22)$$

where $d^k(x_i, U_i)$ is the distance measurement.

Finally, the activation weight of the k th EBR, denoted as w_k , is calculated by

$$w_k = \frac{\theta_k \prod_{i=1}^M (S^k(x_i, U_i))^{\bar{\delta}_i}}{\sum_{l=1}^L (\theta_l \prod_{i=1}^M (S^l(x_i, U_i))^{\bar{\delta}_i})}, \quad \bar{\delta}_i = \frac{\delta_i}{\max_{i=1, \dots, M} \{\delta_i\}} \quad (23)$$

where θ_k is the weight of the k th rule, and δ_i is the weight of the i th antecedent attribute.

Step 2: Integration of activated rules using the ER-C algorithm.

After performing step 1, all activated rules can be integrated using the ER-C algorithm shown in (5) and the integrated belief distribution of the test input data \mathbf{x} can be represented as follows:

$$f(x) = \{(D_n, \beta_n^{\text{ER-C}}); n = 1, \dots, N\}. \quad (24)$$

Afterward, the estimated class for the test input data \mathbf{x} can be obtained as follows:

$$f(x) = D_n, n = \arg \max_{i=1, \dots, N} \{ \beta_i^{\text{ER-C}} \}. \quad (25)$$

Remark 10: Considering that the inference scheme of the Micro-EBRBS is based on the reduced EBRB, which only has \bar{L} rules, the time complexity of the inference scheme regarding the Micro-EBRBS is $O(\bar{L} \times (\sum_{i=1, \dots, M} J_i + N))$ for classifying each test data.

D. Apache Spark-Based Implementation of Micro-EBRBS for Big Data Multiclass Classification

In order to further improve the computing efficiency of the Micro-EBRBS, the Apache Spark is used to achieve the parallel rule generation and inference schemes. As an in-memory big data platform, the Apache Spark has been proven that it supports a much wider range of functionality than the Apache Hadoop [9]. The fundamental data structure of the Apache Spark is the resilient distributed dataset (RDD), which represents a collection of distributed items that can be manipulated across many computing nodes concurrently. Hence, the RDD allows the data cache to be stored in memory and perform computations for the same data directly from memory. After the RDD is constructed, the program can perform the following two operations.

- 1) *Transformations:* This kind of operation is to create a new RDD from existing RDD and the concrete function includes *map* (which is to return a new RDD formed by passing each element of the source through a function), *mapToPair* (which is to return a new RDD

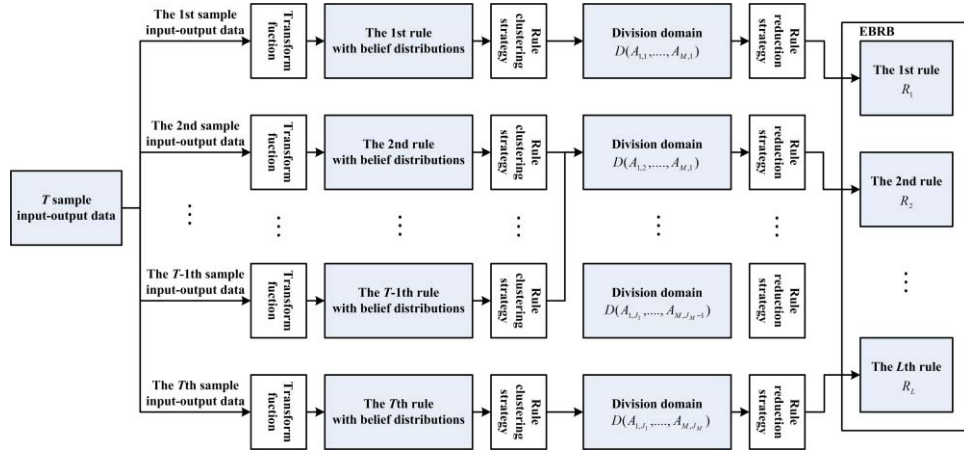


Fig. 11. Illustration of the parallel rule generation scheme.

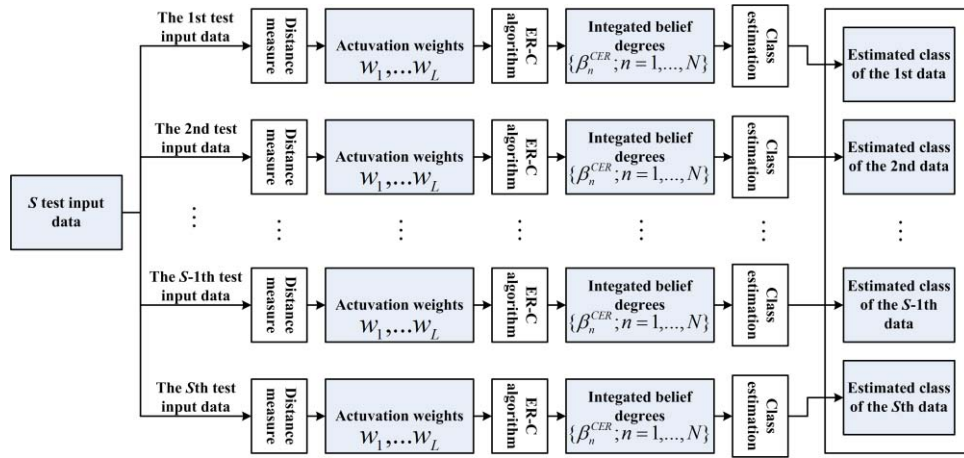


Fig. 12. Illustration of the parallel inference scheme.

of key-value pairs by using a function), *reduceByKey* (which is to return a new RDD of key-value pairs where the values for each key are aggregated using the given reduce function), and so on. The detailed description of those functions can be found in [4].

- 2) *Actions*: This kind of operation returns the final results of RDD computations, and the concrete function includes *reduce* (which is to aggregate the elements of the RDD using a function), *collect* (which is to return all the elements of the RDD as an array at the driver program), and others. The detailed description of those functions can be found in [4].

Based on the functions of transformation and action, the pseudocodes together with Figs. 11 and 12 are provided to illustrate the Apache Spark-based implementation of the parallel rule generation and inference schemes of Micro-EBRBS below, respectively.

IV. EXPERIMENTS

The performance of the Micro-EBRBS is empirically assessed through three different experiments with 14 classification datasets from the well-known UCI repository of machine learning databases [30]. The EBRBS, the conventional FRBCS and machine-learning classifiers, and the big

TABLE I
STATISTICS ON 14 CLASSIFICATION DATASETS

No.	Dataset	No. of data	No. of attributes	No. of classes
1	Diabetes	393	8	2
2	Cancer	569	30	2
3	Transfusion	748	4	2
4	Banknote	1,372	4	2
5	Magic	19,020	10	2
6	Wine	178	13	3
7	Waveform	5,000	21	3
8	Glass	214	9	6
9	Red wine	1,599	11	6
10	Satimage	6,435	36	6
11	Census	95,130	40	2
12	Gas sensors	928,991	10	3
13	Covtype	581,012	54	7
14	Poker	1,025,010	10	10

data FRBCS classifiers are used to compare in terms of the accuracy and computing efficiency, respectively.

A. Datasets and Experiment Conditions

Fourteen classification datasets obtained from UCI are used to evaluate the performance of the Micro-EBRBS. The main characteristics of these datasets are summarized in Table I.

Algorithm 1 Pseudocode of Parallel Rule Generation Scheme of Micro-EBRBS

Input: *SampleDataSet* denotes the set of sample input-output data, each sample input-output data of *SampleDataSet* is denoted as *sampleData*, *rule* denotes the EBR shown in Eq. (20), *tuple1*, *tuple2*, and *tuple3* denote the 2-tuple composing of *rule* and its division domain.

Output: A set of extended belief rules *EBRBSet*

```

01 EBRBSet = new JavaSparkContext().Parallelize(SampleDataSet).mapToPair(sampleData -> {
02     Generate rule from sampleData by using Steps 1 to 2 shown in the rule generation scheme of the Micro- EBRBS;
03     Generate divisionDomain for rule by using Steps 1 to 3 shown in the domain division-based rule reduction method;
04     Return new Tuple2<>(divisionDomain, rule)
05 }).reduceByKey((tuple1, tuple2) -> {
06     Generate tuple3 by using Step 4 shown in the domain division-based rule reduction method;
07     Return tuple3;
08 }).map(tuple3 -> {
09     Obtain rule from tuple3;
10     Return rule;
11 }).collect();

```

Notice that for the datasets Diabetes, Cancer, and Census, we have removed the data with missing attribute values.

To develop the comparison in multiple aspects, k -fold cross-validation is considered in the experiments, where each dataset is divided into k blocks, with $k - 1$ blocks as training data, namely sample input-output data, and the remaining block as testing data. Additionally, the nonparametric statistical analysis is used to assess if significant differences exist among different classifiers at a level of significance of $\alpha = 0.1$. For conducting multiple statistical comparisons over multiple datasets, as suggested in [14], the Friedman and Holm tests are employed.

For the first and the second experiments (Sections IV-B and C), the datasets with relatively small number of data, including the first to the tenth datasets, are used to compare the performance of the Micro-EBRBS with the EBRBS and the conventional FRBCS and machine-learning classifiers. All these classifiers are implemented using Java programming (JDK 1.8.0) and the open source software (Weka and KEEL) on Intel Core i5-4300U CPU at 1.90 GHz and 4GB RAM with Windows 7. For each dataset, the average results of the ten runs of each classifier are used to compare their performances.

For the third experiment (Section IV-D), the datasets with relatively large number of data, including the 11th to the 14th datasets, are used to compare the performance of the Micro-EBRBS with the big data FRBCS classifiers, and all these classifiers are executed in the 17 nodes cluster connected via 8GT/s Ethernet LAN network, where the master node is composed of 1 Intel Xeon E5-2640 4 cores at 2.5 GHz and 16GB RAM and the slave nodes are composed of 2 Intel Xeon E5-2670v2 10 cores at 2.5 GHz and 64GB RAM. The entire cluster runs under Red Hat 7.3 and Apache Spark 2.1.0.

B. Comparative Analysis Between Micro-EBRBS and EBRBS

The first experiment aims to compare the accuracy and computing efficiency of the Micro-EBRBS with the EBRBS, respectively, and the comparisons are based on the 2/4/6/8/10-CV to investigate the influences on the accuracy

and computing efficiency of these EBRBSs by using different numbers of training and testing data.

For the basic parameters of the Micro-EBRBS and EBRBS, suppose that all attribute weights are 1, namely

$$\delta_i = 1; \quad i = 1, \dots, M \quad (26)$$

where M is the number of antecedent attributes. The number of reference values is three for each antecedent attribute, and the utility value of these reference values is defined as follows:

$$\{u(A_{i,j}); \quad j = 1, 2, 3\} = \left\{lb_i, \frac{lb_i + ub_i}{2}, ub_i\right\}; \quad i = 1, \dots, M \quad (27)$$

where $u(A_{i,j})$ denotes the utility value of the reference value $A_{i,j}$, lb_i , and ub_i denote the lower and upper bounds of the i th antecedent attribute, respectively.

Figs. 13 and 14 show the computing time and the accuracy regarding the Micro-EBRBS in comparison with the EBRBS over two and multiclass datasets, respectively, in which the computing time includes the time of rule generation scheme, inference scheme, and total time.

For the two-class datasets, including Diabetes, Cancer, Transfusion, and Banknote, the computing time of rule generation scheme regarding the EBRBS is increasing with the increasing number of training data used to generate EBRs, e.g., 50% data are regarded as the training data in 2-CV and 90% data as the training data in 10-CV. Additionally, there are slight differences between the Micro-EBRBS and the EBRBS in term of the computing time of the inference scheme and the accuracy. As a result, the total computing time of the EBRBS is much more than that of the Micro-EBRBS.

For the multiclass datasets, including Wine, Waveform, Glass, Red Wine, the similar conclusions are obtained in terms of the computing time and the accuracy.

In order to show the detailed comparison of the Micro-EBRBS and the EBRBS, Tables II and III provide the results of rule generation time, inference time, total time, number of rules, number of activated rules, and the accuracy, in which the ratio is the result of the Micro-EBRBS divided by the result of the EBRBS. Hence, the Micro-EBRBS with a larger ratio

Algorithm 2 Pseudocode of Parallel Inference Scheme of Micro-EBRBS

Input: *TestDataSet* denotes the set of test input data, each test input data of *TestDataSet* is denoted as *testData*, *class1* denotes the estimated class of the Micro-EBRBS, and *class2* denotes the actual class of test input data, *a* and *b* denote the integer variable.

Output: The total number of test input data correctly classified by the Micro-EBRBS *totalCorrect*.

```

01 totalCorrect = new JavaSparkContext().parallelize(TestDataSet)
02 .map(testData -> {
03   Generate class1 for testData by using Steps 1 to 2 shown in the inference scheme of the Micro-EBRBS;
04   Obtain class2 from testData;
05   Return class1==class2?1:0;
06 }) .reduce((a, b) -> a + b);

```

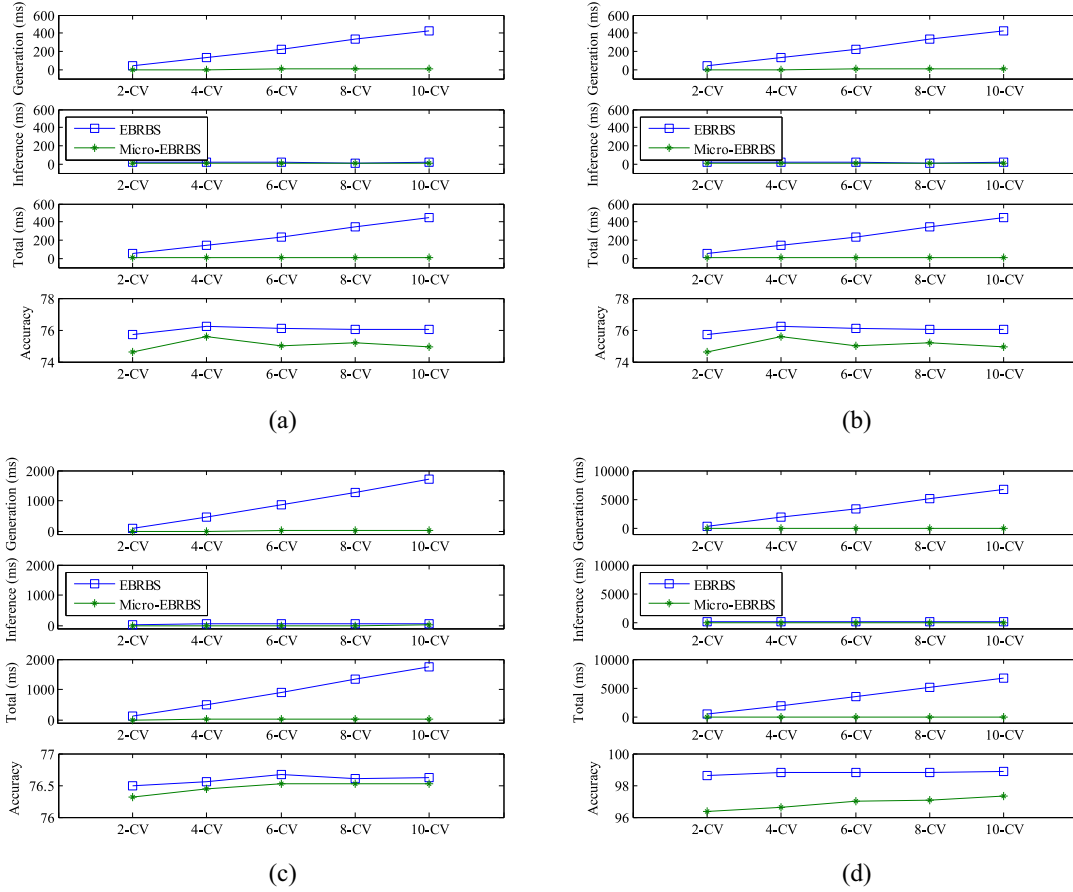


Fig. 13. Comparison of computing time and accuracy between Micro-EBRBS and EBRBS for two-class datasets. (a) Diabetes. (b) Cancer. (c) Transfusion. (d) Banknote.

in term of accuracy and a smaller ratio in term of computing time is better than the EBRBS.

Table II shows that the computing time of the Micro-EBRBS is much less than that of the EBRBS for all two and multi-class datasets, where the minimum ratio of computing time is obtained from the dataset Banknote and its ratios are 0.002, 0.022, and 0.002 for the rule generation, inference, and total time, respectively, and the maximum one is obtained from the datasets Cancer and Wine and their ratio are 0.058 and 0.091 for the rule generation time and total time of Wine and 0.911 for the inference time of Cancer.

Table III shows that the number of rules and activated rules involved in the Micro-EBRBS is less than the EBRBS,

where the minimum ratio is obtained from the datasets transfusion, i.e., 0.019 for the number of rules and 0.014 for the number of activated rules, and the maximum one is obtained from the datasets Cancer, 0.875 for the number of rules and 0.867 for the number of activated rules. Furthermore, it is clear from Table III that the accuracy of the Micro-EBRBS closely approximates that of the EBRBS and the range of the ratio can be expressed as [0.963, 1.000] for eight classification datasets.

In order to further compare the accuracy of the Micro-EBRBS and EBRBS, Friedman and Holm tests are applied to provide the statistical analysis based on two and multiclass datasets, respectively. From Table IV, although the EBRBS can obtain the best accuracy at most of two and multiclass datasets,

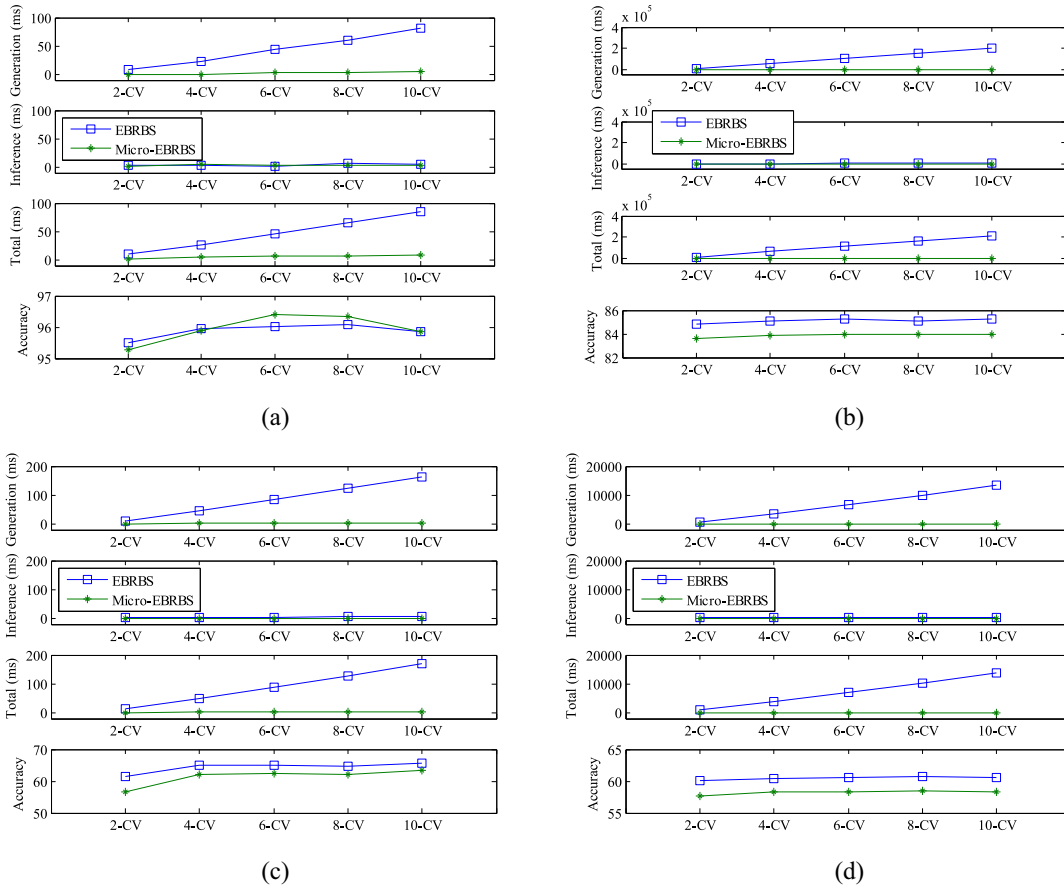


Fig. 14. Comparison of computing time and accuracy between Micro-EBRBS and EBRBS for multiclass datasets. (a) Wine. (b) Waveform. (c) Glass. (d) Red Wine.

TABLE II
COMPARISON OF COMPUTING EFFICIENCY FOR MICRO-EBRBS AND EBRBS

No. of classes	Dataset	Rule generation (ms)			Inference (ms)			Total (ms)		
		Micro-EBRBS	EBRBS	Ratio	Micro-EBRBS	EBRBS	Ratio	Micro-EBRBS	EBRBS	Ratio
Two-class	Diabetes	6.1	429.0	0.014	3.2	15.6	0.205	9.3	444.6	0.021
	Cancer	40.8	1,475.5	0.028	69.9	76.7	0.911	110.7	1,552.2	0.071
	Transfusion	3.2	1,709.6	0.002	3.0	37.5	0.080	6.2	1,747.1	0.004
	Banknote	12.4	6,645.2	0.002	3.2	148.3	0.022	15.6	6,793.5	0.002
Multi-class	Wine	4.6	81.1	0.058	3.2	4.7	0.681	7.8	85.8	0.091
	Waveform	750.5	205,530.3	0.004	3,623.3	8,654.2	0.419	4,373.8	214,184.5	0.020
	Glass	3.0	164.4	0.018	1.1	5.3	0.208	4.1	169.7	0.024
	Red Wine	40.7	13,291.8	0.003	60.2	424.9	0.142	100.9	13,716.7	0.007

TABLE III
COMPARISON OF ACCURACY AND NUMBER RULES AND ACTIVATED RULES FOR MICRO-EBRBS AND EBRBS

No. of classes	Dataset	No. of rules			No. of activated rules			Accuracy (%)		
		Micro-EBRBS	EBRBS	Ratio	Micro-EBRBS	EBRBS	Ratio	Micro-EBRBS	EBRBS	Ratio
Two-class	Diabetes	94.0	353.7	0.266	49.6	208.9	0.237	74.91	76.01	0.986
	Cancer	448.2	512.1	0.875	222.7	256.9	0.867	96.49	96.45	1.000
	Transfusion	12.8	673.2	0.019	7.4	533.1	0.014	76.52	76.62	0.999
	Banknote	30.8	1,234.8	0.025	22.3	955.1	0.023	97.34	98.86	0.985
Multi-class	Wine	126.3	160.2	0.788	45.2	58.6	0.771	95.84	95.84	1.000
	Waveform	2,031.7	4,500.0	0.452	1,492.9	3,523.1	0.424	84.00	85.22	0.986
	Glass	41.9	192.6	0.218	11.8	95.4	0.124	63.32	65.56	0.967
	Red Wine	230.8	1,439.1	0.160	139.6	964.1	0.145	58.36	60.61	0.963

none of hypotheses is in favor of the significant difference between EBRBS and Micro-EBRBS. However, the computing efficiency of Micro-EBRBS is much better than EBRBS.

In summary, for the comparison of the Micro-EBRBS and EBRBS, the experiment results have shown that the former has much less computing time than the latter. Moreover, the

TABLE IV
FRIEDMAN AND HOLM TESTS TO COMPARE THE ACCURACY OF
MICRO-EBRBS AND EBRB ($\alpha = 0.1$)

Indicator	EBRBS (two-class)	EBRBS (Multi-class)
p value	0.3173	0.1336
Critical value	0.1000	0.1000
Hypothesis	Accepted	Accepted

Micro-EBRBS is comparable to the EBRBS in term of accuracy while much less number of rules and activated rules are used to address many classification datasets.

C. Comparative Analysis Between Micro-EBRBS and Conventional Classifiers

The second experiment aims to compare the accuracy of the Micro-EBRBS with the conventional classifiers, which include the FRBCS and the conventional machine-learning classifiers. Apart from the setting of the Micro-EBRBS introduced in (26) and (27), the other classifiers are shown as follows.

- 1) *Chi-FRBCS* [29]: It was proposed by Rio *et al.*, where the penalized certainty factor (PCF) is used to calculate rule weights, the winner rule strategy is used as the fuzzy reasoning method, and the OVO is used to improve the performance of Chi-FRBCS in dealing with multiclass classification problems. Here, assume that the number of fuzzy labels is three for each attribute and these fuzzy labels modeled as triangular membership function.
- 2) *Other FRBCS Classifiers*: SLAVE, FH-GBML, FURIA, and FARC-HD, they are all obtained from KEEL software. The setting of these FRBCS classifiers follows the previous study in [3].
- 3) *Machine-Learning Classifiers*: k nearest neighbor (KNN), naïve Bayes (NB), decision tree (DT), random forest (RF), artificial neural network (ANN), and SVM, are all obtained from WEKA software. Apart from the default setting, 20% number of training data is set as neighbors for the KNN, 5% number of training data is set as the minimum number of data per leaf for the DT, the number of random trees is set as 5 for the RF, and the number of iterations is set as 10 for the ANN.

Table V shows the accuracy of Micro-EBRBS in comparison with five FRBCS classifiers, including SLAVE, FH-GBML, FURIA, FARC-HD, and Chi-FRBCS, over two and multiclass datasets, respectively, where the result of the best accuracy is highlighted in bold-face and the number in brackets denotes the rank of each classifier. For the two-class datasets, the accuracy of FARC-HD is better than other FRBCS classifiers as well as Micro-EBRBS ranked at the third place in term of average rank. For the multiclass datasets, the FARC-HD remains its advantages in dealing with classification problems over other classifiers. Despite the fact that the FARC-HD outperforms the Micro-EBRBS, it is still possible to see a considerable decrease in the average rank of Micro-EBRBS, namely from 4.0 to 2.0. This is so because the distributed belief degree is used in each EBR to express

multiple classes and Micro-EBRBS; therefore, it has excellent abilities to deal with multiclass problems.

In addition to the accuracy and average rank for each dataset and each classifier shown in Table V, Table VI provides the statistical analysis of accuracy while the Micro-EBRBS is selected as the control method for the Friedman and Holm tests. As shown in Table VI, apart from the SLAVE and Chi-FRBCS in the case of multiclass datasets, all hypotheses regarding five FRBCS classifiers are accepted, which means that although some of FRBCS classifiers, such as the FARCHD, FURIA, and FH-GBML, are better than Micro-EBRBS, none of hypotheses is in favor of the significant differences between the FRBCS classifiers and the Micro-EBRBS. For the SLAVE and Chi-FRBCS in the case of multiclass datasets, the hypothesis is rejected, which means that the significant differences can be found to show a better accuracy of Micro-EBRBS comparing to the SLAVE and Chi-FRBCS.

Table VII shows the computing time of Micro-EBRBS in comparison with the SLAVE, FH-GBML, FURIA, FARC-HD, and Chi-FRBCS, where the result of the best computing time is highlighted in bold-face and the number in brackets denotes the rank of each classifier. In both of two and multiclass datasets, the computing time of Micro-EBRBS is close to that of Chi-FRBCS, and is significantly faster than the other FRBCS classifiers. This is because some additional methodologies were used to improve the FRBCS classifiers, e.g., the genetic algorithm, which is an iterative optimization algorithm, and is one of the components of SLAVE and FARC-HD, leading to a time-consuming process while using those FRBCS classifiers.

Hence, in the application of FRBCS classifiers for addressing big data classification problems, the related works introduced in Section II-B were all based on Chi-FRBCS owing to its high efficient process of dealing with data.

In order to further compare with the Micro-EBRBS and Chi-FRBCS, Figs. 15 and 16 show their time of rule generation scheme, inference scheme, and total as well as the accuracy over two and multiclass relatively large datasets, respectively.

For the two-class datasets Banknote and Magic, the computing time of rule generation scheme regarding the Chi-FRBCS closely approximates to that regarding the Micro-EBRBS from 2-CV to 10-CV. Meanwhile, for the dataset Banknote, the Micro-EBRBS is slightly better than the Chi-FRBCS in terms of the computing time of inference scheme and the accuracy. But for the dataset Magic, the Chi-FRBCS is slightly better than the Micro-EBRBS regarding the computing time and accuracy. For the multiclass datasets Waveform and Satimage, apart from the computing time of rule generation scheme, the Micro-EBRBS is much better than the Chi-FRBCS in term of the computing time of the inference scheme and the accuracy.

In order to show the detailed comparison of Micro-EBRBS and Chi-FRBCS, Tables VIII and IX provide the results and their ratio obtained from 10-CV, such as the rule generation time, inference time, total time, number of rules, number of activated rules, and accuracy. Table VIII shows that the Chi-FRBCS has less computing time than the Micro-EBRBS in terms of the rule generation scheme for both two and

TABLE V
COMPARISON OF ACCURACY FOR MICRO-EBRBS AND FIVE FRBCS CLASSIFIERS

No. of classes	Dataset	SLAVE	FH-GBML	FURIA	FARC-HD	Chi-FRBCS	Micro-EBRBS
Two-class	Diabetes	77.10% (1.5)	70.23% (6)	76.59% (3)	77.10% (1.5)	72.80% (5)	74.91% (4)
	Cancer	92.33% (3)	92.26% (5)	90.68% (6)	95.25% (2)	92.32% (2)	96.49% (1)
	Transfusion	76.60% (5)	79.01% (1)	78.74% (2)	77.27% (3)	76.80% (3)	76.52% (6)
	Banknote	91.33% (6)	98.18% (3)	99.13% (2)	99.78% (1)	94.42% (5)	97.34% (4)
	Magic	74.96% (6)	81.30% (3)	84.63% (1)	84.51% (2)	80.62% (2)	77.38% (5)
	Average rank	4.3	3.6	2.8	1.9	4.4	4.0
Multi-class	Wine	89.47% (6)	92.61% (3)	91.88% (4)	94.35% (2)	90.17% (5)	95.84% (1)
	Waveform	81.48% (4)	60.18% (6)	83.10% (3)	83.78% (2)	74.70% (5)	84.00% (1)
	Glass	58.05% (4)	57.99% (5)	58.49% (3)	70.24% (1)	50.37% (6)	63.32% (2)
	Red Wine	55.60% (6)	68.67% (1)	57.72% (4)	59.72% (2)	55.89% (5)	58.36% (3)
	Satimage	81.69% (4)	74.72% (6)	89.15% (1)	87.32% (2)	74.79% (5)	85.41% (3)
	Average rank	4.8	4.2	3.0	1.8	5.2	2.0

TABLE VI
FRIEDMAN AND HOLM TESTS TO COMPARE THE ACCURACY OF MICRO-EBRBS AND FIVE FRBCS CLASSIFIERS ($\alpha = 0.1$)

Item	Indicator	SLAVE	FH-GBML	FURIA	FARC-HD	Chi-FRBCS
Two-class	p value	0.7999	0.7353	0.3105	0.0759	0.7353
	Critical value	0.1000	0.0333	0.0250	0.0200	0.0333
	Hypothesis	Accepted	Accepted	Accepted	Accepted	Accepted
Multi-class	p value	0.0180	0.0630	0.3980	0.8658	0.0068
	Critical value	0.0250	0.0333	0.0500	0.1000	0.0200
	Hypothesis	Rejected	Accepted	Accepted	Accepted	Rejected

TABLE VII
COMPARISON OF COMPUTING TIME IN SECOND FOR MICRO-EBRBS AND FIVE FRBCS CLASSIFIERS

No. of classes	Dataset	SLAVE	FH-GBML	FURIA	FARC-HD	Chi-FRBCS	Micro-EBRBS
Two-class	Diabetes	569 s (5)	2s316 s (6)	4.3 s (3)	56 s (4)	0.0 s (1.5)	0.0 s (1.5)
	Cancer	1200 s (5)	4s947 s (6)	6.8 s (3)	249 s (4)	0.1 s (1.5)	0.1 s (1.5)
	Transfusion	212 s (5)	2s899 s (6)	4.2 s (3)	31 s (4)	0.0 s (1.5)	0.0 s (1.5)
	Banknote	502 s (5)	6s256 s (6)	6.0 s (3)	69 s (4)	0.0 s (1.5)	0.0 s (1.5)
	Magic	31,303 s (5)	387,010 s (6)	715 s (3)	8,655 s (4)	1.0 s (1)	1.1 s (2)
	Average rank	5	6	3	4	1.4	1.6
Multi-class	Wine	268 s (5)	1,394 s (6)	2.4 s (3)	75 s (4)	0.0 s (1.5)	0.0 s (1.5)
	Waveform	73,570 s (6)	45,255 s (5)	184 s (3)	2,490 s (4)	6.5 s (2)	4.0 s (1)
	Glass	611 s (5)	1,184 s (6)	3.2 s (3)	100 s (4)	0.0 s (1.5)	0.0 s (1.5)
	Red Wine	1,049 s (5)	9,434 s (6)	21 s (3)	590 s (4)	0.6 s (2)	0.1 s (1)
	Satimage	29,655 s (5)	62,579 s (6)	346 s (3)	8,876 s (4)	87 s (2)	12 s (1)
	Average rank	5.2	5.8	3	4	1.8	1.2

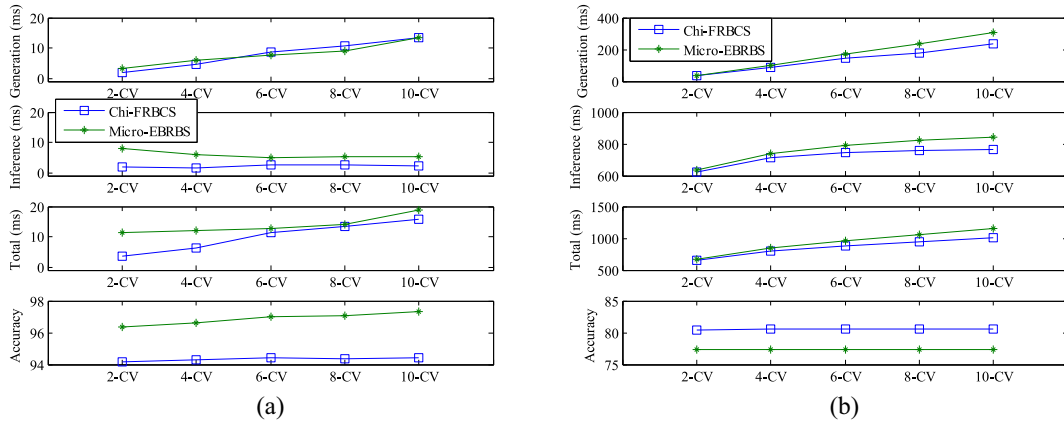


Fig. 15. Comparison of computing efficiency and accuracy between Chi-FRBCS and Micro-EBRBS for two-class datasets. (a) Banknote. (b) Magic.

multiclass datasets and the inference scheme for two-class datasets. However, the Micro-EBRBS has less computing time of the inference scheme than the Chi-FRBCS in the multiclass datasets. As a result, the Chi-FRBCS has better total time and their ratio are 1.184 and 1.142 for the two-class datasets

Banknote and Magic, and the Micro-EBRBS has better total time and their ratio are 0.626 and 0.141 for the multiclass datasets Waveform and Satimage.

Table IX shows that the number of rules in the Chi-FRBCS is the same as the Micro-EBRBS, but the number of activated

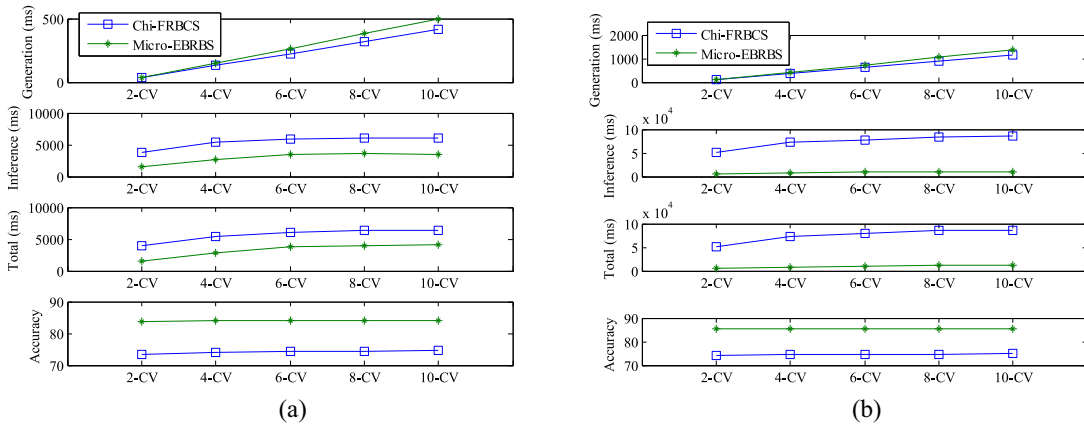


Fig. 16. Comparison of computing efficiency and accuracy between Chi-FRBCS and Micro-EBRBS for multiclass datasets. (a) Waveform. (b) Satimage.

TABLE VIII
COMPARISON OF COMPUTING EFFICIENCY FOR MICRO-EBRBS AND CHI-FRBCS

No. of classes	Dataset	Rule generation (ms)		Ratio	Inference (ms)		Ratio	Total (ms)		Ratio
		Micro-EBRBS	Chi-FRBCS		Micro-EBRBS	Chi-FRBCS		Micro-EBRBS	Chi-FRBCS	
Two-class	Banknote	13.5	13.5	1.000	5.2	2.3	2.261	18.7	15.8	1.184
	Magic	303.5	237.2	1.280	843.6	767.0	1.100	1,147.1	1,004.5	1.142
Multi-class	Waveform	496.7	414.8	1.197	3,542.1	6,034.2	0.587	4,038.8	6,449.0	0.626
	Satimage	1,386.8	1,162.2	1.193	10,864.9	85,759.8	0.127	12,251.7	86,922.0	0.141

TABLE IX
COMPARISON OF ACCURACY AND NUMBER OF RULES AND ACTIVATED RULES FOR MICRO-EBRBS AND CHI-FRBCS

No. of classes	Dataset	No. of rules		Ratio	No. of activated rules		Ratio	Accuracy (%)		Ratio
		Micro-EBRBS	Chi-FRBCS		Micro-EBRBS	Chi-FRBCS		Micro-EBRBS	Chi-FRBCS	
Two-class	Banknote	30.8	30.8	1.000	22.3	12.2	1.828	97.34	94.42	1.031
	Magic	354.1	354.1	1.000	140.9	99.3	1.419	77.38	80.62	0.960
Multi-class	Waveform	2,032.7	2,031.9	1.000	1,494.2	451.9	3.307	83.97	74.66	1.125
	Satimage	3,182.0	3,182.0	1.000	945.6	485.6	1.947	85.41	74.79	1.142

rules in the Chi-FRBCS is smaller than the Micro-EBRBS, where the maximum ratio is obtained from the multiclass dataset Waveform and its ratio is 3.307, and except for the dataset Magic, the accuracy of the Chi-FRBCS is worse than the Micro-EBRBS for all two and multiclass datasets, where the maximum ratio is obtained from the multiclass dataset Satimage and its ratio is 1.142.

In order to compare the accuracy of the Micro-EBRBS with the conventional machine-learning classifiers, Table X shows the accuracy of seven classifiers for ten classification datasets. The number in brackets denotes the rank of each classifier and the best result is marked as bold in Table X. For the two-class datasets, the accuracy of the RF and ANN are better than the Micro-EBRBS ranked at the 3rd place, and are further better than the KNN, NB, DT, and SVM. For the multiclass dataset, the Micro-EBRBS can reach the second best average rank better than the KNN, DT, NB, ANN, and SVM. From Table X, RF obtains the best average rank in both two and multiclass datasets. This is partly because RF is an ensemble learning method that operates by constructing a multitude of DTs. Hence, in one sense, the ensemble learning used in the RF can also use to improve the Micro-EBRBS.

Table XI shows the statistical analysis of accuracy while the Micro-EBRBS is selected as the control method for the Friedman and Holm tests. From Table XI, all hypotheses regarding the six machine-learning classifiers are accepted,

namely the Micro-EBRBS and the machine-learning classifiers have a similar accuracy for two and multiclass datasets without significant differences.

In summary, for the comparison of the Micro-EBRBS and the conventional classifiers, the experiment results have proven that the Micro-EBRBS has the similar accuracies with the conventional FRBCS and the conventional machine-learning classifiers but its computing time is much less than the conventional FRBCS classifiers except for the Chi-FRBCS.

It is worth noting that the Micro-EBRBS has the higher accuracy but less computing time than the Chi-FRBCS for multiclass datasets.

D. Comparative Analysis Between Micro-EBRBS and Big Data FRBCS Classifiers

The third experiment aims to compare the accuracy and the computing efficiency of the Micro-EBRBS with the big data FRBCS classifiers, where the core supporting theory of the big data FRBCS classifiers is shown in Section IV-C, and they include the following two versions [29].

- 1) *Chi-Max*: In this big data classifier, the rule generation scheme searches for the fuzzy rules with the same fuzzy label. Among these fuzzy rules, only the fuzzy rule with the highest rule weight is maintained in the fuzzy rule base.

TABLE X
COMPARISON OF ACCURACY AND NUMBER OF RULES AND ACTIVATED RULES FOR MICRO-EBRBS AND CHI-FRBC

No. of classes	Dataset	KNN	NB	DT	RF	ANN	SVM	Micro-EBRBS
Two-class	Diabetes	74.30% (6)	76.84% (4)	77.61% (3)	77.86% (2)	79.13% (1)	66.92% (7)	74.91% (5)
	Cancer	94.02% (4)	93.15% (6)	93.32% (5)	95.61% (3)	96.31% (2)	62.74% (7)	96.49% (1)
	Transfusion	76.20% (4)	75.40% (5)	78.34% (1)	72.33% (7)	76.34% (3)	75.27% (6)	76.52% (2)
	Banknote	92.93% (5)	84.26% (7)	90.31% (6)	99.20% (2)	98.25% (3)	100.00% (1)	97.34% (4)
	Magic	74.79% (5)	72.69% (6)	81.17% (3)	86.14% (1)	83.73% (2)	65.88% (7)	77.38% (4)
	Average rank	4.8	5.6	3.6	3.0	2.2	5.6	3.2
Multi-class	Wine	97.19% (1.5)	96.63% (3)	92.13% (6)	94.38% (5)	97.19% (1.5)	44.38% (7)	95.84% (4)
	Waveform	82.88% (4)	81.02% (5)	73.26% (7)	80.20% (6)	85.78% (2)	86.10% (1)	84.00% (3)
	Glass	61.21% (5)	47.66% (6)	67.76% (3)	71.50% (1)	39.72% (7)	69.16% (2)	63.32% (4)
	Red Wine	57.22% (5)	54.97% (7)	58.16% (3)	64.35% (1)	57.04% (6)	57.85% (4)	58.36% (2)
	Satimage	74.34% (6)	79.70% (4)	79.63% (5)	88.94% (1)	86.28% (2)	26.08% (7)	85.41% (3)
	Average rank	4.3	5.0	4.8	2.8	3.7	4.2	3.2

TABLE XI
FRIEDMAN AND HOLM TESTS TO COMPARE THE ACCURACY OF MICRO-EBRBS AND SIX MACHINE-LEARNING CLASSIFIERS ($\alpha = 0.1$)

Item	Indicator	KNN	NB	DT	RF	ANN	SVM
Two-class	p value	0.2416	0.0790	0.7697	0.8836	0.4642	0.0790
	Critical value	0.0250	0.0167	0.0500	0.1000	0.0333	0.0167
	Hypothesis	Accepted	Accepted	Accepted	Accepted	Accepted	Accepted
Multi-class	p value	0.4208	0.1877	0.2416	0.7697	0.7144	0.4642
	Critical value	0.0250	0.0167	0.020	0.1000	0.0500	0.0333
	Hypothesis	Accepted	Accepted	Accepted	Accepted	Accepted	Accepted

TABLE XII
COMPARISON OF COMPUTING TIME AND ACCURACY FOR CHI-MAX, CHI-AVE, AND MICRO-EBRBS UNDER FOUR CORES

Indicator	Classifier	Census		Gas sensors		Covtype		Poker	
		Value	Ratio	Value	Ratio	Value	Ratio	Value	Ratio
Rule generation (ms)	Micro-EBRBS	19,817	-	22,269	-	50,062	-	36,099	-
	Chi-Max	20,007	0.991	24,349	0.915	68,246	0.734	388,942	0.093
	Chi-Ave	18,759	1.056	23,000	0.968	67,656	0.740	376,419	0.096
Inference (ms)	Micro-EBRBS	199,324	-	12,879	-	129,858	-	1,818,235	-
	Chi-Max	191,375	1.042	17,845	0.722	8,310,398	0.016	141,730,759	0.013
	Chi-Ave	190,816	1.045	17,223	0.748	8,307,228	0.016	139,971,396	0.013
Total (ms)	Micro-EBRBS	219,141	-	35,148	-	179,920	-	1,854,334	-
	Chi-Max	211,382	1.037	42,194	0.833	8,378,644	0.022	142,119,701	0.013
	Chi-Ave	209,575	1.046	40,223	0.874	8,374,884	0.022	140,347,815	0.013
Accuracy (%)	Micro-EBRBS	89.69	-	48.60	-	70.24	-	57.20	-
	Chi-Max	86.34	1.039	41.34	1.176	65.19	1.078	52.79	1.084
	Chi-Ave	86.14	1.041	41.68	1.166	67.08	1.047	53.53	1.069

2) *Chi-Ave*: In this big data classifier, the rule generation scheme also searches for the fuzzy rules with the same fuzzy label. Afterward, the average rule weight of the fuzzy rules that have the same class is computed. Finally, the fuzzy rules with the greatest average rule weight is kept in the fuzzy rule base.

Fig. 17 shows the computing time and the accuracy of the Micro-EBRBS in comparison with the big data FRBCS classifiers, including Chi-Max and Chi-Ave, over two-class and multiclass datasets, respectively.

From Fig. 17, the Micro-EBRBS has the significantly better computing efficiency for the seven-class dataset Covtype and the ten-class dataset Poker, which is similar to the three-class dataset Gas sensors and worse than the two-class dataset census compared to big data FRBCS classifiers. For the ten, seven, and three-class datasets, the computing efficiency differences between these classifiers diminish gradually from four cores to 128 cores. Additionally, from the datasets census and gas sensors, while 128 cores are used in the cluster computing, the

computing time is more than the result obtained from 64 cores mainly because of the increasing cluster costs. So, determination of the number of cores is important to improve the computing efficiency of the Micro-EBRBS.

From the comparison of accuracy, the Micro-EBRBS reflects a better robustness than the big data FRBCS classifiers because the accuracy is consistent for all two and multiclass datasets when varying the number of cores used in the cluster computing, but the accuracy of the big data FRBCS classifiers are changeable. Therefore, comparing to the big data FRBCS classifiers, the Micro-EBRBS is able to provide exactly the same classifier while implementing by the Apache Spark. More specifically, Table XII shows the value and ratio of the Micro-EBRBS, Chi-Max, and Chi-Ave under four cores, where the best result is marked as bold.

In term of the computing time, the big data FRBCS classifiers have a slight advantage while addressing the two-class dataset Census and all ratios of rule generation, inference, and total time are less than 1.056. However, the computing

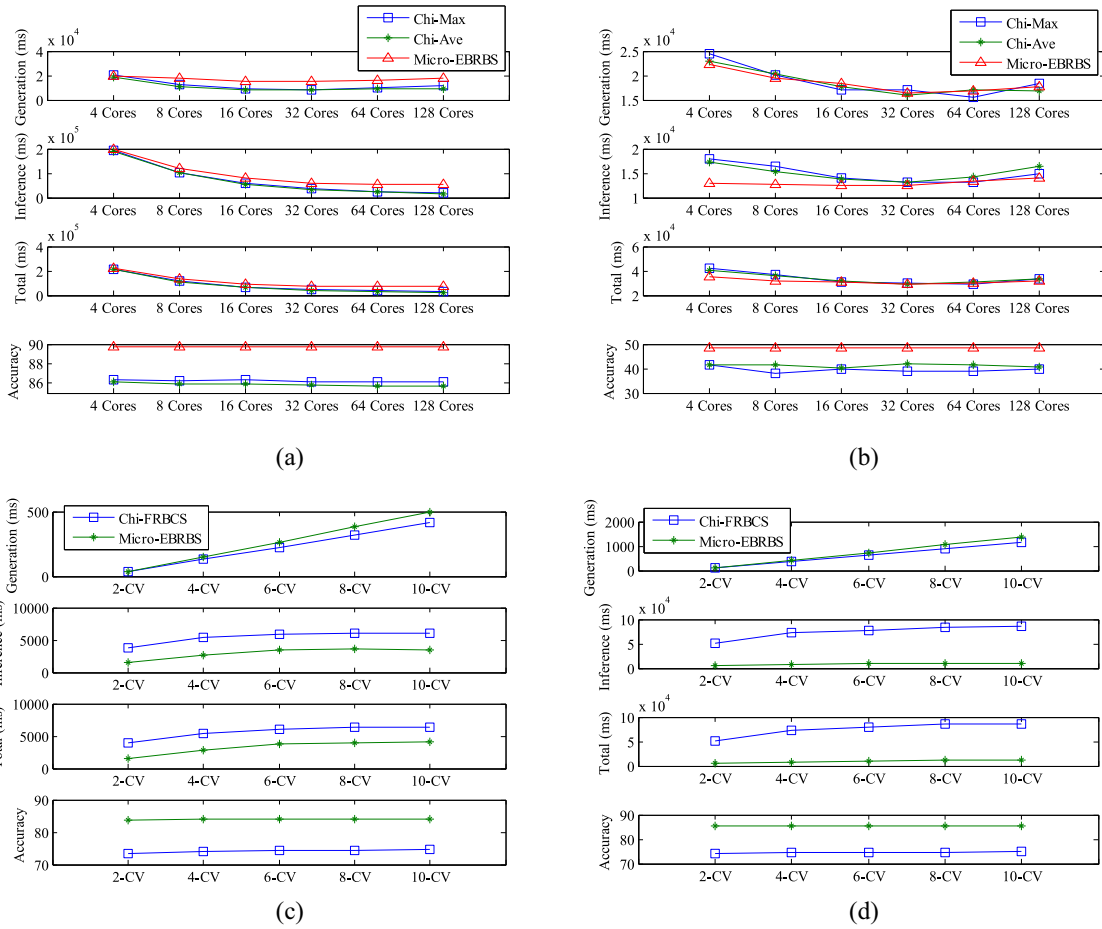


Fig. 17. Comparison of computing efficiency and accuracy for Chi-Max, Chi-Ave, and Micro-EBRBS. (a) Census (two classes). (b) Gas sensors (three classes). (c) Covtype (seven classes). (d) Poker (ten classes).

TABLE XIII
FRIEDMAN AND HOLM TESTS TO COMPARE THE ACCURACY OF
MICRO-EBRBS, CHI-MAX, AND CHI-AVE ($\alpha = 0.1$)

Indicator	Chi-Max	Chi-Ave
p value	0.0133	0.0771
Critical value	0.05	0.1
Hypothesis	Rejected	Rejected

time of the Micro-EBRBS is much better than both Chi-Max and Chi-Ave while increasing number of classes, i.e., ten-class dataset Poker whose ratios are all smaller than 0.1 for the rule generation, inference, and total time. In term of the accuracy, the Micro-EBRBS is better than the big data FRBCS classifiers and the range of the ratio can be expressed as [1.039, 1.176] for four datasets.

Additionally, in order to detect significant differences among the accuracy of the Micro-EBRBS, Chi-Max, and Chi-Ave, the Friedman and Holm tests are carried out. Table XIII shows that two hypotheses are rejected because there are significant differences among the obtained results with a level of significance of $\alpha = 0.1$. Hence, in the datasets Census, Gas sensors, Covtype, and Poker, the accuracy of the Micro-EBRBS is better than that of the Chi-Max and Chi-Ave.

In summary, according to the comparison of the Micro-EBRBS and the big data FRBCS classifiers, it is evident that

the Micro-EBRBS has the advantage of using less computing time and obtaining better accuracy and robustness for the big data multiclass datasets.

E. Time Complexity Comparison

In this section, a comparison of the Micro-EBRBS, EBRBS, and Chi-FRBCS is provided to show which one has a better time complexity to deal with big data multiclass classification problems.

Suppose that there are L rules in EBRB (or sample data), \bar{L} rules in the reduced EBRB, S testing data, M antecedent attribute with J_i reference values, and N classes. The time complexity of different schemes in the EBRBS, Micro-EBRBS, and Chi-FRBCS is shown in Table XIV based on discussions in Sections II-B, III-C, and [29]. Additionally, in order to clearly compare three classifiers, their time complexity can be simplified under the following assumptions: 1) The number of sample data L is much bigger than the square of the number of classes N^2 ; 2) the total number of reference values for antecedent attributes $\sum_{i=1}^M J_i$ is much bigger than the number of classes N .

Remark 11: It is clear from Table XIV that the time complexity of the Micro-EBRBS is better than the EBRBS in both the rule generation and the inference schemes due to the following three reasons.

TABLE XIV
TIME COMPLEXITY OF EBRBS, MICRO-EBRBS, AND CHI-FRBCS

	Rule generation scheme	Inference scheme	Rule generation scheme ($L \gg N^2$)	Inference scheme ($\sum_{i=1}^M J_i \gg N$)
EBRBS	$O(L^2 \cdot \sum_{i=1}^M J_i + L^2 \cdot N)$	$O(S \cdot L \cdot (\sum_{i=1}^M J_i + N))$	$O(L^2 \cdot \sum_{i=1}^M J_i + L^2 \cdot N)$	$O(S \cdot L \cdot \sum_{i=1}^M J_i)$
Micro-EBRBS	$O(L \cdot \bar{L} \cdot \sum_{i=1}^M J_i + L \cdot \bar{L} \cdot N)$	$O(S \cdot \bar{L} \cdot (\sum_{i=1}^M J_i + N))$	$O(L \cdot \bar{L} \cdot \sum_{i=1}^M J_i + L \cdot \bar{L} \cdot N)$	$O(S \cdot \bar{L} \cdot \sum_{i=1}^M J_i)$
Chi-FRBCS	$O(L \cdot \sum_{i=1}^M J_i + L \cdot \bar{L} + N^2 \cdot \bar{L})$	$O(S \cdot \bar{L} \cdot \sum_{i=1}^M N^2)$	$O(L \cdot \sum_{i=1}^M J_i + L \cdot \bar{L})$	$O(S \cdot \bar{L} \cdot \sum_{i=1}^M N^2)$

- 1) The Micro-EBRBS has a simple process in the rule generation scheme because of excluding the calculation of rule weights comparing to the EBRBS.
- 2) The Micro-EBRBS has fewer rules in the reduced EBRB owing to using the proposed rule reduction method to downsize the EBRB comparing to the EBRBS.
- 3) The Micro-EBRBS can be implemented by using the Apache Spark to generate rules and classify test input data in parallel thanks to the better performance of independence in the rule generation and the inference schemes.

Remark 12: It is clear from Table XIV that the Chi-FRBCS is more efficient than the Micro-EBRBS in the rule generation scheme. However, the Micro-EBRBS is more efficient than the Chi-FRBCS in the inference scheme, especially for the multiclass classification problems. Additionally, the computing efficiency of both the two big data classifiers can be further improved by using the Apache Spark.

V. CONCLUSION

In this paper, the analysis of the rule weight calculation and the ER algorithm were carried out to investigate the approach of reducing the time complexity of the EBRBS, a popular advanced rule-based system, followed by an ER-C algorithm and a domain division-based rule reduction method proposed for developing a micro version of EBRBS with high computing efficiency, called Micro-EBRBS. Furthermore, the Apache Spark was introduced to implement the Micro-EBRBS for better dealing with big data multiclass classification problems. Fourteen classification datasets were used to validate the accuracy and computing efficiency of the Micro-EBRBS in comparison with the EBRBS, the conventional FRBCS and machine-learning classifiers, and the big data FRBCS classifiers. The detailed contributions are summarized as follows.

- 1) The necessity of the rule weight calculation and the ER algorithm involved in the EBRBS were investigated, in which the former demonstrates that it is unnecessary to calculate rule weights for each EBR under the assumption of large amount of data, the latter prove that the ER-C algorithm has the same functionality as the ER algorithm under the assumption of classification problems.
- 2) The division point and division domain were defined to divide the input space of the EBRBS into multiple local input spaces. Accordingly, the rule clustering strategy

and rule reduction strategy were further defined to propose a domain division-based rule reduction method to downsize EBRB.

- 3) The Micro-EBRBS, which includes the rule reduction and the ER-C algorithm but excludes the rule weight calculation in comparison with the EBRBS, and its implementation based on the Apache Spark were developed to deal with big data multiclass classification problems, which were then validated through the detailed case studies. The results have shown advantages of the Micro-EBRBS over the existing methods in terms of computing efficiency and classification accuracy.

For the future research, the application of Micro-EBRBS and further improvement to make it more effective to deal with the practical problem with uncertain and imbalance data.

APPENDIX A INFERENCE SCHEME OF EBRBS FOR CLASSIFICATION PROBLEMS

The inference scheme of the EBRBS mainly includes two steps [22]: 1) calculation of activation weights for each EBR using distance measure and 2) integration of activated rules for estimating classes using the ER algorithm.

One thing to note is that the procedure of the first step is the same as the Micro-EBRBS. Hence, the activation weight w_k ($k = 1, \dots, L$) can be obtained by using (23), which shows a positive correlation between the w_k and rule weight θ_k . After calculating activation weights, all activated rules should be integrated using the analytical ER algorithm [32], [37]

$$\beta_n = \left(\prod_{k=1}^L \left(w_k \beta_{n,k} + 1 - w_k \sum_{i=1}^N \beta_{i,k} \right) - \prod_{k=1}^L \left(1 - w_k \sum_{i=1}^N \beta_{i,k} \right) \right) / \left(\sum_{i=1}^N \prod_{k=1}^L \left(w_k \beta_{i,k} + 1 - w_k \sum_{j=1}^N \beta_{j,k} \right) - (N-1) \prod_{k=1}^L \left(1 - w_k \sum_{j=1}^N \beta_{j,k} \right) - \prod_{k=1}^L (1 - w_k) \right). \quad (28)$$

The integrated belief distribution of the test input data \mathbf{x} is

$$f(\mathbf{x}) = \{(D_n, \beta_n); n = 1, \dots, N\}. \quad (29)$$

Algorithm 3 Pseudocode of Rule Generation Scheme of the Chi-FRBCS

Input: $\mu_{y_i}(\mathbf{x}_t)$ and $R_{y_i}(\mathbf{x}_t)$ denote the membership degree and the fuzzy label set of the fuzzy rule which is transformed from the sample input data \mathbf{x}_t and the class y_i , $y_i \in \{D_1, \dots, D_N\}$; $Class_{i,j}(R_k)$ and $w_{i,j}(R_k)$ denote the class and the rule weight of the fuzzy rule R_k while considering the i th and the j th class as a two-class classification problem.

Output: the set of fuzzy rules **FRB**

```

01 Initialize FRB = {};
02 For each sample input data  $\mathbf{x}_t$  in  $\{x_1, \dots, x_L\}$ 
03 Initialize  $\mu_{y_i}(\mathbf{x}_t) = 1$  and  $R_{y_i}(\mathbf{x}_t) = \{\}$ ;
04 For each input data  $x_{t,i}$  in  $\mathbf{x}_t = \{x_{t,1}, \dots, x_{t,M}\}$ 
05 Calculate  $\mu_{y_i}(\mathbf{x}_t) = \mu_{y_i}(\mathbf{x}_t) * \max\{\mu_{A_{i,j}}(x_{t,i}); j = 1, \dots, J_i\}$ ;
06 Add  $R_{y_i}(\mathbf{x}_t) = R_{y_i}(\mathbf{x}_t) \cup \{A_{i,s}\}$ ;  $s = \arg \max_{s=1, \dots, J_i} \{\mu_{A_{i,j}}(x_{t,i})\}$ ;
07 End for
08 If  $\neg R_k \in \mathbf{FRB}$  and  $R_k = R_{y_i}(\mathbf{x}_t)$  then
09 Update  $\mu_{y_i}(R_k) = \mu_{y_i}(R_k) + \mu_{y_i}(\mathbf{x}_t)$ ;
10 Else if  $R_{y_i}(\mathbf{x}_t) \notin \mathbf{FRB}$  then
11 Add FRB = FRB  $\cup \{R_{y_i}(\mathbf{x}_t)\}$ ;
12 Initialize  $\mu_{y_i}(R_k) = \mu_{y_i}(\mathbf{x}_t)$ ;
13 End if
14 End for
15 For each fuzzy rule  $R_k$  in FRB
16 For each class  $D_i$  and  $D_j$  ( $i < j$ ) in  $\{D_1, \dots, D_N\}$ 
17 Initialize  $Class_{i,j}(R_k) = D_s$ ,  $s = \arg \max_{n=i,j} \{\mu_n(R_k)\}$ ;
18 Initialize  $w_{i,j}(R_k) = |\mu_i(R_k) - \mu_j(R_k)| / (\mu_i(R_k) + \mu_j(R_k))$ ;
19 End for
20 End for

```

For classification problems, suppose D_n denotes the n th class, the estimated class of the EBRBS can be obtained by seeking the greatest belief degree

$$f(x) = D_n, n = \arg \max_{i=1, \dots, N} \{\beta_i\}. \quad (30)$$

APPENDIX B

BELIEF DISTRIBUTION GENERATION AND RULE WEIGHT CALCULATION OF EBRBS

The belief distribution generation and the rule weight calculation are important processes in the rule generation scheme of the EBRBS and their details can be refer to [22].

One thing to note is that the detailed procedure of the belief distribution generation is the same as the Micro-EBRBS. Hence, for the k th ($k = 1, \dots, L$) EBR, the belief distributions of the i th antecedent attribute $S_i^k = \{(A_{i,j}, \alpha_{i,j}^k); j = 1, \dots, J_i\}$ ($i = 1, \dots, M$), and the consequent attribute $S^k = \{(D_n, \beta_{n,k}); n = 1, \dots, N\}$ can be obtained by using (16) and (19).

Definition 5 (Similarity of Two Belief Distributions): Suppose two belief distribution $P = (p_1, \dots, p_T)$ and $Q = (q_1, \dots, q_3)$, then the similarity of P and Q can be calculated as follows:

$$\begin{aligned} \text{Sim}(P, Q) &= 1 - d(P, Q) \\ &= 1 - \min \left\{ 1, \sqrt{\sum_{t=1}^T (p_t - q_t)^2} \right\} \end{aligned} \quad (31)$$

where $d(P, Q)$ denotes the distance between P and Q .

Based on Definition 5, for the k th ($k = 1, \dots, L$) EBR, the similarity of rule antecedent (SRA) and the similarity of rule

Algorithm 4 Pseudocode of Inference Scheme of the Chi-FRBCS

Input: $S_{i,j}(\mathbf{x}_t)$ denotes the score of test input data \mathbf{x}_t while considering the i th and the j th class as a two-class classification problem; $\mu(\mathbf{x}_t)$ denotes the membership degree of the test input data \mathbf{x}_t ; $w_{i,j}(R_k)$ denotes the rule weight of the fuzzy rule R_k while considering the i th and the j th class as a two-class classification problem.

Output: The estimated class $Class(\mathbf{x}_t)$ ($t = 1, \dots, S$)

```

01 For each test input data  $\mathbf{x}_t$  in  $\{x_1, \dots, x_S\}$ 
02 Initialize  $S_{i,j}(\mathbf{x}_t) = 0$  ( $i, j = 1, \dots, N$ )
03 For each  $D_i$  and  $D_j$  ( $i < j$ ) in  $\{D_1, \dots, D_N\}$ 
04 For each fuzzy rule  $R_k$  in FRB
05 Initialize  $\mu(\mathbf{x}_t) = 1$ ;
06 For each input data  $x_{t,m}$  in  $\mathbf{x}_t = \{x_{t,1}, \dots, x_{t,M}\}$ 
07 Calculate  $\mu(\mathbf{x}_t) = \mu(\mathbf{x}_t) * \mu_{A_m^k}(x_{t,m})$ ;
08 End for
09 If  $Class_{i,j}(R_k) = D_i$  and  $S_{i,j}(\mathbf{x}_t) < \mu(\mathbf{x}_t) * w_{i,j}(R_k)$  then
10 Calculate  $S_{i,j}(\mathbf{x}_t) = \mu(\mathbf{x}_t) * w_{i,j}(R_k)$ ;
11 Else if  $Class_{i,j}(R_k) = D_j$  and  $S_{j,i}(\mathbf{x}_t) < \mu(\mathbf{x}_t) * w_{i,j}(R_k)$  then
12 Calculate  $S_{j,i}(\mathbf{x}_t) = \mu(\mathbf{x}_t) * w_{i,j}(R_k)$ ;
13 End if
14 End for
15 End for
16 Calculate  $Class(\mathbf{x}_t) = D_n$ ;  $n = \arg \max_{i=1, \dots, N} \{\sum_{j=1}^N S_{i,j}(\mathbf{x}_t)\}$ 
17 End for

```

consequent (SRC) can be calculated as follows:

$$\begin{aligned} \text{SRA}(R_l, R_k) &= \min_{t=1, \dots, M} \left\{ \text{Sim}(S_t^l, S_t^k) \right\} \\ &= \min_{t=1, \dots, M} \left\{ 1 - \min \left\{ 1, \sqrt{\sum_{j=1}^{J_t} (\alpha_{t,j}^l - \alpha_{t,j}^k)^2} \right\} \right\} \end{aligned} \quad (32)$$

$$\begin{aligned} \text{SRC}(R_l, R_k) &= \text{Sim}(S^l, S^k) \\ &= 1 - \min \left\{ 1, \sqrt{\sum_{n=1}^N (\beta_{n,l} - \beta_{n,k})^2} \right\} \end{aligned} \quad (33)$$

where $l = 1, \dots, L$ and $l \neq k$; S_i^k denotes the belief distribution of the i th antecedent attribute in the k th EBR; S^k denotes the belief distribution of the consequent attribute in the k th EBR.

Definition 6 (Consistency of EBRs): Suppose the SRA and the SRC of the l th ($l = 1, \dots, L$) and the k th ($k = 1, \dots, L$; $k \neq l$) EBRs are $\text{SRA}(R_l, R_k)$ and $\text{SRC}(R_l, R_k)$, respectively, then the consistency of the rules R_l and R_k can be calculated as follows:

$$\text{Cons}(R_l, R_k) = \exp \left\{ - \frac{\left(\frac{\text{SRA}(R_l, R_k)}{\text{SRC}(R_l, R_k)} - 1 \right)^2}{\left(\frac{1}{\text{SRA}(R_l, R_k)} \right)^2} \right\}. \quad (34)$$

Based on Definition 6, the inconsistency degree of the k th EBR can be calculated as follows:

$$\begin{aligned} \text{Incons}(R_k) &= \sum_{l=1, l \neq k}^L (1 - \text{Cons}(R_l, R_k)) \\ &= \sum_{l=1, l \neq k}^L \left(1 - \exp \left\{ - \frac{\left(\frac{\text{SRA}(R_l, R_k)}{\text{SRC}(R_l, R_k)} - 1 \right)^2}{\left(\frac{1}{\text{SRA}(R_l, R_k)} \right)^2} \right\} \right). \end{aligned} \quad (35)$$

Finally, the rule weight is calculated as follows:

$$\theta_k = 1 - \frac{\text{Incons}(R_k)}{\sum_{j=1}^L \text{Incons}(R_j)}. \quad (36)$$

APPENDIX C

RULE GENERATION AND INFERENCE SCHEMES OF CHI-FRBCS

A. Time Complexity of the Rule Generation Scheme

The rule generation scheme of the Chi-FRBCS, which consists of the PCF and OVO, is introduced as follows.

As shown in the pseudocode of the rule generation scheme, the time complexity of calculating $\mu_{y_i}(x_i)$ shown in the fifth line and updating the set FRB shown in the 8th to the 13th lines is $O(\sum_{i=1}^M J_i)$ and $O(\bar{L})$, respectively, for each sample data, where \bar{L} is the number of fuzzy rules. In addition, from the 15th to the 20th lines, its time complexity is $O(\bar{L} \times N^2)$. As a result, the time complexity of the rule generation scheme involved in the Chi-FRBCS is shown as follows:

$$O\left(L \times \left(\sum_{i=1}^M J_i + \bar{L}\right) + \bar{L} \times N^2\right). \quad (37)$$

B. Time Complexity of the Inference Scheme

The inference scheme of the Chi-FRBCS, which consists of the winning rule strategy and OVO, is introduced as follows.

From the pseudocode of the rule generation scheme, the time complexity of calculating $\mu(x_i)$ shown in the seventh line is $O(M)$ for each test input data, fuzzy rule, and two-class classification problem. Hence, the time complexity of the inference scheme involved in the Chi-FRBCS is shown as follows:

$$O(S \times \bar{L} \times M \times N^2). \quad (38)$$

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Long-Hao Yang received the B.Eng. and M.Eng. degrees in computer science from Fuzhou University, Fuzhou, China, in 2012 and 2015, respectively, where he is currently pursuing the Ph.D. degree in information management and information system.

He has published over ten articles in belief rule-based (BRB)-related studies. His current research interests include BRB learning, machine learning and artificial intelligence, and BRB applications in complex systems modeling.



Jun Liu (M'09) received the B.Sc. and M.Sc. degrees in applied mathematics and the Ph.D. degree in information engineering from Southwest Jiaotong University, Chengdu, China, in 1993, 1996, and 1999, respectively.

He is currently a Reader in Computer Science with Ulster University, Coleraine, U.K. He has published over 80 papers in journals indexed by the SCI and over 80 contributions in international conferences. His current research interests include logic and reasoning methods for intelligent systems and

formal verification, intelligent decision support systems (DSSs), and information management, with applications in health care, engineering, and industry, as well as applied computational intelligence for uncertainty analysis and optimization.



Ying-Ming Wang received the M.Sc. degree in systems engineering from the Huazhong University of Science and Technology, Wuhan, China, in 1987 and the Ph.D. degree in automatic control theory and application from Southeast University, Nanjing, China, in 1991.

He is currently a Full Distinguished Professor of the Changjiang Scholars Program with Fuzhou University, Fuzhou, China. He has published over 123 SCI and 28 SSCI-indexed journal papers and has been one of the most cited Chinese researchers

since 2014. His current research interests include multiple criteria decision analysis, data envelopment analysis, rule-based inference, and quality function deployment.



Luis Martínez (M'10) received the M.Sc. and Ph.D. degrees in computer science from the University of Granada, Granada, Spain, in 1993 and 1999, respectively.

He is currently a Full Professor with the Computer Science Department, University of Jaén, Jaén, Spain. He has also been a Visiting Professor with the University of Technology Sydney, Sydney, NSW, Australia; the University of Portsmouth, Portsmouth, U.K. (Isambard Kingdom Brunel Fellowship Scheme); and the Wuhan University of Technology, Wuhan, China, (Chutian Scholar); a Guest Professor with Southwest Jiaotong University, Chengdu, China; and an Honourable Professor with Xihua University, Chengdu. He has co-edited 11 journal special issues on fuzzy preference modeling, soft computing, linguistic decision making, and fuzzy sets theory. He has been a Main Researcher in 14 Research and Development projects. His current research interests include decision making, fuzzy logic-based systems, computing with words, and recommender systems. He has also published over 100 papers in journals indexed by the SCI and over 150 contributions in international conferences related to the above areas.

Dr. Martínez was a recipient of the IEEE TRANSACTIONS ON FUZZY SYSTEMS Outstanding Paper Award in 2008 and 2012 (bestowed in 2011 and 2015, respectively) and the *Highly Cited Researcher* Award in Computer Science in 2017. He is the Co-Editor-in-Chief of the *International Journal of Computational Intelligence Systems* and an Associate Editor of the journals IEEE TRANSACTIONS ON FUZZY SYSTEMS, *Information Fusion*, the *International Journal of Fuzzy Systems*, and the *Journal of Intelligent and Fuzzy Systems*. He is a member of the European Society for Fuzzy Logic and Technology.