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# Consensus reaching for social network group decision making with ELICIT information: A perspective from the complex network

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ABSTRACT

Consensus reaching is essential in group decision-making (GDM) since it can mitigate conflicts between expert opinions and promotes the further implementation of decision-making results. Meanwhile, interaction between experts commonly occurs within social networks and in practical GDM problems. Therefore, it neecessary to consider the trust relationship between experts and utilize it to facilitate the consensus-reaching process (CRP). However, most existing social network-based GDM studies mainly use local measures (e.g., degree centrality) to determine the importance of experts, which cannot reflect their actual influence on a global topological structure. To address this issue, we propose a novel consensus-reaching strategy from the perspective of complex network analysis. First, the Extended Comparative Linguistic Expressions with Symbolic Translation (ELICIT) is adopted to flexibly facilitate the expression of experts' uncertain evaluations. The hybrid centrality is then defined to determine the influence of experts in the social network by considering both node importance and edge weight. Since experts with greater influence have stronger information propagation capabilities, hybrid centrality is utilized to guide the CRP, which can better reflect information flows in the social network. Additionally, the BWM-CRITIC weighting method is developed to reflect the significance and relationship among criteria. Finally, we verify the effectiveness and superiority of the proposed method by means of a case study on a sustainable supplier selection problem.

## 1. Introduction

Group decision-making (GDM) problems are pervasive in our lives; they are problems that involve a group of experts evaluating a set of alternatives regarding each criterion to determine an optimal solution [1,2]. GDM has received increasing attention from scholars, and the proposed methodologies have been widely applied in various fields, such as design alternative assessment [3] and urban resettlement project selection [4].

In GDM, qualitative criteria are often ambiguous and imprecise, making it difficult for experts to evaluate them with accurate figures [5]. The fuzzy linguistic variable was proposed and used to describe expert opinions so as to facilitate the expression of human perception [6]. However, a single linguistic term cannot reflect the hesitancy of experts when judging the performance of alternatives. To address this issue, Rodríguez et al. [7] proposed a hesitant linguistic term set (HFLTS) to further assist the evaluation of uncertainty. Recently, Labella et al. [8] generalized the HFLTS by extending the representation of comparative linguistic expressions to a

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continuous domain and proposed the Extended Comparative Linguistic Expressions with Symbolic Translation (ELICIT). Compared with other linguistic expression structures, ELICIT is closer to the human reasoning process and can improve the interpretability and accuracy of results [9,10]. To our knowledge, ELICIT has not yet been utilized for consensus-reaching in GDM. Given its advantages in expressing uncertain information, we have employed ELICIT in this study to represent expert evaluations throughout the GDM process.

As it coordinates different evaluations given by all individuals and ensures that the GDM result can be effectively implemented, consensus building among decision-makers in the GDM process has become a research hotspot in recent years [11]. Consensus reaching refers to a process in which evaluations that deviate largely from others are guided to make appropriate modifications that ultimately bring the group consensus level to a given threshold [12]. At the same time, it is worth noting that experts are not isolated in the group; there are trust relationships between them, through which they exchange information and are influenced by those they trust. Using these relationships between experts to promote consensus brings about social network group decision-making (SNGDM) problems [13]. Researchers have proposed multiple consensus-reaching methods for SNGDM, which can be mainly divided into two categories:

1. The first category belongs to the optimization-based methods. For example, Lu et al. [14] developed a robust optimization-based minimum cost consensus model to coordinate the assessment of experts in a social network. Yuan et al. [15] developed a minimum adjustment consensus approach to determine the modified opinion with incomplete decision information. To analyze the impact of the decision-makers' willingness to adjust on the decision-making result, Liu et al. [16] proposed a bounded confidence-based optimization model for GDM problems. Later, Liu et al. [17] established a maximum consensus-based model for SNGDM with linguistic distribution assessments. Wu et al. [18] developed a maximum self-esteem model to produce personalized suggestions for reaching higher group consensus.
2. The second category is the consensus improvement method based on the identification-direction mechanism, in which the opinions that contribute less to the group consensus are identified, and then the direction rule is introduced to guide their modifications. For instance, in [19], the identified opinions are modified toward the corresponding group opinion. Specifically, the extent of modification is determined by the current individual and group consensus levels. In [20], the identified evaluations are also modified toward the group opinion, but the adjustment coefficient is obtained considering the non-cooperative behavior of the experts involved. In [21], it is suggested that the identified evaluations shifts towards the evaluation closer to the group opinion. In [22], Lu et al. presented an identification mechanism to detect experts' distrust behaviors and gave three modification strategies to manage different types of distrust behaviors while improving the group consensus.

To date, most existing consensus-reaching methods for SNGDM mainly utilize single local measures, such as degree centrality, to determine the importance of experts, which ignores the global information of the network [23,24]. In complex network analysis, identifying influential nodes is indispensable to understanding the process of information propagation, as influencers play a key role in functional and structural aspects [25]. The social network is a type of complex network, therefore, a more reasonable method neesd to be developed to determine the influence of the experts within the group by considering both local and global network information. Then, we can utilize the derived influence degree of experts to better guide CRP. To fill in this gap, we propose a novel consensus-reaching strategy from the perspective of the complex network analysis. Specifically, hybrid centrality is defined to measure the influence of experts by considering node importance and edge weight. It is then suggested to the experts identified with evaluations that do not meet the consensus requirement that they modify their opinions to be closer to those they trust. The greater the influence of the trusted expert in the social network, the greater the degree of reference when modifying the corresponding evaluation. In this way, the consensus-reaching process can be more realistic and can better reflect the flow of information in the network.

After the group consensus level (GCL) satisfies a given threshold, the weight of the criteria should be determined to select an optimal alternative. Criteria weight has a significant impact on the results of SNGDM and therefore should be carefully evaluated. Different objective and subjective weighting methods have been proposed and applied to various GDM problems. For example, in objective methods, Saraswat et al. [26] utilized the entropy method to obtain the criteria weight in the evaluation of sustainable energy alternatives and emergency plans, respectively. In terms of the subjective methods, the weightings of the different criteria were given directly in the gas refinery risk classification problem. Later, Buran et al. [27] extended the traditional AHP to fuzzy environments to obtain the attribute weight in the public transportation business model evaluations. Additionally, in [28], the weights of influencing factors were obtained by maximizing the group consensus level.

However, utilizing a subjective approach alone neglects decision information and will make the results too dependent on expert preferences; while using objective techniques alone cannot reflect experts' subjective inclinations. Therefore, to take advantage of both objective and subjective methods, a combination of approaches was developed for criteria weight distribution. For example, AHP and the entropy method were integrated to compute the weight of evaluation criteria for heavy-duty machine tool remanufacturing analysis [29]. Niu et al. [30] combined the AHP and maximum deviation method to evaluate the operation performance of elevators. However, in AHP, when the number of criteria involved increases, the number of pairwise comparisons increases substantially, which brings a heavy workload for experts and decreases the preference consistency [31]. In fact, most of the existing combination methods ignore the correlations between criteria, which reduces the accuracy of the results. To overcome this limitation, we propose the BWM-CRITIC (Best-Worst method-CRiteria Importance Through Intercriteria Correlation) method. The Best-Worst Method is an extension of AHP, which can reduce the number of pairwise comparisons from $n^2$ to $2n-3$ [32]. Additionally, the weight coefficient has a greater consistency ratio in BWM. Therefore, we extend the classical BWM to the ELICIT environment to better determine the subjective weight. From an objective point-of-view, the Spearman correlation coefficient is introduced to the CRITIC method to obtain the objective weight of criteria. With the BWM-CRITIC method, the subjective aspect, objective aspect, and the correlations between criteria can be considered simultaneously in the weighting process.

Based on the above analysis, we propose a novel consensus-reaching strategy with ELICIT information for SNGDM based on complex networks and apply it to a sustainable supplier selection problem. The main contributions are summarized as follows.

(1) The ELICIT information is first applied to consensus building to express vague evaluations given by experts. Compared with other expression structures of uncertain information, ELICIT is closer to the human reasoning process and can retain more information throughout computations, which can improve the accuracy of decision-making outcomes.
(2) We have developed a novel consensus-reaching strategy from the perspective of complex network analysis. Unlike most current studies that only utilize local measures to characterize the importance of experts in social networks, we defined the hybrid centrality to comprehensively determine experts' influence with both local and global network information. Since the experts with greater influence are more important in the propagation of information, hybrid centrality is used to improve the group consensus, which can better reflect reality.
(3) We have proposed the BWM-CRITIC method to obtain the weight of criteria in a more reasonable way. The weight of criteria is more reasonably obtained by developing the BWM-CRITIC method. On the one hand, to overcome the limitations of AHP, the traditional BWM method is extended to the ELICIT environment to determine the subjective weight. On the other hand, the Spearman correlation coefficient is incorporated into the CRITIC method to obtain the objective weight considering the criteria correlation. In this way, coordination between subjectivity and objectivity can be achieved during the weight determination process.
(4) We have applied the proposed SNGDM method to address a sustainable supplier selection problem, in which a set of criteria involving social, economic, and environmental aspects is constructed. Sensitivity, validity, and comparative analysis demonstrate the robustness and effectiveness of our method in handling real-life group decision-making problems.

The rest of this paper is organized as follows. Section 2 briefly reviews the basic concepts of the ELICIT information and complex network analysis. Section 3 develops the consensus-reaching strategy in detail. Section 4 validates the effectiveness of the proposed method with a case study on a sustainable supplier selection problem. Discussions are presented in Section 5 to illustrate the robustness, rationality, and superiority of our method. Finally, Section 6 provides the conclusions and future directions.

## 2. Preliminaries

In this section, the basic concepts of the ELICIT information and complex network analysis are briefly reviewed.

### 2.1. The ELICIT information

To facilitate the interpretability and precision of computing with words, Labella et al. [8] proposed a flexible linguistic representation model called ELICIT. By utilizing the symbolic translation concept associated with the 2-tuple linguistic model, ELICIT extends the definition of the comparative linguistic expressions into a continuous domain built from a context free grammar source [33]. Since it can retain more information during the process of computing with words, ELICIT was utilized in this study to model expert evaluations.

**Definition 1.** [8]. Let $S = \{s_0, s_1, \ldots, s_g\}$ be a set of linguistic terms, and $g+1$ is the granularity of $S$. The possible ELICIT expressions can be represented as: "$at\,least(s_i, \alpha)^{\gamma}$", "$at\,most(s_j, \alpha)^{\gamma}$", or "$between(s_i, \alpha_1)^{\gamma_1}\,and(s_j, \alpha_2)^{\gamma_2}$", where $\alpha$is the symbolic translation parameter with $\alpha \in [-0.5, 0.5)$, $\gamma$is the adjustment parameter with $\gamma \in \left(-\frac{1}{2g}, \frac{1}{2g}\right)$ for $i,j = 1, 2, \ldots, g$.

The process of computing with words for ELICIT involves three stages: translation, manipulation, and re-translation, which are shown as follows.

**Definition 2.** [8]. Let $x_{EL}$ be an ELICIT expression and $Tr(a, b, c, d)$ denotes a trapezoidal fuzzy number (TrFN). The transformation function $\lambda^{-1}$ can be defined as:

$$\lambda^{-1} : x_{EL} \rightarrow Tr(a, b, c, d) \tag{1}$$

This function can be defined in various ways according to the specific ELICIT expression. For details, please refer to reference [9].

**Definition 3.** [8]. The manipulation process is about carrying out the fuzzy arithmetic computations with the TrFNs derived from the transformation stage. Let $Tr_A(a_1, b_1, c_1, d_1)$ and $Tr_B(a_2, b_2, c_2, d_2)$ represent two fuzzy envelops modeled by two TrFNs. The addition of these two fuzzy envelops can be defined by a shape function $\mu_{A+B}$ as:

$$\mu_{A+B} = \begin{cases} \dfrac{(x-(a_1+a_2))^n}{(b_1+b_2)-(a_1+a_2)} & a_1+a_2 \leqslant x \leqslant b_1+b_2 \\ 1 & b_1+b_2 \leqslant x \leqslant c_1+c_2 \\ \dfrac{((d_1+d_2)-x)^n}{(d_1+d_2)-(c_1+c_2)} & c_1+c_2 \leqslant x \leqslant d_1+d_2 \\ 0 & otherwise \end{cases} \tag{2}$$

**Definition 4.** [8]. The subtraction of the two fuzzy envelops by two TrFNs $Tr_A(a_1, b_1, c_1, d_1)$ and $Tr_B(a_2, b_2, c_2, d_2)$ can be defined with a shape function $\mu_{A-B}$ as:

$$\mu_{A-B} = \begin{cases} \dfrac{(x-(a_1-d_2))^n}{(b_1-a_1)+(d_2-c_2)} & a_1-d_2 \leqslant x \leqslant b_1-c_2 \\ 1 & b_1-c_2 \leqslant x \leqslant c_1-b_2 \\ \dfrac{((d_1-a_2)-x)^n}{(d_1-c_1)+(b_2-a_2)} & c_1-b_2 \leqslant x \leqslant d_1-a_2 \\ 0 & otherwise \end{cases} \tag{3}$$

**Definition 5.** [8]. In the re-translation stage, the TrFN $\widetilde{\beta}$obtained in the manipulation process is transformed into an equivalent ELICIT expression by the inverse function $\lambda$. Therefore, the re-translation function $\lambda : \widetilde{\beta} \rightarrow \widetilde{x_{EL}}$ is a mapping defined as follows:

(1) If $\widetilde{\beta} = Tr(a, b, 1, 1)$, then $\lambda\left(\widetilde{\beta}\right) = at\,least\,(s_i, \alpha)^{\gamma}$.
(2) If $\widetilde{\beta} = Tr(0, 0, c, d)$, then $\lambda\left(\widetilde{\beta}\right) = at\,most\,(s_j, \alpha)^{\gamma}$.
(3) If $\widetilde{\beta} = Tr(a, b, c, d)$, then $\lambda\left(\widetilde{\beta}\right) = between(s_i, \alpha_1)^{\gamma_1} and (s_j, \alpha_2)^{\gamma_2}$.

**Definition 6.** [8]. Let $x_{EL1}$ and $x_{EL2}$ be two ELICIT expressions, and let $\widetilde{\beta_1} = Tr_1(a_1, b_1, c_1, d_1)$ and $\widetilde{\beta_2} = Tr_2(a_2, b_2, c_2, d_2)$ be their equivalent fuzzy numbers obtained from $\lambda^{-1}(x_{EL1})$ and $\lambda^{-1}(x_{EL2})$, respectively. Then, the distance between $x_{EL1}$ and $x_{EL2}$ can be determined as:

$$d(x_{EL1}, x_{EL2}) = d\left(\widetilde{\beta}_1, \widetilde{\beta}_2\right) = \sqrt{\frac{(a_1-a_2)^2 + (b_1-b_2)^2 + (c_1-c_2)^2 + (d_1-d_2)^2}{4}} \tag{4}$$

**Definition 7.** [34]. The expectation of an ELICIT expression $x_{EL}$ can be obtained by calculating the magnitude of its equivalent fuzzy number $\widetilde{\beta} = Tr(a, b, c, d)$, which is defined as:

$$E(x_{EL}) = Mag\left(\widetilde{\beta}\right) = \frac{1}{12}(a + 5b + 5c + d) \tag{5}$$

### 2.2. Complex network analysis

Complex network analysis is the study of the nature and behavior of systems considering the interaction between network elements. Any complex system can be studied as a complex network by abstracting its constituent units into nodes and abstracting the interrelationships between units as edges. Research on complex networks has received a lot of attention from scholars lately [35]. Many real-life problems can be boiled down to problems in network science, and many real-world systems can be characterized as complex networks, such as power networks, biological networks, and social networks.

In complex networks, influencers play an essential role in the flow of information. Therefore, identifying influential nodes is critical from both functional and structural perspectives in network science.

In this subsection, several basic centrality measures and the k-shell method are first given to present some existing typical methods for determining the node importance. We then briefly revise the *Effective Distance* that can reveal the hidden geometry of complex networks. Let $G = (E, L)$ denote an undirected graph, in which $E$ denotes the set of nodes and $L$ denotes the set of edges. The number of nodes in $G = (E, L)$ is $N$. The adjacency matrix of this network can be represented as $T = [t_{ij}]_{N \times N}$, where $t_{ij} = 1$ if there is an edge between node $i$ and $j$, otherwise $t_{ij} = 0$. In the context of social networks, the experts are considered nodes and the trust relationships between them are considered edges.

#### 2.2.1. Centrality measures

There are various centrality measures to determine the importance of nodes in a network. Typical measures include degree centrality (DC), betweenness centrality (BC), and closeness centrality (CC), which are introduced as follows.

**Definition 8.** [36]. (Degree centrality) The degree centrality is the most widely used measure of centrality, which determines the importance of nodes by comparing their degrees. The degree centrality (DC) of node $i$ can be computed as:

$$DC(i) = \sum_{j=1, j\neq i}^{N} t_{ij} \tag{6}$$

where $N$ denotes the number of nodes in the network. $t_{ij} = 1$ if there is an edge between node $i$ and $j$, otherwise $t_{ij} = 0$. The degree centrality $DC(i)$ can also be represented as $k(i)$.

**Definition 9.** [36]. (Betweenness centrality) The betweenness centrality is concerned with the concentration degree of the path. It measures the importance of a node by the number of the shortest paths that run through it. The betweenness centrality (BC) of node $i$ can be defined as follows.

$$BC(i) = \frac{\sum_{j\neq k\neq i} P_{jk}(i)}{\sum_{j\neq k} P_{jk}} \tag{7}$$

where $P_{jk}$ denotes the number of the shortest paths from node $j$ to node $k$, and $P_{jk}(i)$ denotes the number of the shortest paths from node $j$ to node $k$ through node $i$.

**Definition 10.** [36]. (Closeness centrality) The closeness centrality first determines the sum of the shortest distance from one node to the others. Then the influence of nodes are computed by the reciprocal of the sum of the shortest path between nodes. The closeness centrality (CC) of node $i$ is defined as:

$$CC(i) = \frac{1}{\sum_{j\neq i} d_{ij}} \tag{8}$$

where $d_{ij}$ represents the shortest distance between node $i$ and $j$, which can be computed by the number of edges in the geodesic linking node $i$ and $j$.

#### 2.2.2. K-shell method

The k-shell (KS) decomposition method, proposed by Kitsak [25], is a technique in graph theory and has been used as a visualization tool for studying networks such as the Internet. Thus, the node importance is determined by separating all nodes into different shells. The k-shell technique begins by removing all nodes in the network with degree centrality $DC(i) = 1$, then this process is performed iteratively until there are no nodes with one degree in the network. All the removed nodes constitute a 1-shell.In the same manner, we recursively remove all nodes with degree centrality $DC(i) = 2$, creating a 2-shell. Then, we continue this process, increasing $k$ until all nodes in the network have been assigned to one of the shells. In this way, we can derive the 3-shell, 4-shell and so on, until, finally, the k-shell value of every node can be determined. The nodes with larger k-shell values are located more centrally in the network and are more important.

#### 2.2.3. Effective distance

*Effective Distance*, proposed by Dirk Brockmann and Dirk Helbing, is a probabilistically motivated distance measure. It can disclose the hidden pattern geometry of complex networks. The essence of *Effective Distance* is to discover the most probable path between two nodes by calculating the probability using information from the network. The original definition of *Effective Distance* is given as follows.

**Definition 11.** [37]. Let $0 \leqslant P_{mn} \leqslant 1$ represent the fraction of travelers that leave node $n$ and arrive at node $m$, we define the *Effective Distance* from a node $n$ to a connected node $m$ as:

$$d_{mn} = 1 - \log_2 P_{mn} \tag{9}$$

where $P_{mn} = \frac{F_{mn}}{F_n}$ and $F_n = \sum_m F_{mn}$. $F_{mn}$ reflects the traveler flux from node $n$ to node $m$. The adjacency matrix, also called connection matrix, is a square matrix used to describe a complex network or a finite graph. Let $T = [t_{ij}]_{N\times N}$ represent a complex network with $N$ nodes. For an unweighted network, $t_{ij} = 1$ indicates that there is a connection from node $i$ to node $j$, otherwise $t_{ij} = 0$. Therefore, it is reasonable to extend the original definition of *Effective Distance* to the following.

**Definition 12.** The *Effective Distance* from node $i$ to node $j$ which are directly connected can be computed as:

$$ED_{ij} = 1 - \log_2\left(\frac{t_{ij}}{k(i)}\right) \tag{10}$$

where $t_{ij}$ is the element in the adjacency matrix $T$ of the complex network, which can reflect the flux from node $i$ to node $j$. $k(i)$ is the degree centrality of node $i$, which can reflect the sum of fluxes from node $i$ to the other nodes in the network. Therefore, $\frac{t_{ij}}{k(i)}$ quantifies the fraction of flux from node $i$ to node $j$, which is a reasonable extension of $P_{mn} = \frac{F_{mn}}{F_n}$ in Definition 11. For indirectly connected nodes, the *Effective Distance* can be determined based on transitivity. For instance, the *Effective Distance* from node $p$ to node $q$ can be obtained using $ED_{pq} = ED_{pi} + ED_{iq}$, where node $p$ and node $i$, node $i$ and node $q$ are directly connected.

Unlike the traditional Euclidean distance, the *Effective Distance* is directional and asymmetric. If there are several paths from node $i$ to node $j$, the shortest *Effective Distance* between them will be chosen as the ultimate *Effective Distance*.

## 3. Consensus reaching with ELICIT based on complex network analysis

This section introduces a novel consensus-reaching strategy with ELICIT based on complex network analysis. First, we provide a succinct description of typical SNGDM problems. In subSection 3.2, the consensus-reaching strategy is elaborated, subsection 3.3 details the weighting method for criteria and the framework of the proposed method is then illustrated in subSection 3.4.

### 3.1. Problem description

Suppose there are $m$ alternatives denoted as $A = \{a_i | i = 1, 2, \ldots, m\}$, $n$ criteria denoted as $C = \{c_j | j = 1, 2, \ldots, n\}$, and $r$ invited experts represented as $E = \{e_k | k = 1, 2, \ldots, r\}$. The weight of criteria can be expressed as $w_j (j = 1, 2, \ldots, n)$ with $w_j \in (0, 1)$ and $\sum_{j=1}^{n} w_j = 1$. Due to the uncertainty of human perception, the ELICIT expression is utilized to evaluate the performance of alternatives regarding each criterion with a 7-scale linguistic term set $S = \{s_0 = \text{very bad}, s_1 = \text{bad}, s_2 = \text{slightly bad}, s_3 = \text{medium}, s_4 = \text{slightly good}, s_5 = \text{good}, s_6 = \text{very good}\}$. $X = \left[x_{ij}^k\right]_{m \times n}$ is the decision information matrix, where $x_{ij}^k$ is an ELICIT representing expert $e_k$'s evaluation towards alternative $a_i$ regarding criterion $c_j$.

$$X^k = \left[x_{ij}^k\right]_{m \times n} = \begin{array}{c|c} & \begin{matrix} c_1 & c_2 & \cdots & c_n \end{matrix} \\ \begin{matrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{matrix} & \begin{bmatrix} x_{11}^k & x_{12}^k & \cdots & x_{1n}^k \\ x_{21}^k & x_{22}^k & \cdots & x_{2n}^k \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1}^k & x_{m2}^k & \cdots & x_{mn}^k \end{bmatrix} \end{array} \left(k = 1, 2, \ldots, r\right) \tag{11}$$

The social network among $r$ experts is portrayed using a directed graph $G = (E, L)$, in which $E$ is the set of experts and $L$ denotes the set of trust relationships between them. The adjacency matrix can be represented as $T = [t_{kl}]_{r \times r}$, where $t_{kl}$ is a 0–1 variable indicating whether there is a trust relationship from $e_k$ to $e_l$.

We aim to improve the group consensus for SNGDM and determine an acceptable alternative classification based on the decision information expressed with ELICIT and the social network adjacency matrix.

### 3.2. The consensus-reaching strategy in social networks

First, the consensus measurement of the evaluation, expert and group levels is introduced. The feedback mechanism is then presented to improve group consensus based on the complex network perspective.

#### 3.2.1. Consensus measurement

In SNGDM, it is essential to ensure that the group consensus can reach a certain level of satisfaction before making the final decisions. The consensus measurement can be used to determine the current level of agreement within the group. Palomares et al. [38] presented a taxonomy in which consensus can be quantified in two ways: the first is according to the deviation between personal opinions and group assessment, and the other is based on the divergence between personal evaluations. Here, we consider the second approach to define the three levels of consensus measurement.

**Definition 13.** (Consensus at the evaluation level) Let $x_{ij}^k$ and $x_{ij}^l$ be the evaluation from $e_k$ and $e_l (k, l = 1, 2, \ldots, r)$ on $a_i (i = 1, 2, \ldots, m)$ regarding $c_j (j = 1, 2, \ldots, n)$, respectively. Then, the consensus measurement at the evaluation level is defined as:

$$CL_{ij}^k = \frac{1}{r-1} \sum_{l=1, l \neq k}^{r} \left(1 - d\left(x_{ij}^k, x_{ij}^l\right)\right) \tag{12}$$

where $d\left(x_{ij}^k, x_{ij}^l\right)$ is the deviation between evaluations $x_{ij}^k$ and $x_{ij}^l$.

**Definition 14.** (Consensus at the expert level) The consensus level of $e_k(k = 1, 2, \ldots, r)$ on all alternatives regarding all criteria is defined as:

$$CL^k = \frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n} CL_{ij}^k \tag{13}$$

**Definition 15.** (Consensus at the group level) The group consensus level is defined as:

$$GCL = \frac{1}{r}\sum_{k=1}^{r} CL^k \tag{14}$$

with $GCL \in [0, 1]$, and a larger GCL indicates a higher consensus among the group members. To ensure the current evaluations are acceptable, the group consensus level will be compared with a predefined threshold $\vartheta$. If $GCL \geqslant \vartheta$, then we can carry out the criteria weighting procedure; otherwise, the feedback strategy should be implemented to improve group consensus.

*3.2.2. The feedback mechanism*

The feedback mechanism is built to help the group of experts to reach the preset soft consensus. First, the evaluations that contribute less to a sufficient GCL are identified using two levels.

(1) The experts with inadequate consensus degrees are identified as:

$$EPS = \left\{k \middle| CL^k < \vartheta\right\} \tag{15}$$

(2) For the experts in the set of *EPS*, the evaluations whose consensus levels are below the threshold are also identified as:

$$EVS = \left\{(i, j, k) \middle| k \in EPS \wedge CL_{ij}^k < \vartheta\right\} \tag{16}$$

Suppose the evaluations to be modified are represented as $x_{ij}^k(i, j, k \in EVS)$, subsequently, the direction rules will be generated to guide them in making modifications to the social network.

In real-life complex networks, the nodes with greater influence have a greater ability to regulate the information propagation. Therefore, determining the influence degree of nodes is an essential task in network science. A social network is a type of complex network and the consensus-reaching process in the network structure relies on the flow of information. In this study, we define hybrid centrality to determine the influence of experts from both local and global perspectives and utilize this influence to guide the consensus-reaching process.

**Step 1.** Compute the *Effective Distance* between the connected experts in the social network. Unlike the traditional Euclidean distance that only focuses on the static topological distance of the nodes, the *Effective Distance* can discover the hidden pattern geometry between two nodes by determining the probability using network information. In the original definition of *Effective Distance* in [37], $P_{mn}$ refers to the fraction of travelers or passengers that leave node $n$ and arrive at node $m$, which is also called the fraction of flux. In the Oxford dictionary, flux is defined as a flow or an act of flowing. In [37], it refers to the passenger flux or the traveler flux. In our paper, the specific type of complex network is a social network, where the adjacency matrix describes the trust relationships between experts. $t_{kl}$ reflects the trust from expert $e_k$ to expert $e_l$, and $DC(e_k)$ reflects the sum of trust from expert $e_k$ to the others. Therefore, $\frac{t_{kl}}{DC(e_k)}$ refers to the fraction of trust from expert $e_k$ to $e_l$, which is a reasonable extension.

Therefore, the idea of *Effective Distance* is introduced in this consensus-reaching strategy to calculate the distance between two experts in the social network. The *Effective Distance* from $e_k(k = 1, 2, \ldots, r)$ to a connected expert $e_l(l = 1, 2, \ldots, r)$ can be calculated as:

$$ED_{kl} = 1 - \log_2\left(\frac{t_{kl}}{DC(e_k)}\right) \tag{17}$$

where $t_{kl}$ is the element in adjacency matrix $T = [t_{kl}]_{r\times r}$ and $DC(e_k)$ is the degree centrality of expert $e_k$.

**Step 2.** Calculate the diffusion importance of the edge connecting pairs of experts.

Edge weighting plays an important part in complex network analysis and should be differentiated based on the connected nodes. Take the traffic network as an example, the road between two metropolises is more important than the link between two villages. Therefore, the edge connecting pairs of experts in the social network should be considered when determining their influence degree. The k-shell decomposition method shows that the nodes with higher k-shell values are located more centrally in the network, which captures the global structure of the network and can reflect the node importance from a global perspective. Meanwhile, the degree centrality of the nodes reflects the local characteristics of the network. Therefore, it can be reasonably considered that the edge weight is positively correlated with k-shell values and the degree centrality of the connected experts, and negatively correlated with the *Effective Distance* between them. In this way, both local and global network topological structures can be considered. The calculation formula is shown as follows.

$$\omega_{kl} = \frac{\alpha(DC(e_k) + DC(e_l)) + (KS(e_k) + KS(e_l))}{ED_{kl}} \tag{18}$$

where $ED_{kl}$ is the *Effective Distance* from $e_k$ to $e_l$. $DC(e_k)$ denotes the degree centrality of $e_k$, which can be calculated using Eq. (6). $KS(e_k)$ denotes the k-shell value of $e_k$, which can be determined based on subSection 2.2.2. Given a social network of $r$ experts, though the value range of degree centrality and k-shell is $[0, r-1]$ and $[1, r-1]$, respectively, the KS value of the nodes is mainly distributed over a range of small values. Therefore, the KS value and the degree centrality are not comparable. So, $\alpha = \frac{\overline{KS}}{\overline{DC}}$, i.e., the ratio of the average k-shell value and average degree centrality of all nodes in the network, is utilized to bring the DC and KS measures to a uniform scale.

**Step 3.** Determine the hybrid centrality (HC) of experts.

The influence of a node in the network should not only be determined by itself but also depends on the importance of its connected neighbor nodes and the propagation capabilities of the connected edges. Therefore, we propose hybrid centrality to determine the comprehensive influence of an expert by considering their neighbor's importance and the weight of their relationship. The edge weight is utilized to adjust the impact of their neighbors' centrality. The hybrid centrality of an expert can be obtained as follows.

$$HC\left(e_k\right) = KS(e_k) + \sum_{l \in N(e_k)} \frac{\omega_{kl}}{\overline{\omega}} \times KS(e_l) \tag{19}$$

where $KS(e_k)$ denotes the k-shell value of $e_k (k = 1, 2, \ldots, r)$. $N(e_k)$ denotes the set of neighbors of $e_k$ in the social network. $\omega_{kl}$ is the weight of the edge between $e_k$ and $e_l$, and $\overline{\omega}$is the average weight of all the edges.

**Step 4.** Generate the modified evaluations.

In practice, the identified experts who need to modify their opinions will refer to the corresponding evaluations of the experts they trust. The greater the expert's influence in the social network, the greater the degree of reference.

$$\lambda^{-1}\left(\overline{x_{ij}^k}\right) = \frac{HC(e_k)}{\sum_{l=1}^{r} t_{kl} HC(e_l)} \lambda^{-1}\left(x_{ij}^k\right) + \sum_{l=1, l \neq k}^{r} t_{kl} \left(\frac{HC(e_l)}{\sum_{l=1}^{r} t_{kl} HC(e_l)}\right) \lambda^{-1}\left(x_{ij}^l\right), (i, j, k \in EVS) \tag{20}$$

where $x_{ij}^k$ and $\overline{x_{ij}^k}$ is the original evaluation that need to be modified and the modified one, respectively. $\lambda^{-1}(\bullet)$ represents the mapping defined in Eq. (1). $t_{kl}$ is the element in the adjacency matrix, which denotes the trust relationship between $e_k$ and $e_l$. $HC(e_k)(k = 1, 2, \ldots, r)$ is the hybrid centrality of $e_k$, which reflects the influence of the expert in the social network.

It is worth noting that each individual has their own level of acceptable compromise $\varepsilon_k$ [39,40], which reflects the amount of decrease or increase an individual can accept on their evaluations without supervision. If the above adjustment amount exceeds the threshold of their compromise levels, i.e., $\left|\overline{x_{ij}^k} - x_{ij}^k\right| \rangle \varepsilon_k$, then the expert will be unwilling to accept the calculated modified evaluations. In this case, to consider the adjustment willingness of experts and their individual concern, we will invite experts to give their revised assessment. It should be noted that a newly given evaluation will only be considered reasonable if its consensus level is higher than the current one. If the amount of adjustment is within the acceptable range, i.e., $\left|\overline{x_{ij}^k} - x_{ij}^k\right| \leqslant \varepsilon_k$, the expert will follow the recommendations to improve the efficiency of the consensus-reaching process.

The above process will perform iteratively until the group consensus reaches the preset threshold $\vartheta$. Then, the group evaluation $x_{ij}^g$ can be obtained with the hybrid centrality as:

$$\lambda^{-1}\left(x_{ij}^g\right) = \sum_{k=1}^{r} w_{e_k} \lambda^{-1}\left(x_{ij}^k\right) \tag{21}$$

where $w_{e_k}$ denotes the importance degree of experts with $w_{e_k} = \frac{HC(e_k)}{\sum_{k=1}^{r} HC(e_k)}$. $\lambda^{-1}(\bullet)$ represents the mapping defined in Eq. (1).

### 3.3. Determine the criteria weight with BWM-CRITIC and rank the alternatives

In this subsection, a comprehensive criteria weighting method is developed by coordinating the subjective aspect, objective aspect, and criteria correlations. Then, the ranking of alternatives can be determined with the obtained criteria weight.

#### 3.3.1. Obtain the subjective weight based on the ELICIT-BWM method

Experts from different fields have a different awareness about the importance of criteria. Therefore, utilizing the subjective weighting method can fully consider the knowledge, experience, and understanding of the specific problem of the experts. Compared with AHP, the BWM method requires fewer pairwise comparisons, which improves the efficiency of weight calculation and helps to achieve a more consistent result [32]. However, the traditional BWM cannot address the problems with ELICIT information. Therefore, we extend the BWM to the ELICIT environment to obtain the subjective weight of the criteria.

**Step 1.** Each expert uses ELICIT expressions to evaluate the importance of each criterion and construct the criteria evaluation matrix $S = \left[s_j^k\right]_{r\times n}$, where $s_j^k$ denotes the importance of $c_j(j = 1,2,\ldots,n)$ assessed by $e^k(k = 1,2,\ldots,r)$.

**Step 2.** Determine the group evaluation of criteria importance.

$$\lambda^{-1}\left(s_j^g\right) = \sum_{k=1}^{r} w_{e_k}\lambda^{-1}\left(s_j^k\right) \tag{22}$$

where $w_{e_k}$ denotes the importance degree of experts. $\lambda^{-1}(\bullet)$ represents the mapping defined in Eq. (1).

**Step 3.** Calculate the expectation value of $s_j^g(j = 1,2,\ldots,n)$ using Eq. (5). The best criterion $c_B$ is the one with the highest expectation value, whereas the worst criterion $c_W$ is the one with the lowest expectation value.

**Step 4.** Then, each expert $e^k(k = 1,2,\ldots,r)$ makes pairwise comparisons of the best and worst criterion with all other criteria to establish the Best-to-Others vectors and the Worst-to-Others vectors, which can be expressed as $P_{BO}^k = (p_{B1}^k, p_{B2}^k, \ldots, p_{Bn}^k)$ and $P_{WO}^k = (p_{1W}^k, p_{2W}^k, \ldots, p_{nW}^k)^T$, respectively. $p_{Bj}^k$ reflects $e^k$'s preference of the best criterion over $c_j$ and $p_{jW}^k$ indicates $e^k$'s preference of $c_j$ over the worst criterion. The pairwise comparisons are expressed using ELICIT based on a 7-scale linguistic term set $S = \{s_0 = \text{equal importance}, s_1 = \text{weak importance}, s_2 = \text{less importance}, s_3 = \text{fairly importance}, s_4 = \text{importance}, s_5 = \text{very importance}, s_6 = \text{extreme importance}\}$. Then, the group Best-to-Others vector $P_{BO}^g = (p_{B1}^g, p_{B2}^g, \ldots, p_{Bn}^g)$ and group Worst-to-Others vector $E(P_{WO}^g) = (E(p_{1W}^g), E(p_{2W}^g), .., E(p_{nW}^g))^T$ can be obtained with $w_{e_k}(k = 1,2,\ldots,r)$ To calculate the subjective weight, the numerical vectors are determined with the expectation function $E(P_{BO}^g) = (E(p_{B1}^g), E(p_{B2}^g), \ldots, E(p_{Bn}^g))$ and $E(P_{WO}^g) = (E(p_{1W}^g), E(p_{2W}^g), \ldots, E(p_{nW}^g))^T$.

**Step 5.** The aim of this step is to obtain the optimal subjective weight of criteria $w_j^{sub}$, such that the maximum differences $\left|\frac{w_B^{sub}}{w_j^{sub}} - E\left(p_{Bj}^g\right)\right|$ and $\left|\frac{w_j^{sub}}{w_W^{sub}} - E\left(p_{jW}^g\right)\right|$ for all $j$ are minimized, which can be represented in the following model.

$$\min_j \max\left\{\left|\frac{w_B^{sub}}{w_j^{sub}} - E(p_{Bj}^g)\right|, \left|\frac{w_j^{sub}}{w_W^{sub}} - E(p_{jW}^g)\right|\right\} s.t. \begin{cases} \sum_{j=1}^{n} w_j^{sub} = 1 \\ w_j^{sub} \geqslant 0, j \in \{1,2,\ldots,n\} \end{cases} \tag{23}$$

where $w_B^{sub}$ and $w_W^{sub}$ are the subjective weights of the best and the worst criteria, respectively. $w_j^{sub}$ represents the subjective weight of $c_j(j = 1,2,\ldots,n)$.

The above model is equivalent to the following one:

$$\min \xi\, s.t. \begin{cases} \left|\frac{w_B^{sub}}{w_j^{sub}} - E\left(p_{Bj}^g\right)\right| \leqslant \xi \\ \left|\frac{w_j^{sub}}{w_W^{sub}} - E\left(p_{jW}^g\right)\right| \leqslant \xi \\ \sum_{j=1}^{n} w_j^{sub} = 1 \\ w_j^{sub} \geqslant 0, j \in \{1,2,\ldots,n\} \end{cases} \tag{24}$$

where $w_B^{sub}$ and $w_W^{sub}$ is the subjective weight of the best and the worst criteria, respectively. $w_j^{sub}$ represents the subjective weight of $c_j(j = 1,2,\ldots,n)$.

By solving model (24), the optimal subjective weight of criteria can be obtained.

### 3.3.2. Calculate the objective weight with the extended CRITIC method

The correlation between criteria often exists in SNGDM problems. If we take the issue of selecting sustainable suppliers as an example, product quality and innovation have a positive impact on customer satisfaction, while excessive emissions of pollutants and waste can have a negative impact on a company's reputation. Compared with the Pearson correlation coefficient, the Spearman correlation coefficient is applicable to a wider range of situations and does not limit the distribution of the data [41]. Consequently, we introduced Spearman's correlation coefficient into the CRITIC method to derive the objective weight of the criteria.

**Step 1.** Treat each column of the group decision matrix $\left[x_{ij}^g\right]_{m\times n}$ as a vector to calculate the Spearman correlation coefficient between criteria. Suppose $Y_1 = (x_{1u}^g, x_{2u}^g, \ldots, x_{mu}^g)^T$ and $Y_2 = (x_{1v}^g, x_{2v}^g, \ldots, x_{mv}^g)^T(u,v = 1,2,\ldots,n)$ are two ELICIT vectors, the expectation value of each ELICIT expression can be obtained. Then, the vectors can be rewritten with the ranking of the elements as $\widehat{Y_1} = \left(R\left(x^g_{1u}\right), R\left(x^g_{2u}\right), \dots, R\left(x^g_{mu}\right)\right)^T$ and $\widehat{Y_2} = \left(R\left(x^g_{1v}\right), R\left(x^g_{2v}\right), \dots, R\left(x^g_{mv}\right)\right)^T$. Therefore, the Spearman correlation coefficient $\sigma_{uv}$ between criteria $c_u$ and $c_v$ can be calculated as:

$$\sigma_{uv} = 1 - \frac{6 \times \sum_{i=1}^{m} \left[R\left(x^g_{iu}\right) - R\left(x^g_{iv}\right)\right]^2}{m \times (m^2 - 1)} \tag{25}$$

**Step 2.** The standard deviation $SD(c_u)$ of criterion $c_u(u = 1, 2, \dots, n)$ is obtained as:

$$SD(c_u) = \sqrt{\frac{\sum_{i=1}^{m} \left(x^g_{iu} - \frac{1}{m}\sum_{i=1}^{m} x^g_{iu}\right)^2}{m - 1}} \tag{26}$$

**Step 3.**By introducing the Spearman correlation coefficient into CRITIC, the objective weight $w^{obj}_u$ of criterion $c_u(u = 1, 2, \dots, n)$ can be determined as:

$$w^{obj}_u = \frac{SD(c_u)\sum_{v=1}^{n}(1 - \sigma_{uv})}{\sum_{u=1}^{n}\left(SD(c_u)\sum_{v=1}^{n}(1 - \sigma_{uv})\right)} \tag{27}$$

*3.3.3. Determine the comprehensive weight with minimum relative entropy*

In real problems, if only the preference of the experts on the importance of the criteria is taken into account, the results would be too subjective, while if only the information about the decision is taken into account, the personal inclinations of the experts would be ignored. Therefore, coordination between objective and subjective aspects must be achieved. The Kullback–Leibler divergence (also called relative entropy) is an effective measure to quantify the difference between two probability distributions, which has been widely utilized in various fields, such as fault diagnosis and pattern recognition. Since the subjective, objective and the overall weight of the criteria can be regarded as probability distributions, the Kullback–Leibler divergence is introduced to measure the distance between different weight vectors. The minimum relative entropy is employed to construct an optimization model to obtain the overall criteria weight. In this way, the final weight can be close to both the subjective and objective weights.

$$\begin{aligned} &\min : (1-\gamma)D_{KL}\left(w, w^{sub}\right) + \gamma D_{KL}\left(w, w^{obj}\right) \\ &s.t. \begin{cases} \sum_{j=1}^{n} w_j = 1, w_j \in (0,1) \\ D_{KL}\left(w, w^{sub}\right) = \sum_{j=1}^{n} w_j \ln \frac{w_j}{w^{sub}_j} \\ D_{KL}\left(w, w^{obj}\right) = \sum_{j=1}^{n} w_j \ln \frac{w_j}{w^{obj}_j} \end{cases} \end{aligned} \tag{28}$$

where $w^{sub} = \left(w^{sub}_1, w^{sub}_2 \dots, w^{sub}_j, \dots, w^{sub}_n\right)$ and $w^{obj} = \left(w^{obj}_1, w^{obj}_2 \dots, w^{obj}_j, \dots, w^{obj}_n\right)$ denote the vectors of subjective and objective weights, respectively. $w = \left(w_1, w_2 \dots, w_j, \dots, w_n\right)$ is the overall criteria weight. $D_{KL}\left(w, w^{sub}\right)$ denotes the Kullback–Leibler divergence between the overall weight and the subjective weight. $D_{KL}\left(w, w^{obj}\right)$ denotes the Kullback–Leibler divergence between the overall weight and the objective weight. The balance coefficient $\gamma$reflects the relative importance between subjectivity and objectivity, which can be selected according to the specific situation. Since the objective function and the constraints are convex, Model (28) is a convex optimization problem that can be solved with Lingo software.

Then, with the obtained overall criteria weights $w_j(j = 1, 2, \dots, n)$ and the consensual group evaluation $\left[x^g_{ij}\right]_{m \times n}$, the final ranking value of alternatives $a_i(i = 1, 2, \dots, m)$ can be calculated as:

$$RV(a_i) = E\left(\sum_{j=1}^{n} w_j x^g_{ij}\right) \tag{29}$$

where $E(\bullet)$ is the expectation function given in Eq. (5).

By sorting the $RV(a_i)$ from large to small, we can determine the ranking of the alternatives and select the optimal one.

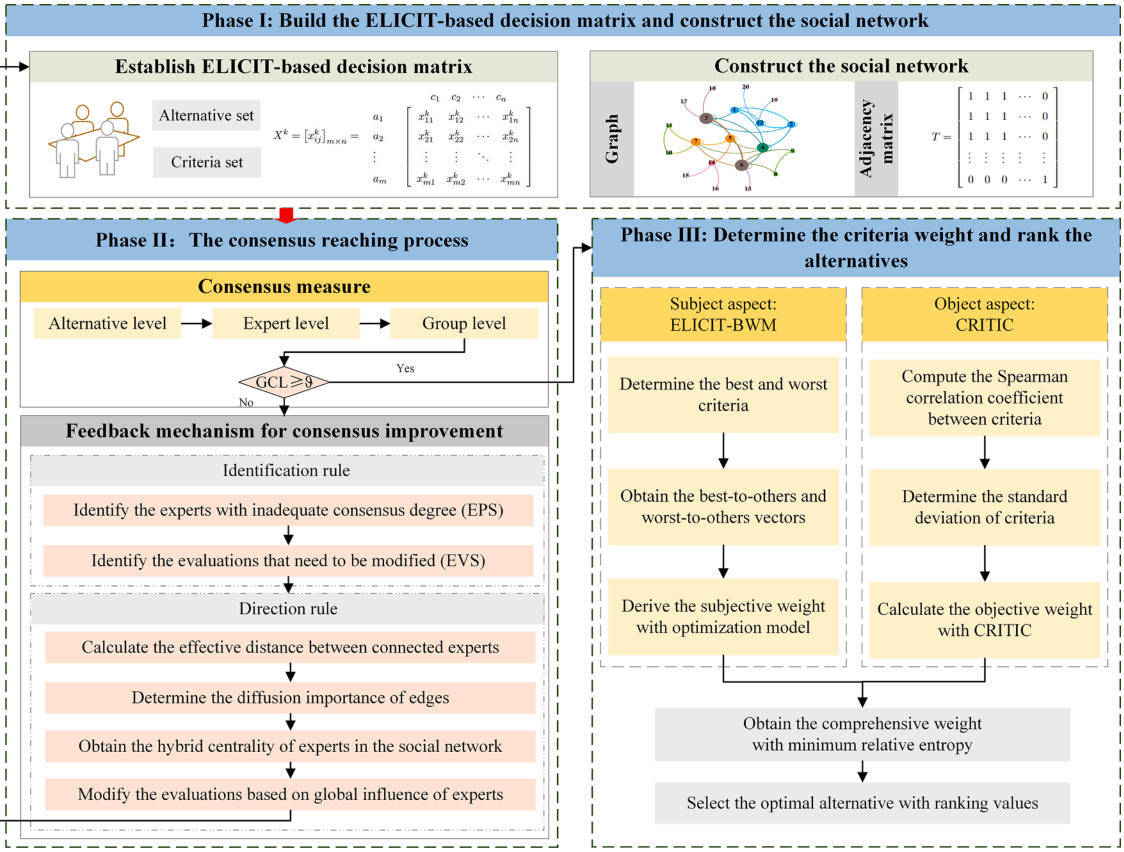


**Fig. 1.** The framework of the proposed social network group decision-making method.

### *3.4. The structure of the proposed SNGDM approach*

Fig. 1 shows the framework of the proposed method and the specific steps are as follows.

**Phase I:Build the ELICIT-based decision matrix and construct the social network among experts.**

**Step 1.1:** Sort the potential alternatives $A = \{a_i|i = 1, 2, \ldots, m\}$ and construct a set of criteria $C = \{c_j|j = 1, 2, \ldots, n\}$.

**Step 1.2:** Invite a team of experts $E = \{e_k|k = 1, 2, \ldots, r\}$ from different professional fields to evaluate the performance of alternatives with ELICIT information and establish the decision matrix $X = \left[x_{ij}^k\right]_{m\times n}$.

**Step 1.3:** Collect the trust relationships between experts and construct the adjacency matrix $T = [t_{kl}]_{r\times r}$ of the social network.

**Phase II: The consensus-reaching process.**

**Step 2.1:** Measure the consensus at evaluation level $CL_{ij}^k$, expert level $CL^k$, and the group level $GCL$ using Eqs. (12)–(14).

**Step 2.2:** If the current group consensus level (GCL) reaches the given threshold $GCL \geqslant \vartheta$, go to Phase III; otherwise, the feedback mechanism should be activated to promote the group consensus.

**Step 2.3:** Identify the evaluations in conflict via Eqs. (15)–(16).

**Step 2.4:** Calculate the *Effective Distance* between the connected experts in the social network using Eq. (17).

**Step 2.5:** Determine the diffusion importance of the edge connecting pairs of experts via Eq. (18).

**Step 2.6:** Compute experts' hybrid centrality (HC) using Eq. (19).

**Step 2.7:** Calculate the modified evaluations with Eq. (20). If the amount of modification exceeds the range that the expert can accept, the expert will be asked to re-evaluate the corresponding alternative. Then, go back to Step 2.1.

**Phase III: Determine the weight of the criteria with BWM-CRITIC and rank the alternatives.**

**Step 3.1:** Select the best and worst criteria with ELICIT-based evaluations and determine the Best-to-Others vectors and the Worst-to-Others vectors.

**Step 3.2:** Calculate the subjective criteria weight with Model (24).

**Step 3.3:** Determine the Spearman correlation coefficient between criteria using Eq. (25).

**Step 3.4:** Obtain the objective criteria weight by introducing the Spearman correlation coefficient into the traditional CRITIC method via Eqs. (26)–(27).

**Step 3.5:** Generate the comprehensive weight with minimum relative entropy using Model (28).

**Table 1**
The evaluation criteria for a sustainable supplier.

| Aspect | Symbol | Criterion | Literature source |
|---|---|---|---|
| Environmental | Green technologies | $c_1$ | [42] |
| | Environmental regulations | $c_2$ | [43] |
| Economic | Quality | $c_3$ | [44] |
| | Flexibility | $c_4$ | [42] |
| Social | Customer satisfaction | $c_5$ | [42] |

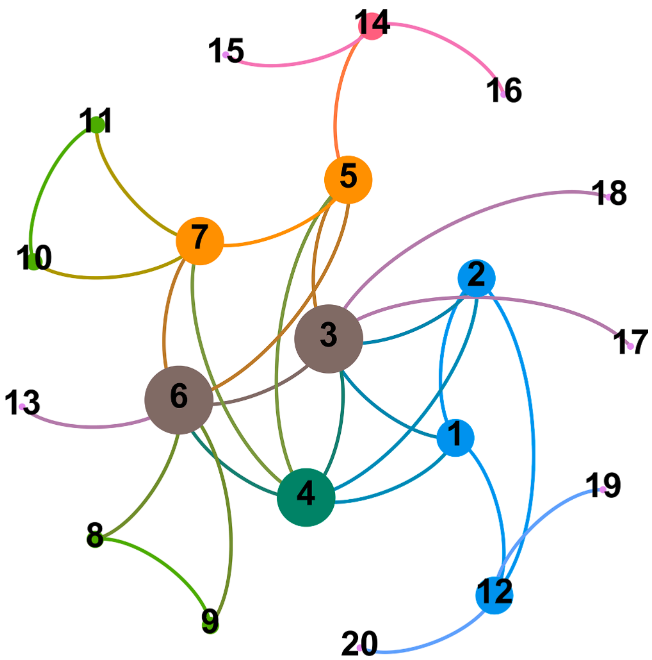


**Fig. 2.** The social network of twenty experts.

**Step 3.6:** With the criteria weight and the consensual group evaluation, the ranking of alternatives can be determined via Eq. (29).

## 4. Case analysis on the sustainable supplier selection problem

In this section, a case study on a sustainable supplier selection problem is presented to illustrate the effectiveness of our SNGDM method.

### 4.1. Problem background

As people become more conscious of environmental preservation and social responsibility, the issue of sustainable supply chain management is receiving increased attention. An efficient sustainable supply chain has a positive impact on reducing costs, improving corporate reputation, and controlling risks. Meanwhile, supply chains cannot be truly sustainable without partnering with the right suppliers. An excellent sustainable supplier should have a good overall performance in terms of economic effect, environmental impact, and social responsibility. Therefore, choosing a suitable supplier plays an increasingly important role in sustainable supply chain management.

Although scholars have made many contributions to research on sustainable supplier selection [42–44], some aspects still need to be studied further. First, the trust relationships among decision-makers are rarely considered in existing studies, let alone the exploitation of the topological structure of the social network to help experts reach consensus on the selection problem. Second, most methods either use objective or subjective methods to determine the criteria weights. Although a few approaches utilize combinatorial weighting methods, they ignore the correlation between criteria, which reduces the rationality of the results. To fill these gaps, we will apply the proposed SNGDM method to solve a practical sustainable supplier selection problem of Chinese company X. Company X is a

**Table 2**
The original consensus on the expert level.

| $k$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $CL^k$ | 0.864 | 0.853 | 0.855 | 0.694 | 0.871 | 0.852 | 0.682 | 0.859 | 0.693 | 0.864 |
| $k$ | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| $CL^k$ | 0.854 | 0.698 | 0.873 | 0.679 | 0.866 | 0.879 | 0.664 | 0.852 | 0.861 | 0.867 |

**Table 3**
The degree centrality, k-shell value, and the hybrid centrality of experts in the network.

| $k$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $DC(e_k)$ | 4.000 | 4.000 | 7.000 | 6.000 | 5.000 | 7.000 | 5.000 | 2.000 | 2.000 | 2.000 |
| $KS(e_k)$ | 3.000 | 3.000 | 3.000 | 3.000 | 3.000 | 3.000 | 3.000 | 2.000 | 2.000 | 2.000 |
| $HC(e_k)$ | 13.874 | 13.874 | 18.880 | 19.981 | 15.776 | 17.882 | 15.114 | 7.874 | 7.874 | 7.365 |
| $k$ | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| $DC(e_k)$ | 2.000 | 4.000 | 1.000 | 3.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| $KS(e_k)$ | 2.000 | 2.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| $HC(e_k)$ | 7.365 | 8.312 | 8.104 | 4.664 | 2.184 | 2.184 | 7.085 | 7.085 | 4.212 | 4.212 |

**Fig. 3.** The k-shell decomposition diagram.

well-known machinery and equipment manufacturer, which places great importance on environmental protection and social responsibility when setting up new production lines. Due to production and operation needs, company X needs to select a suitable parts supplier.

### *4.2. Application to a sustainable supplier selection problem*

**Phase I. Build the ELICIT-based decision matrix and construct the social network among experts.**

**Step 1.1:** After a preliminary screening, four suppliers are selected for expert evaluation, which can be denoted as $a_i(i = 1, 2, 3, 4)$. By investigating relevant literature, five beneficial criteria related to environmental, economic, and social aspects are selected, as shown in Table 1. A team of twenty experts $e_k(k = 1, 2, \ldots, 20)$ from company management, R&D, marketing, and other departments are invited to evaluate the four alternatives regarding each criterion. Appendix A provides a brief profile of these experts.

**Table 4**
The weight of the edges connecting pairs of experts.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $\omega_{12}$ | $\omega_{13}$ | $\omega_{14}$ | $\omega_{1,12}$ | $\omega_{21}$ | $\omega_{23}$ | $\omega_{24}$ | $\omega_{2,12}$ | $\omega_{31}$ | $\omega_{32}$ |
| 3.733 | 4.383 | 4.267 | 3.400 | 3.733 | 4.383 | 4.167 | 3.400 | 3.454 | 3.454 |
| $\omega_{34}$ | $\omega_{35}$ | $\omega_{36}$ | $\omega_{3,17}$ | $\omega_{3,18}$ | $\omega_{41}$ | $\omega_{42}$ | $\omega_{43}$ | $\omega_{45}$ | $\omega_{46}$ |
| 3.796 | 3.625 | 3.966 | 2.417 | 2.416 | 3.487 | 3.487 | 4.031 | 3.668 | 4.031 |
| $\omega_{47}$ | $\omega_{53}$ | $\omega_{54}$ | $\omega_{56}$ | $\omega_{57}$ | $\omega_{5,14}$ | $\omega_{63}$ | $\omega_{64}$ | $\omega_{65}$ | $\omega_{67}$ |
| 3.668 | 4.154 | 3.958 | 4.154 | 3.763 | 2.769 | 3.966 | 3.796 | 3.625 | 3.625 |
| $\omega_{68}$ | $\omega_{69}$ | $\omega_{6,13}$ | $\omega_{74}$ | $\omega_{75}$ | $\omega_{76}$ | $\omega_{7,10}$ | $\omega_{7,11}$ | $\omega_{86}$ | $\omega_{89}$ |
| 2.850 | 2.850 | 2.417 | 3.958 | 3.763 | 4.154 | 2.875 | 2.875 | 5.425 | 3.300 |
| $\omega_{96}$ | $\omega_{98}$ | $\omega_{10,7}$ | $\omega_{10,11}$ | $\omega_{11,7}$ | $\omega_{11,10}$ | $\omega_{12,1}$ | $\omega_{12,2}$ | $\omega_{12,19}$ | $\omega_{12,20}$ |
| 5.425 | 3.300 | 4.775 | 3.300 | 4.775 | 3.300 | 3.400 | 3.400 | 2.083 | 2.083 |
| $\omega_{13,6}$ | $\omega_{14,5}$ | $\omega_{14,15}$ | $\omega_{14,16}$ | $\omega_{15,14}$ | $\omega_{16,14}$ | $\omega_{17,3}$ | $\omega_{18,3}$ | $\omega_{19,12}$ | $\omega_{20,12}$ |
| 9.200 | 3.339 | 1.779 | 1.770 | 4.600 | 4.600 | 9.200 | 9.200 | 6.250 | 6.250 |

**Table 5**
The calculated modified evaluations.

| | | |
|---|---|---|
| $\overline{x_{23}^{4}}$ | $\overline{x_{25}^{4}}$ | $\overline{x_{45}^{4}}$ |
| $bet(s_3,0.176)^{-0.103}\&(s_4,-0.162)^{-0.029}$ | $bet(s_2,-0.401)^{0.012}\&(s_3,0.429)^{-0.016}$ | $bet(s_3,0.692)^{0.167}\&(s_4,0.525)^{0.033}$ |
| $\overline{x_{13}^{7}}$ | $\overline{x_{23}^{7}}$ | $\overline{x_{24}^{7}}$ |
| $bet(s_3,0.096)^{-0.010}\&(s_4,0.014)^{-0.153}$ | $bet(s_4,0.367)^{-0.172}\&(s_5,0.024)^{-0.233}$ | $bet(s_1,-0.127)^{-0.058}\&(s_2,-0.439)^{-0.074}$ |
| $\overline{x_{45}^{7}}$ | $\overline{x_{14}^{9}}$ | $\overline{x_{22}^{9}}$ |
| $bet(s_3,0.452)^{-0.213}\&(s_4,0.376)^{-0.168}$ | $bet(s_1,-0.285)^{0.014}\&(s_2,0.126)^{-0.057}$ | $bet(s_4,-0.427)^{0.081}\&(s_5,-0.573)^{0.145}$ |
| $\overline{x_{44}^{9}}$ | $\overline{x_{25}^{12}}$ | $\overline{x_{31}^{12}}$ |
| $bet(s_3,-0.110)^{0.318}\&(s_4,-0.429)^{0.058}$ | $bet(s_2,0.423)^{0.657}\&(s_3,0.541)^{0.726}$ | $bet(s_1,0.132)^{0.003}\&(s_2,0.256)^{0.017}$ |
| $\overline{x_{24}^{14}}$ | $\overline{x_{34}^{14}}$ | $\overline{x_{43}^{17}}$ |
| $bet(s_4,-0.045)^{-0.017}\&(s_5,-0.513)^{-0.001}$ | $bet(s_3,0.492)^{0.013}\&(s_4,0.128)^{0.075}$ | $bet(s_2,0.028)^{-0.085}\&(s_3,0.496)^{-0.066}$ |

**Step 1.2:** Considering the inherent ambiguity in human perception and the qualitative characteristics of the criteria, ELICIT is adopted to express experts' evaluations on the performance of suppliers with a 7-scale linguistic term set $S=\{s_0=\text{very bad},s_1=\text{bad},s_2=\text{slightlybad},s_3=\text{medium},s_4=\text{slightlygood},s_5=\text{good},s_6=\text{very good}\}$ For brevity, individual evaluations are given in Appendix A. The compromise levels of experts are given as: $\varepsilon=(0.65,0.80,0.60,0.80,0.55,0.80,0.75,0.70,0.85,0.50,0.65,0.80,0.60,0.75,0.75,0.55,0.85,0.60,0.65,0.70)$.

**Step 1.3:** Based on the previous interactions between the experts, their trust relationships are depicted in Fig. 2 and the adjacency matrix is given in Appendix B.

**Phase II. The consensus-reaching process.**

**Step 2.1:** Based on the original decision information and the consensus measures, the current consensus can be determined on all three levels by means of Eqs. (12)–(14). Table 2 shows the consensus degree on the expert level. The current group consensus level is calculated as: $GCL=0.809$.

**Step 2.2:** To ensure the final decision is acceptable to the majority of experts, the consensus threshold is set at $\vartheta=0.85$. Since the group consensus is currently below the threshold, the consensus-reaching strategy should be implemented to help the experts improve the GCL.

**Step 2.3:** Following the identification rule on the expert and evaluation levels, the specific evaluations that need to be modified are:

$$EVS=\left\{\begin{array}{l}(2,3,4),(2,5,4),(4,5,4),(1,3,7),(2,3,7),(2,4,7),(4,5,7),(1,4,9),\\(2,2,9),(4,4,9),(2,5,12),(3,1,12),(2,4,14),(3,4,14),(4,3,17)\end{array}\right\}$$

**Step 2.4:** In our consensus-reaching strategy, the identified experts who need to modify their corresponding opinions are encouraged to refer to the people they trust in the social network. The more influential the individual is in the network, the more they will be referenced in the CRP. First, the degree centrality and k-shell value of each expert can be determined, which are shown in Table 3. The k-shell decomposition diagram is shown in Fig. 3.

**Step 2.5:**To reflect the diffusion importance of the edges in information propagation, the weight of the edges is calculated using Eq. (18), and the results are shown in Table 4.

**Step 2.6:** Then, the hybrid centrality of experts can be determined using Eq. (19), which is presented in Table 3. Take $e_4$ and $e_6$ as examples, the k-shell values of $e_4$ and $e_6$ are the same and the degree centrality of $e_6$ is larger than that of $e_4$, but $e_4$'s hybrid centrality is larger than that of $e_6$. The reason is that our method not only considers the node itself when determining its influence, but also considers the importance of its connected neighbors. $e_4$ is expected to have more influence since it is connected with more important neighbors compared with $e_6$. Therefore, our method is more reasonable and can better reflect the global topological structure of the network.

**Step 2.7:**After obtaining the hybrid centrality that reflects the expert's influence in the group, we will guide the modifications of the identified evaluations to improve group consensus. Take $x_{23}^{4}$ as an example, since the experts that $e_4$ trusts are $e_1$,$e_2$,$e_3$,$e_5$,$e_6$, and $e_7$, based on the direction rule given by Eq. (20), the calculated modified evaluation can be obtained as:

**Table 6**
Consensus on the expert level after the first iteration.

| | $e_1$ | $e_2$ | $e_3$ | $e_4$ | $e_5$ | $e_6$ | $e_7$ | $e_8$ | $e_9$ | $e_{10}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $CL^{k(1)}$ | 0.869 | 0.861 | 0.863 | 0.768 | 0.884 | 0.861 | 0.725 | 0.867 | 0.746 | 0.879 |
| | $e_{11}$ | $e_{12}$ | $e_{13}$ | $e_{14}$ | $e_{15}$ | $e_{16}$ | $e_{17}$ | $e_{18}$ | $e_{19}$ | $e_{20}$ |
| $CL^{k(1)}$ | 0.866 | 0.737 | 0.882 | 0.729 | 0.875 | 0.881 | 0.697 | 0.858 | 0.865 | 0.871 |

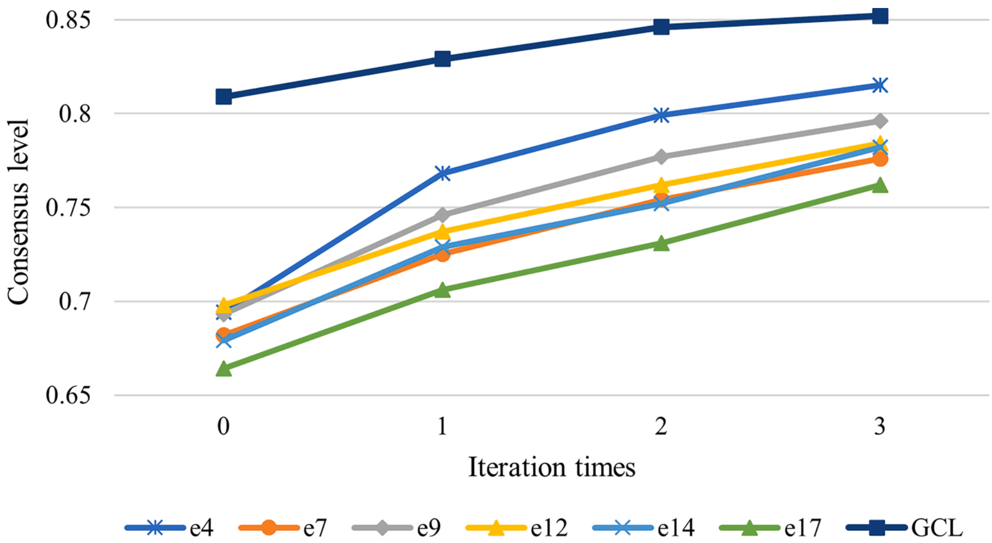


**Fig. 4.** The improvement of consensus on $e_4, e_7, e_9, e_{12}, e_{14}, e_{17}$, and group level.

$$\lambda^{-1}\left(\overline{x_{23}^{4}}\right) = \frac{HC(e_4)}{\sum_{k=1}^{7} HC(e_k)}\lambda^{-1}\left(x_{23}^{4}\right) + \sum_{k=1,k\neq 4}^{7}\frac{HC(e_k)}{\sum_{k=1}^{7} HC(e_k)}\lambda^{-1}\left(x_{23}^{k}\right)$$

The other identified evaluations can be adjusted in the same way and the results are shown in Table 5. Then, we calculate the amount of modification to check whether the expert-defined compromise levels $\varepsilon_k$ were exceeded. If the modification amount exceeds the expert's acceptability threshold, additional human supervision is needed; otherwise, the calculated modifications are automatically applied. According to the modified evaluations given in Table 5, all modifications are within the acceptable range of the respective experts. Therefore, the calculated recommendations are adopted to improve the efficiency of the consensus-reaching process.

Then, the consensus degree on the evaluation, expert, and group levels are recalculated using Eqs. (12)–(14). Table 6 shows the updated consensus on the expert level.

Then, the group consensus level is recalculated as: $GCL^{(1)} = 0.829$. The degree of group consensus has been clearly improved, but the preset threshold has not been reached. Therefore, we repeat the above steps until the group consensus reaches the threshold requirement. After another two iterations, we obtain $GCL^{(3)} = 0.857 > 0.85$. The specific process is omitted here for simplicity. The consensus improvement of the identified experts and the group is presented in Fig. 4.

The final group evaluation that meets the consensus requirement is shown in Table 7.

**Phase III. Determine the weight of criteria and select the optimal alternative.**

**Step 3.1:**Each expert evaluates the importance of criteria using ELICIT. Then, the group evaluation of criteria importance can be obtained and the results are shown in Table 8. By calculating the expectation value of each ELICIT expression we can determine that the best criterion is $c_1$ and the worst criterion is $c_4$.

The group of experts $e^k (k = 1, 2, \ldots, 20)$ will make pairwise comparisons of the best criterion $c_1$ and the worst criterion $c_4$ with all other criteria. With the normalized hybrid centrality of experts, the group Best-to-Others vector $P_{BO}^{g} = \left(p_{B2}^{g}, p_{B3}^{g}, p_{B4}^{g}, p_{B5}^{g}\right)$ and Worst-to-Others vector $P_{WO}^{g} = \left(p_{1W}^{g}, p_{2W}^{g}, p_{3W}^{g}, p_{5W}^{g}\right)^{T}$ can be obtained, which are shown in Table 9.

**Step 3.2:** By solving the optimization model given in Model (24), the subjective weight of criteria is obtained as: $w_1^{sub} = 0.235$, $w_2^{sub} = 0.218, w_3^{sub} = 0.197, w_4^{sub} = 0.154, w_5^{sub} = 0.196$.

**Step 3.3:** Based on the group evaluation, the Spearman correlation coefficient between different criteria can be obtained using Eq. (25), which is shown in Table 10.

**Step 3.4:** Via Eqs. 26,27, the objective weight of the criteria can be obtained as: $w_1^{obj} = 0.249, w_2^{obj} = 0.208, w_3^{obj} = 0.232, w_4^{obj} =$

**Table 7**
The consensual group evaluation of different suppliers regarding each criterion.

| | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ |
|---|---|---|---|---|---|
| $a_1$ | $bet(s_2, 0.498)^{0.150}(s_3, 0.072)^{0.269}$ | $bet(s_2, 0.846)^{0.063}\ (s_3, 0.634)^{0.051}$ | $bet(s_1, 0.102)^{-0.003}(s_2, 0.144)^{-0.043}$ | $bet(s_3, 0.630)^{0.150}\ (s_3, 0.072)^{0.269}$ | $bet(s_2, -0.018)^{0.173}\ (s_3, -0.092)^{0.145}$ |
| $a_2$ | $bet(s_3, 0.823)^{0.174}\ (s_4, 0.756)^{0.152}$ | $bet(s_2, 0.330)^{0.157}\ (s_3, 0.458)^{0.124}$ | $bet(s_2, -0.724)^{-0.063}\ (s_3, -0.598)^{-0.152}$ | $bet(s_2, 0.026)^{-0.012}\ (s_3, 0.439)^{-0.007}$ | $bet(s_2, -0.013)^{-0.167}\ (s_3, -0.156)^{-0.257}$ |
| $a_3$ | $bet(s_4, 0.654)^{0.067}\ (s_5, 0.558)^{0.137}$ | $bet(s_3, 0.296)^{-0.183}\ (s_4, 0.435)^{-0.234}$ | $bet(s_2, 0.368)^{0.125}\ (s_3, 0.475)^{0.167}$ | $bet(s_4, 0.702)^{0.017}\ (s_5, 0.330)^{0.033}$ | $bet(s_2, 0.402)^{0.173}\ (s_3, 0.439)^{0.268}$ |
| $a_4$ | $bet(s_3, -0.367)^{-0.142}\ (s_4, -0.425)^{-0.174}$ | $bet(s_3, 0.542)^{0.051}\ (s_4, 0.402)^{-0.146}$ | $bet(s_1, 0.841)^{0.105}\ (s_2, 0.739)^{-0.147}$ | $bet(s_1, 0.189)^{0.141}\ (s_2, 0.072)^{0.167}$ | $bet(s_2, -0.396)^{-0.267}(s_3, -0.402)^{-0.105}$ |

**Table 8**
The group evaluation of criteria importance.

| | The group evaluation | Expectation value | |
|---|---|---|---|
| $c_1$ | $bet(s_4, 0.177)^{-0.046}$ | $(s_5, 0.136)^{-0.090}$ | 0.742 |
| $c_2$ | $bet(s_3, 0.012)^{0.003}$ | $(s_4, 0.439)^{0.055}$ | 0.685 |
| $c_3$ | $bet(s_2, 0.451)^{0.094}$ | $(s_3, -0.067)^{0.134}$ | 0.449 |
| $c_4$ | $bet(s_1, 0.219)^{0.037}$ | $(s_2, -0.440)^{-0.001}$ | 0.238 |
| $c_5$ | $bet(s_2, -0.290)^{0.075}$ | $(s_3, 0.124)^{-0.011}$ | 0.408 |

**Table 9**
The group Best-to-Others vector and Worst-to-Others vector.

| The best criterion | $c_2$ | $c_3$ | $c_4$ | $c_5$ |
|---|---|---|---|---|
| $c_1$ | $bet(s_3, 0.176)^{-0.019}$ $(s_4, -0.061)^{-0.033}$ | $bet(s_2, 0.453)^{0.105}$ $(s_3, -0.067)^{0.127}$ | $bet(s_4, 0.312)^{-0.067}$ $(s_5, 0.274)^{-0.104}$ | $bet(s_2, -0.413)^{0.013}$ $(s_3, -0.427)^{-0.006}$ |
| The worst criterion | $c_1$ | $c_2$ | $c_3$ | $c_5$ |
| $c_4$ | $bet(s_3, 0.024)^{0.048}$ $(s_4, 0.368)^{0.074}$ | $bet(s_2, 0.382)^{-0.154}$ $(s_3, 0.594)^{-0.182}$ | $bet(s_3, -0.430)^{0.013}$ $(s_4, 0.125)^{0.022}$ | $bet(s_2, 0.027)^{-0.012}$ $(s_3, 0.429)^{-0.016}$ |

**Table 10**
The Spearman correlation coefficient between criteria.

| | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ |
|---|---|---|---|---|---|
| $c_1$ | – | −0.8 | 0.8 | 0.4 | 0.6 |
| $c_2$ | −0.8 | – | −0.4 | 0 | −0.4 |
| $c_3$ | 0.8 | −0.4 | – | 0.2 | 0.4 |
| $c_4$ | 0.4 | 0 | 0.2 | – | −0.8 |
| $c_5$ | 0.6 | −0.4 | 0.4 | −0.8 | – |

**Table 11**
The group evaluation, expectation value, and ranking result of the suppliers.

| | The group evaluation | Expectation value | Ranking result |
|---|---|---|---|
| $a_1$ | $bet(s_1, 0.199)^{0.034}\&(s_2, -0.429)^{-0.008}$ | 0.229 | 4 |
| $a_2$ | $bet(s_2, -0.278)^{0.077}\&(s_3, 0.126)^{-0.021}$ | 0.408 | 2 |
| $a_3$ | $bet(s_4, -0.107)^{0.020}\&(s_5, -0.318)^{-0.049}$ | 0.723 | 1 |
| $a_4$ | $bet(s_2, -0.274)^{0.018}\&(s_3, -0.043)^{0.030}$ | 0.315 | 3 |

$0.129, w_5^{obj} = 0.182$.

**Step 3.5:**Then, the comprehensive criteria weight can be determined using Model (28) with $w_1 = 0.243, w_2 = 0.214, w_3 = 0.217, w_4 = 0.142, w_5 = 0.184$.

**Step 3.6:**With the obtained criteria weight, we can calculate the group evaluation on each supplier and rank the alternatives according to their expectation values. The results are shown in Table 11, where we can conclude that the optimal supplier is $a_3$.

## 5. Discussions

### *5.1. Sensitivity analysis*

To ensure the group decision-making method is effective and scientific, the ranking of alternatives needs to be robust to the changes of the criteria weight within a certain range. Therefore, we have to conduct a sensitivity analysis on the comprehensive weight of criteria $w_j$ and the balance coefficient $\gamma$ to illustrate the robustness of our method.

First, the perturbation method is utilized to investigate the impact of the criteria weight on the alternative ranking when it changes within ±25% [45]. Since the original weight of $c_u$ is $w_u$, the weight after perturbation can be expressed as $\widetilde{w_u} = \varsigma w_u$, where $\widetilde{w_u} \in (0, 1)$ and $\varsigma$is the perturbation coefficient. Because the weight of criteria satisfies $\sum_{j=1}^{n} w_j = 1$, when the weight of $c_u$ is disturbed, the weight of other attributes will be affected, and it can be denoted as $\widetilde{w_v} = \tau w_v, v \neq u, v = 1, 2, \ldots, n$.

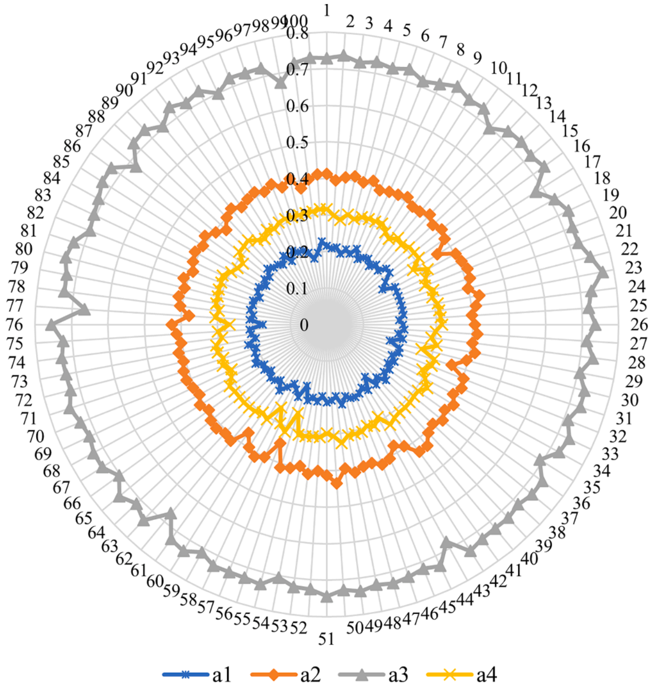


**Fig. 5.** The ranking values of alternatives with perturbed attribute weights.

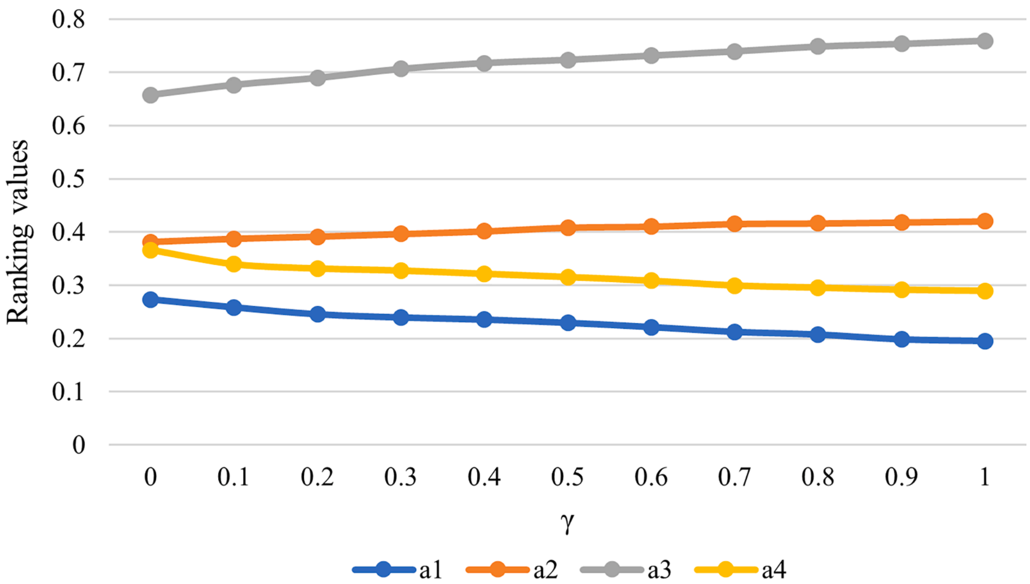


**Fig. 6.** The ranking of alternatives when $\gamma$varies from 0 to 1.

Since $\widetilde{w_u} + \sum_{v=1,v\neq u}^{n}\widetilde{w_v} = 1$, we can get $\tau = \frac{1-\varsigma w_u}{1-w_u}$. When $\varsigma$takes different values, the weight of criteria will change accordingly. Then, we run 100 perturbance simulations on the criteria weight and the ranking values of alternatives are shown in Fig. 5.

Fig. 5 shows that no matter how much the criteria weight varies within a range of $\pm 25\%, a_3$ is always identified as the optimal supplier and the performance of $a_1$ is always the worst. This illustrates the robustness of our method and indicates that the ranking is determined based on the advantages and disadvantages of the alternatives themselves.

Then, when determining the comprehensive weight of the criteria based on the minimum relative entropy, the balance coefficient $\gamma$ is utilized to reflect the relative importance of subjectivity and objectivity. Therefore, it is necessary to analyze the effect of this balance coefficient on alternative ranking. Fig. 6 shows the ranking of alternatives when $\gamma$ varies from 0 to 1.

As $\gamma$ varies between 0 and 1, the alternative ranking remains at $a_3 \succ a_2 \succ a_4 \succ a_1$, which further illustrates the effectiveness and

**Table 12**
The original and changed group evaluation of $a_2$.

| | The original evaluation | The changed evaluation |
|---|---|---|
| $c_1$ | $bet(s_3, 0.823)^{0.174} \& (s_4, 0.756)^{0.153}$ | $bet s_1 \& s_2$ |
| $c_2$ | $bet(s_2, 0.330)^{0.157} \& (s_3, 0.458)^{0.124}$ | $atmost s_2$ |
| $c_3$ | $bet(s_2, -0.724)^{-0.063} \& (s_3, -0.598)^{-0.152}$ | $bet s_1 \& s_2$ |
| $c_4$ | $bet(s_2, 0.026)^{-0.012} \& (s_3, 0.439)^{-0.007}$ | $bet s_1 \& s_2$ |
| $c_5$ | $bet(s_2, -0.013)^{-0.167} \& (s_3, -0.156)^{-0.257}$ | $atmost s_2$ |

**Table 13**
The ranking of alternatives in the subsets.

| Subsets | Ranking values of alternatives | Ranking |
|---|---|---|
| $\{a_1, a_2, a_3\}$ | $RV(a_1) = 0.382, RV(a_2) = 0.549, RV(a_3) = 0.715$ | $a_3 > a_2 > a_1$ |
| $\{a_1, a_3, a_4\}$ | $RV(a_1) = 0.211, RV(a_3) = 0.693, RV(a_4) = 0.289$ | $a_3 > a_4 > a_1$ |
| $\{a_1, a_2, a_4\}$ | $RV(a_1) = 0.205, RV(a_2) = 0.564, RV(a_4) = 0.397$ | $a_2 > a_4 > a_1$ |
| $\{a_2, a_3, a_4\}$ | $RV(a_2) = 0.416, RV(a_3) = 0.687, RV(a_4) = 0.343$ | $a_3 > a_2 > a_4$ |

robustness of our method. Specifically, with the increase of $\gamma$, the ranking values of $a_3$ show an upward trend, increasing by 13.44%, indicating that the advantage of $a_3$ is becoming more and more obvious. On the contrary, the ranking values of $a_4$ and $a_1$ show a downward trend, decreasing by 21.04% and 28.57%, respectively, and the ranking value of $a_2$ does not change significantly. The results show that as we put more emphasis on the objective aspect of the weight, the differences between the performance of the alternatives will increase. Therefore, it is necessary to consider the coordination between objectivity and subjectivity.

### 5.2. *Validity analysis*

In this subsection, we will verify the validity of the proposed decision-making method on the basis of the following three points. First, the optimal alternative obtained by the proposed method should not be changed when substituting a non-optimal alternative with another worse alternative. Second, a valid GDM method should satisfy the property of transitivity. Third, if a GDM problem is decomposed into several sub-problems and the same method is utilized to address these sub-problems, the ranking of the alternatives should be consistent with the original problem.

In terms of the first point, we have changed the original group evaluation of alternative $a_2$, which is represented in Table 12. The group assessment for the remaining alternatives remains the same. Based on the proposed method, we can obtain the ranking $a_3 \succ a_4 \succ a_1 \succ a_2$. Compared with the original ranking determined in subSection 4.2, the optimal alternative is the same, which demonstrates the validity of our method in terms of the first point.

Regarding the second and the third points, we have decomposed the original set of alternatives into four subsets as shown in Table 13. Based on the proposed method, the ranking of alternatives in the subsets can be obtained. We can observe that the rankings in these subsets are consistent with the ranking of the original alternative set, which further illustrates the validity of our method.

### 5.3. *Comparative analysis*

To illustrate the superiority of our method, we have qualitatively and quantitatively compared our method with some typical methods in this subsection.

First, we qualitatively compared our method with some of the latest SNGDM methods from the following four perspectives: (i) the expression structure of experts' evaluations; (ii) the consensus-reaching strategy; (iii) the criteria weighting method; and (iv) whether correlations between criteria are considered. The specific information is summarized in Table 14.

For the expression of expert evaluations, methods in [24,45,46] utilize crisp numbers or interval numbers to describe experts' judgements concerning the performance of alternatives, which cannot capture the uncertainty and vagueness of human perception. Different preference relations are introduced into SNGDM to represent expert evaluations [4,19,47]. However, requiring each expert to carry out pairwise evaluations of the alternatives according to each criterion causes a lot of work when the number of alternatives is large. For example, if a GDM problem involves ten alternatives and five criteria, each expert needs to perform 500 pairwise comparisons, which is very time-consuming. In addition, the pairwise comparison may lead to a decrease in evaluation consistency, therefore additional consistency checks and improvements are often required. Although scholars have utilized expressions containing linguistic variables to further characterize the ambiguity of expert assessments, such as linguistic distribution assessments (LDAs) [48], the linguistic variables in LDAs are expressed discretely. However, the ELICIT employed in our method extends the representation of comparative linguistic expressions to a continuous domain to better model the expert preferences. Therefore, ELICIT is closer to a human reasoning process and can improve the interpretability and accuracy of the results.

**Table 14**
Qualitative comparisons of various social network group decision-making methods.

| Method | Evaluation expression | Consensus-reaching strategy | Criteria weighting method | Correlation between criteria |
|---|---|---|---|---|
| [24] | Crisp numbers | Minimum cost model | Not considered | Not considered |
| [46] | Crisp numbers | Modified toward the expert with highest consensus level | Given in advance | Not considered |
| [47] | Interval values | Minimum adjustment model | Not considered | Not considered |
| [19] | Fuzzy preference relations | Modified toward group opinion | Not considered | Not considered |
| [4] | Additive/multiplicative preference relations | Minimum cost model | Not considered | Not considered |
| [48] | Distributed preference relation | Modified toward the opinion of group leader | Given in advance | Not considered |
| [49] | Linguistic distribution assessment | Minimum adjustment model | Assumed to be equal | Not considered |
| This paper | ELICIT | Hybrid centrality-based method | BWM-CRITIC model | Considered |

The consensus-reaching strategy for SNGDM problems can be mainly divided into two types, one is the optimization-based method (the most representative of which are the minimum adjustment model [45,48] and minimum cost model [4,24]), and the other is the identification-direction-based method [19,46,47].

Although the optimization-based methods are more efficient, they do not fully utilize trust relationships to guide experts for consensus improvement. For example, in the minimum cost model, the modified evaluations are obtained by minimizing the cost to reach consensus, ignoring the fact that expert opinions are susceptible to being influenced by others in the network.

As for the identification-direction-based methods, different references for modification have been proposed for the identified assessments. In [46], experts are required to modify towards the individual with the highest consensus degree, which ignores the trust relationship between experts and does not make good use of social network information. In [19], the modification reference is group assessment, which is very commonly used. In practice, however, decision makers tend to refer to the opinions of people they trust, while ignoring the ideas of the rest. In [47], the identified experts are encouraged to modify their opinions according to the group leader, who is determined based on degree centrality. However, degree centrality is a local measure that cannot reflect the real influence exerted by an individual from a global network structure.

In this study, we consider the locality of the individual and the global topology of the social network simultaneously and propose hybrid centrality. This measure can determine the influence of the individual by considering the importance of the individual itself, the importance of neighbor nodes, and the diffusion importance of the connected edges. Then, hybrid centrality is utilized to guide the consensus-reaching process, which can better reflect the information propagation in the network.

Many existing SNGDM methods fail to consider the criteria weight [19,24,45], they either give the weight in advance [46,47] or assume they are equal [48]. However, the weight of the criteria plays an important role in group decision-making problems, which directly affects the final alternative ranking. In our study, we develop the BWM-CRITIC method, which not only considers subjective and objective aspects, but also correlations between criteria. Specifically, the traditional BWM method is extended into the ELICIT environment to calculate the subjective weight; the Spearman correlation coefficient is introduced into the CRITIC method to determine the objective weight. Finally, the minimum relative entropy is applied to obtain the final weight, which ensures that the final weight is as close as possible to both the subjective and objective weights.

After various qualitative comparisons, we selected several typical methods for quantitative comparisons. Since different approaches employ different evaluation expression structures, we uniformly used crisp numbers to express expert opinions, and applied the same example to calculate the alternative ranking. The results are shown in Table 15.

Table 15 shows that the ranking values of alternatives calculated with the various methods are different, but the optimal and worst alternatives are consistent, which illustrates the effectiveness of the proposed method.

Then, we calculated the differentiation degree of the different methods to examine their ability to distinguish the alternatives. The higher the differentiation degree, the easier it is for experts to identify the most suitable choice. The formulas are given as follows.

**Table 15**
The ranking of alternatives obtained using different methods.

| Alternative | [24] | | [46] | | [19] | | This paper | |
|---|---|---|---|---|---|---|---|---|
| | Ranking value | Rank | Ranking value | Rank | Ranking value | Rank | Ranking value | Rank |
| $a_1$ | 0.813 | 4 | 0.382 | 4 | 0.494 | 4 | 0.229 | 4 |
| $a_2$ | 1.016 | 2 | 0.488 | 3 | 0.765 | 2 | 0.408 | 2 |
| $a_3$ | 1.142 | 1 | 0.835 | 1 | 0.943 | 1 | 0.723 | 1 |
| $a_4$ | 0.878 | 3 | 0.569 | 2 | 0.628 | 3 | 0.315 | 3 |

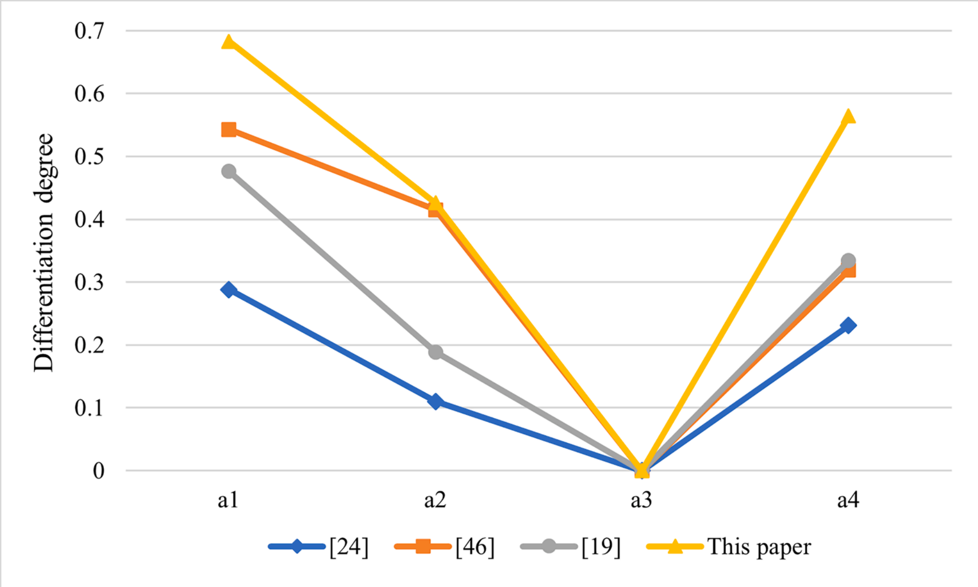


**Fig. 7.** The differentiation degree of alternatives using different methods.

$$Dif_i^\chi = \frac{\max_i\left(RV_i^\chi\right) - RV_i^\chi}{\max_i\left(RV_i^\chi\right)} \tag{30}$$

$$TDif^\chi = \sum_{i=1}^{4} Dif_i^\chi \tag{31}$$

where $RV_i^\chi (i = 1, 2, 3, 4)$ represents the ranking value of $a_i$ in method $\chi$ and $TDif^\chi$ denotes the total differentiation degree of method $\chi$.

Then, the differentiation degree and the total differentiation degree of the alternative ranking using different methods are given in Fig. 7 and Fig. 8, respectively.

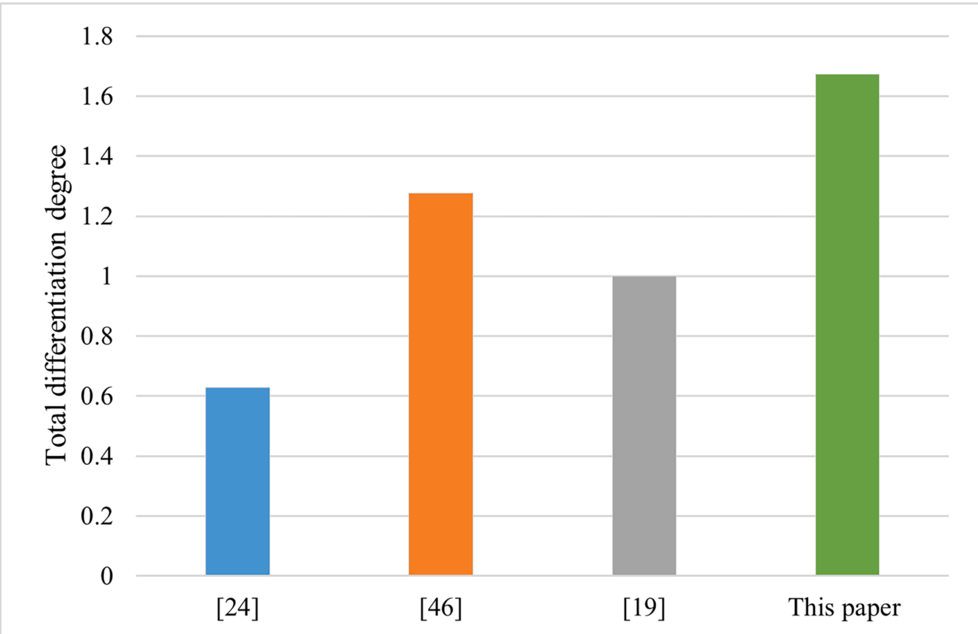


**Fig. 8.** The total differentiation degree of different methods.

As for future research, we believe that there are three directions worth exploring.

## 6. Conclusion

In group decision-making, the consensus-reaching process is usually adopted to mitigate conflict between the opinions of group members and to ensure that an agreed final solution can be implemented efficiently and effectively. Meanwhile, with the continuous advancement of information technology, social networks are becoming an effective tool to facilitate group communication. Therefore, how to mine the network topology with network information to facilitate consensus improvement is still an open research problem. In this study, we have proposed a novel consensus-building strategy from a complex network perspective for group decision-making with ELICIT information. By means of theoretical analysis and case verification, we can draw the following conclusions:

(1) Decision experts can use ELICIT to evaluate the performance of alternatives in a more flexible manner, which facilitates the elicitation of their preferences and improves the practicality of the method.
(2) A novel consensus-improving strategy is proposed from the perspective of complex networks. Specifically, we define hybrid centrality to characterize the influence of individuals by considering both local and global network information. Then, hybrid centrality is utilized to guide the consensus improvement, which can better reflect the information flow in the network.
(3) The weight of criteria is more comprehensively determined by constructing the BWM-CRITIC model. This model can coordinate the objective aspect, subjective aspect, and the correlations between criteria in criteria weighting, thereby improving the rationality of the ranking result.
(4) By applying the proposed method to a sustainable supplier selection problem, we verify the effectiveness, robustness, and validity of our method. Additionally, the qualitative and quantitative comparisons with typical methods illustrate the superiority of our approach.

As for future research, we believe that there are three directions worth exploring.

(1) As the decision-making environment becomes more complex, the need for large-scale group decision-making (LSGDM) is increasing. One of the keys to addressing LSGDM problems is to cluster a large number of decision-makers to reduce dimensionality and improve efficiency. There are multiple community detection algorithms in complex network analysis, such as the Louvain algorithm and BigCLAM. Therefore, figuring out how to extend and improve the traditional cluster algorithms to facilitate large-scale group decision-making will become a worthy research direction.
(2) Most of the existing GDM methods (as well as LSGDM) are constructed based on the principle of majority. However, minority opinions can be beneficial to the decision-making process. Therefore, it is necessary to effectively identify valuable minority opinions and protect the interests of minorities to achieve a more rational decision outcome.
(3) Last but not least, most of the current research assumes that the trust relationships between experts remain unchanged during the consensus-reaching process. However, these degrees of trust may evolve over time with variations in evaluation similarities. Therefore, figuring out how to address the consensus issue in a dynamic network needs to be explored further.

## CRediT authorship contribution statement

**Zhen Hua:** Conceptualization, Methodology, Software, Validation, Formal analysis, Writing - original draft, Writing - review & editing, Visualization. **Xiaochuan Jing:** Validation, Formal analysis, Investigation, Writing - review & editing. **Luis Martínez:** Conceptualization, Methodology, Validation, Formal analysis, Writing - original draft, Writing - review & editing, Supervision.

## Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Data availability

Data will be made available on request.

## Appendix A. The information about experts and their original evaluations on alternatives.

Tables 16–18.

**Table 16**
The information about the experts involved in the case study.

| Expert | Job Title | Educational background | Years of experience | Department |
|---|---|---|---|---|
| $e_1$ | Chief engineer | MEng | 10 | General management |
| $e_2$ | Deputy general manager | MEng | 12 | General management |
| $e_3$ | Marketing specialist | BBA | 5 | Marketing department |
| $e_4$ | Marketing specialist | BSc | 6 | Marketing department |
| $e_5$ | Supply chain manager | BEng | 9 | Purchase department |
| $e_6$ | Procurement officer | BEng | 4 | Purchase department |
| $e_7$ | Business operations manager | BBA | 8 | Operations department |
| $e_8$ | Operations specialist | Ph.D | 4 | Operations department |
| $e_9$ | Operations consultant | BEng | 5 | Operations department |
| $e_{10}$ | Finance director | BFin | 11 | Finance department |
| $e_{11}$ | Financial analyst | BFin | 9 | Finance department |
| $e_{12}$ | Sales manager | BEng | 12 | Sales department |
| $e_{13}$ | Customer service representative | BBA | 4 | Sales department |
| $e_{14}$ | Sales representative | BBA | 4 | Sales department |
| $e_{15}$ | Chief manufacturing executive | BEng | 10 | Production department |
| $e_{16}$ | Assembly supervisor | BEng | 5 | Production department |
| $e_{17}$ | Director of quality management | BEng | 11 | Production department |
| $e_{18}$ | R&D project manager | BEng | 9 | R&D department |
| $e_{19}$ | Senior R&D project manager | Ph.D | 9 | R&D department |
| $e_{20}$ | R&D project coordinator | BEng | 5 | R&D department |

**Table 17**
The original evaluations of experts $e_1$ to $e_{10}$.

| | | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ |
|---|---|---|---|---|---|---|
| $e_1$ | $a_1$ | $s_3$ | $bets_1\&s_2$ | $bets_3\&s_5$ | $bets_1\&s_2$ | $atmost s_2$ |
| | $a_2$ | $atmost s_2$ | $bets_2\&s_3$ | $bets_4\&s_5$ | $atleast s_4$ | $atleast s_4$ |
| | $a_3$ | $bets_2\&s_3$ | $bets_2\&s_3$ | $s_3$ | $atleast s_4$ | $bets_3\&s_5$ |
| | $a_4$ | $bets_1\&s_2$ | $atmost s_2$ | $atleast s_4$ | $bets_4\&s_5$ | $s_3$ |
| $e_2$ | $a_1$ | $bets_2\&s_3$ | $atmost s_2$ | $s_3$ | $atmost s_2$ | $bets_1\&s_2$ |
| | $a_2$ | $s_3$ | $bets_1\&s_2$ | $bets_3\&s_4$ | $bets_1\&s_2$ | $bets_2\&s_3$ |
| | $a_3$ | $atmost s_2$ | $atmost s_2$ | $atleast s_4$ | $s_3$ | $atleast s_5$ |
| | $a_4$ | $bets_1\&s_2$ | $atmost s_2$ | $s_3$ | $bets_4\&s_5$ | $bets_3\&s_5$ |
| $e_3$ | $a_1$ | $bets_2\&s_3$ | $atleast s_4$ | $bets_1\&s_2$ | $atmost s_2$ | $atleast s_4$ |
| | $a_2$ | $s_3$ | $bets_1\&s_2$ | $atmost s_2$ | $atmost s_2$ | $atleast s_4$ |
| | $a_3$ | $atleast s_5$ | $atleast s_4$ | $bets_2\&s_3$ | $s_3$ | $s_3$ |
| | $a_4$ | $atleast s_4$ | $bets_3\&s_4$ | $atmost s_2$ | $bets_1\&s_2$ | $atmost s_2$ |
| $e_4$ | $a_1$ | $s_3$ | $atleast s_4$ | $bets_2\&s_3$ | $bets_2\&s_3$ | $s_3$ |
| | $a_2$ | $bets_1\&s_2$ | $s_3$ | $bets_2\&s_3$ | $atmost s_2$ | $atmost s_2$ |
| | $a_3$ | $atleast s_4$ | $atleast s_5$ | $atmost s_2$ | $bets_3\&s_4$ | $s_3$ |
| | $a_4$ | $bets_2\&s_3$ | $bets_3\&s_5$ | $atmost s_2$ | $bets_2\&s_3$ | $bets_2\&s_3$ |
| $e_5$ | $a_1$ | $atmost s_2$ | $atleast s_4$ | $bets_2\&s_3$ | $bets_2\&s_3$ | $bets_2\&s_3$ |
| | $a_2$ | $bets_1\&s_2$ | $s_3$ | $bets_1\&s_2$ | $atmost s_2$ | $bets_3\&s_4$ |
| | $a_3$ | $atmost s_2$ | $atleast s_5$ | $bets_2\&s_3$ | $atleast s_4$ | $bets_1\&s_2$ |
| | $a_4$ | $bets_3\&s_4$ | $bets_3\&s_4$ | $bets_1\&s_2$ | $bets_2\&s_3$ | $atleast s_4$ |
| $e_6$ | $a_1$ | $atmost s_2$ | $bets_2\&s_3$ | $bets_3\&s_4$ | $bets_2\&s_3$ | $bets_1\&s_2$ |
| | $a_2$ | $bets_2\&s_3$ | $atmost s_2$ | $atleast s_4$ | $bets_2\&s_3$ | $bets_2\&s_3$ |
| | $a_3$ | $bets_1\&s_2$ | $bets_3\&s_5$ | $s_3$ | $s_3$ | $s_3$ |
| | $a_4$ | $s_3$ | $atmost s_2$ | $s_3$ | $bets_4\&s_5$ | $bets_4\&s_5$ |
| $e_7$ | $a_1$ | $atmost s_2$ | $bets_2\&s_3$ | $bets_3\&s_4$ | $bets_1\&s_2$ | $s_3$ |
| | $a_2$ | $bets_2\&s_3$ | $atmost s_2$ | $s_3$ | $atleast s_5$ | $bets_2\&s_3$ |
| | $a_3$ | $s_3$ | $s_3$ | $s_3$ | $bets_1\&s_2$ | $s_3$ |
| | $a_4$ | $bets_1\&s_2$ | $atleast s_4$ | $bets_4\&s_5$ | $atmost s_2$ | $atmost s_2$ |
| $e_8$ | $a_1$ | $bets_2\&s_3$ | $bets_1\&s_2$ | $atmost s_2$ | $bets_3\&s_4$ | $atmost s_2$ |
| | $a_2$ | $bets_3\&s_4$ | $s_3$ | $atmost s_2$ | $s_3$ | $bets_1\&s_2$ |
| | $a_3$ | $bets_2\&s_3$ | $bets_4\&s_5$ | $bets_4\&s_5$ | $atleast s_4$ | $s_3$ |
| | $a_4$ | $s_3$ | $bets_2\&s_3$ | $atleast s_5$ | $s_3$ | $atleast s_4$ |
| $e_9$ | $a_1$ | $atleast s_4$ | $atleast s_4$ | $atleast s_4$ | $atmost s_2$ | $atmost s_2$ |
| | $a_2$ | $s_3$ | $atleast s_4$ | $bets_3\&s_5$ | $atmost s_2$ | $bets_2\&s_3$ |
| | $a_3$ | $atleast s_5$ | $s_3$ | $s_3$ | $atmost s_2$ | $bets_1\&s_2$ |
| | $a_4$ | $bets_3\&s_5$ | $bets_4\&s_5$ | $bets_3\&s_5$ | $bets_2\&s_3$ | $s_3$ |
| $e_{10}$ | $a_1$ | $s_3$ | $s_3$ | $s_3$ | $s_3$ | $bets_1\&s_2$ |
| | $a_2$ | $bets_4\&s_5$ | $bets_3\&s_5$ | $bets_4\&s_5$ | $bets_1\&s_2$ | $s_3$ |
| | $a_3$ | $atleast s_5$ | $bets_4\&s_5$ | $s_3$ | $bets_3\&s_4$ | $bets_1\&s_2$ |
| | $a_4$ | $s_3$ | $bets_1\&s_2$ | $bets_2\&s_3$ | $s_3$ | $atleast s_4$ |

**Table 18**
The original evaluations of experts $e_{11}$ to $e_{20}$.

| | | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ |
|---|---|---|---|---|---|---|
| $e_{11}$ | $a_1$ | $bets_4\&s_5$ | $atmosts_2$ | $s_3$ | $atmosts_2$ | $atleasts_4$ |
| | $a_2$ | $bets_4\&s_5$ | $bets_2\&s_3$ | $bets_4\&s_5$ | $bets_2\&s_3$ | $s_3$ |
| | $a_3$ | $atmosts_2$ | $atmosts_2$ | $atmosts_2$ | $s_3$ | $bets_4\&s_5$ |
| | $a_4$ | $s_3$ | $bets_2\&s_3$ | $bets_2\&s_3$ | $atmosts_2$ | $bets_3\&s_4$ |
| $e_{12}$ | $a_1$ | $atleasts_5$ | $atleasts_4$ | $bets_2\&s_3$ | $bets_2\&s_3$ | $bets_3\&s_5$ |
| | $a_2$ | $atleasts_4$ | $bets_3\&s_4$ | $atmosts_2$ | $bets_1\&s_2$ | $atmosts_2$ |
| | $a_3$ | $atmosts_2$ | $bets_3\&s_4$ | $s_3$ | $bets_3\&s_5$ | $atleasts_4$ |
| | $a_4$ | $s_3$ | $atmosts_2$ | $atmosts_2$ | $s_3$ | $atleasts_5$ |
| $e_{13}$ | $a_1$ | $bets_4\&s_5$ | $bets_2\&s_3$ | $bets_2\&s_3$ | $atmosts_2$ | $bets_3\&s_4$ |
| | $a_2$ | $s_3$ | $atmosts_2$ | $s_3$ | $atleasts_4$ | $bets_2\&s_3$ |
| | $a_3$ | $bets_2\&s_3$ | $bets_3\&s_5$ | $atmosts_2$ | $bets_2\&s_3$ | $bets_3\&s_4$ |
| | $a_4$ | $bets_1\&s_2$ | $bets_3\&s_4$ | $bets_1\&s_2$ | $s_3$ | $s_3$ |
| $e_{14}$ | $a_1$ | $s_3$ | $atmosts_2$ | $bets_2\&s_3$ | $bets_3\&s_4$ | $bets_2\&s_3$ |
| | $a_2$ | $atmosts_2$ | $atleasts_5$ | $s_3$ | $atleasts_4$ | $bets_2\&s_3$ |
| | $a_3$ | $atleasts_4$ | $bets_1\&s_2$ | $atmosts_2$ | $bets_2\&s_3$ | $bets_3\&s_4$ |
| | $a_4$ | $bets_3\&s_4$ | $s_3$ | $s_3$ | $atleasts_4$ | $bets_3\&s_5$ |
| $e_{15}$ | $a_1$ | $bets_1\&s_2$ | $bets_3\&s_4$ | $bets_1\&s_2$ | $bets_2\&s_3$ | $atmosts_2$ |
| | $a_2$ | $S_3$ | $bets_2\&s_3$ | $bets_3\&s_4$ | $bets_2\&s_3$ | $atmosts_2$ |
| | $a_3$ | $bets_3\&s_5$ | $s_3$ | $atmosts_2$ | $bets_1\&s_2$ | $bets_3\&s_5$ |
| | $a_4$ | $bets_4\&s_5$ | $bets_2\&s_3$ | $bets_1\&s_2$ | $bets_2\&s_3$ | $bets_3\&s_4$ |
| $e_{16}$ | $a_1$ | $bets_2\&s_3$ | $bets_2\&s_3$ | $bets_2\&s_3$ | $atmosts_2$ | $bets_3\&s_5$ |
| | $a_2$ | $bets_1\&s_2$ | $atmosts_2$ | $bets_3\&s_4$ | $bets_4\&s_5$ | $bets_2\&s_3$ |
| | $a_3$ | $bets_2\&s_3$ | $atleasts_4$ | $bets_1\&s_2$ | $bets_2\&s_3$ | $bets_1\&s_2$ |
| | $a_4$ | $bets_1\&s_2$ | $s_3$ | $atmosts_2$ | $atmosts_2$ | $bets_2\&s_3$ |
| $e_{17}$ | $a_1$ | $atmosts_2$ | $s_3$ | $atmosts_2$ | $bets_2\&s_3$ | $bets_3\&s_5$ |
| | $a_2$ | $atmosts_2$ | $atmosts_2$ | $atleasts_4$ | $bets_2\&s_3$ | $bets_2\&s_3$ |
| | $a_3$ | $s_3$ | $atmosts_2$ | $atmosts_2$ | $s_3$ | $atmosts_2$ |
| | $a_4$ | $bets_1\&s_2$ | $atmosts_2$ | $bets_3\&s_5$ | $s_3$ | $bets_2\&s_3$ |
| $e_{18}$ | $a_1$ | $atmosts_2$ | $atmosts_2$ | $s_3$ | $atmosts_2$ | $bets_4\&s_5$ |
| | $a_2$ | $s_3$ | $bets_2\&s_3$ | $atmosts_2$ | $atmosts_2$ | $bets_2\&s_3$ |
| | $a_3$ | $s_3$ | $bets_2\&s_3$ | $atmosts_2$ | $bets_3\&s_5$ | $atleasts_4$ |
| | $a_4$ | $bets_1\&s_2$ | $s_3$ | $atleasts_4$ | $bets_1\&s_2$ | $atleasts_4$ |
| $e_{19}$ | $a_1$ | $atleasts_4$ | $bets_1\&s_2$ | $atmosts_2$ | $bets_2\&s_3$ | $atmosts_2$ |
| | $a_2$ | $bets_1\&s_2$ | $atmosts_2$ | $atmosts_2$ | $s_3$ | $bets_1\&s_2$ |
| | $a_3$ | $atleasts_4$ | $bets_2\&s_3$ | $s_3$ | $s_3$ | $bets_2\&s_3$ |
| | $a_4$ | $s_3$ | $atmosts_2$ | $bets_4\&s_5$ | $bets_2\&s_3$ | $bets_3\&s_4$ |
| $e_{20}$ | $a_1$ | $atmosts_2$ | $atleasts_4$ | $s_3$ | $bets_1\&s_2$ | $bets_2\&s_3$ |
| | $a_2$ | $atmosts_2$ | $s_3$ | $bets_4\&s_5$ | $bets_2\&s_3$ | $bets_2\&s_3$ |
| | $a_3$ | $atleasts_4$ | $bets_2\&s_3$ | $bets_4\&s_5$ | $bets_2\&s_3$ | $bets_2\&s_3$ |
| | $a_4$ | $s_3$ | $bets_2\&s_3$ | $atmosts_2$ | $atleasts_5$ | $atleasts_4$ |

## Appendix B. The adjacency matrix of twenty experts

.

$$T=\begin{bmatrix}
0&1&1&1&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0\\
1&0&1&1&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0\\
1&1&0&1&1&1&0&0&0&0&0&0&0&0&0&0&1&1&0&0\\
1&1&1&0&1&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&1&0&1&1&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&1&1&1&0&1&1&1&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&1&1&1&0&0&0&1&1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&1&0&0&0&0&0&0&0&0&0&0\\
1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&1&1\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&1&1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0
\end{bmatrix}$$

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