Information Sciences 716 (2025) 122222

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Automatic consensus models to balance consensus cost, consistency level and consensus degree with attitudinal trust mechanism

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ARTICLE INFO

Keywords:
Group decision making
Consensus reaching process
Consistency level
Consensus cost

ABSTRACT

In light of the inevitable consensus costs incurred by preference adjustments of decision makers during the consensus reaching process (CRP), multiple minimum cost driven consensus models have been developed, which either prioritize the attainment of a high consensus degree, or focus on the consistency maintenance of individual opinions. However, the strategic equilibrium of consensus cost, consistency level and consensus degree, which shapes the cogency of the decision-making outcome, becomes one of the main challenges which should be overcome in the CRP. To address this scenario, this study proposes three novel trust attitude-based consensus models to balance these three factors. These consensus models are implemented through optimization models, tailored to distinct primary objectives, resulting in outputs that encompass attitudinal parameters to realize the balance of consensus cost, consistency level and consensus degree. Correspondingly, the proposed consensus models have been applied to solve severe air pollution emergency management decision problems. Comparative analysis with existing works is provided to show the validity of the proposed models.

1. Introduction

In the context of real-world decision making, the results generated through group decision making (GDM) processes exhibit significantly greater reliability compared to those based on the preferences of a single individual. This is particularly true when decision makers exhibit insufficient domain-specific expertise or contextual awareness [1,2]. In a common GDM process, preferences collected from multiple decision makers are fused either to obtain the ranking of alternatives or choose the best alternative [3]. Most of GDM processes encompass three key stages: an information collection process, an aggregation process and an exploitation process [4]. The aggregation and exploitation processes are often collectively regarded as a selection process. However, direct aggregation of decision makers' preferences tends to elicit dissent among group members, as some may feel that their opinions have been disregarded. To attain a decision result that is accepted by the group, there is a compelling need to integrate a consensus reaching process (CRP) into GDM [5,6], before the selection process. In essence, CRP can be regarded as a negotiation process wherein the opinions of decision

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https://doi.org/10.1016/j.ins.2025.122222
Received 2 October 2024; Received in revised form 15 March 2025; Accepted 18 April 2025

Available online 23 April 2025

makers converge towards each other achieving a high consensus degree, aiming to balance multiple interests within the group, eliminate conflicts, improve the quality of GDM outcomes, and increase the satisfaction of decision makers with the final decision [7,8]. Implementing consensus models is of critical importance for augmenting decision making effectiveness [9]. In addition to consensus degree, consistency level and consensus cost are also two important factors that may influence the satisfaction of decision makers in CRPs. The consideration of consistency level is to ensure the preference expression is reasonable, while the consideration of consensus cost aids in reducing the interest loss for decision makers and maintain their willingness to cooperate in the CRP. Over recent decades, consensus models that have considered consensus cost or consistency level of preferences relations have been widely studied [10–12].

When decision makers adjust their preferences in CRP, they typically aim to preserve as much initial preference as possible. Consequently, consensus cost embedded in consensus models has garnered significant attention [13]. For instance, Guo et al. [14] explored minimum cost consensus models based on linear uncertainty distribution preferences. Wu et al. [15] proposed some minimum cost consensus models considering the trust relationship among decision makers and moderators. Ben-Arieh and Easton [16] applied the minimum distance between individual and the group opinion to measure the consensus degree, and proposed a minimum cost consensus model. Zhang et al. [17] presented a similar minimum cost consensus model, measuring group consensus degree based on different aggregation operators. Wu et al. [18] proposed an attitudinal trust recommendation mechanism to balance consensus degree and harmony degree, in which harmony degree is highly related to the cost and the concept of attitudinal parameter was first used to measure the trust relationship between decision makers.

Meanwhile, consensus models that address the consistency of preference relations have also been extensively studied in the literature. For instance, Herrera et al. [19] proposed a consensus model with consistency control, which consists of both an consistency enhancement process and a CRP. Chiclana et al. [20] developed a consistency-driven consensus model that generates recommendations for modifying preferences of decision makers to improve consistency. Tang et al. [21] developed two consensus models by applying personalized numerical scales of linguistic terms, while the computation of personalized numerical scales are based on consistency-driven methodology. Wan et al. [22] established a personalized individual semantic-based consensus model to deal with large-scale GDM problem, which constructed several minimum-adjustment-based models and modeling personalized individual semantics based on the consistency of a linguistic preference relation. Wu and Xu [23] proposed an automatic consensus enhancement process that considers individual consistency. Dong et al. [24] proposed two consistency-driven consensus models based on the analytic hierarchy process under the row geometric mean prioritization method. Leyva-López [25] proposed a sequence based consensus model that considers the consistency of preference relations while pursuing consensus.

Eventhough, consensus models considering consistency and cost have been independently and rigorously studied in the literature, the strategy to balance the consistency level, consensus cost, and consensus degree in CRPs is still limited. Some extant studies emphasized the significance of balancing two of these three factors in CRPs [18], nevertheless, they fail to furnish specific strategies to achieve such an objective. Building on the preceding analysis, the motivations of this study are:

(1) Intricate relationships exist between consistency levels, consensus degrees, and consensus costs, which stemming from the preference relation adjustments during the CRP. However, limited research has focused on exploring such relationships among these three factors, potentially leading to an inability to fully harness the underlying decision making information, and, consequently, affecting the rationality of decision result.
(2) An ideal automatic preference relation adjustment mechanism that cooperates with CRP should concurrently maximize the group consensus degree, minimize the consensus cost, and keeps or increases the level of consistency after preference relation adjustments. However, in real-world GDM processes, these three objectives are usually in conflict. There is a notable scarcity of effective strategies to balance consensus degree, consistency level and consensus cost.
(3) Existing consensus models that pursue high consistency levels usually fail to consider consensus cost, while minimum cost consensus model ignores the consistency of preference relation. There is a lack of research on consensus models that concurrently consider of consistency level, consensus degree, and consensus cost.

To address these research limitations, this study endeavors to leverage the concept of attitudinal parameter, as introduced by Wu et al. [18], to develop effective automated consensus models. This integration is designed to harmonize the interplay between consensus degree, consistency level and consensus cost, achieved by means of in-depth exploration of intricate relationships among these three factors, facilitated by the utilization of attitudinal parameters. The main contributions of the current research are stated as below.

(1) Intricate relationships between attitudinal parameter and consensus degree, consistency level, and consensus cost have been investigated, respectively. The linkage among consensus degree, consistency level, and consensus cost is explored based on the attitudinal parameter.
(2) Several automatic preference relation adjustment mechanisms are introduced, which offer a systematic approach to ascertain the optimal combination of attitudinal parameter and adjustment range, thereby facilitating the equilibrium of consensus degree, consistency level and consensus cost in the CRP.
(3) Three types of automatic consensus models are proposed to reduce consensus cost while striving for an elevated consistency level of preference relations. It provides flexibility to choose suitable consensus models based on the most urgent demands in different decision making scenarios.

The remainder of this paper is structured of as follows. Section 2 reviews some basis knowledge related to this study. Section 3 presents the GDM framework and delineates the related formulations essential for the consensus models. Section 4 focuses on the automatic preference relation adjustment mechanisms to determine adjustment parameters and attitudinal parameters during the CRP, and establishes three novel automatic consensus models. In Sections 5 and 6, we provide a numerical example and conduct some simulation experiments to validate the proposed models. Section 7 concludes the paper.

2. Preliminaries

In this section, some knowledge pertinent to the subsequent study is revised, including the additive consistency of a fuzzy preference relation (FPR), consensus cost, and attitudinal trust degree-based preference aggregation.

2.1. Additive consistency of an FPR

To standardize notational conventions and facilitate the discussion, the following definitions have been adapted to fit within the context of GDM. Let $G = \{g_1, g_2, \ldots, g_f\}$ be the set of decision makers and $g_h \in G$.

Definition 1. *[26,27] Let $X = \{x_1, x_2, \ldots, x_n\}$ be an alternative set, the FPR provided by decision maker g_h is defined as a matrix $R^h = (r^h_{ij})_{n\times n}$, such that*

$$r^h_{ij} \geq 0, r^h_{ij} + r^h_{ji} = 1, i, j = 1, 2, \ldots, n, \tag{1}$$

where r^h_{ij} indicates the preference of x_i over x_j for g_h. In particular, $r^h_{ij} = 0.5$ indicates that x_i is indifferent to x_j, $0 \leq r^h_{ij} < 0.5$ indicates that x_j is preferred to x_i, while $0.5 < r^h_{ij} \leq 1$ indicates that x_i is preferred to x_j.

To ensure the rationality of decision results, scholars have proposed numerous definitions of the consistency of FPRs [28], which can be primarily categorized into multiplicative consistency [29], ordinal consistency [30], and additive consistency [31]. In the current research, we embrace additive consistency without loss of generality.

Definition 2. *[27] An FPR $R^h = (r^h_{ij})_{n\times n}$ provided by g_h is additively consistent if $r^h_{ij} = r^h_{ik} + r^h_{kj} - 0.5$ for all $i, j, k = 1, 2, \ldots, n$.*

Definition 3. *[32] Let $R^h = (r^h_{ij})_{n\times n}$ be the FPR provided by g_h, then the additive consistency level of R^h is computed by*

$$CI_h = 1 - \frac{4}{n(n-1)(n-2)} \sum\nolimits_{i=1}^{n} \sum\nolimits_{j=i+1}^{n} \sum\nolimits_{k=j+1}^{n} |r^h_{ij} + r^h_{jk} - r^h_{ik} - 0.5|. \tag{2}$$

2.2. Consensus cost

Consensus cost denotes the expenditure required to realign non-consensus decision makers' perspectives with the group viewpoint. In existing literature, individual consensus cost is often directly proportional to, and sometimes directly measured by, the deviation between the individual viewpoint before and after the CRP [33,34]. In the current research, we employ the measure proposed in [33] without loss of generality.

Definition 4. *[33] Let $G = \{g_1, g_2, \ldots, g_f\}$ be the set of decision makers and $R^h = (r^h_{ij})_{n\times n}$ be the preference relation provided by g_h ($h = 1, 2, \ldots, f$). If $R^h = (r^h_{ij})_{n\times n}$ is adjusted to $\overline{R}^h = (\overline{r}^h_{ij})_{n\times n}$ after the CPR, the consensus cost of g_h ($h = 1, 2, \ldots, f$) is defined by*

$$TC_h = \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} |r^h_{ij} - \overline{r}^h_{ij}|}{n(n-1)}. \tag{3}$$

It is evident that $TC_h \in [0, 1]$.

2.3. Attitudinal trust model

To quantify the trust relationship among experts, attitudinal trust model was proposed in [18]. The core principle of this model posits that the degree of trust between two decision makers is commensurate with the proximity of their respective opinions. Trust relationship matrix is defined as follows.

Definition 5. *[18] Suppose that $R^h = (r^h_{ij})_{n\times n}$ is the preference relation provided by decision maker g_h, $R^l = (r^l_{ij})_{n\times n}$ is the preference relation provided by decision maker g_l, their trust relationship is described by*

$$Tr_{hl} = Sim(R^h, R^l). \tag{4}$$

Here, $Sim(R^h, R^l) = 1 - \frac{1}{n^2}\sum_{i,j\in\{1,2,\cdots,n\}} |r^h_{ij} - r^l_{ij}|$. The trust relation matrix is constructed as $Tr = (Tr_{hl})_{f\times f}$.[1]

The attitudinal trust degree (ATD) was introduced in [35] to aggregate decision makers' preferences, which allows their weights to be determined by attitudinal trust parameter.

Definition 6. *[18] Suppose that the trust relationship degree of decision maker g_h to decision maker g_l is $\{Tr_{hl}|h,l = 1,2,\cdots,f\}$, σ : $\{1,2,\dots,f\} \to \{1,2,\dots,f\}$ is a mapping that satisfies $Tr_{h\sigma(f)} < Tr_{h\sigma(f-1)} < \cdots < Tr_{h\sigma(1)}$. The ATD is calculated using an OWA operator guided by a basic unit-monotonic function Q such that*

$$w^{ATD}_{\sigma(v)} = Q\left(\frac{v}{f}\right) - Q\left(\frac{v-1}{f}\right), \quad v = 1,2,\dots,f. \tag{5}$$

The parameterized family of regular increasing monotone quantifiers $Q(r) = r^\eta$ [36] is employed in the rest of the paper, with $\eta \in [0,1]$ as an attitudinal parameter that reflects the decision maker's attitude. Denote $W^{ATD} = (w^{ATD}_{\sigma(1)},\dots,w^{ATD}_{\sigma(f)})$, if $\eta \in [0,1]$, we have ***orness*** $(W^{ATD}) = 1/(1+\eta)$ [37], a lower η results in an aggregation guided by $Q(r)$ closer to maximum aggregation operator. Chiclana et al. [38] demonstrated that $w^{ATD}_{\sigma(f)} < w^{ATD}_{\sigma(f-1)} \cdots < w^{ATD}_{\sigma(1)}$.

The collected preference relation is computed to ascertain the direction of adjustment, the ATD based OWA operator is utilized to allocate weights for decision makers, with different attitudinal parameters η. In particular, three types of trust policies were defined by Wu et al. [39], according to the attitude of non-consensus decision maker to others.

(1) if $Q(r) = r^0$, then $W^{ATD} = (1,0,\dots,0)$, which means that a non-consensus decision maker only trust decision maker with closest opinions, i.e., himself/herself. This strategy is termed "optimistic trust policy" in [18].[2]

(2) if $Q(r) = r$, then $W^{ATD} = (\frac{1}{f},\dots,\frac{1}{f})$, which means that a non-consensus decision maker trusts all decision makers equally, that is "indifferent trust policy", corresponding to arithmetic mean operator.

(3) "bounded trust policy" pertains to a decision maker selecting an attitudinal parameter that ensures the consensus degree reaches the threshold value.

The ATD method can be considered a dependable technique here for aggregating individual preference relations into a collective preference relation, i.e.,

$$R^c = \sum_{v=1}^{f} w^{ATD}_{\sigma(v)} R^v. \tag{6}$$

The weights $w^{ATD}_{\sigma(v)}(v \in \{1,2,\cdots,f\})$ are computed by using the linguistic quantifier (see Eq. (5)), the mapping $\sigma : [1,f] \to [1,f]$ is applied to carry out the mechanism that the more trusted decision maker (by non-consensus decision makers) gains the higher weight value.

Fig. 1 shows that W^{ATD} is influenced by both the attitudinal parameter η and the number of decision makers, denoted as f. Specifically, Figs. 1(a)-1(b) show how experts' weights change as the number of decision-makers increases from 2 to 10, and then from 10 to 50, assuming a pre-determined attitudinal parameter of 7/8. Figs. 1(b)-1(f) further demonstrate how the weights vary with the number of decision-makers for different attitudinal parameters.

3. Consensus group decision making formulation

In this section, the consensus group decision making process is formulated, the measures to compute individual and group consensus degree are introduced and the identification rule for non-consensus decision makers is presented.

3.1. The description of consensus group decision making

Usually, a consensus group decision making can be described by four stages: information collection stage, CRP, information aggregation stage, and exploitation stage. In the current research, an automatic consensus model is used in the CRP, prior to the information aggregation and exploitation stage. Specifically, the preference relations of non-consensus decision makers will be automatically adjusted during the CRP, in accordance with the preference relation adjustment mechanisms derived from the proposed optimal models. A set of alternatives is denoted by $X = \{x_1,x_2,\dots,x_n\}$, a set of decision makers is denoted by $\{g_1,g_2,\dots,g_f\}$. Suppose that the preference of decision maker g_h on x_i over x_j is r^h_{ij}, the preference relation is denoted by $R^h = (r^h_{ij})_{n\times n}$.

For the sake of clarity, we present the list of mathematical notations in Table 1.

[1] The definition has been refined from a decision matrix framework to a preference relation scenario.

[2] Eq. (5) to compute ATD-OWA has been adjusted, from $n-1$ dimension in [18] to n dimension in the current work. Correspondingly, the way to state trust policies have also been adjusted, from $n-1$ dimension to n dimension computation. For instance, in [18], "optimistic trust" policy means that a decision maker only trust the decision maker with the most similar opinion, in the current proposal it is the decision maker only trust himself/herself.

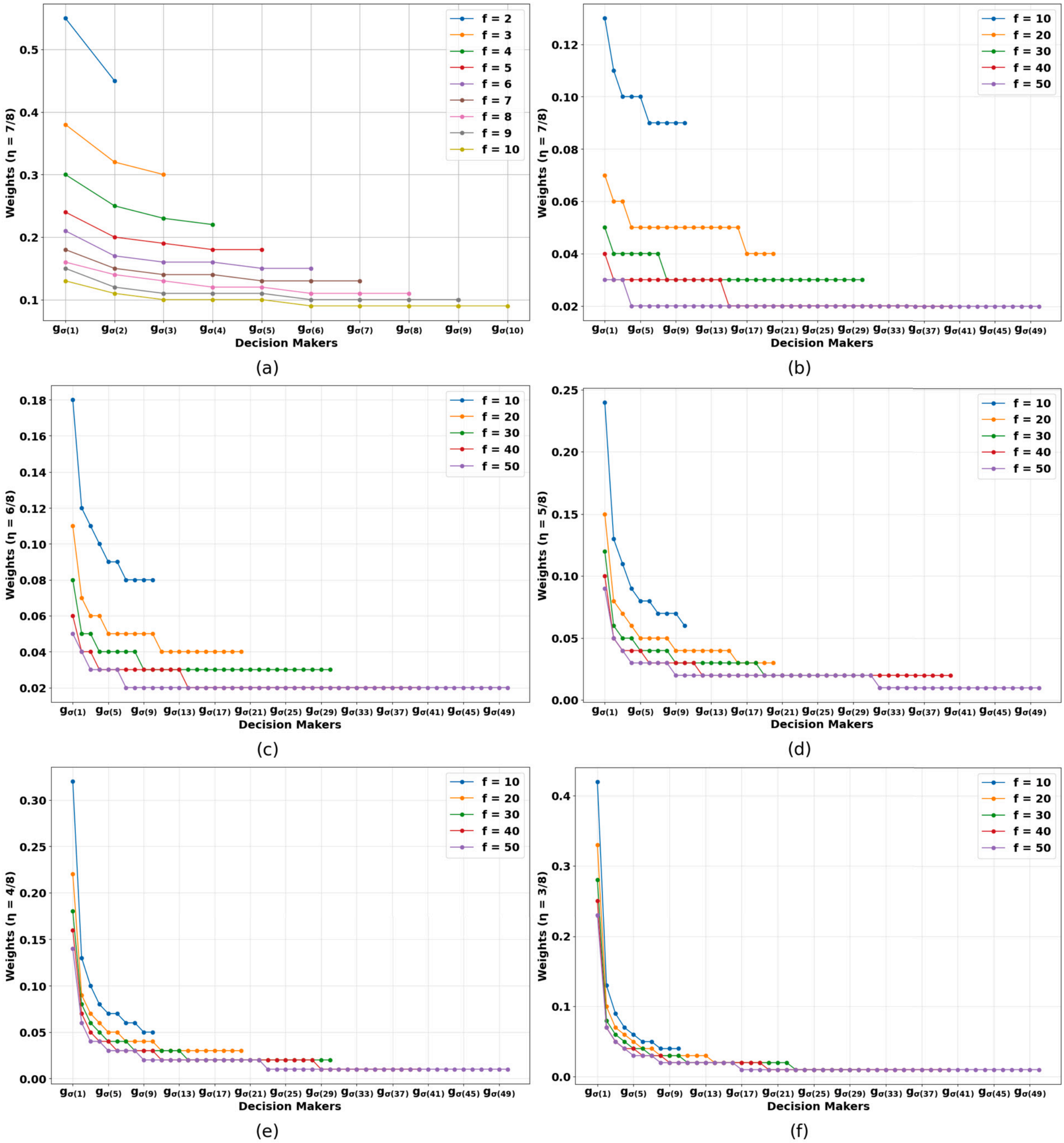

Fig. 1. W^{ATD} with various amounts of decision makers.

3.2. Consensus degree measurement

We measure the consensus degree by computing the deviation between individual preference relation and group average preference relation. The consensus degree of individual g_h is computed by:

$$CL_h = 1 - |R^h - R^a| = 1 - \frac{\sum_{i \neq j} |r_{ij}^h - r_{ij}^a|}{n(n-1)},\ h = 1, 2 \ldots, f, \tag{7}$$

where R^a is the average of decision makers' individual preference relations:

$$R^a = \frac{1}{f} \sum_{h=1}^{f} R^h \tag{8}$$

Table 1
List of mathematical notations.

Mathematical symbol	Meaning of symbols
$X=\{x_1,x_2,\dots,x_n\}$	alternative set
$G=\{g_1,g_2,\dots,g_f\}$	decision maker set
R^h	preference relation of g_h before the CRP
R^c	collective preference relation before the CRP
$\overline{R}^h$	preference relation of g_h after the CRP
θ	group consensus threshold
θ'	individual consensus threshold
$G_1=\{g_h\in G\mid CL_h<\theta'\}$	decision makers with an unacceptable consensus level
$G_2=\{g_h\in G\mid CL_h\geq\theta'\}$	decision makers with an acceptable consensus level
CL	consensus degree of the group before CRP
CI	consistency level of the group before CRP
$\overline{CL}$	consensus degree of the group after CRP
$\overline{CI}$	consistency level of the group after CRP
TC	consensus cost of the group
CL_h	consensus level of g_h before CRP
CI_h	consistency level of g_h before CRP
$\overline{CL}_h$	consensus level of g_h after CRP
$\overline{CI}_h$	consistency level of g_h after CRP
TC_h	consensus cost of g_h
η	attitudinal parameter
δ	adjustment parameter
α, β and μ	the balance parameters to control the importance of two different factors
γ_1	flexible requirement for consensus degree
γ_2	holistic requirement for consensus degree and consensus cost
γ_3	holistic requirement for consistency level and consensus cost
γ_4	holistic requirement for consensus degree and consistency level
$R^*=\sum_{g_h\in G_1}R^h/\#G_1=(r^*_{ij})_{n\times n}$	the average preference relation of the non-consensus decision makers
$R^a=\frac{1}{f}\sum_{h=1}^{f}R^h=(r^a_{ij})_{n\times n}$	the average preference relation of all the decision makers

The consensus degree of the group is computed by

$$CL=\frac{1}{f}\sum_{h=1}^{f}CL_h. \tag{9}$$

3.3. Non-consensus decision maker identification

The objective to identify the non-consensus decision makers is to determine the preference relation of whom to be adjusted in order to reach the consensus. The non-consensus decision makers are identified based on the following principles.

Identification Principle 1. If the consensus degree $CL<\theta$, then preference relation of every decision maker whose individual consensus degree satisfies $CL_h<\theta'$ will be automatically adjusted.

Identification Principle 2. Threshold values θ' and θ could be different. Without loss of generality, we use $\theta'=\theta$ in the current work (Some common methods to fix such thresholds can be found in [20][40], etc.).

Various automatic adjustment principles can be employed by decision makers during the CRP. Specific principles are outlined as below.

Adjustment Principle 1. It is assumed that the group consensus could be reached after automatic preference relations adjustments in accordance with the proposed optimal models.

Adjustment Principle 2. All the decision makers whose individual consensus degree fail to meet the threshold could be automatically adjusted according to the proposed optimal models.

In future research, these principles could be relaxed by permitting only preference relations of a subset of decision makers, whose individual consensus degrees fall below the threshold, to be modified aimed at achieving the consensus, contingent upon decision makers' respective consensus degrees.

3.4. Decision makers' weights allocation with the ATD-OWA operator

Based on the principle "the greater the alignment in the viewpoints of two experts, the higher the degree of trust that will exist between them" [18], by quantifying the level of trust among experts, we can assign corresponding weights to each expert. Denote the average preference of the non-consensus decision makers by $R^*=\sum_{g_{h_1}\in G_1}R^{h_1}/\#G_1$, where $\#G_1$ represents the number of decision makers in G_1. The similarity between R^h and R^* can be computed by

$$Sim(R^h,R^*)=1-\frac{1}{n^2}\sum\nolimits_{i,j\in\{1,2,\cdots,n\}}|r^h_{ij}-r^*_{ij}|,\ h\in\{1,2,\cdots,f\}, \tag{10}$$

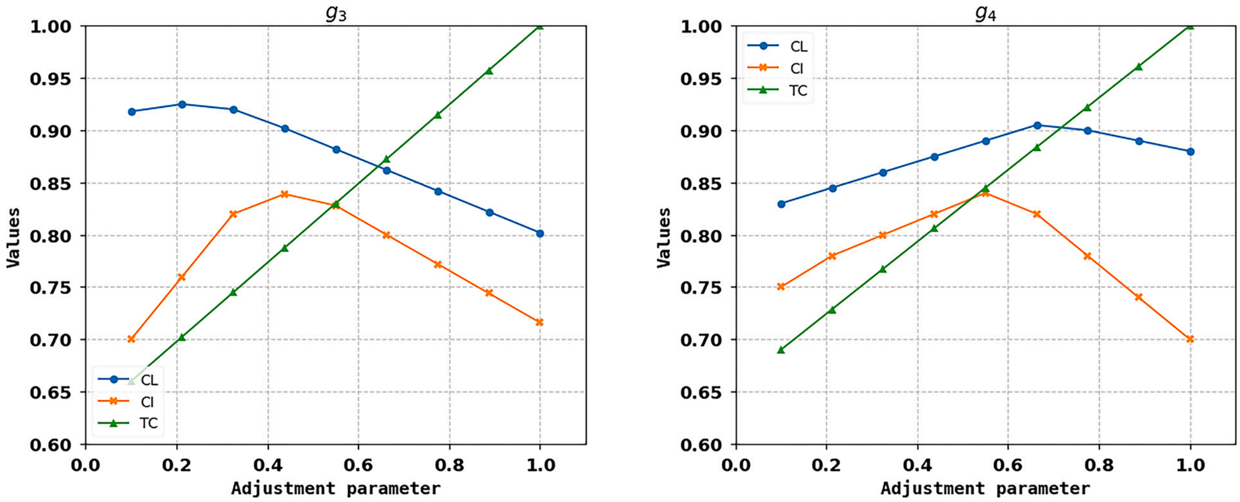

Fig. 2. Conflicts among CI, CL and TC for g_3 and g_4.

where r_{ij}^h and r_{ij}^* denote elements in the preference relation R^h and the average preference relation of non-consensus decision makers R^*, respectively. R^* can be regarded as the sub-group opinion of non-consensus decision makers, if $Sim(R^l, R^*) > Sim(R^h, R^*)$ for $l, h \in \{1, 2, \cdots, f\}$, then $w_{\sigma(l)}^{ATD} > w_{\sigma(h)}^{ATD}$.

3.5. Automatic preference relation adjustment

With the weights allocation scheme introduced in Section 3.4, in the preference relation adjustment process, the preference relations of non-consensus decision makers should be modified according to the following rules to expedite the convergence of the consensus process.

$$\overline{R}^h = (1-\delta)R^h + \delta \sum_{v=1}^{f} w_{\sigma(v)}^{ATD} R^v,\ g_h \in G_1. \tag{11}$$

$\delta \in [0,1]$ represents an adjustment parameter quantifying the change from original preference to preferences of other decision makers. The decision makers' preference relations should be automatically adjusted according to the opinions of decision makers he/she trusts, to make the automatic adjustment mechanism more in line with expectations of participants in the GDM, avoiding the outcomes of automated adjustments that are too psychologically unpalatable to decision makers. The aggregation based on ATD-OWA is employed to address the trust difference among decision makers. To achieve this goal, the adjustment parameter will be determined based on the proposed optimal models in the current research.

3.6. The necessity to balance consensus degree, consistency level and consensus cost in CRP

In the following, an example is provided to illustrate the conflict among consensus cost, consistency level and consensus degree, emphasizing the critical need to balance these three factors.

Example 1. *Suppose that the preference relations provided by decision makers $G = \{g_1, g_2, \cdots, g_4\}$ are provided as below, the decision makers g_3 and g_4 are identified as non-consensus individuals whose preference relations should be modified. If the attitudinal parameter is predetermined as $1/8$, Fig. 2 demonstrates the trends in consensus degree, consistency level and consensus cost for g_3 and g_4 under varying adjustment parameter. For g_3, increasing the adjustment parameter from 0.2 to 0.4 demonstrates contrasting patterns: the consistency level enhances while the consensus degree declines, and the consensus cost escalates. For g_4, raising the adjustment parameter from 0.2 to 0.4 simultaneously elevates both consensus degree and consistency level, albeit accompanied by an increase in consensus cost.*

$$R^1 = \begin{bmatrix} 0.5 & 0.3 & 0.8 & 0.6 \\ 0.7 & 0.5 & 0.6 & 0.4 \\ 0.2 & 0.4 & 0.5 & 0.3 \\ 0.4 & 0.6 & 0.7 & 0.5 \end{bmatrix}, R^2 = \begin{bmatrix} 0.5 & 0.7 & 0.6 & 0.3 \\ 0.3 & 0.5 & 0.5 & 0.4 \\ 0.4 & 0.5 & 0.5 & 0.7 \\ 0.7 & 0.6 & 0.3 & 0.5 \end{bmatrix},$$

$$R^3 = \begin{bmatrix} 0.5 & 0.1 & 0.9 & 0.8 \\ 0.9 & 0.5 & 0.7 & 0.4 \\ 0.1 & 0.3 & 0.5 & 0.9 \\ 0.2 & 0.6 & 0.1 & 0.5 \end{bmatrix}, R^4 = \begin{bmatrix} 0.5 & 0.4 & 0.3 & 0.9 \\ 0.6 & 0.5 & 0.4 & 0.8 \\ 0.7 & 0.6 & 0.5 & 0.2 \\ 0.1 & 0.2 & 0.8 & 0.5 \end{bmatrix}.$$

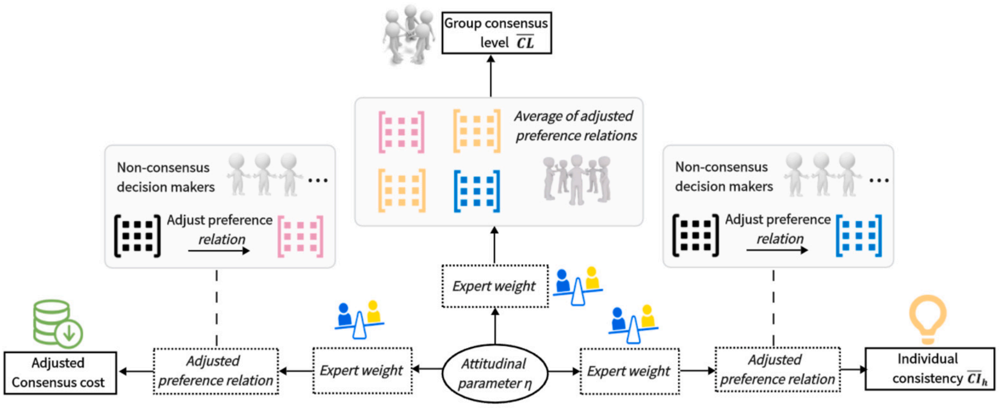

Fig. 3. Impacts of attitudinal parameter on consistency level, consensus cost and consensus degree.

From this numerical example, the adjustment degree of preference relation influences consensus degree, consistency level and consensus cost of decision makers. Furthermore, it is challenging to prevent conflicts among such three factors during the CRP. Consequently, it is essential to develop effective mechanisms for determine the adjustments of preference relations, along with corresponding consensus models, to achieve a balance among these tree factors, tailored to the specific contexts of decision making situations.

4. ATD-based consensus models

In this section, the interrelationships among consistency level, group consensus degree, and consensus cost are discussed. Subsequently, three ATD-based consensus models are presented namely i) the consistency driven model, ii) the consensus driven model and iii) the cost-driven model, respectively.

4.1. Relationships between attitudinal parameter and three factors

The consensus degree of the decision maker g_h before and after adjustment is denoted by CL_h and $\overline{CL}_h$, respectively. The group consensus degree before and after adjustment is denoted by CL and $\overline{CL}$, respectively. The preference relation provided by decision maker g_h before and after adjustment is denoted by R^h and $\overline{R}^h$, respectively. The consensus cost for decision maker g_h and the group is denoted by TC_h and TC, respectively. The consistency of preference provided by decision maker g_h before and after adjustment is denoted by CI_h and $\overline{CI}_h$, respectively. The group consensus degree, consistency level, and consensus cost are connected by the attitudinal parameter, as described by Fig. 3.

The relationships among attitudinal parameter and consistency level, consensus cost and consensus degree can be described as follows.

(1) An attitudinal parameter determines the weights assigned to decision makers, thereby influencing the collective preference, which in turn impacts the modified preference relation and the consistency of the modified preference relation.

(2) An attitudinal parameter determines the decision makers' weights, thereby influencing the collective preference and the modified preference relation. This subsequently affects the average of the modified preference relations and ultimately impacts the degree of group consensus.

(3) An attitudinal parameter determines the decision makers weights, subsequently affects the collective preference, consequently affects the modified preference relation, ultimately affects the cost calculated from the discrepancy between the preference relations prior to and following the adjustment.[3]

For the sake of facilitating discussion, three additional indicators are introduced: $\overline{TCL}$ (the combination of group consensus degree and consensus cost), $\overline{TCI}$ (the combination of group consistency level and consensus cost), and $\overline{CLI}$ (the combination of group consensus degree and consistency).

$$\overline{TCL} = \alpha\overline{CL} + (1-\alpha)(1-TC). \tag{12}$$

$$\overline{TCI} = \beta\overline{CI} + (1-\beta)(1-TC). \tag{13}$$

$$\overline{CLI} = \mu\overline{CL} + (1-\mu)\overline{CI}. \tag{14}$$

Here, $\alpha \in (0,1)$(or β, or μ) are parameters to control the importance of different factors among consensus cost, consensus degree, and consistency of preference relation.

[3] Quantitative analysis on these relationships can be found in Sections 6.4-6.5, shown by Fig. 9 and Fig. 11.

4.2. A consistency driven consensus model: SIS-DRI model

In the real-life CRP, if the GDM problem demands higher reliability of the solution (reflected by consistency of adjusted preference relation), rather than agreement among the group (reflected by consensus degree) and reduced consensus cost, the following preference relation adjustment mechanism is recommended. The optimization objective of this model is to achieve the highest level of consistency in all decision makers' preference relations after reaching group consensus. At the same time, the consensus cost can be restricted to some extent to prevent excessive modification of preference relations during the consensus reaching process. Consistency, consensus degree and consensus cost can be taken into account at the same time, with the balance among these three factors achieved by applying the attitudinal parameter as one of the outputs for the optimization model.

$$
\begin{aligned}
&\max \quad \sum_{g_h\in G_1} \overline{CI}_h\\
&s.t.\begin{cases}
\overline{CI}_h = 1-\frac{4}{n(n-1)(n-2)}\sum_{i=1}^{n}\sum_{j=i+1}^{n}\sum_{k=j+1}^{n}|\overline{r}_{ij}^h+\overline{r}_{jk}^h-\overline{r}_{ik}^h-0.5|,\ h=1,2,\dots,f\\
\overline{CI}=\frac{1}{f}\sum_{h=1}^{f}\overline{CI}_h\\
TC_h=\frac{\sum_{i=1}^{n}\sum_{j=1}^{n}|r_{ij}^h-\overline{r}_{ij}^h|}{n(n-1)},\ \forall g_h\in G_1\\
TC=\frac{\sum_{h\in G_1}TC_h}{\#G_1}\\
\overline{CL}_h=1-|\overline{R}^h-\overline{R}^a|=1-\frac{\sum_{i\neq j}|\overline{r}_{ij}^h-\overline{r}_{ij}^a|}{n(n-1)},\ h=1,2,\dots,f\\
\overline{CL}=\frac{1}{f}\sum_{h=1}^{f}\overline{CL}_h\geq\gamma_1\geq\theta\\
\overline{TCL}=\alpha\overline{CL}+(1-\alpha)(1-TC)\geq\gamma_2\\
\overline{R}^a=\frac{1}{f}\sum_{h=1}^{f}\overline{R}^h\\
\overline{R}^h=(1-\delta)R^h+\delta\sum_{v=1}^{f}w_{\sigma(v)}^{ATD}R^v,\ \forall g_h\in G_1\\
\overline{R}^h=R^h,\ \forall g_h\in G_2\\
\delta\in[0,1],\eta\in O,\alpha\in(0,1)
\end{cases}
\end{aligned}
\tag{15}
$$

In this optimization model, the existence of a value γ_1 demonstrates that the consensus can be reached. γ_2 serves as a threshold that encapsulates the holistic requirements for consensus degree and consensus cost, θ indicates the group consensus threshold. Parameter α is employed to harmonize the significance of group consensus degree and consensus cost, which can be fine-tuned in accordance with the demanding of decision makers. The parameter α can be predetermined or simply acknowledged as existent. Notably, a pre-set α will yield a larger consensus cost compared to the scenario where α merely exist. In detail, when α is pre-set, the constraint $\overline{TCL}=\alpha\overline{CL}+(1-\alpha)(1-TC)\geq\gamma_2$ becomes more rigorous compared to the scenario where α is allowed to vary flexibly within interval $[0,1]$. This requires more significant adjustments to non-consensus decision makers' preference relations to meet the requirement, resulting in higher consensus costs. The model's inputs are decision makers' preference relations. Intermediate outputs include preference relations of decision makers with consensus degrees below the threshold. The ultimate outputs are the optimal adjustment parameter δ and the trust attitudinal parameter η, which maximize the consistency. Since the attitudinal parameter determines decision makers' weights, it is discretized in this paper, assuming uniform distribution within (0,1), while the adjust parameter is a continuous variable in (0,1). Denote $O=\{1/o,2/o,\cdots,o-1/o\}$, where o denotes a positive integer, then $\eta\in O\subseteq(0,1)$. This model is tailored for GDM problems in which the rationality of preference relations of decision makers are vital, and decision makers exhibit a pronounced inclination towards keeping the consistency of the preference relations during the CRP, significantly surpassing the requirements for consensus degree and consensus cost. Anyway, it is assumed that automatic preference relation adjustments can be implemented for consensus achievement. The CRP framework with this adjustment mechanism is named the SIS-DRI consensus model and is illustrated in Fig. 4. The corresponding pseudocode for the SIS-DRI consensus model is provided in Algorithm 1.

4.3. A consensus model aiming for maximum consensus degree: SEN-DRI model

In certain specific GDM scenarios, GDM problem needs more reliability on agreements, which can be reflected by consensus degree. To ensure decision result rationality, the consensus cost and the consistency level of preference relation after adjustment must also be considered in such cases. Therefore, the requirements for consistency level and consensus cost serve as constraints, whereas maximizing group consensus degree becomes the objective for optimization. In this way, an optimization model can be constructed to generate preference relation adjustment mechanisms. To achieve maximum group consensus, subject to the constraint that the consistency and the consensus cost are maintained within acceptable limits, it is recommended to employ the SEN-DRI consensus model, in conjunction with an adjustment mechanism, as delineated by Eq. (16).

In this model, the existence of a specific γ_1 value guarantees consensus attainment, ensures that the consensus can be reached. Hyper-parameter γ_3 serves as a threshold that encompasses the comprehensive demands for consistency level and consensus cost. The inputs of this model are the preference relations provided by decision makers. The output is the optimal adjustment parameter δ and trust attitudinal parameter η. Parameter η controls the balance, and the adjustment parameter determines the adjustment degree for the non-consensus decision makers. This model is designed for GDM scenarios where maximal consensus attainment is imperative, significantly outweighing considerations of preference consistency and the associated costs of consensus. The consensus framework

Algorithm 1: The SIS-DRI consensus model.

Input: Decision makers' preference relations R^h, where $h = 1, 2, \dots, f$; Consensus thresholds θ and θ'; Holistic requirement for consensus degree and consensus cost γ_2; Balance parameter α.
Output: Optimal combination of adjustment parameter δ and trust attitudinal parameter η; Maximum consistency level; Adjusted preference relations of non-consensus decision makers.
begin
  for $g_h \in G$ **do**
    Compute: The consensus degree CL_h for each decision maker g_h using Eq. (7).
  end
  Compute: The group consensus level CL using Eq. (9).
  if $CL < \theta$ **then**
    for $g_h \in G$ *with* $CL_h < \theta'$ **do**
      Identify: Non-consensus decision makers whose preference relations need adjustment, i.e., $g_h \in G_1$.
    end
  end
  else
    Terminate: End the algorithm.
  end
  for $\eta \in (0, 1)$ **do**
    Compute: Derive experts' weights $w^{ATD}_{\sigma(v)}$ using Eq. (5).
  end
  Compute: The optimal adjustment parameter δ and attitudinal parameter η using Eq. (15) to maximize consistency level.
  for $g_h \in G_1$ **do**
    Update: Adjust the preference relations of g_h using Eq. (11).
  end
  return *The adjusted preference relations of non-consensus decision makers.*
end

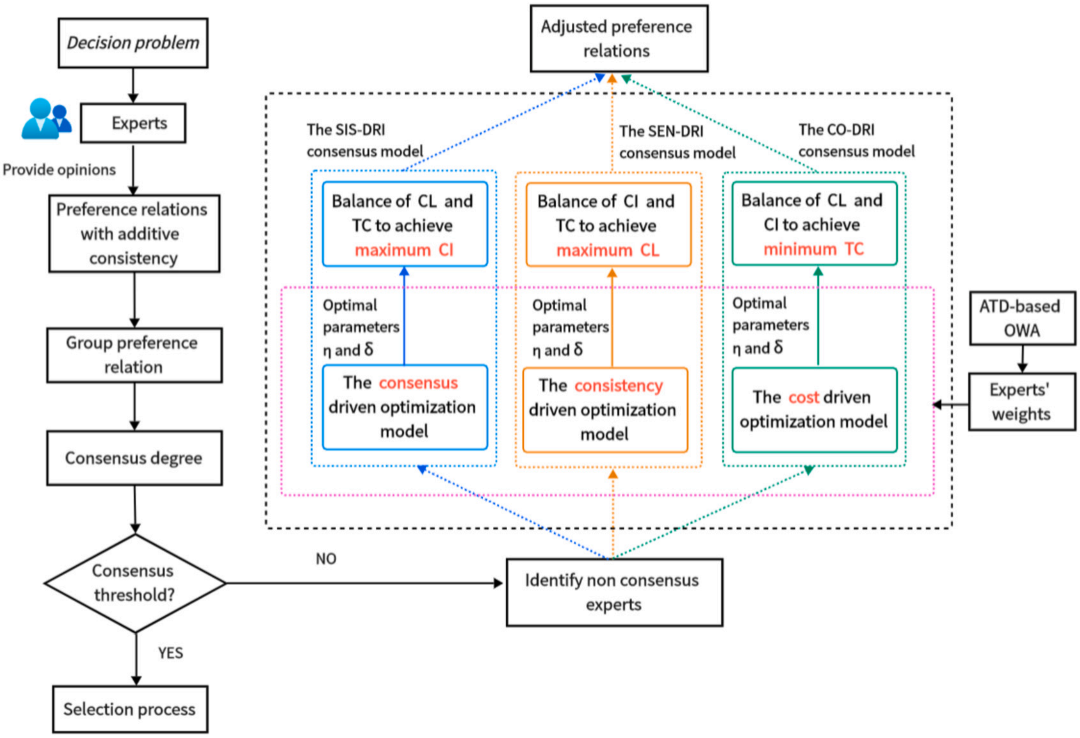

Fig. 4. The SIS-DRI, SEN-DRI and CO-DRI consensus models.

for this maximum consensus degree driven adjustment generation model, the SEN-DRI consensus model, is presented in Fig. 4. To save space, the pseudocode for the SEN-DRI consensus model is provided in Appendix B.

$$\max \quad \sum_{g_h \in G_1} \overline{CL}_h$$

$$s.t.\begin{cases} \overline{CL}_h = 1 - |\overline{R}^h - \overline{R}^a| = 1 - \dfrac{\sum\limits_{i\neq j} |\overline{r}_{ij}^h - \overline{r}_{ij}^a|}{n(n-1)}, \ h = 1,2,\ldots,f \\ \overline{CL} = \frac{1}{f} \sum_{h=1}^{f} \overline{CL}_h \ge \gamma_1 \ge \theta \\ TC_h = \frac{\sum_{i=1}^n \sum_{j=1}^n |r_{ij}^h - \overline{r}_{ij}^h|}{n(n-1)}, \ \forall g_h \in G_1 \\ TC = \frac{\sum_{h\in G_1} TC_h}{\# G_1} \\ \overline{CI}_h = 1 - \frac{4}{n(n-1)(n-2)} \sum_{i=1}^n \sum_{j=i+1}^n \sum_{k=j+1}^n |\overline{r}_{ij}^h + \overline{r}_{jk}^h - \overline{r}_{ik}^h - 0.5|, \ h = 1,2,\ldots,f \\ \overline{CI} = \frac{1}{f} \sum_{h=1}^f \overline{CI}_h \\ \overline{TCI} = \beta \overline{CI} + (1-\beta)(1-TC) \ge \gamma_3 \\ \overline{R}^a = \frac{1}{f} \sum_{h=1}^f \overline{R}^h \\ \overline{R}^h = (1-\delta) R^h + \delta \sum_{v=1}^f w_{\sigma(v)}^{ATD} R^v, \ \forall g_h \in G_1 \\ \overline{R}^h = R^h, \ \forall g_h \in G_2 \\ \delta \in [0,1], \eta \in O, \alpha \in (0,1) \end{cases} \tag{16}$$

4.4. A minimum consensus cost-driven consensus model: CO-DRI model

Although extensive literature focuses on minimum cost consensus model, there are few studies that consider consistency at the same time. If a GDM problem demands cost reduction beyond solution reliability and group agreement, yet still requires some preference relation consistency, the CO-DRI consensus model cooperates with the following adjustment mechanism is recommended for application.

$$\min \quad \sum_{g_h \in G_1} TC_h$$

$$s.t.\begin{cases} TC_h = \frac{\sum_{i=1}^n \sum_{j=1}^n |r_{ij}^h - \overline{r}_{ij}^h|}{n(n-1)}, \ \forall g_h \in G_1 \\ TC = \frac{\sum_{h\in G_1} TC_h}{\# G_1} \\ \overline{CL}_h = 1 - |\overline{R}^h - \overline{R}^a| = 1 - \dfrac{\sum\limits_{i\neq j} |\overline{r}_{ij}^h - \overline{r}_{ij}^a|}{n(n-1)}, \ h = 1,2,\ldots,f \\ \overline{CL} = \frac{1}{f} \sum_{h=1}^{f} \overline{CL}_h \ge \gamma_1 \ge \theta \\ \overline{CI}_h = 1 - \frac{4}{n(n-1)(n-2)} \sum_{i=1}^n \sum_{j=i+1}^n \sum_{k=j+1}^n |\overline{r}_{ij}^h + \overline{r}_{jk}^h - \overline{r}_{ik}^h - 0.5|, \ h = 1,2,\ldots,f \\ \overline{CI} = \frac{1}{f} \sum_{h=1}^f \overline{CI}_h \\ \overline{CLI} = \mu \overline{CL} + (1-\mu) \overline{CI} \ge \gamma_4 \\ \overline{R}^a = \frac{1}{f} \sum_{h=1}^f \overline{R}^h \\ \overline{R}^h = (1-\delta) R^h + \delta \sum_{v=1}^f w_{\sigma(v)}^{ATD} R^v, \ \forall g_h \in G_1 \\ \overline{R}^h = R^h, \ \forall g_h \in G_2 \\ \delta \in [0,1], \eta \in O, \alpha \in (0,1) \end{cases} \tag{17}$$

In this model, γ_4 serves as a threshold that encapsulates the holistic requirements for consensus degree and consistency level. Consensus attainment requires the presence of γ_1. Parameter μ is utilized to harmonize the significance of consensus degree and consistency level, its value can be determined in accordance with other information in the decision making process. The parameter μ may be predetermined or simply acknowledged as existent. The model inputs comprise decision makers' preference relations. The intermediate outputs are preference relations of decision makers whose individual consensus degree cannot reach the threshold. The ultimate outputs are the optimal adjustment parameter δ and trust attitudinal parameter η. This model suits GDM scenarios where achieving the minimum consensus cost is crucial, surpassing preference consistency and consensus degree considerations. Nonetheless, there is a fundamental rule that the consensus must be achievable. The optimization model controls the balance between consistency level and group consensus degree by determining the optimal η. The consensus framework related to this optimization model, named CO-DRI consensus model, and it is presented in Fig. 4. The pseudocode for the CO-DRI consensus model is provided in Appendix B.

Remark 1. *For the proposed optimization models (15)-(17), clearly, the parameter space is bounded and satisfies the compactness condition. The continuity of the objective functions ensures the convergence of the models and guarantees the existence of at least one global optimal solution. But the solution for each model is not restricted to be unique. The optimization models (15)-(17) can be easily transformed to linear programming models, which are shown in Appendix A.*

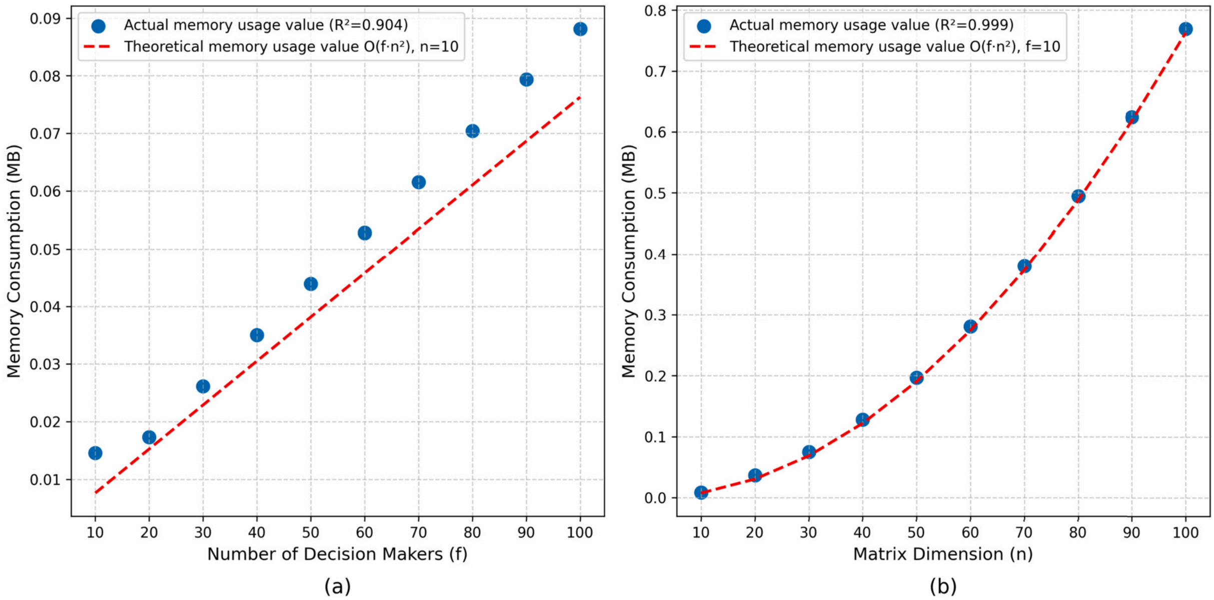

Fig. 5. Scalability Fitting Curve.

4.5. Complexity analysis for the consensus models

The following presents the time and space complexity analysis of the ATD-based consensus models: In terms of time complexity, the time complexity for computing the degree of group consensus is $O(f \cdot n^2)$, while the time complexity for automatically preferences adjustment generation mechanisms is $O(f^2 \cdot n^2)$. Therefore, the overall time complexity of the proposed consensus models is $O(f^2 \cdot n^2)$. Regarding the space complexity, the size of the preference relation R_h is $n \times n$, which results in a space complexity of $O(f \cdot n^2)$ for all decision-makers. The size of the collective preference relation R_c is also $n \times n$, and the space complexity is $O(n^2)$. Considering both factors, the overall space complexity of the consensus models is $O(f \cdot n^2 + n^2)$. Because $f \geq 1$, the space complexity can be simplified as $O(f \cdot n^2)$. Simulation experiments have been added to verify the growth of memory usage with varing f (number of decision makers) and n (matrix dimension), to demonstrate the computational performance of the algorithm. Since the SIS-DRI model, the CO-DRI model and SEN-DRI model share identical space complexity, we select the SIS-DRI model as the representative case for comparison with the theoretical memory consumption model derived from $f \cdot n^2$, and the accuracy of memory consumption prediction is evaluated by examining the goodness-of-fit between predicted and actual values. The experimental data is presented as below.

(1) $n = 10$, with varying number of decision makers $f = \{10, 20, 30, \cdots, 100\}$, the change of space complexity is show by Fig. 5(a).
(2) $f = 10$, with varying dimension of preference matrix $n = \{10, 20, 30, \cdots, 100\}$, the change of space complexity is shown by Fig. 5(b). Here, without loss of generality, the complex numbers in matrices are random created.

From Fig. 5, the acceptable goodness-of-fit between the SIR model and the theoretical memory consumption model based on $f \cdot n^2$ indicates that the SIR model is capable of accurately predicting memory usage, thereby validating its feasibility.

5. Applications in severe air pollution emergency management

Under severe air pollution conditions, the meticulous implementation of emergency management measures holds paramount importance for ensuring the quality of life of the population and the sustainable development of urban areas. A pivotal emergency strategy is to adopt production restriction measures for specific enterprises. Given the multi-stakeholder interests that this strategy encompasses, including enterprises, the government, and the general public, it is imperative to adopt a group consensus decision making model in selecting the appropriate production restriction mechanisms. The selection of emergency production restriction schemes in heavy air pollution weather requires consideration of the rationality of preference relations, the public satisfaction with the decision outcomes, the possible economic costs and emotional resistance caused by the automated change of opinions in emergency situations. Consequently, applying the proposed consensus models to balance preference consistency, consensus degree, and consensus costs is essential. Assume that there are four production restriction plans $\{x_1, x_2, x_3, x_4\}$. Representatives from the government, enterprises, environmental organizations, and the general public are $\{g_1, g_2, g_3, g_4\}$, they are requested to provide preferences on the plans, to determine the final strategy from the four alternative solutions. Suppose the decision makers' preference relations are as follows:

Table 2
Weights of decision makers with different parameter η.

η	Decision maker weight
1/8	(0.08, 0.03, 0.84, 0.05)
2/8	(0.13, 0.07, 0.71, 0.09)
3/8	(0.18, 0.1, 0.59, 0.13)
4/8	(0.21, 0.13, 0.5, 0.16)
5/8	(0.23, 0.16, 0.42, 0.19)
6/8	(0.24, 0.19, 0.35, 0.22)
7/8	(0.25, 0.22, 0.3, 0.23)

$$R^1 = \begin{bmatrix} 0.5 & 0.3 & 0.8 & 0.6 \\ 0.7 & 0.5 & 0.6 & 0.4 \\ 0.2 & 0.4 & 0.5 & 0.3 \\ 0.4 & 0.6 & 0.7 & 0.5 \end{bmatrix}, R^2 = \begin{bmatrix} 0.5 & 0.7 & 0.6 & 0.3 \\ 0.3 & 0.5 & 0.5 & 0.4 \\ 0.4 & 0.5 & 0.5 & 0.7 \\ 0.7 & 0.6 & 0.3 & 0.5 \end{bmatrix},$$

$$R^3 = \begin{bmatrix} 0.5 & 0.4 & 0.3 & 0.9 \\ 0.6 & 0.5 & 0.4 & 0.8 \\ 0.7 & 0.6 & 0.5 & 0.2 \\ 0.1 & 0.2 & 0.8 & 0.5 \end{bmatrix}, R^4 = \begin{bmatrix} 0.5 & 0.1 & 0.9 & 0.8 \\ 0.9 & 0.5 & 0.7 & 0.4 \\ 0.1 & 0.3 & 0.5 & 0.9 \\ 0.2 & 0.6 & 0.1 & 0.5 \end{bmatrix}.$$

5.1. The problem solving process by applying the SIS-DRI model

(1) Non-consensus decision maker identification. Prior to the CRP, the consensus degree of the decision makers is:$CL_1 = 0.891, CL_2 = 0.825, CL_3 = 0.767, CL_4 = 0.783$. The group consensus degree is 0.816. Given a consensus threshold of 0.8, some experts' consensus degrees fall below this threshold. Therefore, it is necessary to adjust the preference relations of decision makers with lower consensus degrees. The decision makers with consensus degree less than 0.8 should be automatically adjusted. Thus, the preference relations of decision makers g_3 and g_4 are determined to be adjusted.

(2) Weight calculation for decision makers. To compute R^* as the average of the preference relation of all non-consensus decision makers, and to measure the similarity between all decision makers and R^*. R^* is presented below.

$$R^* = \begin{bmatrix} 0.5 & 0.25 & 0.6 & 0.85 \\ 0.75 & 0.5 & 0.55 & 0.6 \\ 0.4 & 0.45 & 0.5 & 0.55 \\ 0.15 & 0.4 & 0.45 & 0.5 \end{bmatrix}.$$

The similarity of decision makers to R^* are 0.676, 0.618, 0.733, and 0.656, respectively. Based on $\eta = 7/8$, the weights of the four decision makers can be obtained as $(\omega_1, \omega_2, \omega_3, \omega_4) = (0.25, 0.22, 0.3, 0.23)$(see Table 2).

(3) By solving the consistency driven optimization model with the parameters $\delta = 0.5$ and $\eta = 7/8$, the preference relation of g_3 and g_4 need to be modified.

$$\overline{R}^3 = \begin{bmatrix} 0.5 & 0.3895 & 0.4785 & 0.769 \\ 0.6105 & 0.5 & 0.4755 & 0.646 \\ 0.5215 & 0.5245 & 0.5 & 0.3545 \\ 0.281 & 0.554 & 0.6455 & 0.5 \end{bmatrix}.$$

$$\overline{R}^4 = \begin{bmatrix} 0.5 & 0.2395 & 0.7785 & 0.719 \\ 0.7605 & 0.5 & 0.6255 & 0.446 \\ 0.2215 & 0.3745 & 0.5 & 0.7045 \\ 0.281 & 0.554 & 0.2955 & 0.5 \end{bmatrix}.$$

The non-consensus decision makers g_3 and g_4 are identified. Without loss of generality, suppose that the threshold γ_2 for TCL is 0.5. The optimal attitudinal and adjustment parameters, $\eta = 7/8$, and $\delta = 0.143$, are determined via a consistency-driven optimization model. At this moment, the individual consistency of g_3 after modification is 0.904, the individual consensus level is 0.947, the individual consensus cost is 0.78. The individual consistency of g_4 is 0.847, the individual consensus level is 0.947, the individual consensus cost is 0.82. The SIS-DRI model implementation ensures the highest consistency in preference relations for non-consensus decision makers.

5.2. The problem solving process by applying the SEN-DRI model

The non-consensus decision makers, denoted as g_3 and g_4, are identified following the same methodology delineated in Section 5.1. It is supposed that the threshold γ_3 for TCI is 0.5. By using the SEN-DRI model, the optimal combination for attitudinal parameter and adjust parameter is $\eta = 3/8$ and $\delta = 0.126$. The individual consistency of g_3 after modification is 0.894, the individual consensus level

Table 3
Consistency levels of decision makers after adjustments.

Model	$\overline{CI_2}$	$\overline{CI_3}$
The SIS-DRI model	0.967	0.957
Chiclana et al's model [20]	0.94	0.92
Li et al's model [41]	0.9	0.9
Zhang et al's model [42]	0.95	0.95
Wu et al's model [23]	0.907	0.9

is 0.948, the individual consensus cost is 0.779. The individual consistency of g_4 is 0.863, the individual consensus level is 0.960, the individual consensus cost is 0.802. The SEN-DRI consensus model ensures the highest level of group consensus while maintaining a high level of consistency in preference relations for non-consensus decision makers and a relatively low consensus cost.

5.3. The problem solving process by applying the CO-DRI model

The non-consensus decision makers are g_3 and g_4, identified in the same manner as in Section 5.1. It is assumed that the CLI threshold γ_4 is 0.8. By using the cost driven optimization model cooperates with the CO-DRI consensus model, the optimal attitudinal and adjustment parameters are $\eta = 1/8$, and $\delta = 0.1$ determined by the optimization model related to the CO-DRI consensus model. The individual consistency of g_3 after modification is 0.869, the individual consensus level is 0.948, the individual consensus cost is 0.768. The individual consistency of g_4 is 0.85, the individual consensus level is 0.95, the individual consensus cost is 0.79. The CO-DRI model ensures the lowest consensus cost while achieving a high group consensus degree and a relatively high consistency level.[4]

6. Comparative study

This section compares the three different kinds of models presented in this paper with classical consensus models through some experimental studies. Simulation experiments are carried out to illustrate the performances of the proposed models.

6.1. Comparison on the SIS-DRI model and existing proposals

Existing consensus models, such as those [20,23,41,42], which consider not only consensus degree, but also consistency level, are compared with the proposed SIS-DRI model. The comparison is carried out based on the numerical example shared across these studies (Example 3 in [20], Example 1 in [41], Example 2 in [23] and [42]). The initial consistency levels of $G = \{g_1, \dots, g_4\}$ are $CI_1 = 1, CI_2 = 0.7667, CI_3 = 0.65, CI_4 = 0.8333$, while the consensus degrees are $CL_1 = 838, CL_2 = 0.796, CL_3 = 0.796, CL_4 = 0.858$. Multiple experiments are conducted using the SIS-DRI model, and the final results are presented using the data mean. The individual consensus degree of g_2 and g_3 cannot reach the threshold, therefore the preferences of decision makers g_2, and g_3 should be adjusted. If the SIS-DRI model is applied, the consensus degree of g_2 and g_3 can reach the threshold after adjustments, while the consistency level also increases to $\overline{CI_2} = 0.967, \overline{CI_3} = 0.957$. The comparison on different proposals is shown in Fig. 6, and the specific values are shown in Table 3. Multiple trials were carried out with different values of parameters γ_2 and α when SIS-DRI is applied to deal with the problem (see Table 4), and the data presented in Table 3 is the average consistency level of g_3 and g_4 after the CRP. Through this numerical comparison, the proposed SIS-DRI model ensures the consistency level reaches a higher extent, while the group consensus degree is also relative higher. Among these models, the proposed SIS-DRI model is the only one accounts the consensus cost, which is also one of its major advantages. The proposed SIS-DRI model provides a way to achieve higher consistency without significantly compromising consensus degree and consensus cost.

6.2. Comparison on the SEN-DRI model and existing proposals

Continued from the numerical example in Section 6.1. Some existing consensus models such as [23,41] not only ensure the process of consistency improvement, but also the process of reaching consensus. The SEN-DRI model is compared with models in [23,41]. The comparison results on consensus degree of each decision maker are shown in Table 5. Multiple trials are carried out with different values of parameters γ_3 and α when applying the SEN-DRI model to deal with the problem (see Table 6), and the data presented in Table 5 is the average consensus degree of decision makers after the CRP, i.e., $\overline{CL_1} = 0.867, \overline{CL_2} = 0.949, \overline{CL_3} = 0.958, \overline{CL_4} = 0.892$, with the group consensus level being $CL_c = 0.917$. In contrast to the results of Li et al.'s model [41] and Wu et al.'s model [23], the SEN-DRI model can achieve a higher consensus degree while maintaining a relatively high consistency level (see Fig. 7). Through this numerical comparison, it is evident that if the consensus degree is the most urgent issue in a real-life GDM case, the SEN-DRI model can be considered to catch a higher consensus degree while keeping a relative higher level of consistency.

[4] The code is accessible at the website: https://github.com/yayaliu-118/2024-CRP-LYY/blob/main/README.md.

Table 4
Multiple trials with different α and γ_2 when SIS-DRI model is applied.

		$\gamma_2=0.1$	$\gamma_2=0.2$	$\gamma_2=0.3$	$\gamma_2=0.4$	$\gamma_2=0.5$	$\gamma_2=0.6$	$\gamma_2=0.7$	$\gamma_2=0.8$	$\gamma_2=0.9$	mean	standard deviation
$\alpha=0.5$	$\overline{CI}_2$	0.974	0.974	0.974	0.974	0.974	0.937	0.937	0.937	-	0.96	0.0374
	$\overline{CI}_3$	0.965	0.965	0.965	0.965	0.965	0.923	0.923	0.923	-	0.949	0.02
$\alpha=0.6$	$\overline{CI}_2$	0.974	0.974	0.974	0.974	0.974	0.974	0.937	0.937	-	0.965	0.0289
	$\overline{CI}_3$	0.965	0.965	0.965	0.965	0.965	0.965	0.923	0.923	-	0.955	0.0038
$\alpha=0.7$	$\overline{CI}_2$	0.974	0.974	0.974	0.974	0.974	0.974	0.974	0.937	-	0.969	0.00179
	$\overline{CI}_3$	0.965	0.965	0.965	0.965	0.965	0.965	0.965	0.923	-	0.96	0.02
$\alpha=0.8$	$\overline{CI}_2$	0.974	0.974	0.974	0.974	0.974	0.974	0.974	0.937	-	0.969	0.00179
	$\overline{CI}_3$	0.965	0.965	0.965	0.965	0.965	0.965	0.965	0.923	-	0.96	0.02
$\alpha=0.9$	$\overline{CI}_2$	0.974	0.974	0.974	0.974	0.974	0.974	0.974	0.974	-	0.974	0
	$\overline{CI}_3$	0.965	0.965	0.965	0.965	0.965	0.965	0.965	0.965	-	0.965	0

"-" indicates that no combination of consensus degree and consensus cost can reach the holistic requirement r_2, regardless of parameter adjustments.

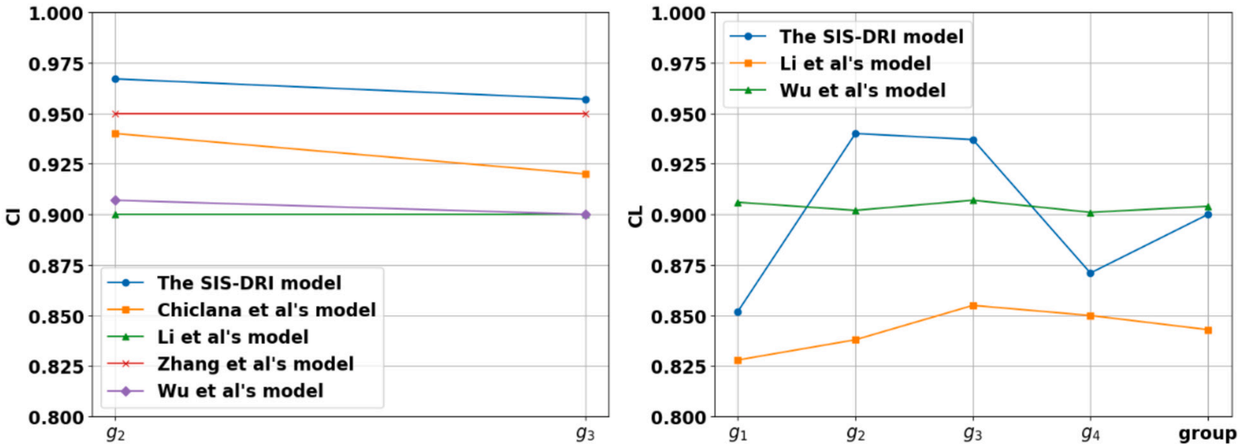

Fig. 6. Comparison on consensus degree (and consistency level) between the SIS-DRI model and other proposals.

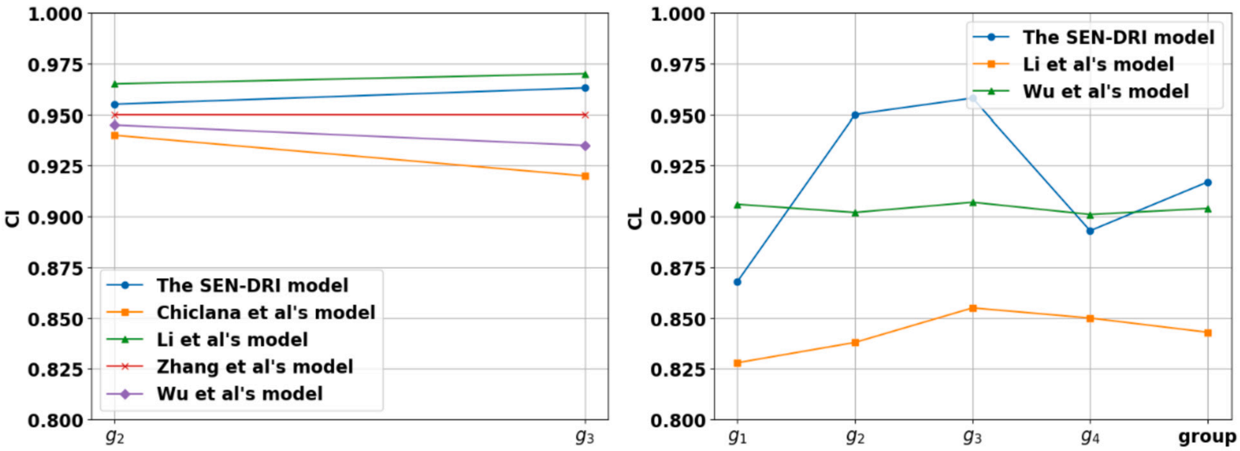

Fig. 7. Comparison on consensus degree (and consistency level) between the SEN-DRI model and other proposals.

6.3. Comparison on the CO-DRI model and existing proposals

The advantages of the CO-DRI model will be illustrated by comparing it with existing minimum-cost-driven consensus models [43–46]: by applying Zhang et al's [43], Zhong et al.'s [45] and Yu et al.'s [46] models, all the decision makers' preferences will be adjusted to reach the consensus. In contrast, the proposed CO-DRI model, only preference relations of non-consensus decision makers

Table 5
Comparison on consensus degree between the SEN-DRI model and other proposals.

Model	$\overline{CL_1}$	$\overline{CL_2}$	$\overline{CL_3}$	$\overline{CL_4}$	$\overline{CL}$
The SEN-DRI model	0.867	0.949	0.958	0.892	0.917
Li et al's model [41]	0.828	0.838	0.855	0.85	0.843
Wu et al's model [23]	0.906	0.902	0.907	0.901	0.904

Table 6
Multiple trials with different α and γ_3 when SEN-DRI model is applied.

		$\gamma_3=0.1$	$\gamma_3=0.2$	$\gamma_3=0.3$	$\gamma_3=0.4$	$\gamma_3=0.5$	$\gamma_3=0.6$	$\gamma_3=0.7$	$\gamma_3=0.8$	$\gamma_3=0.9$	mean	standard deviation
$\alpha=0.3$	$\overline{CL_1}$	0.868	0.868	0.868	0.868	0.868	0.868	0.859	0.859	-	0.866	0.02135
	$\overline{CL_2}$	0.95	0.95	0.95	0.95	0.95	0.95	0.943	0.943	-	0.948	0.0154
	$\overline{CL_3}$	0.958	0.958	0.958	0.958	0.958	0.958	0.952	0.952	-	0.957	0.00468
	$\overline{CL_4}$	0.893	0.893	0.893	0.893	0.893	0.893	0.882	0.882	-	0.89	0.00859
$\alpha=0.4$	$\overline{CL_1}$	0.868	0.868	0.868	0.868	0.868	0.868	0.859	0.859	-	0.866	0.02135
	$\overline{CL_2}$	0.95	0.95	0.95	0.95	0.95	0.95	0.943	0.943	-	0.948	0.0154
	$\overline{CL_3}$	0.958	0.958	0.958	0.958	0.958	0.958	0.952	0.952	-	0.957	0.00468
	$\overline{CL_4}$	0.893	0.893	0.893	0.893	0.893	0.893	0.882	0.882	-	0.89	0.00859
$\alpha=0.5$	$\overline{CL_1}$	0.868	0.868	0.868	0.868	0.868	0.868	0.868	0.859	-	0.867	0.00323
	$\overline{CL_2}$	0.95	0.95	0.95	0.95	0.95	0.95	0.95	0.943	-	0.949	0.00263
	$\overline{CL_3}$	0.958	0.958	0.958	0.958	0.958	0.958	0.958	0.952	-	0.957	0.00212
	$\overline{CL_4}$	0.893	0.893	0.893	0.893	0.893	0.893	0.893	0.882	-	0.892	0.00386
$\alpha=0.6$	$\overline{CL_1}$	0.868	0.868	0.868	0.868	0.868	0.868	0.868	0.859	-	0.867	0.00323
	$\overline{CL_2}$	0.95	0.95	0.95	0.95	0.95	0.95	0.95	0.943	-	0.949	0.00263
	$\overline{CL_3}$	0.958	0.958	0.958	0.958	0.958	0.958	0.958	0.952	-	0.957	0.00212
	$\overline{CL_4}$	0.893	0.893	0.893	0.893	0.893	0.893	0.893	0.882	-	0.892	0.00386
$\alpha=0.7$	$\overline{CL_1}$	0.868	0.868	0.868	0.868	0.868	0.868	0.868	0.859	-	0.867	0.00323
	$\overline{CL_2}$	0.95	0.95	0.95	0.95	0.95	0.95	0.95	0.943	-	0.949	0.00263
	$\overline{CL_3}$	0.958	0.958	0.958	0.958	0.958	0.958	0.958	0.952	-	0.957	0.00212
	$\overline{CL_4}$	0.893	0.893	0.893	0.893	0.893	0.893	0.893	0.882	-	0.892	0.00386
$\alpha=0.8$	$\overline{CL_1}$	0.868	0.868	0.868	0.868	0.868	0.868	0.868	0.868	-	0.868	0
	$\overline{CL_2}$	0.95	0.95	0.95	0.95	0.95	0.95	0.95	0.95	-	0.95	0
	$\overline{CL_3}$	0.958	0.958	0.958	0.958	0.958	0.958	0.958	0.958	-	0.958	0
	$\overline{CL_4}$	0.893	0.893	0.893	0.893	0.893	0.893	0.893	0.893	-	0.893	0
$\alpha=0.9$	$\overline{CL_1}$	0.868	0.868	0.868	0.868	0.868	0.868	0.868	0.868	-	0.868	0
	$\overline{CL_2}$	0.95	0.95	0.95	0.95	0.95	0.95	0.95	0.95	-	0.95	0
	$\overline{CL_3}$	0.958	0.958	0.958	0.958	0.958	0.958	0.958	0.958	-	0.958	0
	$\overline{CL_4}$	0.893	0.893	0.893	0.893	0.893	0.893	0.893	0.893	-	0.893	0

"-" indicates that no combination of consistency level and consensus cost can reach the holistic requirement r_3, regardless of parameter adjustments.

should be adjusted. Consequently, the CO-DRI model is capable of circumventing certain redundant adjustments, thereby mitigating the need for consensus costs. This model is compared with the consensus model proposed by Herrera et al. [44], which estimated the missing elements in incomplete preference relation and proposed an indicator called consensus/consistency level (they named it as CCL) to balance group consensus degree and consistency. The CCL in Herrera et al.'s model is methodologically and conceptually equivalent to the CLI in this research, enabling direct comparative analysis between the two models. Calculating the consensus cost of Herrera et al.'s consensus model (see Eq.(3)), and applying the CO-DRI model to solve the Example 12 in [44], we compare the values of CI, CL, TC, and CLI under the same consensus cost (the results are shown in Fig. 8). The CO-DRI model yields a lower consensus cost than Herrera et al.'s model while maintaining similar CLI values. It indicates that with a similar comprehensive evaluation of consistency level and consensus degree, the consensus cost of CO-DRI model is lower.

The F-test is carried out based on the data collected from multiple experiments. Consensus degree, consistency level, and consensus cost of individuals before and after the CRP are collected when three types of consensus models are applied, respectively, as shown in Table 7. Table 7 demonstrates that all F values are lower than $F_{0.05}(3,3)$. As a result, it can be concluded that the SIS-DRI model, the SEN-DRI model and the CO-DRI model maintain stable data distributions for individual consistency level and consensus degree of decision makers.

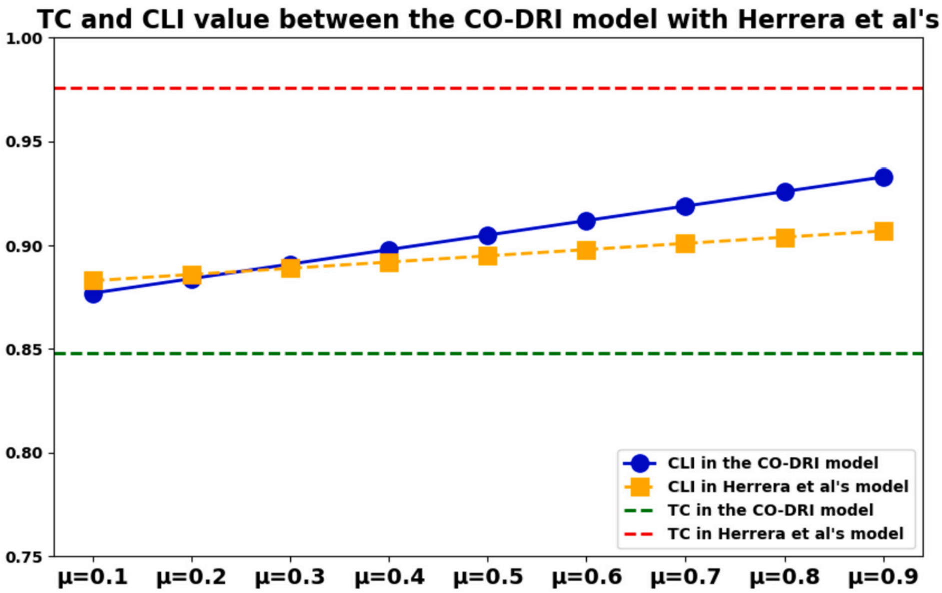

Fig. 8. Comparison on CO-DRI model and Herrera et al.'s model [44].

Table 7
F-test for SIS-DRI model, SEN-DRI model, and CO-DRI model.

Model	Index	g_1	g_2	g_3	g_4	F value	$F_{0.05}(3,3)$
SIS-DRI model	CL_h	0.838	0.796	0.796	0.858	1.3493	9.2766
	$\overline{CL_h}$	0.852	0.94	0.937	0.871		
	CI_h	1	0.7667	0.65	0.8333	0.4016	9.2766
	$\overline{CI_h}$	1	0.967	0.957	0.8333		
SEN-DRI model	CL_h	0.838	0.796	0.796	0.858	2.0975	9.2766
	$\overline{CL_h}$	0.868	0.95	0.958	0.893		
	CI_h	1	0.7667	0.65	0.8333	0.3935	9.2766
	$\overline{CI_h}$	1	0.955	0.963	0.8333		
CO-DRI model	CL_h	0.838	0.796	0.796	0.858	1.0825	9.2766
	$\overline{CL_h}$	0.845	0.936	0.919	0.862		
	CI_h	1	0.932	0.907	0.8333	0.2847	9.2766
	$\overline{CI_h}$	1	0.932	0.907	0.8333		

6.4. Theoretical comparison with proposals in the literature

To demonstrate the characteristics of the proposed models, the theoretical comparison will be conducted with existing consensus models, encompassing aspects such as consistency measurement, consensus degree computation, and the incorporation of consensus cost.

The comparison results are shown in Table 8. Among these models, only Rodríguez et al.'s model in [50] and the proposed three types of models can comprehensively consider consistency, consensus and consensus cost at the same time. Each of the proposed models shows superior performance in specific scenarios, therefore in practical GDM problem solving process, it is imperative to consider practical factors to select the most suitable model. Furthermore, through the application of the proposed models, the balance mechanisms are provided through the application of attitudinal parameter. By applying the current proposals, if the attitudinal parameter is determined in a prior, the proposed adjustment mechanism offers a method to achieve the balance of consistency level, consensus degree and consensus cost by determining optimal adjustment parameter. As depicted in Fig. 9, varying the attitudinal parameter results in the computation of diverse optimal adjustment parameter values, thereby influencing the performance of the consensus model, as evidenced by distinct consistency levels, group consensus degrees and consensus costs. The attitudinal parameter should not be confined a prior, the current proposals provide a more flexible method to determine both the optimal adjustment mechanism and attitudinal parameter, enabling the realization of various primary objectives in the CRP.

Table 8
A comprehensive comparison of the consensus models.

Consensus models	Consistency measure	Consensus measure	Consensus cost	Optimized variables
The SIS-DRI model	Additive (maximum value)	Distance between DMs	Degree of deviation before and after the change of opinion	Attitudinal parameter and adjustment parameter
The SEN-DRI model	Additive	Distance between DMs	Degree of deviation before and after the change of opinion	Attitudinal parameter and adjustment parameter
The CO-DRI model	Additive	Distance between DMs	Degree of deviation before and after the change of opinion	Attitudinal parameter and adjustment parameter
Wu et al's model [18]	-	Distance between DMs	Degree of deviation before and after the change of opinion	Attitudinal parameter and adjustment parameter
Li et al's model [41]	Additive	Distance between DMs	-	-
Wu et al's model [47]	-	Distance to group matrix	Harmony degree	adjustment parameters
Labella et al's model [48]	-	Distance to group opinions	Distance to global opinion and consensus degree	Parameters for measuring the distance between decision maker and collective opinion
Zhang et al's model [49]	Multiplicative	Distance between DMs	-	Adjusted preferences
Rodríguez et al's model [50]	Multiplicative	Distance to group opinion	Distance between decision makers and the collective opinion	-

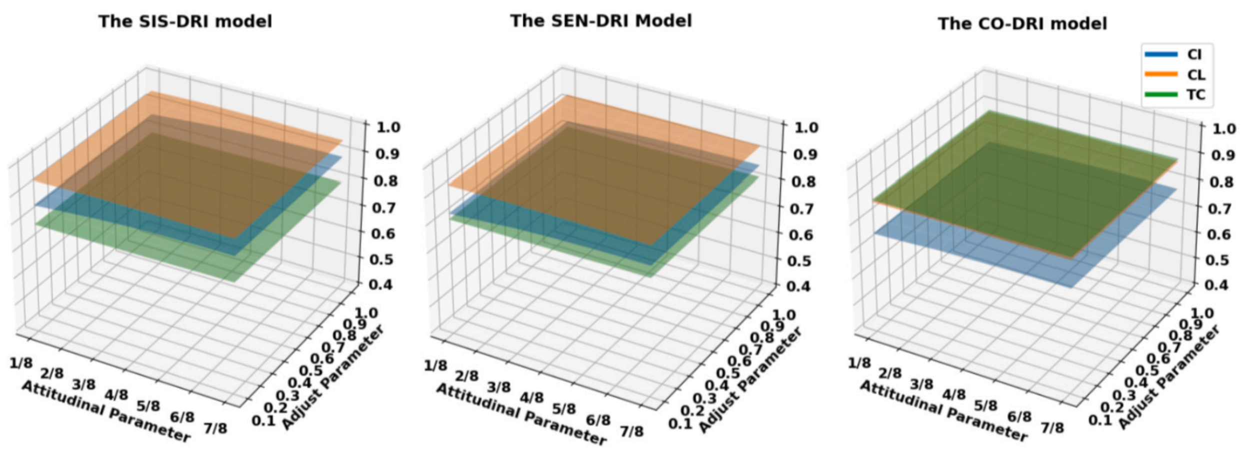

Fig. 9. The influence of attitudinal and adjustment parameters on three factors in the proposed models.

6.5. Sensitive analysis

This section will conduct a sensitivity analysis, the CI, CL and TC will be calculated and a sensitive analysis will be conducted on the impact of different parameters on the CI, CL, and TC. Without loss of generality, it is assumed that both the attitudinal parameter and the adjustment parameter are discretized. The values of the adjustment parameter range from 0.1 to 1 with an increment of 0.1, while the values of the attitudinal parameter span from 1/8 to 7/8, with a step size of 1/8.[5] Figs. 10-11 illustrate the variations in CI, CL, and TC for non-consensus experts (g_3 and g_4) as the attitude parameters and adjustment parameters vary. For the SIS-DRI model, if the attitudinal parameter η is pre-determined as 7/8, the best choice for adjustment parameter δ is 0.1; and if δ is pre-determined as 0.1, the best choice for η is 6/8, with which the highest consistency level can be reached; for the SEN-DRI model, if η is pre-determined as 3/8, the best choice for δ should be 0.1, if $\delta = 0.1$, the best choice for η is 3/8, with which the highest consensus degree can be achieved. For the CO-DRI model, if η is pre-determined as 1/8, the best set for δ is 0.1, and if δ is 0.1, the best η should be 1/8, with

[5] Previously mentioned were the scenarios where setting the value to 0 implies solely relying on the opinion of one individual, while setting it to 1 indicates that each individual's opinion holds equal weight. To avoid these two extremes, we have chosen not to adopt either of these values.

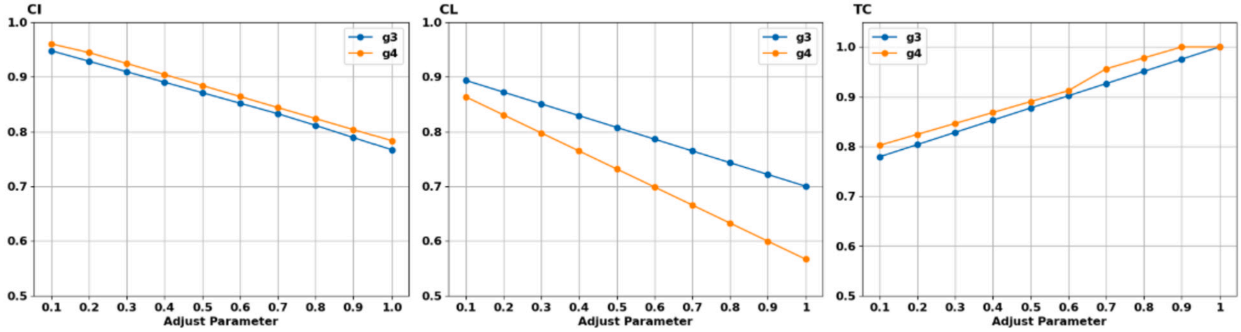

Fig. 10. The influence of adjustment parameters on three factors (CI, CL and TC) in three models.

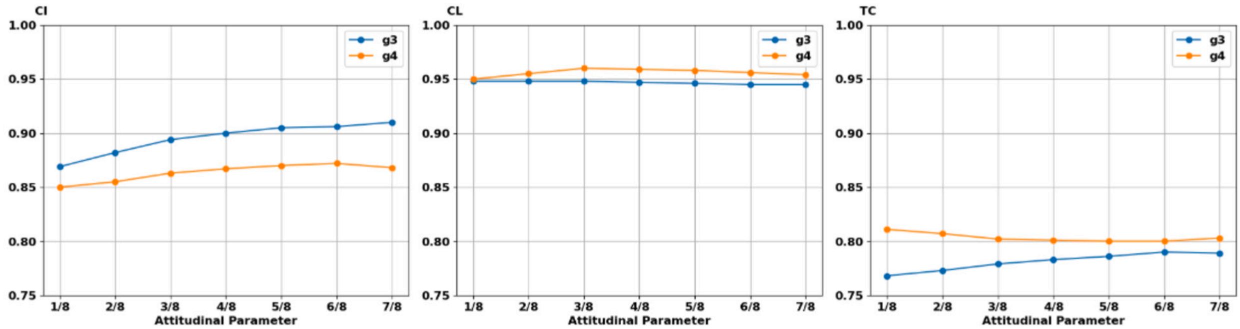

Fig. 11. The influence of attitudinal parameters on three factors (CI, CL and TC) in three models.

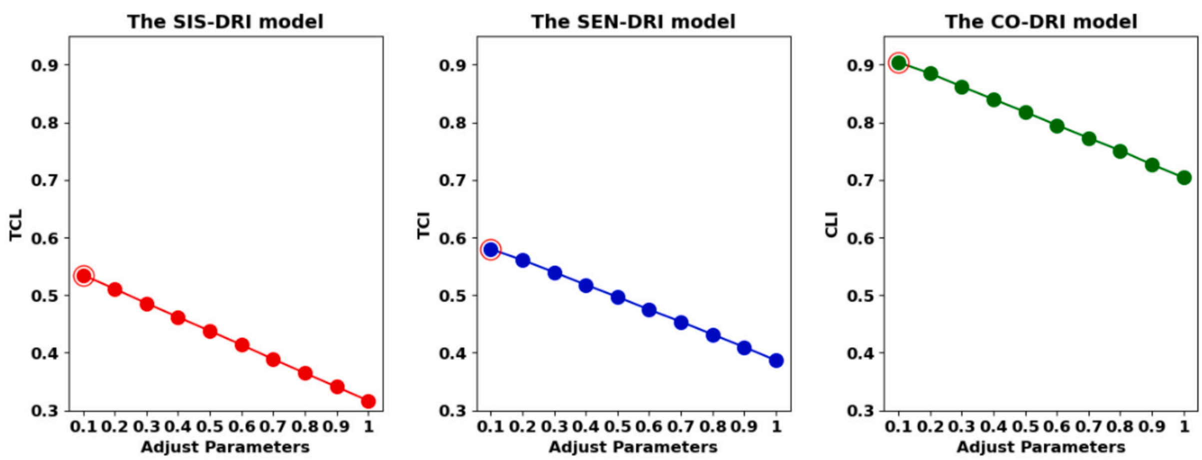

Fig. 12. TCL, TCI, and CLI with three proposed models under different adjustment parameters.

which the lowest consensus cost can be reached (see Figs. 10-11). From the sensitivity analysis, it becomes evident that the impact on CI, CL, and TC varies with changes in the attitudinal and adjustment parameters. For different decision-makers, the changes in CI, CL, and TC caused by the same attitudinal or adjustment parameter can also be differentiated. The principal advantage of the proposed consensus models lies in their ability to determine an optimal combination of attitudinal and adjustment parameters, which presents a method to balance CI, CL and TC under different primary objectives.

Besides, Figs. 12-13 illustrate the evolving trends of TCL, TCI and CLI across varying adjustment and attitudinal parameters. In different decision making scenarios, the significance of consensus degree, consistency level, and consensus cost can be modulated by adjusting the values of α, β and μ, respectively, to achieve the most appropriate solutions.

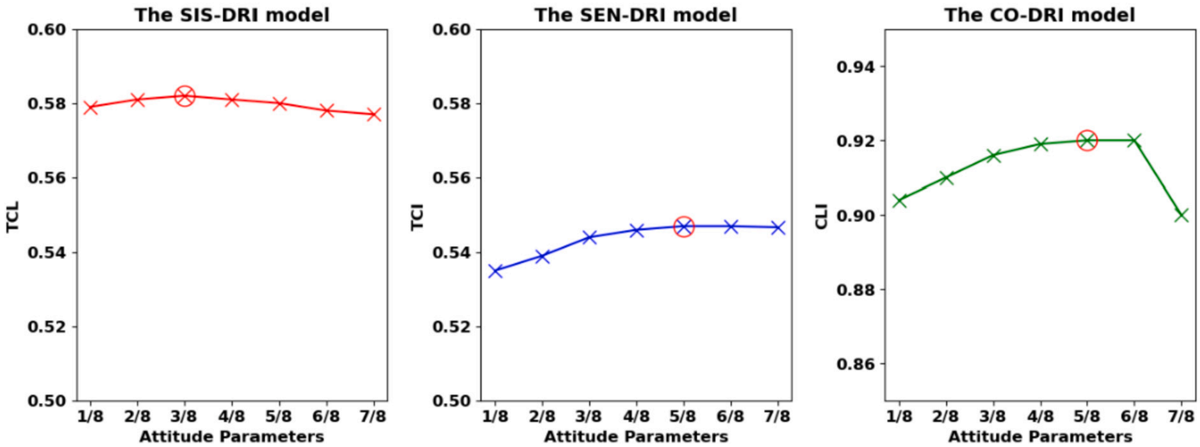

Fig. 13. TCL, TCI, and CLI with three proposed models under different attitudinal parameters.

7. Conclusion

A desirable consensus mechanism is proposed to seek to maximize CL, minimize TC, and thereby achieve a high value for CI. However, the interrelations among these factors are non-linear and, to some extent, constitute a challenge in finding solutions to the game. Consequently, the challenge in developing consensus model resides in establishing an appropriate preference relation adjustment mechanism that can harmonize the balance among CI, CL and TC. Through investigation of the intrinsic relationships between attitudinal parameter and these three factors, this study has developed three preference relation adjustment mechanisms, to achieve this equilibrium and derive three distinct types of corresponding consensus models. These consensus models could be respectively applied to satisfy the diverse and most urgent expectations of decision makers. Comparative analysis with existing consensus models has been carried out to demonstrate the effectiveness of the proposed models.

In future research, more strategies to balance CL, CI and TC will be further explored, to fit for the demanding of GDM in different practical situations. Subsequently, the equivalence mechanism of CL, CI, and TC will be explored in more complex GDM scenarios, such as in large-scale GDM, linguistic GDM, multi-attribute GDM problems, etc. Furthermore, the proposed consensus models will be further extended or adapted to accommodate more intricate GDM scenarios.

CRediT authorship contribution statement

Yaya Liu: Writing – review & editing, Writing – original draft. **Yue Wang:** Writing – review & editing, Writing – original draft. **Rosa M. Rodríguez:** Writing – review & editing. **Zhen Zhang:** Writing – review & editing. **Luis Martínez:** Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgements

This work was supported in part by the National Natural Science Foundation of China (62006154, 72371049, 71971039), the Funds for Humanities and Social Sciences of Ministry of Education of China (23YJC630219), the Natural Science Foundation of Liaoning Province under Grant No. 2024-MSBA-26, the Fundamental Research Funds for the Central Universities of China (DUT23RW406), and the Andalucía Excellence Research Program (ProyExcel-00257).

Appendix A. The transformed linear programming models for Eqs. (15)-(17)

The transformed linear programming model for Eq. (15):

$$\text{Max} \quad \sum_{g_h \in G_1} \overline{CI}_h$$

$$s.t.\begin{cases} \overline{CI}_h = 1 - \frac{4}{n(n-1)(n-2)} \sum_{i=1}^{n} \sum_{j=i+1}^{n} \sum_{k=j+1}^{n} z_{ijk}^h, \quad h = 1,2,\dots,f \\ z_{ijk}^h \geq \overline{r}_{ij}^h + \overline{r}_{jk}^h - \overline{r}_{ik}^h - 0.5, \quad \forall i,j,k,h \\ z_{ijk}^h \geq -(\overline{r}_{ij}^h + \overline{r}_{jk}^h - \overline{r}_{ik}^h - 0.5), \quad \forall i,j,k,h \\ TC_h = \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} y_{ij}^h}{n(n-1)}, \quad \forall g_h \in G_1 \\ y_{ij}^h \geq r_{ij}^h - \overline{r}_{ij}^h, \quad \forall i,j,h \\ y_{ij}^h \geq -(r_{ij}^h - \overline{r}_{ij}^h), \quad \forall i,j,h \\ \overline{CL}_h = 1 - \frac{\sum_{i \neq j} x_{ij}^h}{n(n-1)}, \quad h = 1,2,\dots,f \\ x_{ij}^h \geq \overline{r}_{ij}^h - \overline{r}_{ij}^a, \quad \forall i,j \\ x_{ij}^h \geq -(\overline{r}_{ij}^h - \overline{r}_{ij}^a), \quad \forall i,j \\ \overline{CL} = \frac{1}{f} \sum_{h=1}^{f} \overline{CL}_h \geq \gamma_1 \geq \theta \\ \overline{TCI} = \alpha \overline{CL} + (1-\alpha)(1-TC) \geq \gamma_2 \\ \overline{R}^h = (1-\delta) R^h + \delta \sum_{v=1}^{f} w_{\sigma(v)}^{ATD} R^v, \quad \forall g_h \in G_1 \\ \overline{R}^h = R^h, \quad \forall g_h \in G_2 \\ 0 \leq \delta \leq 1, \quad 1/o \leq \eta \leq (o-1)/o, \quad 0 < \alpha < 1, \quad \forall o \in N^+ \\ x_{ij}^h, y_{ij}^h, z_{ijk}^h \geq 0, \quad \forall i,j,k,h \end{cases} \tag{A.1}$$

The transformed linear programming model for Eq. (16):

$$\text{Max} \quad \sum_{g_h \in G_1} \overline{CL}_h$$

$$s.t.\begin{cases} \overline{CL}_h = 1 - \frac{\sum_{i \neq j} x_{ij}^h}{n(n-1)}, \quad h = 1,2,\dots,f \\ x_{ij}^h \geq \overline{r}_{ij}^h - \overline{r}_{ij}^a, \quad \forall i,j \\ x_{ij}^h \geq -(\overline{r}_{ij}^h - \overline{r}_{ij}^a), \quad \forall i,j \\ \overline{CL} = \frac{1}{f} \sum_{h=1}^{f} \overline{CL}_h \geq \gamma_1 \geq \theta \\ TC_h = \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} y_{ij}^h}{n(n-1)}, \quad \forall g_h \in G_1 \\ y_{ij}^h \geq r_{ij}^h - \overline{r}_{ij}^h, \quad \forall i,j,h \\ y_{ij}^h \geq -(r_{ij}^h - \overline{r}_{ij}^h), \quad \forall i,j,h \\ \overline{CI}_h = 1 - \frac{4}{n(n-1)(n-2)} \sum_{i=1}^{n} \sum_{j=i+1}^{n} \sum_{k=j+1}^{n} z_{ijk}^h, \quad h = 1,2,\dots,f \\ z_{ijk}^h \geq \overline{r}_{ij}^h + \overline{r}_{jk}^h - \overline{r}_{ik}^h - 0.5, \quad \forall i,j,k,h \\ z_{ijk}^h \geq -(\overline{r}_{ij}^h + \overline{r}_{jk}^h - \overline{r}_{ik}^h - 0.5), \quad \forall i,j,k,h \\ \overline{TCL} = \alpha \overline{CL} + (1-\alpha)(1-TC) \geq \gamma_3 \\ \overline{R}^h = (1-\delta) R^h + \delta \sum_{v=1}^{f} w_{\sigma(v)}^{ATD} R^v, \quad \forall g_h \in G_1 \\ \overline{R}^h = R^h, \quad \forall g_h \in G_2 \\ 0 \leq \delta \leq 1, 1/o \leq \eta \leq (o-1)/o, 0 < \alpha < 1, \quad \forall o \in N^+ \\ x_{ij}^h, y_{ij}^h, z_{ijk}^h \geq 0, \quad \forall i,j,k,h \end{cases} \tag{A.2}$$

The transformed linear programming model for Eq. (17):

$$\text{Min} \quad \sum_{g_h \in G_1} TC_h$$

$$s.t.\begin{cases} TC_h = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} y_{ij}^h}{n(n-1)}, \quad \forall g_h \in G_1 \\ y_{ij}^h \geq r_{ij}^h - \overline{r}_{ij}^h, \quad \forall i,j,h \\ y_{ij}^h \geq -(r_{ij}^h - \overline{r}_{ij}^h), \quad \forall i,j,h \\ \overline{CL}_h = 1 - \frac{\sum_{i \neq j} x_{ij}^h}{n(n-1)}, \quad h = 1,2,\dots,f \\ x_{ij}^h \geq \overline{r}_{ij}^h - \overline{r}_{ij}^a, \quad \forall i,j \\ x_{ij}^h \geq -(\overline{r}_{ij}^h - \overline{r}_{ij}^a), \quad \forall i,j \\ \overline{CL} = \frac{1}{f}\sum_{h=1}^{f} \overline{CL}_h \geq \gamma_1 \geq \theta \\ \overline{CI}_h = 1 - \frac{4}{n(n-1)(n-2)} \sum_{i=1}^{n}\sum_{j=i+1}^{n}\sum_{k=j+1}^{n} z_{ijk}^h, \quad h = 1,2,\dots,f \\ z_{ijk}^h \geq \overline{r}_{ij}^h + \overline{r}_{jk}^h - \overline{r}_{ik}^h - 0.5, \quad \forall i,j,k \\ z_{ijk}^h \geq -(\overline{r}_{ij}^h + \overline{r}_{jk}^h - \overline{r}_{ik}^h - 0.5), \quad \forall i,j,k \\ \overline{CLI} = \beta\overline{CI} + (1-\beta)(1-TC) \geq \gamma_4 \\ \overline{R}^h = (1-\delta)R^h + \delta\sum_{v=1}^{f} w_{\sigma(v)}^{ATD} R^v, \quad \forall g_h \in G_1 \\ \overline{R}^h = R^h, \quad \forall g_h \in G_2 \\ 0 \leq \delta \leq 1, 1/o \leq \eta \leq (o-1)/o, 0 < \beta < 1, \quad \forall o \in N^+ \\ x_{ij}^h, y_{ij}^h, z_{ijk}^h \geq 0, \quad \forall i,j,k,h \end{cases} \tag{A.3}$$

Appendix B. The pseudocodes for the SEN-DRI and CO-DRI consensus models

Algorithm 2: The SEN-DRI consensus model.

Input: Decision makers' preference relations R^h, where $h = 1, 2, \dots, f$; Consensus thresholds θ and θ'; Holistic requirement for consensus degree and consensus cost γ_3; Balance parameter β.
Output: Optimal combination of adjustment parameter δ and trust attitudinal parameter η; Maximum consensus level; Adjusted preference relations of non-consensus decision makers.
begin
  for $g_h \in G$ **do**
    Compute: The consensus degree CL_h for each decision maker g_h using Eq. (7).
  end
  Compute: The group consensus level CL using Eq. (9).
end
if $CL < \theta$ **then**
  for $g_h \in G$ *with* $CL_h < \theta'$ **do**
    Identify: Non-consensus decision makers whose preference relations need adjustment, i.e., $g_h \in G_1$.
  end
end
else
  Terminate: End the algorithm.
end
for $\eta \in (0, 1)$ **do**
  Compute: Derive experts' weights $w_{\sigma(v)}^{ATD}$ using Eq. (5).
end
Compute: The optimal adjustment parameter δ and attitudinal parameter η using Eq. (16) to maximize consensus level.
for $g_h \in G_1$ **do**
  Update: Adjust the preference relations of g_h using Eq. (11).
end
return *The adjusted preference relations of non-consensus decision makers.*

Algorithm 3: The CO-DRI consensus model.

Input: Decision makers' preference relations R^h, where $h = 1, 2, \dots, f$; Consensus thresholds θ and θ'; Holistic requirement for consensus degree and consensus cost γ_4; Balance parameter μ.
Output: Optimal combination of adjustment parameter δ and trust attitudinal parameter η; Minimum consensus cost; Adjusted preference relations of non-consensus decision makers.
begin
  for $g_h \in G$ **do**
    Compute: The consensus degree CL_h for each decision maker g_h using Eq. (7).
  end
  Compute: The group consensus level CL using Eq. (9).
  if $CL < \theta$ **then**
    for $g_h \in G$ *with* $CL_h < \theta'$ **do**
      Identify: Non-consensus decision makers whose preference relations need adjustment, i.e., $g_h \in G_1$.
    end
  end
  else
    Terminate: End the algorithm.
  end
  for $\eta \in (0, 1)$ **do**
    Compute: Derive experts' weights $w^{ATD}_{\sigma(v)}$ using Eq. (5).
  end
  Compute: The optimal adjustment parameter δ and attitudinal parameter η using Eq. (17) to minimum consensus cost.
  for $g_h \in G_1$ **do**
    Update: Adjust the preference relations of g_h using Eq. (11).
  end
  return *The adjusted preference relations of non-consensus decision makers.*
end

Data availability

Data will be made available on request.

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