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Robust multi-view fuzzy clustering with exponential transformation and automatic view weighting

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ABSTRACT

Multi-view fuzzy clustering has gained widespread attention due to its unique capability to handle uncertainty through flexible membership assignment, allowing samples to belong to multiple clusters with varying supports, thereby providing a comprehensive understanding of multi-view data. This capability is particularly relevant to knowledge-driven systems that require interpretable integration of multi-view data. However, existing multi-view fuzzy clustering algorithms often struggle with handling noise and incorporating flexible weighting strategies for different views effectively. To address these challenges, this paper proposes four robust multi-view fuzzy clustering algorithms (RMFC-ET-VS, RMFC-ET-VP, RMFC-ET-MS, RMFC-ET-MP), which leverage an exponential transformation of Euclidean distance to effectively mitigate the impact of noise and outliers in the data, thereby enhancing clustering stability. Moreover, we introduce vector-based and matrix-based view weighting strategies, employing sum-to-1 and product-to-1 constraints to ensure that the most informative views contribute more effectively during clustering. The proposed algorithms offer a dual emphasis on robust distance metrics and adaptable view weighting, resulting in more accurate and resilient clustering outcomes. Extensive experiments on multiple real-world datasets demonstrate that the proposed algorithms significantly outperform existing multi-view clustering algorithms, both in terms of clustering performance and robustness.

1. Introduction

Clustering is a core paradigm in unsupervised learning, widely used in fields like pattern recognition and machine learning [1–4]. The classic k-means clustering [5] is one of the most popular algorithms, which minimizes the distance between samples and their assigned cluster centers. Despite its popularity, k-means is limited by its hard assignment strategy, where each sample is assigned exclusively to one cluster. This rigid strategy is often inadequate for data with complex structures, as it fails to capture samples that may belong to multiple clusters to varying degrees. For flexibility, fuzzy c-means (FCM) [6] based on fuzzy theory [7–9] was developed, enabling soft clustering by allowing samples to have partial membership across multiple clusters. FCM is particularly advantageous when dealing with data uncertainty or ambiguity, providing a more nuanced model for cluster assignments. Nonetheless, both k-means and FCM are constrained by their reliance on Euclidean distance, making them sensitive to noise and outliers. In response to these limitations, Wu and Yang [10] developed alternative k-means and alternative fuzzy c-means (AFCM) based on exponential transformation of Euclidean distance, thereby enhancing robustness against noise. However, despite these advancements, they are generally designed for single-view data, where only one type of feature representation or data source is available.

With the rapid growth of multi-view data in various fields, multi-view clustering has become an essential tool in data analysis [11–13]. Multi-view data arises in numerous real-world scenarios, where different views represent diverse and complementary aspects of the same entities [14–16]. For instance, in multimedia, an object can be described by its visual features, audio annotations and textual tags. In biomedical analysis, a patient can be characterized by genetic data, medical imaging and clinical records. Leveraging such diverse

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information, multi-view clustering integrates these multiple perspectives to provide more accurate and comprehensive clustering results than single-view algorithms [17–19]. Existing multi-view clustering algorithms can be generally categorized into hard clustering and soft clustering [20].

Hard clustering involves assigning each sample to a unique cluster without overlapping memberships, ensuring distinct separation between clusters, and mainly includes k-means-based, graph-based and spectral clustering algorithms. For instance, Bickel and Scheffer [21] developed one of the earlier algorithms based on k-means for processing multi-view data. Chen et al. [22] introduced a two-level weighted k-means (TW-k-means) that incorporates view and attribute weights, improving clustering performance by effectively capturing the unique importance of different views and attributes. The introduction of view and attribute weighting allowed the algorithm to focus more on influential views and attributes, thereby enhancing clustering quality. Jiang et al. [23] presented SMVF, a multi-view k-means algorithm that integrates attribute and view weights for more precise clustering. Zhang et al. [24] further developed TW-Co-k-means, a collaborative extension that combines double-level weighted multi-view k-means clustering to enhance consistency between views and achieve more stable clustering outcomes. Xu et al. [25] introduced re-weighted discriminatively embedded k-means, which incorporates discriminative subspaces into the clustering process. Deng et al. [26] further advanced multi-view clustering techniques by proposing an approach that captures interactions between visible and hidden views, with the latter generated via nonnegative matrix factorization.

There are also other clustering algorithms for multi-view data. Graph clustering algorithms are popular for multi-view clustering as they effectively capture both local and global structures within the data. These algorithms typically construct graphs to represent pairwise similarities between data points across different views. Wang et al. [27] introduced a graph learning-based multi-view clustering algorithm, where a unified graph matrix is constructed by integrating information from all views. This unified representation captures the overall structure of the data, leading to improved clustering performance. Jing et al. [28] introduced a robust affinity graph model that mitigates noise and handles view heterogeneity effectively, while Tang et al. [29] employed a cross-view graph diffusion algorithm to ensure a coherent fusion of complementary information from multiple graphs. These algorithms leverage graph theory to efficiently model relationships across views, making them well-suited for multi-view data with complex interdependencies. Spectral clustering has also been a popular choice for multi-view clustering due to its ability to discover the underlying manifold structure of data. Zhu et al. [30] developed one-step multi-view spectral clustering that integrates all these stages into a unified framework, providing a more streamlined and effective approach to multi-view clustering. Shi et al. [31] proposed a flexible multi-view spectral clustering with adaptive weighting for each view, ensuring the contributions of the views are appropriately balanced based on their quality. Although multi-view hard clustering provides effective tools for leveraging the complementary information across multiple views. These algorithms enhance the robustness and effectiveness of clustering, capturing both local and global structures within the data. However, the hard clustering property that assigns each sample to a specific cluster may be less suitable for handling complex or overlapping boundaries often present in real-world datasets.

Multi-view soft clustering, on the other hand, allows samples to belong to multiple clusters with different degrees of membership, thereby capturing the uncertainty in real-world datasets. Fuzzy clustering, as a prominent form of soft clustering, has gained significant attention in multi-view clustering. For instance, Cleuziou et al. [32] introduced Co-FKM, a centralized fuzzy clustering algorithm. However, Co-FKM assigns equal importance to each view, which may not be optimal when views differ significantly in quality. To address this, Jiang et al. [33] developed weighted view collaborative fuzzy c-means (WV-Co-FCM), which dynamically assign weights to each view during the clustering process, leading to more robust and accurate results. Yang and Sinaga [34] further extended these ideas by incorporating both view and feature weights, and by adding feature dimensionality reduction to enhance clustering efficiency. Recent developments include MVASM, introduced by Han et al. [35], which uses fuzzy partition strategies to effectively extract shared information from different views. Zhang et al. [36] introduced one-step multi-view fuzzy clustering (OMFC-CS), which integrates common and view-specific information in a unified step, simplifying the process and improving clustering quality. Zhang et al. [37] proposed a fuzzy clustering that incorporates cross-view anchor graphs and latent information, effectively reducing computational complexity while ensuring robust clustering by capturing latent relationships between views. Additionally, Hu et al. [38] addressed privacy concerns by developing federated multi-view fuzzy c-means clustering, which ensures data privacy while performing distributed clustering.

Despite the progress made in multi-view clustering, existing algorithms face several limitations that motivate the need of developing more robust and adaptable approaches:

- First, real-world multi-view datasets are often noisy and contain outliers, which can significantly distort clustering results. Most current multi-view clustering algorithms rely on Euclidean distance as the primary measure of similarity. However, it is highly sensitive to noise, which leads to unstable clustering outcomes in noisy environments. While the exponential transformation of Euclidean distance has shown effectiveness in improving robustness of single-view clustering (*e.g.* AFCM), this approach has not been applied in the multi-view clustering context. This gap motivates the introduction of an exponential transformation of Euclidean distance in multi-view clustering, a strategy that enhances stability and robustness against noise and outliers in multi-view data.
- Second, many existing algorithms assign a single weight to each view, treating all clusters within a view equally. However, in real-world applications, different clusters may require different levels of contribution from each view. For instance, in image data, certain views (*e.g.* color features) may be highly relevant for distinguishing one cluster (*e.g.* a group of red images) but less informative for others (*e.g.* a group of green images). This lack of flexibility in weighting views leads to suboptimal clustering performance. The limitation suggests the need for more adaptable algorithms that allow different weights for each view at the cluster level, thus enhancing clustering performance.

The purpose of this paper is to address the limitations of existing algorithms by proposing a set of novel multi-view fuzzy clustering algorithms, considering vector-based and matrix-based view weightings with different weight constraints (*i.e.* the constraints that the sum of weights equals 1 and the product of weights equals 1), which improves the robustness to noise while incorporating flexible view weights. Specifically, we explore an exponential transformation of Euclidean distance to reduce the influence of noise and ensure stable clustering outcomes. Additionally, we address the limitations of treating all clusters within a view equally by introducing view-level and cluster-level weight adjustments. This flexibility allows the proposed algorithms to better adapt to the specific characteristics of each view and the requirements of each cluster, thereby improving clustering performance and robustness across various datasets. The main contributions of this study are as follows:

- We present four novel multi-view fuzzy clustering algorithms: RMFC-ET-VS, RMFC-ET-MS, RMFC-ET-MP and RMFC-ET-VP. These algorithms leverage exponential transformation to mitigate the impact of noise and enhance the robustness of the clustering process.

- We propose both vector-based and matrix-based weighting strategies, incorporating sum-to-1 and product-to-1 constraints to balance view contributions effectively. These weighting mechanisms ensure that the most informative views contribute more to the clustering process, leading to more reliable clustering outcomes.
- We provide a comprehensive evaluation of the proposed algorithms on different real-world datasets, demonstrating their superiority over existing state-of-the-art algorithms in clustering performance.

The rest of this paper is organized as follows. Section 2 reviews fuzzy c-means and alternative fuzzy c-means. Section 3 details the proposed algorithms, including the mathematical formulation and optimization strategies. Section 4 presents the experimental results, comparing our algorithms with existing algorithms on several benchmark datasets. Finally, Section 5 concludes the paper.

2. Preliminaries

In this section, we provide an overview of fuzzy c-means clustering and alternative fuzzy c-means clustering. These two algorithms lay the groundwork for understanding the developments presented in our framework.

2.1. Fuzzy c-means clustering

The fuzzy c-means (FCM) [6] is a classic clustering algorithm that allows each sample to belong to multiple clusters. Its objective is to minimize the intra-cluster variance while maximizing the inter-cluster separation. Consider a dataset $X = \{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_{\mathcal{N}}\} \in \mathbb{R}^{\mathcal{N}\times p}$. The goal is to partition X into $\mathcal{K}$ clusters defined by the discernment framework $\Theta = \{\theta_1, \theta_2, \ldots, \theta_{\mathcal{K}}\}$, achieved by minimizing the following objective function:

$$J_{FCM}(\mathbf{U},\mathbf{Z}) = \sum_{i=1}^{\mathcal{N}}\sum_{k=1}^{\mathcal{K}} u_{ik}^{\beta}\|\mathbf{x}_i - \mathbf{z}_k\|^2 \quad s.t. \sum_{k=1}^{\mathcal{K}} u_{ik} = 1, u_{ik} \geq 0 \tag{1}$$

where

- $\mathbf{U} = [u_{ik}]_{\mathcal{N}\times\mathcal{K}}$: The fuzzy membership matrix, where u_{ik} represents the membership degree of $\mathbf{x}_i$ to cluster θ_k.
- $\mathbf{Z} = [\mathbf{z}_k]$: The set of cluster centers, where $\mathbf{z}_k$ represents the center of cluster θ_k.
- β is the fuzzification parameter, controlling the degree of fuzziness in cluster assignment.

The update rules *w.r.t.* u_{ik} and $\mathbf{z}_k$ are defined as follows:

- *Fuzzy membership update*:

$$u_{ik} = \frac{\left(\|\mathbf{x}_i - \mathbf{z}_k\|^2\right)^{-\frac{1}{\beta-1}}}{\sum_{k'=1}^{\mathcal{K}}\left(\|\mathbf{x}_i - \mathbf{z}_{k'}\|^2\right)^{-\frac{1}{\beta-1}}} \tag{2}$$

- *Cluster center update*:

$$\mathbf{z}_k = \frac{\sum_{i=1}^{\mathcal{N}} u_{ik}^{\beta}\mathbf{x}_i}{\sum_{i=1}^{\mathcal{N}} u_{ik}^{\beta}} \tag{3}$$

Despite its simplicity and effectiveness, FCM has several limitations. For instance, the Euclidean distance used in the objective function makes the algorithm prone to being influenced by noise or outliers. The Euclidean distance assumption may not always capture the underlying structure of complex datasets.

2.2. Alternative fuzzy c-means clustering

To address the shortcomings of FCM, the alternative Fuzzy c-Means (AFCM) [10] was developed. Consider a dataset $X = \{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_{\mathcal{N}}\} \in \mathbb{R}^{\mathcal{N}\times p}$. To partition the dataset X into $\mathcal{K}$ clusters, the objective function of AFCM is defined as:

$$J_{AFCM}(\mathbf{U},\mathbf{Z}) = \sum_{i=1}^{\mathcal{N}}\sum_{k=1}^{\mathcal{K}} u_{ik}^{\beta}\left(1 - \exp\left(-\lambda\|\mathbf{x}_i - \mathbf{z}_k\|^2\right)\right) \quad s.t. \sum_{k=1}^{\mathcal{K}} u_{ik} = 1, u_{ik} \geq 0 \tag{4}$$

where

- λ is defined as:

$$\lambda = \left(\frac{\sum_{i=1}^{\mathcal{N}}\|\mathbf{x}_i - \bar{\mathbf{x}}\|^2}{\mathcal{N}}\right)^{-1} \text{ with } \bar{\mathbf{x}} = \frac{\sum_{i=1}^{\mathcal{N}}\mathbf{x}_i}{\mathcal{N}} \tag{5}$$

The update rules *w.r.t.* u_{ik} and $\mathbf{z}_k$ are defined as follows:

- *Fuzzy membership update*:

$$u_{ik} = \frac{\left(1 - \exp\left(-\lambda\|\mathbf{x}_i - \mathbf{z}_k\|^2\right)\right)^{-\frac{1}{\beta-1}}}{\sum_{k'=1}^{\mathcal{K}}\left(1 - \exp\left(-\lambda\|\mathbf{x}_i - \mathbf{z}_{k'}\|^2\right)\right)^{-\frac{1}{\beta-1}}} \tag{6}$$

- *Cluster center update*:

$$\mathbf{z}_k = \frac{\sum_{i=1}^{\mathcal{N}} u_{ik}^{\beta}\exp\left(-\lambda\|\mathbf{x}_i - \mathbf{z}_k\|^2\right)\mathbf{x}_i}{\sum_{i=1}^{\mathcal{N}} u_{ik}^{\beta}\exp\left(-\lambda\|\mathbf{x}_i - \mathbf{z}_k\|^2\right)} \tag{7}$$

3. Enhancing multi-view fuzzy clustering: A robust multi-view fuzzy clustering framework

In existing multi-view fuzzy clustering algorithms [32–34], Euclidean distance is a standard measure of similarity. However, this metric can be sensitive to noise and outliers, which often leads to distorted cluster assignment, especially in high-dimensional multi-view data. Although AFCM has obvious advantages in dealing with noise and outliers, it is only suitable for single-view data and is difficult to meet the needs of multi-view data. To address these limitations, we propose a robust multi-view fuzzy clustering framework by introducing an exponential transformation of Euclidean distance. The exponential transformation suppresses the impact of noise and emphasizes significant separations, improving the clustering performance in multi-view data. Such transformation is integrated into the objective functions of all proposed algorithms, which provides a consistent foundation for improving clustering robustness. The unique aspect of each algorithm lies in how it incorporates view weighting strategies to adapt to the multi-view context. The following subsections describe in further detail each algorithm.

3.1. Robust multi-view fuzzy clustering with exponential transformation and vector weighting of sum constraint (RMFC-ET-VS)

In multi-view clustering, each view may contribute differently to the clustering process, with some views offering more discriminative power while others introduce noise. To address this, we introduce a vector-based view weighting scheme, where a global weight is assigned to each view based on its relevance. This scheme ensures that the most informative views are emphasized while less relevant views contribute minimally. The weights are constrained to sum to 1, ensuring that views are normalized in their contributions to the clustering process. Given a multi-view dataset $\mathbf{X} = [X^{(1)}, X^{(2)}, \ldots, X^{(\mathcal{G})}]$, and gth view is defined as

$X^{(g)} = [\mathbf{x}_1^{(g)}, \mathbf{x}_2^{(g)}, \dots, \mathbf{x}_{\mathcal{N}}^{(g)}] \in \mathbb{R}^{\mathcal{N} \times p^{(g)}}$. The goal is to partition the $\mathbf{X}$ into $\mathcal{K}$ clusters. The objective function of RMFC-ET-VS can be defined as:

$$J_{RMFC-ET-VS}(\mathbf{U},\mathbf{Z},\mathbf{v}) = \sum_{g=1}^{\mathcal{G}} \sum_{i=1}^{\mathcal{N}} \sum_{k=1}^{\mathcal{K}} v_g^{\alpha} u_{ik}^{\beta} \left(1 - \exp\left(-\lambda^{(g)} \left\|\mathbf{x}_i^{(g)} - \mathbf{z}_k^{(g)}\right\|^2\right)\right)$$
$$s.t. \sum_{k=1}^{\mathcal{K}} u_{ik} = 1, u_{ik} \geq 0, \ \sum_{g=1}^{\mathcal{G}} v_g = 1, v_g \geq 0 \tag{8}$$

where

- $\mathbf{U} = [u_{ik}]_{\mathcal{N}\times\mathcal{K}}$: The fuzzy membership matrix, where u_{ik} represents the degree to the ith sample belonging to the kth cluster.
- $\mathbf{Z} = \left[\mathbf{z}_k^{(g)}\right]$: The set of cluster centers, where $\mathbf{z}_k^{(g)}$ represents the center of kth cluster in gth view.
- $\mathbf{v} = \left[v_g\right]$: The weight vector of the view, where v_g represents the weight of gth view.
- α: A regularization parameter controlling the distribution of the view weight.
- β: A fuzzification parameter controlling the influence of cluster membership.
- $\lambda^{(g)}$ can be defined as:

$$\lambda^{(g)} = \left(\frac{\sum_{i=1}^{\mathcal{N}} \|\mathbf{x}_i^{(g)} - \bar{\mathbf{x}}_i^{(g)}\|^2}{\mathcal{N}}\right)^{-1} \text{ with } \bar{\mathbf{x}}_i^{(g)} = \frac{\sum_{i=1}^{\mathcal{N}} \mathbf{x}_i^{(g)}}{\mathcal{N}}$$

Remark 1. RMFC-ET-VS introduces a vector weighting scheme to assign a single importance weight to each view. The sum-to-1 constraint ($\sum_{g=1}^{\mathcal{G}} v_g = 1$) ensures global interpretability of view contributions, making it ideal for scenarios requiring explicit prioritization of views. For example, in medical diagnosis, genomic data (assigned $v_1 = 0.7$) might dominate over clinical notes ($v_2 = 0.3$) for cancer subtyping. The exponential transformation of Euclidean distance metric enhances robustness against noise. The fuzzy membership matrix enables soft partition, capturing the degree to which each sample belongs to different clusters. This flexibility is especially useful for datasets with ambiguous boundaries.

Theorem 1. *The necessary conditions to minimize the objective function of RMFC-ET-VS are:*

- *Fuzzy membership update:*

$$u_{ik} = \frac{\left(\sum_{g=1}^{\mathcal{G}} v_g^{\alpha}\left(1 - \exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)} - \mathbf{z}_k^{(g)}\right\|^2\right)\right)\right)^{-\frac{1}{\beta-1}}}{\sum_{k'=1}^{\mathcal{K}}\left(\sum_{g=1}^{\mathcal{G}} v_g^{\alpha}\left(1 - \exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)} - \mathbf{z}_{k'}^{(g)}\right\|^2\right)\right)\right)^{-\frac{1}{\beta-1}}} \tag{9}$$

- *Cluster centers update:*

$$\mathbf{z}_k^{(g)} = \frac{\sum_{i=1}^{\mathcal{N}} u_{ik}^{\beta} \exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)} - \mathbf{z}_k^{(g)}\right\|^2\right)\mathbf{x}_i^{(g)}}{\sum_{i=1}^{\mathcal{N}} u_{ik}^{\beta} \exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)} - \mathbf{z}_k^{(g)}\right\|^2\right)} \tag{10}$$

- *View weights update:*

$$v_g = \frac{\left(\sum_{i=1}^{\mathcal{N}}\sum_{k=1}^{\mathcal{K}} u_{ik}^{\beta}\left(1 - \exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)} - \mathbf{z}_k^{(g)}\right\|^2\right)\right)\right)^{-\frac{1}{\alpha-1}}}{\sum_{g'=1}^{\mathcal{G}}\left(\sum_{i=1}^{\mathcal{N}}\sum_{k=1}^{\mathcal{K}} u_{ik}^{\beta}\left(1 - \exp\left(-\lambda^{(g')}\left\|\mathbf{x}_i^{(g')} - \mathbf{z}_k^{(g')}\right\|^2\right)\right)\right)^{-\frac{1}{\alpha-1}}} \tag{11}$$

Proof. To derive the necessary conditions for minimizing the objective function of RMFC-ET-VS, we start by focusing on each variable in turn: the fuzzy membership matrix $\mathbf{U}$, the cluster centers $\mathbf{Z}$, and the view weights $\mathbf{v}$. Since the optimization process involves constraints, Lagrange multipliers are used to handle these effectively.

- *Fuzzy membership update*:

To derive the update rule for u_{ik}, we first incorporate the constraint $\sum_{k=1}^{\mathcal{K}} u_{ik} = 1$ using a Lagrange multiplier γ_i. The Lagrangian function becomes:

$$\mathcal{L}_{RMFC-ET-VS}^{\mathbf{U}} = J_{RMFC-ET-VS} + \sum_{i=1}^{\mathcal{N}} \gamma_i \left(\sum_{k=1}^{\mathcal{K}} u_{ik} - 1\right) \tag{12}$$

Taking the partial derivative of $\mathcal{L}_{RMFC-ET-VS}^{\mathbf{U}}$ *w.r.t* u_{ik} gives:

$$\frac{\partial \mathcal{L}_{RMFC-ET-VS}^{\mathbf{U}}}{\partial u_{ik}} = \sum_{g=1}^{\mathcal{G}} \beta v_g^{\alpha} u_{ik}^{\beta-1}\left(1 - \exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)} - \mathbf{z}_k^{(g)}\right\|^2\right)\right) + \gamma_i \tag{13}$$

To find the optimal u_{ik}, set $\frac{\partial \mathcal{L}_{RMFC-ET-VS}^{\mathbf{U}}}{\partial u_{ik}} = 0$:

$$u_{ik} = \left(-\frac{\gamma_i}{\beta}\right)^{\frac{1}{\beta-1}} \left(\frac{1}{\sum_{g=1}^{\mathcal{G}} v_g^{\alpha}\left(1 - \exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)} - \mathbf{z}_k^{(g)}\right\|^2\right)\right)}\right)^{\frac{1}{\beta-1}} \tag{14}$$

To ensure the constraint $\sum_{k=1}^{\mathcal{K}} u_{ik} = 1$ is satisfied, we obtain:

$$\left(-\frac{\gamma_i}{\beta}\right)^{\frac{1}{\beta-1}} = \sum_{k=1}^{\mathcal{K}} \left(\frac{1}{\sum_{g=1}^{\mathcal{G}} v_g^{\alpha}\left(1 - \exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)} - \mathbf{z}_k^{(g)}\right\|^2\right)\right)}\right)^{\frac{1}{\beta-1}} \tag{15}$$

Returning (14), the final expression for the membership degree is obtained.

- *Cluster centers update*:

For the cluster centers, we focus on the terms in the objective function involving $\mathbf{z}_k^{(g)}$. Differentiating $J_{RMFC-ET-VS}^{\mathbf{Z}}$ *w.r.t* $\mathbf{z}_k^{(g)}$ yields:

$$\frac{\partial J_{RMFC-ET-VS}^{\mathbf{Z}}}{\partial \mathbf{z}_k^{(g)}} = \sum_{i=1}^{\mathcal{N}} v_g^{\alpha} u_{ik}^{\beta} \frac{\partial}{\partial \mathbf{z}_k^{(g)}}\left(1 - \exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)} - \mathbf{z}_k^{(g)}\right\|^2\right)\right) \tag{16}$$

For the constant 1, the derivative is zero. For the exponential term, the derivative is:

$$\frac{\partial}{\partial \mathbf{z}_k^{(g)}} \exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)} - \mathbf{z}_k^{(g)}\right\|^2\right) = -\lambda^{(g)} \exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)} - \mathbf{z}_k^{(g)}\right\|^2\right) \cdot \frac{\partial}{\partial \mathbf{z}_k^{(g)}}\left\|\mathbf{x}_i^{(g)} - \mathbf{z}_k^{(g)}\right\|^2 \tag{17}$$

The derivative of the squared Euclidean distance is:

$$\frac{\partial}{\partial \mathbf{z}_k^{(g)}}\left\|\mathbf{x}_i^{(g)} - \mathbf{z}_k^{(g)}\right\|^2 = 2\left(\mathbf{z}_k^{(g)} - \mathbf{x}_i^{(g)}\right) \tag{18}$$

Substituting back, we get:

$$\frac{\partial}{\partial \mathbf{z}_k^{(g)}} \exp\left(-\lambda^{(g)} \left\|\mathbf{x}_i^{(g)} - \mathbf{z}_k^{(g)}\right\|^2\right) = - 2\lambda^{(g)} \exp\left(-\lambda^{(g)} \left\|\mathbf{x}_i^{(g)} - \mathbf{z}_k^{(g)}\right\|^2\right)\left(\mathbf{z}_k^{(g)} - \mathbf{x}_i^{(g)}\right) \tag{19}$$

Now substitute this into the derivative of $J^{\mathbf{Z}}_{RMFC-ET-VS}$:

$$\frac{\partial J^{\mathbf{Z}}_{RMFC-ET-VS}}{\partial \mathbf{z}_k^{(g)}} = \sum_{i=1}^{\mathcal{N}} v_g^{\alpha} u_{ik}^{\beta} 2\lambda^{(g)} \exp\left(-\lambda^{(g)} \left\|\mathbf{x}_i^{(g)} - \mathbf{z}_k^{(g)}\right\|^2\right)(\mathbf{z}_k^{(g)} - \mathbf{x}_i^{(g)}) \tag{20}$$

To find the optimal $\mathbf{z}_k^{(g)}$, set $\frac{\partial J^{\mathbf{Z}}_{RMFC-ET-VS}}{\partial \mathbf{z}_k^{(g)}} = 0$. Distribute $\mathbf{z}_k^{(g)} - \mathbf{x}_i^{(g)}$ and rearrange terms:

$$\mathbf{z}_k^{(g)} \sum_{i=1}^{\mathcal{N}} v_g^{\alpha} u_{ik}^{\beta} 2\lambda^{(g)} \exp\left(-\lambda^{(g)} \left\|\mathbf{x}_i^{(g)} - \mathbf{z}_k^{(g)}\right\|^2\right) = \sum_{i=1}^{\mathcal{N}} v_g^{\alpha} u_{ik}^{\beta} 2\lambda^{(g)} \exp\left(-\lambda^{(g)} \left\|\mathbf{x}_i^{(g)} - \mathbf{z}_k^{(g)}\right\|^2\right)\mathbf{x}_i^{(g)} \tag{21}$$

Dividing by the normalization term can obtain the optimal cluster centers.

- *View weights update*:

To incorporate the constraint $\sum_{g=1}^{\mathcal{G}} v_g = 1$, we define the Lagrangian function as:

$$\mathcal{L}^{\mathbf{v}}_{RMFC-ET-VS} = J_{RMFC-ET-VS} + \eta\left(\sum_{g=1}^{\mathcal{G}} v_g - 1\right) \tag{22}$$

where η is the Lagrange multiplier for the constraint.

Taking the partial derivative of $\mathcal{L}^{\mathbf{v}}_{RMFC-ET-VS}$ *w.r.t.* v_g:

$$\begin{aligned}\frac{\partial \mathcal{L}^{\mathbf{v}}_{RMFC-ET-VS}}{\partial v_g} &= \frac{\partial J_{RMFC-ET-VS}}{\partial v_g} + \frac{\partial}{\partial v_g}\eta\left(\sum_{g=1}^{\mathcal{G}} v_g - 1\right) \\ &= \sum_{i=1}^{\mathcal{N}}\sum_{k=1}^{\mathcal{K}} \alpha v_g^{\alpha-1} u_{ik}^{\beta}\left(1 - \exp\left(-\lambda^{(g)} \left\|\mathbf{x}_i^{(g)} - \mathbf{z}_k^{(g)}\right\|^2\right)\right) + \eta\end{aligned} \tag{23}$$

To find the optimal v_m, set $\frac{\partial \mathcal{L}^{\mathbf{v}}_{RMFC-ET-VS}}{\partial v_g} = 0$:

$$v_g = \left(-\frac{\eta}{\alpha}\right)^{\frac{1}{\alpha-1}}\left(\frac{1}{\sum_{i=1}^{\mathcal{N}}\sum_{k=1}^{\mathcal{K}} u_{ik}^{\beta}\left(1 - \exp\left(-\lambda^{(g)} \left\|\mathbf{x}_i^{(g)} - \mathbf{z}_k^{(g)}\right\|^2\right)\right)}\right)^{\frac{1}{\alpha-1}} \tag{24}$$

The constraint $\sum_{g=1}^{\mathcal{G}} v_g = 1$ ensures proper normalization of v_g, we then get:

$$\left(-\frac{\eta}{\alpha}\right)^{\frac{1}{\alpha-1}} = \sum_{g=1}^{\mathcal{G}}\left(\frac{1}{\sum_{i=1}^{\mathcal{N}}\sum_{k=1}^{\mathcal{K}} u_{ik}^{\beta}\left(1 - \exp\left(-\lambda^{(g)} \left\|\mathbf{x}_i^{(g)} - \mathbf{z}_k^{(g)}\right\|^2\right)\right)}\right)^{\frac{1}{\alpha-1}} \tag{25}$$

Returning (24), the final expression for the view weight is obtained.

The RMFC-ET-VS algorithm is outlined in Algorithm 1.[1]

3.2. Robust multi-view fuzzy clustering with exponential transformation and vector weighting of product constraint (RMFC-ET-VP)

While RMFC-ET-VS ensures the sum of view weights equals 1, the RMFC-ET-VP algorithm instead constrains the product of view weights to 1. This strategy reduces the number of parameter requiring tuning, further simplifying the algorithm. Given a multi-view dataset $\mathbf{X} = [X^{(1)}, X^{(2)}, \ldots, X^{(\mathcal{G})}]$ to be grouped to $\mathcal{K}$ clusters, and gth view is defined as $X^{(g)} = [\mathbf{x}_1^{(g)}, \mathbf{x}_2^{(g)}, \ldots, \mathbf{x}_{\mathcal{N}}^{(g)}] \in \mathbb{R}^{\mathcal{N}\times p^{(g)}}$. The objective function of RMFC-ET-VP can be defined as:

$$\begin{aligned} &J_{RMFC-ET-VP}(\mathbf{U}, \mathbf{Z}, \mathbf{v}) = \sum_{g=1}^{\mathcal{G}}\sum_{i=1}^{\mathcal{N}}\sum_{k=1}^{\mathcal{K}} v_g u_{ik}^{\beta}\left(1 - \exp\left(-\lambda^{(g)} \left\|\mathbf{x}_i^{(g)} - \mathbf{z}_k^{(g)}\right\|^2\right)\right) \\ &s.t. \sum_{k=1}^{\mathcal{K}} u_{ik} = 1, u_{ik} \geq 0, \prod_{g=1}^{\mathcal{G}} v_g = 1, v_g > 0\end{aligned} \tag{26}$$

Remark 2. RMFC-ET-VP builds on RMFC-ET-VS by replacing the sum-to-1 constraint with a product-to-1 constraint ($\prod_{g=1}^{\mathcal{G}} v_g = 1$), which avoids extreme weighting by balancing contributions across views, further improving the algorithm robustness without the need for additional tuning. For instance, in environmental monitoring, temperature ($v_1 = 1.25$), humidity ($v_2 = 1.0$), and air quality ($v_3 = 0.8$) sensors jointly contribute to pollution level clustering. The membership matrix continues to provide soft cluster assignment, allowing the algorithm to effectively handle uncertain or ambiguous samples.

Theorem 2. *The necessary conditions to minimize the objective function of RMFC-ET-VP are:*

- *Fuzzy membership update:*

$$u_{ik} = \frac{\left(\sum_{g=1}^{\mathcal{G}} v_g\left(1 - \exp\left(-\lambda^{(g)} \left\|\mathbf{x}_i^{(g)} - \mathbf{z}_k^{(g)}\right\|^2\right)\right)\right)^{-\frac{1}{\beta-1}}}{\sum_{k'=1}^{\mathcal{K}}\left(\sum_{g=1}^{\mathcal{G}} v_g\left(1 - \exp\left(-\lambda^{(g)} \left\|\mathbf{x}_i^{(g)} - \mathbf{z}_{k'}^{(g)}\right\|^2\right)\right)\right)^{-\frac{1}{\beta-1}}} \tag{27}$$

- *Cluster center update:*

$$\mathbf{z}_k^{(g)} = \frac{\sum_{i=1}^{\mathcal{N}} u_{ik}^{\beta} \exp\left(-\lambda^{(g)} \left\|\mathbf{x}_i^{(g)} - \mathbf{z}_k^{(g)}\right\|^2\right)\mathbf{x}_i^{(g)}}{\sum_{i=1}^{\mathcal{N}} u_{ik}^{\beta} \exp\left(-\lambda^{(g)} \left\|\mathbf{x}_i^{(g)} - \mathbf{z}_k^{(g)}\right\|^2\right)} \tag{28}$$

- *View weight update:*

$$v_g = \frac{\left(\prod_{g'}^{\mathcal{G}}\sum_{i=1}^{\mathcal{N}}\sum_{k=1}^{\mathcal{K}} u_{ik}^{\beta}\left(1 - \exp\left(-\lambda^{(g')} \left\|\mathbf{x}_i^{(g')} - \mathbf{z}_k^{(g')}\right\|^2\right)\right)\right)^{\frac{1}{\mathcal{G}}}}{\sum_{i=1}^{\mathcal{N}}\sum_{k=1}^{\mathcal{K}} u_{ik}^{\beta}\left(1 - \exp\left(-\lambda^{(g)} \left\|\mathbf{x}_i^{(g)} - \mathbf{z}_k^{(g)}\right\|^2\right)\right)} \tag{29}$$

Proof. The proofs of (27) and (28) are similar to Theorem 1. We only give the proof of (29).

- *View weight update*:

To solve the constrained optimization, introduce a Lagrange multiplier η for the product constraint and construct the Lagrangian:

$$\mathcal{L}^{\mathbf{v}}_{RMFC-ET-VP} = J_{RMFC-ET-VP} + \eta\left(\prod_{g=1}^{\mathcal{G}} v_g - 1\right) \tag{30}$$

[1] To reduce the sensitivity caused by random initialization, we use the initialization method in [39] to set $\mathbf{z}_k^{(g)}$ for each view.

Algorithm 1 RMFC-ET-VS

Require: $\mathbf{X} = \{X^{(1)}, X^{(2)}, \ldots, X^{(\mathcal{G})}\}$: Multi-view dataset with $\mathcal{G}$ views, $\mathcal{K}$: Number of clusters, β, α: Fuzzification and regularization parameters, max_iter, ϵ: Maximum iterations and convergence tolerance

Ensure: $\mathbf{U}$: Membership matrix ($\mathcal{N} \times \mathcal{K}$), $\mathbf{Z}$: Cluster centers, $\mathbf{v}$: View weights

1: **Initialization:**
2: Initialize cluster centers $\mathbf{Z}$ for each view g
3: Initialize $\mathbf{v}$ such that $\sum_{g=1}^{\mathcal{G}} v_g = 1$
4: Set iter = 0, prev_$J_{RMFC-ET-VS} = \infty$
5: **while** iter < max_iter **do**
6: iter ← iter + 1

▷ Update Membership Matrix

7: **for all** samples i and clusters k **do**

8: $$u_{ij} \leftarrow \frac{\left(\sum_{g=1}^{\mathcal{G}} v_g^{\alpha}\left(1-\exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)}-\mathbf{z}_k^{(g)}\right\|^2\right)\right)\right)^{-\frac{1}{\beta-1}}}{\sum_{k'=1}^{\mathcal{K}}\left(\sum_{g=1}^{\mathcal{G}} v_g^{\alpha}\left(1-\exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)}-\mathbf{z}_{k'}^{(g)}\right\|^2\right)\right)\right)^{-\frac{1}{\beta-1}}}$$

▷ Update Cluster Centers

9: **for all** clusters k and views g **do**

10: $$\mathbf{z}_k^{(g)} \leftarrow \frac{\sum_{i=1}^{\mathcal{N}} u_{ik}^{\beta} \exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)}-\mathbf{z}_k^{(g)}\right\|^2\right)\mathbf{x}_i^{(g)}}{\sum_{i=1}^{\mathcal{N}} u_{ik}^{\beta} \exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)}-\mathbf{z}_k^{(g)}\right\|^2\right)}$$

▷ Update View Weights

11: **for all** views g **do**

12: $$v_g \leftarrow \frac{\left(\sum_{i=1}^{\mathcal{N}}\sum_{k=1}^{\mathcal{K}} u_{ik}^{\beta}\left(1-\exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)}-\mathbf{z}_k^{(g)}\right\|^2\right)\right)\right)^{-\frac{1}{\alpha-1}}}{\sum_{g'=1}^{\mathcal{G}}\left(\sum_{i=1}^{\mathcal{N}}\sum_{k=1}^{\mathcal{K}} u_{ik}^{\beta}\left(1-\exp\left(-\lambda^{(g')}\left\|\mathbf{x}_i^{(g')}-\mathbf{z}_k^{(g')}\right\|^2\right)\right)\right)^{-\frac{1}{\alpha-1}}}$$

▷ Compute Objective Function

13: $$J_{RMFC-ET-VS} \leftarrow \sum_{g=1}^{\mathcal{G}}\sum_{i=1}^{\mathcal{N}}\sum_{k=1}^{\mathcal{K}} v_g^{\alpha} u_{ik}^{\beta}\left(1-\exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)}-\mathbf{z}_k^{(g)}\right\|^2\right)\right)$$

▷ Check Convergence

14: **if** $|$prev_$J_{RMFC-ET-VS} - J_{RMFC-ET-VS}| < \epsilon$ **then**
15: **break**
16: prev_$J_{RMFC-ET-VS} \leftarrow J_{RMFC-ET-VS}$
17: **Return:** $\mathbf{U}, \mathbf{Z}, \mathbf{v}$

Take the partial derivative of $\mathcal{L}^{\mathbf{v}}_{RMFC-ET-VP}$ *w.r.t.* v_m:

$$\begin{aligned}
&\frac{\partial \mathcal{L}^{\mathbf{v}}_{RMFC-ET-VP}}{\partial v_g} \\
&= \frac{\partial J_{RMFC-ET-VP}}{\partial v_g} + \frac{\partial}{\partial v_g}\eta\left(\prod_{g=1}^{\mathcal{G}} v_g - 1\right) \\
&= \sum_{i=1}^{\mathcal{N}}\sum_{k=1}^{\mathcal{K}} u_{ik}^{\beta}\left(1-\exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)}-\mathbf{z}_k^{(g)}\right\|^2\right)\right) \\
&\quad + \eta\frac{\partial}{\partial v_g}\left(\prod_{g=1}^{\mathcal{G}} v_g\right) \\
&= \sum_{i=1}^{\mathcal{N}}\sum_{k=1}^{\mathcal{K}} u_{ik}^{\beta}\left(1-\exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)}-\mathbf{z}_k^{(g)}\right\|^2\right)\right) + \eta\left(\frac{\prod_{g=1}^{\mathcal{G}} v_g}{v_g}\right) \\
&= \sum_{i=1}^{\mathcal{N}}\sum_{k=1}^{\mathcal{K}} u_{ik}^{\beta}\left(1-\exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)}-\mathbf{z}_k^{(g)}\right\|^2\right)\right) + \frac{\eta}{v_g}
\end{aligned} \tag{31}$$

Set the derivative to zero for optimization:

$$v_g = \frac{-\eta}{\sum_{i=1}^{\mathcal{N}}\sum_{k=1}^{\mathcal{K}} u_{ik}^{\beta}\left(1-\exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)}-\mathbf{z}_k^{(g)}\right\|^2\right)\right)} \tag{32}$$

To satisfy the product constraint $\prod_{g=1}^{\mathcal{G}} v_g = 1$, we can obtain:

$$-\eta = \left(\prod_{g=1}^{\mathcal{G}}\sum_{i=1}^{\mathcal{N}}\sum_{k=1}^{\mathcal{K}} u_{ik}^{\beta}\left(1-\exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)}-\mathbf{z}_k^{(g)}\right\|^2\right)\right)\right)^{\frac{1}{\mathcal{G}}} \tag{33}$$

Returning in (32), the final update rule is obtained.

The RMFC-ET-VP algorithm is displayed in Algorithm 2.

3.3. Robust multi-view fuzzy clustering with exponential transformation and matrix weighting of sum constraint (RMFC-ET-MS)

In many real-world datasets, the contributions of views may vary not only globally but also across clusters. For example, one view may be important for separating several clusters but irrelevant for others. To address this, we introduce matrix-based view weighting, where a unique weight is assigned to each view-cluster pair. These weights are constrained to sum to 1 for each cluster, ensuring localized relevance of views. Given a multi-view dataset $\mathbf{X} = [X^{(1)}, X^{(2)}, \ldots, X^{(\mathcal{G})}]$, and gth view is defined as $X^{(g)} = [\mathbf{x}_1^{(g)}, \mathbf{x}_2^{(g)}, \ldots, \mathbf{x}_{\mathcal{N}}^{(g)}] \in \mathbb{R}^{\mathcal{N}\times p^{(g)}}$. The goal is to partition $\mathbf{X}$ into $\mathcal{K}$ clusters. The objective function of RMFC-ET-MS can

Algorithm 2 RMFC-ET-VP

Require: $\mathbf{X} = \{X^{(1)}, X^{(2)}, \dots, X^{(\mathcal{G})}\}$: Multi-view dataset with $\mathcal{G}$ views, $\mathcal{K}$: Number of clusters, β: Fuzzification parameter, max_iter, ϵ: Maximum iterations and convergence tolerance

Ensure: $\mathbf{U}$: Membership matrix ($\mathcal{N} \times \mathcal{K}$), $\mathbf{Z}$: Cluster centers, $\mathbf{v}$: View weights

Initialization:
Initialize cluster centers $\mathbf{Z}$ for each view g
Initialize $\mathbf{v}$ such that $\prod_{g=1}^{\mathcal{G}} v_g = 1$
Set iter = 0, prev_$J_{RMFC-ET-VS} = \infty$
while iter < max_iter **do**
    iter ← iter + 1
    ▷ Update Membership Matrix
    for all samples i and clusters k **do**

$$u_{ij} \leftarrow \frac{\left(\sum_{g=1}^{\mathcal{G}} v_g \left(1-\exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)}-\mathbf{z}_k^{(g)}\right\|^2\right)\right)\right)^{-\frac{1}{\beta-1}}}{\sum_{k'=1}^{\mathcal{K}}\left(\sum_{g=1}^{\mathcal{G}} v_g \left(1-\exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)}-\mathbf{z}_{k'}^{(g)}\right\|^2\right)\right)\right)^{-\frac{1}{\beta-1}}}$$

    ▷ Update Cluster Centers
    for all clusters k and views g **do**

$$\mathbf{z}_k^{(g)} \leftarrow \frac{\sum_{i=1}^{\mathcal{N}} u_{ik}^{\beta} \exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)}-\mathbf{z}_k^{(g)}\right\|^2\right)\mathbf{x}_i^{(g)}}{\sum_{i=1}^{\mathcal{N}} u_{ik}^{\beta} \exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)}-\mathbf{z}_k^{(g)}\right\|^2\right)}$$

    ▷ Update View Weights
    for all views g **do**

$$v_g \leftarrow \frac{\left(\prod_{g'}^{\mathcal{G}} \sum_{i=1}^{\mathcal{N}} \sum_{k=1}^{\mathcal{K}} u_{ik}^{\beta}\left(1-\exp\left(-\lambda^{(g')}\left\|\mathbf{x}_i^{(g')}-\mathbf{z}_k^{(g')}\right\|^2\right)\right)\right)^{\frac{1}{\mathcal{G}}}}{\sum_{i=1}^{\mathcal{N}} \sum_{k=1}^{\mathcal{K}} u_{ik}^{\beta}\left(1-\exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)}-\mathbf{z}_k^{(g)}\right\|^2\right)\right)}$$

    ▷ Compute Objective Function

$$J_{RMFC-ET-VP} \leftarrow \sum_{g=1}^{\mathcal{G}} \sum_{i=1}^{\mathcal{N}} \sum_{k=1}^{\mathcal{K}} v_g u_{ik}^{\beta}\left(1-\exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)}-\mathbf{z}_k^{(g)}\right\|^2\right)\right)$$

    ▷ Check Convergence
    if $|\text{prev\_}J_{RMFC-ET-VP} - J_{RMFC-ET-VP}| < \epsilon$ **then**
        break
    prev_$J_{RMFC-ET-VP} \leftarrow J_{RMFC-ET-VP}$
Return: $\mathbf{U}, \mathbf{Z}, \mathbf{v}$

be defined as:

$$J_{RMFC-ET-MS}(\mathbf{U}, \mathbf{Z}, \mathbf{V}) = \sum_{g=1}^{\mathcal{G}} \sum_{i=1}^{\mathcal{N}} \sum_{k=1}^{\mathcal{K}} v_{kg}^{\alpha} u_{ik}^{\beta}\left(1-\exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)}-\mathbf{z}_k^{(g)}\right\|^2\right)\right)$$
$$s.t. \sum_{k=1}^{\mathcal{K}} u_{ik} = 1, u_{ik} \geq 0,\ \sum_{g=1}^{\mathcal{G}} v_{kg} = 1, v_{kg} \geq 0 \quad (34)$$

where

- $\mathbf{V} = [v_{kg}]$: The weight vector of the view, where v_{kg} represents the weight of gth view to kth cluster.

Remark 3. RMFC-ET-MS extends the vector weighting scheme to a matrix-based approach, assigning cluster-specific weights v_{kg} for each view-cluster pair. This enables the algorithm to adaptively tailor the importance of each view based on the characteristics of individual clusters, where different views may play varying roles across clusters. The sum-to-1 constraint within each cluster ensures interpretability and prevents overfitting. For example, in social media analysis, image features (view 1) may dominate "landscape" cluster ($v_{k1} = 0.8$), while text features (view 2) are pivotal for "news" cluster ($v_{k'2} = 0.7$).

Theorem 3. *The necessary conditions to minimize the objective function of RMFC-ET-MS are as follows:*

- *Fuzzy membership update:*

$$u_{ik} = \frac{\left(\sum_{g=1}^{\mathcal{G}} v_{kg}^{\alpha}\left(1-\exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)}-\mathbf{z}_k^{(g)}\right\|^2\right)\right)\right)^{-\frac{1}{\beta-1}}}{\sum_{k'=1}^{\mathcal{K}}\left(\sum_{g=1}^{\mathcal{G}} v_{k'g}^{\alpha}\left(1-\exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)}-\mathbf{z}_{k'}^{(g)}\right\|^2\right)\right)\right)^{-\frac{1}{\beta-1}}} \quad (35)$$

- *Cluster center update:*

$$\mathbf{z}_k^{(g)} = \frac{\sum_{i=1}^{\mathcal{N}} u_{ik}^{\beta} \exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)}-\mathbf{z}_k^{(g)}\right\|^2\right)\mathbf{x}_i^{(g)}}{\sum_{i=1}^{\mathcal{N}} u_{ik}^{\beta} \exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)}-\mathbf{z}_k^{(g)}\right\|^2\right)} \quad (36)$$

- *View weight update:*

$$v_{kg} = \frac{\left(\sum_{i=1}^{\mathcal{N}} u_{ik}^{\beta}\left(1-\exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)}-\mathbf{z}_k^{(g)}\right\|^2\right)\right)\right)^{-\frac{1}{\alpha-1}}}{\sum_{g'=1}^{\mathcal{G}}\left(\sum_{i=1}^{\mathcal{N}} u_{ik}^{\beta}\left(1-\exp\left(-\lambda^{(g')}\left\|\mathbf{x}_i^{(g')}-\mathbf{z}_k^{(g')}\right\|^2\right)\right)\right)^{-\frac{1}{\alpha-1}}} \quad (37)$$

Proof. The proofs of (35), (36) and (37) are similar to Theorem 1.

Algorithm 3 RMFC-ET-MS

Require: $\mathbf{X} = \{X^{(1)}, X^{(2)}, \dots, X^{(\mathcal{G})}\}$: Multi-view dataset with $\mathcal{G}$ views, $\mathcal{K}$: Number of clusters, β, α: Fuzzification and regularization parameters, max_iter, ϵ: Maximum iterations and convergence tolerance

Ensure: $\mathbf{U}$: Membership matrix ($\mathcal{N} \times \mathcal{K}$), $\mathbf{Z}$: Cluster centers, $\mathbf{V}$: View weights

Initialization:

Initialize cluster centers $\mathbf{Z}$ for each view g

Initialize $\mathbf{v}$ such that $\sum_{g=1}^{\mathcal{G}} v_{kg} = 1$

Set iter = 0, prev_$J_{RMFC-ET-MS} = \infty$

while iter < max_iter **do**

iter ← iter + 1

▷ Update Membership Matrix

for all samples i and clusters k **do**

$$u_{ij} \leftarrow \frac{\left(\sum_{g=1}^{\mathcal{G}} v_{kg}^{\alpha}\left(1-\exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)}-\mathbf{z}_k^{(g)}\right\|^2\right)\right)\right)^{-\frac{1}{\beta-1}}}{\sum_{k'=1}^{\mathcal{K}}\left(\sum_{g=1}^{\mathcal{G}} v_{k'g}^{\alpha}\left(1-\exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)}-\mathbf{z}_{k'}^{(g)}\right\|^2\right)\right)\right)^{-\frac{1}{\beta-1}}}$$

▷ Update Cluster Centers

for all clusters k and views g **do**

$$\mathbf{z}_k^{(g)} \leftarrow \frac{\sum_{i=1}^{\mathcal{N}} u_{ik}^{\beta}\exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)}-\mathbf{z}_k^{(g)}\right\|^2\right)\mathbf{x}_i^{(g)}}{\sum_{i=1}^{\mathcal{N}} u_{ik}^{\beta}\exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)}-\mathbf{z}_k^{(g)}\right\|^2\right)}$$

▷ Update View Weights

for all clusters k and views g **do**

$$v_{kg} \leftarrow \frac{\left(\sum_{i=1}^{\mathcal{N}} u_{ik}^{\beta}\left(1-\exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)}-\mathbf{z}_k^{(g)}\right\|^2\right)\right)\right)^{-\frac{1}{\alpha-1}}}{\sum_{g'=1}^{\mathcal{G}}\left(\sum_{i=1}^{\mathcal{N}} u_{ik}^{\beta}\left(1-\exp\left(-\lambda^{(g')}\left\|\mathbf{x}_i^{(g')}-\mathbf{z}_k^{(g')}\right\|^2\right)\right)\right)^{-\frac{1}{\alpha-1}}}$$

▷ Compute Objective Function

$$J_{RMFC-ET-MS} \leftarrow \sum_{g=1}^{\mathcal{G}}\sum_{i=1}^{\mathcal{N}}\sum_{k=1}^{\mathcal{K}} v_{kg}^{\alpha} u_{ik}^{\beta}\left(1-\exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)}-\mathbf{z}_k^{(g)}\right\|^2\right)\right)$$

▷ Check Convergence

if $|$prev_$J_{RMFC-ET-MS} - J_{RMFC-ET-MS}| < \epsilon$ **then**

break

prev_$J_{RMFC-ET-MS} \leftarrow J_{RMFC-ET-MS}$

Return: $\mathbf{U}, \mathbf{Z}, \mathbf{V}$

The RMFC-ET-MS algorithm is shown in Algorithm 3.

3.4. Robust multi-view fuzzy clustering with exponential transformation and matrix weighting of product constraint (RMFC-ET-MP)

RMFC-ET-MP further enhances RMFC-ET-MS by imposing a product constraint on view weights for each cluster. This ensures a more balanced weight allocation across views within individual clusters. Given a multi-view dataset $\mathbf{X} = [X^{(1)}, X^{(2)}, \dots, X^{(\mathcal{G})}]$ to be divided into $\mathcal{K}$ clusters, and gth view is defined as $X^{(g)} = [\mathbf{x}_1^{(g)}, \mathbf{x}_2^{(g)}, \dots, \mathbf{x}_{\mathcal{N}}^{(g)}] \in \mathbb{R}^{\mathcal{N}\times p^{(g)}}$. The objective function of RMFC-ET-MP can be defined as:

$$J_{RMFC-ET-MP}(\mathbf{U},\mathbf{Z},\mathbf{V}) = \sum_{g=1}^{\mathcal{G}}\sum_{i=1}^{\mathcal{N}}\sum_{k=1}^{\mathcal{K}} v_{kg} u_{ik}^{\beta}\left(1-\exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)}-\mathbf{z}_k^{(g)}\right\|^2\right)\right) \tag{38}$$

$$s.t. \sum_{k=1}^{\mathcal{K}} u_{ik} = 1, u_{ik} \geq 0, \prod_{g=1}^{\mathcal{G}} v_{kg} = 1, v_{kg} > 0$$

Remark 4. MV-FCM-MP further refines the matrix weighting scheme by imposing a product-to-1 constraint for view-cluster weights. The product-to-1 constraint ($\prod_{g=1}^{\mathcal{G}} v_{kg} = 1$) preserves weak but non-trivial view contributions, critical for applications like remote sensing, where even noisy views (*e.g.* low-resolution infrared for night scenes) retain utility and does not require additional parameters to control the weight distribution.

Theorem 4. *The necessary conditions to minimize the objective function of RMFC-ET-MP are as follows:*

- *Fuzzy membership update:*

$$u_{ik} = \frac{\left(\sum_{g=1}^{\mathcal{G}} v_{kg}\left(1-\exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)}-\mathbf{z}_k^{(g)}\right\|^2\right)\right)\right)^{-\frac{1}{\beta-1}}}{\sum_{k'=1}^{\mathcal{K}}\left(\sum_{g=1}^{\mathcal{G}} v_{k'g}\left(1-\exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)}-\mathbf{z}_{k'}^{(g)}\right\|^2\right)\right)\right)^{-\frac{1}{\beta-1}}} \tag{39}$$

- *Cluster center update:*

$$\mathbf{z}_k^{(g)} = \frac{\sum_{i=1}^{\mathcal{N}} u_{ik}^{\beta}\exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)}-\mathbf{z}_k^{(g)}\right\|^2\right)\mathbf{x}_i^{(g)}}{\sum_{i=1}^{\mathcal{N}} u_{ik}^{\beta}\exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)}-\mathbf{z}_k^{(g)}\right\|^2\right)} \tag{40}$$

- *View weight update:*

$$v_{kg} = \frac{\left(\prod_{g'}^{\mathcal{G}}\sum_{i=1}^{\mathcal{N}} u_{ik}^{\beta}\left(1-\exp\left(-\lambda^{(g')}\left\|\mathbf{x}_i^{(g')}-\mathbf{z}_k^{(g')}\right\|^2\right)\right)\right)^{\frac{1}{\mathcal{G}}}}{\sum_{i=1}^{\mathcal{N}} u_{ik}^{\beta}\left(1-\exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)}-\mathbf{z}_k^{(g)}\right\|^2\right)\right)} \tag{41}$$

Proof. The proofs of (39), (40) and (41) are similar to Theorem 2.

Algorithm 4 RMFC-ET-MP

Require: $\mathbf{X} = \{X^{(1)}, X^{(2)}, \ldots, X^{(\mathcal{G})}\}$: Multi-view dataset with $\mathcal{G}$ views, $\mathcal{K}$: Number of clusters, β: Fuzzification parameter, max_iter, ϵ: Maximum iterations and convergence tolerance
Ensure: $\mathbf{U}$: Membership matrix ($\mathcal{N} \times \mathcal{K}$), $\mathbf{Z}$: Cluster centers, $\mathbf{V}$: View weights

Initialization:
Initialize cluster centers $\mathbf{Z}$ for each view g
Initialize $\mathbf{v}$ such that $\prod_{g=1}^{\mathcal{G}} v_{kg} = 1$
Set iter = 0, prev_$J_{RMFC-ET-MP} = \infty$
while iter < max_iter **do**
    iter ← iter + 1
    ▷ Update Membership Matrix
    for all samples i and clusters k **do**

$$u_{ij} \leftarrow \frac{\left(\sum_{g=1}^{\mathcal{G}} v_{kg}\left(1-\exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)}-\mathbf{z}_k^{(g)}\right\|^2\right)\right)\right)^{-\frac{1}{\beta-1}}}{\sum_{k'=1}^{\mathcal{K}}\left(\sum_{g=1}^{\mathcal{G}} v_{k'g}\left(1-\exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)}-\mathbf{z}_{k'}^{(g)}\right\|^2\right)\right)\right)^{-\frac{1}{\beta-1}}}$$

    ▷ Update Cluster Centers
    for all clusters k and views g **do**

$$\mathbf{z}_k^{(g)} \leftarrow \frac{\sum_{i=1}^{\mathcal{N}} u_{ik}^{\beta} \exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)}-\mathbf{z}_k^{(g)}\right\|^2\right)\mathbf{x}_i^{(g)}}{\sum_{i=1}^{\mathcal{N}} u_{ik}^{\beta} \exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)}-\mathbf{z}_k^{(g)}\right\|^2\right)}$$

    ▷ Update View Weights
    for all clusters k and views g **do**

$$v_{kg} \leftarrow \frac{\left(\prod_{g'}^{\mathcal{G}} \sum_{i=1}^{\mathcal{N}} u_{ik}^{\beta}\left(1-\exp\left(-\lambda^{(g')}\left\|\mathbf{x}_i^{(g')}-\mathbf{z}_k^{(g')}\right\|^2\right)\right)\right)^{\frac{1}{\mathcal{G}}}}{\sum_{i=1}^{\mathcal{N}} u_{ik}^{\beta}\left(1-\exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)}-\mathbf{z}_k^{(g)}\right\|^2\right)\right)}$$

    ▷ Compute Objective Function

$$J_{RMFC-ET-MP} \leftarrow \sum_{g=1}^{\mathcal{G}}\sum_{i=1}^{\mathcal{N}}\sum_{k=1}^{\mathcal{K}} v_{kg} u_{ik}^{\beta}\left(1-\exp\left(-\lambda^{(g)}\left\|\mathbf{x}_i^{(g)}-\mathbf{z}_k^{(g)}\right\|^2\right)\right)$$

    ▷ Check Convergence
    if |prev_$J_{RMFC-ET-MP} - J_{RMFC-ET-MP}| < \epsilon$ **then**
        break
    prev_$J_{RMFC-ET-MP} \leftarrow J_{RMFC-ET-MP}$
Return: $\mathbf{U}, \mathbf{Z}, \mathbf{V}$

The RMFC-ET-MP algorithm is shown in Algorithm 4.

3.5. Computational complexity analysis

Time complexity: The time complexity of the proposed algorithms, including RMFC-ET-VS, RMFC-ET-VP, RMFC-ET-MS and RMFC-ET-MP, is analyzed as follows. While the algorithms differ in their specific weighting strategies (vector-based vs. matrix-based, sum-to-1 constraint vs. product-to-1 constraint), they share the same overall computational structure. The primary operations in each iteration include updating the fuzzy membership matrix, updating cluster centers and updating view weights. In each step, the complexity is $O(\mathcal{G}\mathcal{N}\mathcal{K})$, where $\mathcal{G}$, $\mathcal{N}$ and $\mathcal{K}$ are the numbers of views, samples and clusters, respectively. Assuming the algorithm converges after $\mathcal{T}$ iterations, the total complexity for all algorithms is $O(\mathcal{T}\mathcal{G}\mathcal{N}\mathcal{K})$. The proposed framework scales linearly with the number of views ($\mathcal{G}$), samples ($\mathcal{N}$), and clusters ($\mathcal{K}$), making the algorithms computationally efficient for moderate-sized datasets. Empirical results show that the number of iterations required for convergence ($\mathcal{T}$) is typically small, further enhancing the efficiency of the algorithms.

Space complexity: The space complexity of the proposed algorithms is determined by the storage requirements for three main variables: the fuzzy membership matrix, the cluster centers and the view weights. These are the core components required for the algorithms' computations. The fuzzy membership matrix requires storage for the degree of membership of each sample in each cluster and the space complexity is: $O(\mathcal{N}\mathcal{K})$. The space complexity for the cluster centers is $O(p_g\mathcal{K}\mathcal{G})$, where p_g is the dimensionality of each sample in gth view. The view weights have a complexity of either $O(\mathcal{G})$ or $O(\mathcal{K}\mathcal{G})$, depending on whether they are vector-based or matrix-based. Overall, the space complexity is $O(\mathcal{N}\mathcal{K} + p_g\mathcal{K}\mathcal{G} + \mathcal{K}\mathcal{G})$.

4. Experiments

In this section, we provide a comprehensive evaluation of the proposed multi-view fuzzy clustering framework, assessing the performance of its four algorithms in comparison to related algorithms on multi-view datasets. Before performing the clustering, the attributes of all samples are normalized to the [0, 1] range. The experiments are conducted on a PC with an Intel Core i7-1165G7 processor and 16 GB of RAM, using MATLAB 2021a. Section 4.1 describes the related algorithms, parameter settings and evaluation metrics in detail. Section 4.2 compares the proposed algorithms with related algorithms on multi-view datasets. Section 4.3 examines the sensitivity of the proposed algorithms to parameter variations and their convergence behavior. Section 4.4 analyzes the statistical significance of the differences between the proposed and related algorithms. Lastly, Section 4.5 presents an ablation study to evaluate the effect of the exponential transformation of the Euclidean distance on overall performance.

4.1. Experimental settings

To establish a thorough understanding of the performance of our proposed four algorithms, we select ten related and state-of-the-art

Table 1
Description of real-world multi-view datasets.

Dataset	Samples	Clusters	Views	View name	Features
Iris	150	3	3	Sepal dimension	2
				Mixed dimension	2
				Petal dimension	2
IS	2310	7	2	Shape information	9
				RGB information	10
MF	1000	5	3	Profile correlations	216
				Karhunen–Love coefficients	64
				Pixel averages	240
Pima	768	2	4	Demographic	2
				Blood Glucose and Pressure	2
				Body measurement	3
				Genetic risk	1
MSRCv1	210	7	4	Color moment	24
				GIST	512
				LBP	256
				CENTRIST	254
ProP	551	4	3	Gabor	393
				Wavelet moments	3
				CENTRIST	438
Seeds	210	3	2	Morphological	3
				Geometric	4

multi-view clustering algorithms as the benchmark algorithms for comparison. These include Co-FKM [32], TW-k-means [22], Co-FCM [33], WV-Co-FCM [33], SMVF [23], TW-Co-k-means [24], Co-FW-MVFCM [34], MVASM [35], OMFC-CS [36] and FedMVFCM [38]. Among them, TW-k-means, SMVF and TW-Co-k-means stand out as notable examples of multi-view clustering via hard partition, while others represent a group of prominent multi-view clustering via fuzzy partition.

For each algorithm, including the proposed and benchmark algorithms, we optimize the parameters using a grid-search strategy, based on the recommended ranges from relevant literature, to determine the best configurations. Specifically, for our proposed algorithms, we vary a common parameter, β, within the range of $[1.1, 2]$ with a step size of 0.1. Additionally, two algorithms, RMFC-ET-VS and RMFC-ET-MS, include an extra parameter α. To evaluate its influence on clustering performance, we test α across the values $\{1.1, 2, 4, 6, \ldots, 16, 18, 20\}$.

To evaluate the performance of the clustering algorithms, we employ several widely used criteria, including Accuracy (ACC) [40], Rand Index (RI) [41], Normalized Mutual Information (NMI) [16] and Fowlkes–Mallows Index (FMI) [20]. These metrics quantitatively measure the similarity between the clustering results and the ground-truth labels. Higher values of ACC, RI, NMI and FMI correspond to better clustering performance. These metrics are provided below:

- Accuracy (ACC): This metric measures the proportion of correctly assigned samples to their respective clusters compared to the total number of samples. It is calculated as:

$$ACC = \frac{\mathcal{N}_c}{\mathcal{N}} \tag{42}$$

where $\mathcal{N}_c$ is the number of correctly assigned samples, and $\mathcal{N}$ is the total number of samples.

- Rand Index (RI): This metric assesses the level of agreement between the clustering results and the ground truth. It is computed as the ratio of correctly paired samples, both within the same cluster and across different clusters, to the total number of sample pairs:

$$RI = \frac{TP + TN}{TP + FP + TN + FN} \tag{43}$$

where TP represents true positives, TN true negatives, FP false positives and FN false negatives.

- Normalized Mutual Information (NMI): This metric evaluates the similarity between predicted clusters and the ground truth, with the results normalized for comparison. It is expressed as:

$$NMI = \frac{\sum_{k=1}^{\mathcal{K}} \sum_{l=1}^{\mathcal{K}} \mathcal{N}_{k,l} \log\left(\frac{\mathcal{N} \times \mathcal{N}_{k,l}}{\mathcal{N}_k \times \mathcal{N}_l}\right)}{\sqrt{\left(\sum_{k=1}^{\mathcal{K}} \mathcal{N}_k \log \frac{\mathcal{N}_k}{\mathcal{N}}\right)\left(\sum_{l=1}^{\mathcal{K}} \mathcal{N}_l \log \frac{\mathcal{N}_l}{\mathcal{N}}\right)}} \tag{44}$$

where $N_{k,l}$ denotes the number of samples in the kth class assigned to the lth cluster, and $\mathcal{N}_k$ and N_l are the total samples in the kth class and lth cluster, respectively.

- Fowlkes–Mallows Index (FMI): This metric provides the geometric mean of precision and recall in the context of clustering evaluation. It is defined as:

$$FMI = \sqrt{\frac{TP}{TP + FP} \times \frac{TP}{TP + FN}} \tag{45}$$

4.2. Results on real-world datasets

To compare the performance of four proposed algorithms against ten benchmark algorithms, we conduct an extensive experimental analysis using seven real-world multi-view datasets. These datasets include Iris,[2] Image Segmentation (IS),[3] Multiple features (MF) [20], Pima,[4] MSRCv1 [42], Prokaryotic Phyla (ProP) [43] and Seeds.[5] A detailed description of each multi-view dataset is provided in Table 1. We comprehensively evaluate clustering performance across four metrics: ACC, RI, NMI and FMI. The detailed results are presented in Tables 2 and 3, with the four top-performing algorithms emphasized in bold. By meticulously analyzing these results, we uncover several notable insights:

- Across all datasets, the proposed algorithms consistently outperform the other algorithms in terms of ACC, RI, NMI and FMI. Specifically, compared to OMFC-CS, RMFC-ET-VS achieves average enhancements of 4.68% in ACC, 2.24% in RI, 4.90% in NMI, and 6.40% in FMI, while RMFC-ET-VP shows improvements of 4.25% in ACC, 2.26% in RI, 4.72% in NMI, and 6.06% in FMI. Likewise, RMFC-ET-MS exhibits average gains of 5.21% in ACC, 2.63% in RI, 5.42% in NMI, and 6.80% in FMI,

[2] https://archive.ics.uci.edu/dataset/53/iris.
[3] https://archive.ics.uci.edu/ml/datasets/image+segmentation.
[4] https://www.kaggle.com/datasets/uciml/pima-indians-diabetes-database.
[5] https://archive.ics.uci.edu/dataset/236/seeds.

Table 2
Clustering results of the real-world multi-view datasets on ACC and RI.

Algorithm	Dataset													
	Iris		IS		MF		Pima		MSRCv1		ProP		Seeds	
	ACC	RI	ACC	RI	ACC	RI	ACC	RI	ACC	RI	ACC	RI	ACC	RI
Co-FKM	0.8987	0.8847	0.4406	0.7568	0.6405	0.8287	0.6238	0.5356	0.4330	0.7540	0.4717	0.5990	0.8133	0.8173
TW-k-means	0.9173	0.9036	0.6106	0.8506	0.8214	0.9059	0.6551	0.5476	0.6047	0.8391	0.5502	0.6160	0.8400	0.8382
Co-FCM	0.8960	0.8822	0.5014	0.7615	0.7915	0.8794	0.6268	0.5367	0.5859	0.8246	0.5550	0.6138	0.8400	0.8330
WV-Co-FCM	0.9033	0.8891	0.5935	0.8576	0.8888	0.9233	0.6665	0.5558	0.6097	0.8277	0.5575	0.6213	0.8648	0.8524
SWVF	0.9160	0.9258	0.6235	0.8623	0.8163	0.8958	0.6424	0.5439	0.6050	0.8478	0.5556	0.6249	0.8567	0.8436
TW-Co-k-means	0.9593	0.9488	0.6375	0.8613	0.8543	0.9163	0.6461	0.5461	0.6234	0.8495	0.5598	0.6277	0.8795	0.8559
Co-FW-MVFCM	**0.9600**	**0.9495**	0.5291	0.8400	0.6612	0.8229	0.6237	0.5300	0.5955	0.8345	0.5395	0.6230	0.7905	0.7896
MVASM	0.9587	0.9479	0.5245	0.8054	0.8397	0.9048	0.6241	0.5327	0.5714	0.8342	0.5008	0.5716	0.8619	0.8437
OMFC-CS	0.9333	0.9195	0.6357	0.8694	0.8983	0.9338	0.6458	0.5419	0.5861	0.8240	0.5451	0.6280	0.8762	**0.8572**
FedMVFCM	**0.9600**	**0.9495**	0.5808	0.8573	0.7459	0.8673	0.6488	0.5442	0.4188	0.7776	0.5423	0.6244	0.8557	0.8425
RMFC-ET-VS	**0.9667**	**0.9575**	**0.6723**	**0.8706**	**0.9660**	**0.9737**	**0.6745**	**0.5603**	**0.7143**	**0.8768**	**0.5735**	**0.6403**	**0.8810**	**0.8587**
RMFC-ET-VP	**0.9667**	**0.9575**	**0.6442**	**0.8720**	**0.9630**	**0.9715**	**0.6771**	**0.5621**	**0.7143**	**0.8778**	**0.5717**	**0.6490**	**0.8810**	0.8571
RMFC-ET-MS	**0.9600**	**0.9495**	**0.6814**	**0.8734**	**0.9670**	**0.9745**	**0.6875**	**0.5698**	**0.7190**	**0.8800**	**0.5844**	**0.6473**	**0.8857**	**0.8625**
RMFC-ET-MP	**0.9667**	**0.9575**	**0.6874**	**0.8719**	**0.9650**	**0.9731**	**0.6862**	**0.5688**	**0.7143**	**0.8786**	**0.5717**	**0.6288**	**0.8857**	**0.8625**

Table 3
Clustering results of the real-world multi-view datasets on NMI and FMI.

Algorithm	Dataset													
	Iris		IS		MF		Pima		MSRCv1		ProP		Seeds	
	NMI	FMI	NMI	FMI	NMI	FMI	NMI	FMI	NMI	FMI	NMI	FMI	NMI	FMI
Co-FKM	0.7549	0.8265	0.4198	0.4241	0.6738	0.6822	0.0365	0.5691	0.3135	0.3591	0.2075	0.4252	0.5939	0.7299
TW-k-means	0.7821	0.8544	0.6336	0.5685	0.7398	0.7518	0.0345	0.6045	0.4958	0.4433	0.2806	0.4335	0.6431	0.7785
Co-FCM	0.7491	0.8226	0.5219	0.5092	0.7308	0.7383	0.0347	0.5735	0.4249	0.4290	0.2727	0.4411	0.6103	0.7478
WV-Co-FCM	0.7572	0.8325	0.5966	0.5311	0.7771	0.8111	0.0373	0.6642	0.4754	0.4590	0.2815	0.4506	0.6444	0.7769
SWVF	0.8354	0.8962	0.5974	0.5533	0.7442	0.7582	0.0354	0.6399	0.5242	0.4892	0.2840	0.4433	0.6450	0.7689
TW-Co-k-means	0.8624	0.9221	0.6368	0.5873	0.7709	0.7926	0.0369	0.5949	0.5105	0.4728	0.2870	0.4557	0.6468	0.7826
Co-FW-MVFCM	0.8642	0.9233	0.5714	0.4881	0.5995	0.6365	0.0196	0.5745	0.4499	0.4402	0.2750	0.4414	0.5454	0.6902
MVASM	0.8648	0.9209	0.4602	0.4782	0.7384	0.7630	0.0322	0.5636	0.4505	0.4309	0.2713	0.4388	0.6436	0.7676
OMFC-CS	0.8038	0.8776	0.6129	0.5799	0.8095	0.8286	0.0362	0.5810	0.4877	0.4808	0.2834	0.4373	**0.6656**	**0.7885**
FedMVFCM	**0.8705**	**0.9234**	0.6138	0.5336	0.6813	0.6977	0.0341	0.6000	0.2859	0.2955	0.2620	0.4375	0.6357	0.7731
RMFC-ET-VS	**0.8851**	**0.9355**	**0.6435**	**0.6007**	**0.8993**	**0.9341**	**0.0374**	**0.7166**	**0.6128**	**0.5948**	**0.3241**	**0.4646**	**0.6529**	**0.7866**
RMFC-ET-VP	**0.8851**	**0.9355**	**0.6515**	**0.5992**	**0.8889**	**0.9284**	**0.0382**	**0.7022**	**0.6095**	**0.5962**	**0.3433**	**0.4755**	0.6471	0.7841
RMFC-ET-MS	**0.8705**	**0.9234**	**0.6671**	**0.6133**	**0.9015**	**0.9361**	**0.0530**	**0.6996**	**0.6144**	**0.6034**	**0.3124**	**0.4806**	**0.6578**	**0.7923**
RMFC-ET-MP	**0.8851**	**0.9355**	**0.6403**	**0.6015**	**0.8978**	**0.9324**	**0.0501**	**0.6929**	**0.6111**	**0.5983**	**0.3087**	**0.4612**	**0.6578**	**0.7923**

and RMFC-ET-MP achieves increases of 5.09% in ACC, 2.18% in RI, 4.54% in NMI, and 5.96% in FMI. This superior performance highlights the effectiveness of the proposed algorithms in handling diverse multi-view datasets.

- Among the proposed four algorithms, RMFC-ET-MS and RMFC-ET-MP generally exhibit superior performance compared to RMFC-ET-VS and RMFC-ET-VP across most datasets. For example, RMFC-ET-MS achieves average improvements of 0.53% in ACC, 0.39% in RI, 0.52% in NMI, and 0.40% in FMI compared to RMFC-ET-VS. Similarly, RMFC-ET-MP exhibits gains of 0.85% in ACC over RMFC-ET-VP. For other metrics, while RMFC-ET-VP shows better performance than RMFC-ET-MP on average performance, RMFC-ET-MP consistently outperforms RMFC-ET-VP on most datasets. These results highlight the advantage of using matrix-based view weighting scheme, as seen in RMFC-ET-MS and RMFC-ET-MP, which allow each cluster to have a unique view weight distribution. In contrast, RMFC-ET-VS and RMFC-ET-VP use vector-based scheme, where all clusters share the same view weight.
- The superior performance of our proposed algorithms is largely due to the incorporation of two key innovations: the exponential transformation of Euclidean distance and automatic learning of view weights. The exponential transformation reduces the influence of noise while amplifying meaningful separations between clusters, leading to improved clustering quality. Meanwhile, the automatic view weight learning scheme enables our algorithms to dynamically adjust the contribution of each view based on its relevance to the clustering process. This enables more effective utilization of multi-view information, as demonstrated by the improvements in ACC, RI, NMI and FMI across all datasets.

To demonstrate the effectiveness of the view-weighting scheme in the proposed algorithms, we present the weight assignment process for different views on Iris, Pima and Seeds datasets. As illustrated in Fig. 1(a)–(f), RMFC-ET-VS and RMFC-ET-VP adopt a vector-based weighting strategy to define view weights. Under this scheme, all clusters share the same view weights, treating the importance of each view as uniform across clusters. While this strategy simplifies the computation and management of weights, it may fail to capture subtle differences among clusters, particularly in datasets with complex structures. In contrast, RMFC-ET-MS and RMFC-ET-MP, as shown in Fig. 1(g)–(l), employ a matrix-based weighting strategy, allowing unique view weight distributions for each cluster. This flexibility enables the importance of each view to vary across clusters, thereby capturing the intrinsic structure and characteristics of the dataset more precisely. Specifically, views that exhibit stronger discriminative capabilities are assigned higher weights, playing a more critical role in the clustering process.

4.3. Parameter and convergence analysis

In this subsection, we examine the effects of the parameters α and β on clustering performance and explore the convergence behavior of the proposed algorithms.

First, we analyze the impact of β on the performance of RMFC-ET-VP and RMFC-ET-MP, evaluating the algorithms using four metrics: ACC, RI, NMI and FMI on the IS, Pima and Seeds datasets. As shown in Fig. 2, β is varied within the range of $[1.1, 2]$ with a step of 0.1.

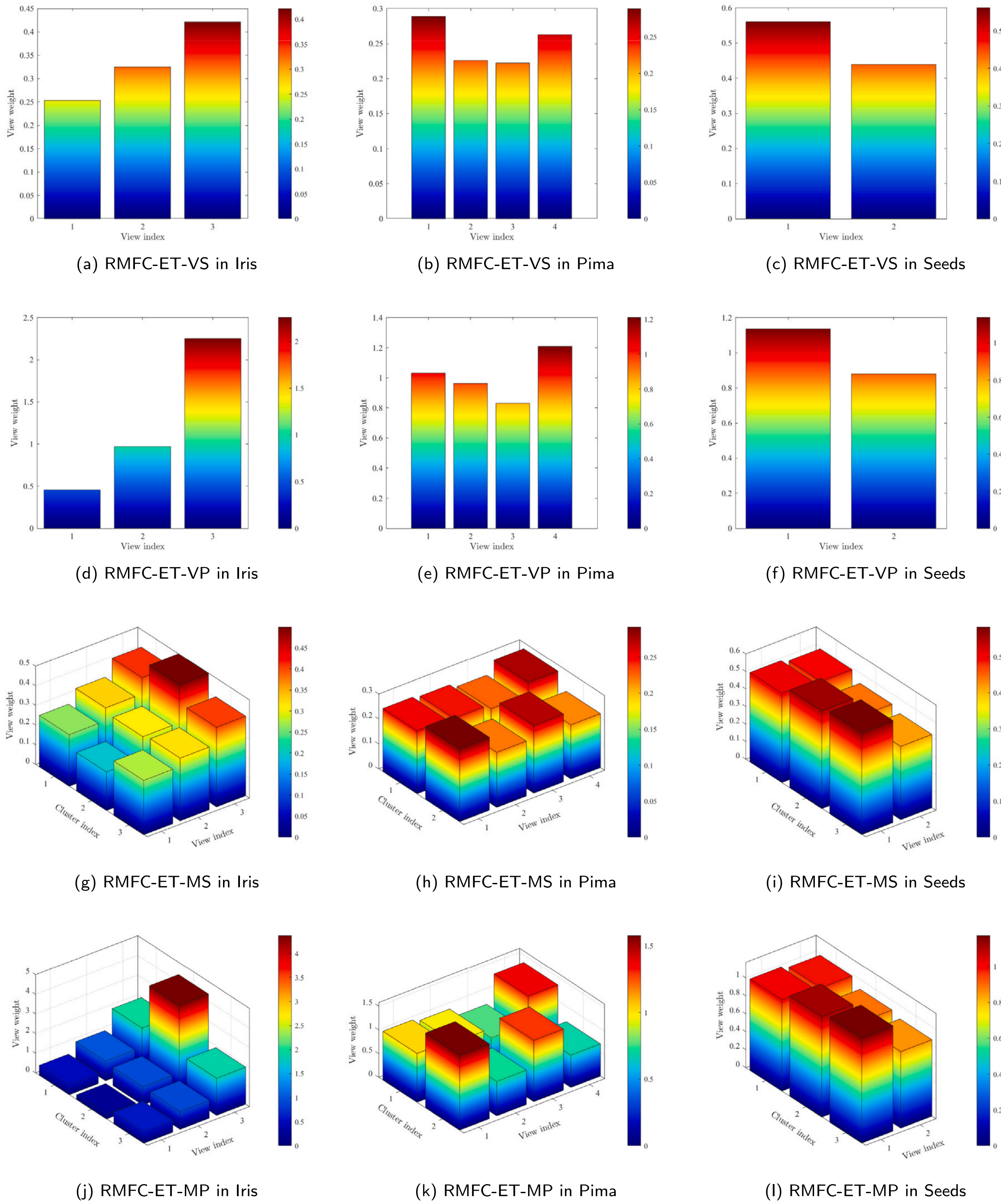
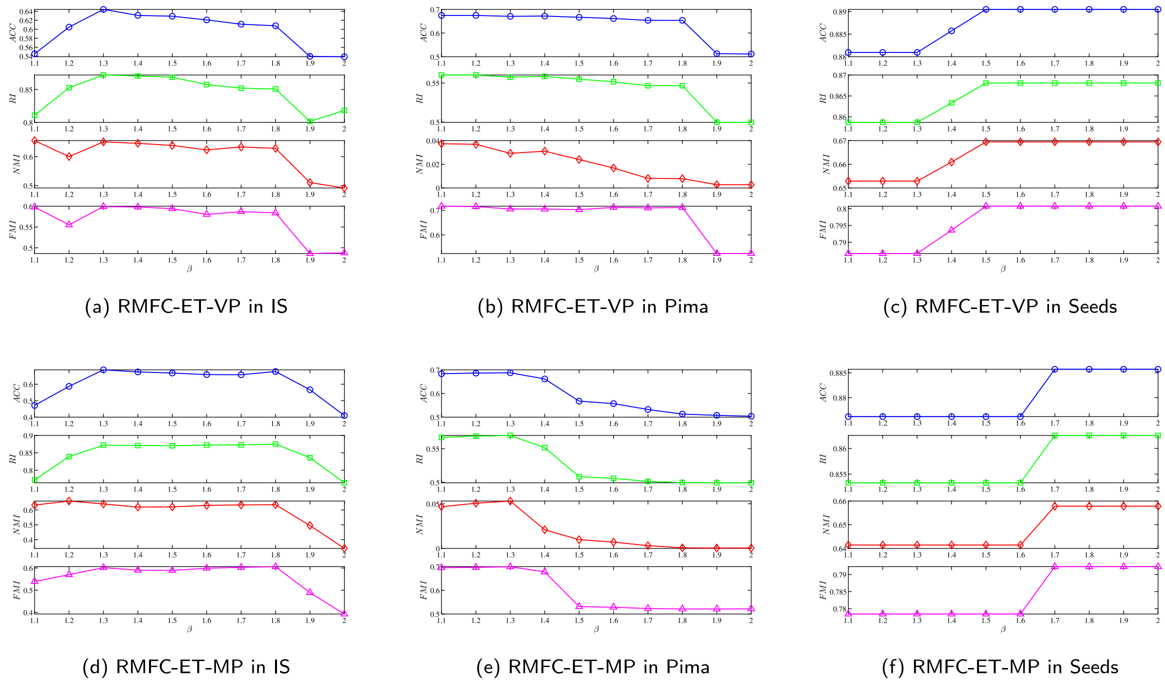


Fig. 1. View weight distributions of the proposed algorithms on Iris, Pima and Seeds datasets.

While performance metrics exhibit some minor fluctuations across this range, the overall performance of both RMFC-ET-VP and RMFC-ET-MP remains relatively stable, indicating that the algorithms can maintain relatively stable clustering results with parameter settings. Next, we further analyze the joint effect of both α and β parameters on the performance of RMFC-ET-VS and RMFC-ET-MS. As shown in Figs. 3 and 4, we set a wide range for α, starting from 1.1 and increasing by increments of 2 until 20, while keeping the range for β consistent. The results show that the performance of these algorithms is also robust to parameter variations. While certain parameter combinations result in optimal or near-optimal performance, the changes in parameters do not lead to significant performance degradation or large fluctuations. While the algorithms demonstrate robustness, selecting optimal values for α and β can further enhance their performance across different datasets.

To better understand the convergence behavior of the proposed algorithms, we analyze the objective functions curves across iterations on the IS, MF and MSRCv1 datasets, as shown in Fig. 5. Initially, the objective function value decreases rapidly with each iteration. However, after approximately ten iterations, the rate of decrease slows, indicating that the optimization process is stabilizing. At this point, the objective function changes become negligible with minimal fluctuations. This behavior suggests that the algorithms converge effectively within a small number of iterations, reaching a stable and reliable clustering solution.



(a) RMFC-ET-VP in IS (b) RMFC-ET-VP in Pima (c) RMFC-ET-VP in Seeds

(d) RMFC-ET-MP in IS (e) RMFC-ET-MP in Pima (f) RMFC-ET-MP in Seeds

Fig. 2. The performance of RMFC-ET-VP and RMFC-ET-MP with different β in terms of ACC, RI, NMI and FMI.

4.4. Statistical analysis

To assess whether the proposed algorithms exhibit statistically significant differences in clustering performance compared to the benchmark algorithms, we perform a comprehensive Friedman–Nemenyi test analysis. The detailed procedure for this analysis is described in [44, 45]. Table 4 shows the average rankings of 14 algorithms (including the proposed and the benchmark algorithms) across all multi-view datasets, evaluated using four performance metrics: ACC, RI, NMI and FMI.

First, the τ value for the Friedman test is computed, followed by the formulas (46) and (47).

$$\tau_{\chi^2} = \frac{12Q_F}{P_F\left(P_F+1\right)}\left(\sum_{i=1}^{P_F} r_i^2 - \frac{P_F\left(P_F+1\right)^2}{4}\right) \tag{46}$$

$$\tau_F = \frac{(Q_F-1)\tau_{\chi^2}}{Q_F(P_F-1)-\tau\chi^2} \tag{47}$$

where P_F represents the number of algorithms, and Q_F refers to the number of datasets. We evaluate the performance of fourteen algorithms on seven different datasets, with the parameters $P_F = 14$ and $Q_F = 7$. To determine statistical significance, we use the τ_F value based on the τ_{χ^2} distribution, considering the corresponding degrees of freedom (P_F-1) and $(P_F-1)(Q_F-1)$. At a 95% confidence level, the critical value for the F-distribution is $F_{0.05}((14-1),(14-1)*(7-1)) = 1.8478$. Notably, this critical value is significantly lower than the observed τ_F values. Specifically, the observed τ_F values are $\tau_F = 13.4384$ *w.r.t.* ACC, $\tau_F = 13.4420$ *w.r.t.* RI, $\tau_F = 13.4425$ *w.r.t.* NMI, and $\tau_F = 13.4426$ *w.r.t.* FMI. These results indicate that, at a 5% significance level, the performance differences among the fourteen algorithms are statistically significant.

Subsequently, we apply the Nemenyi test as a post-hoc analysis following the Friedman test. The results of the Nemenyi test are presented in a graphical format, as shown in Fig. 6. In this figure, the midpoint of each line represents the average rank of a specific clustering algorithm across all datasets, while the length of the line indicates the magnitude of the critical difference (CD) value. The CD value is computed as follows:

$$CD = q_\alpha\sqrt{\frac{P_F(P_F+1)}{6Q_F}} \tag{48}$$

where the critical value q_α is a parameter associated with the Friedman test statistic and depends on the number of algorithms P_F. When $P_F = 14$ and the significance level α is set to 0.05, we calculate q_α to be 7.6841. Using this critical value, the CD value is determined to be 6.2740. This result provides an essential criterion: if the performance rank difference between two algorithms exceeds the CD value (*i.e.* 6.2740), their performance differences are statistically significant. Based on this criterion, we can clearly conclude that the proposed algorithms significantly outperform benchmark algorithms in the comparison. This indicates that proposed algorithms demonstrate superior performance in solving specific problems or datasets compared to the alternatives.

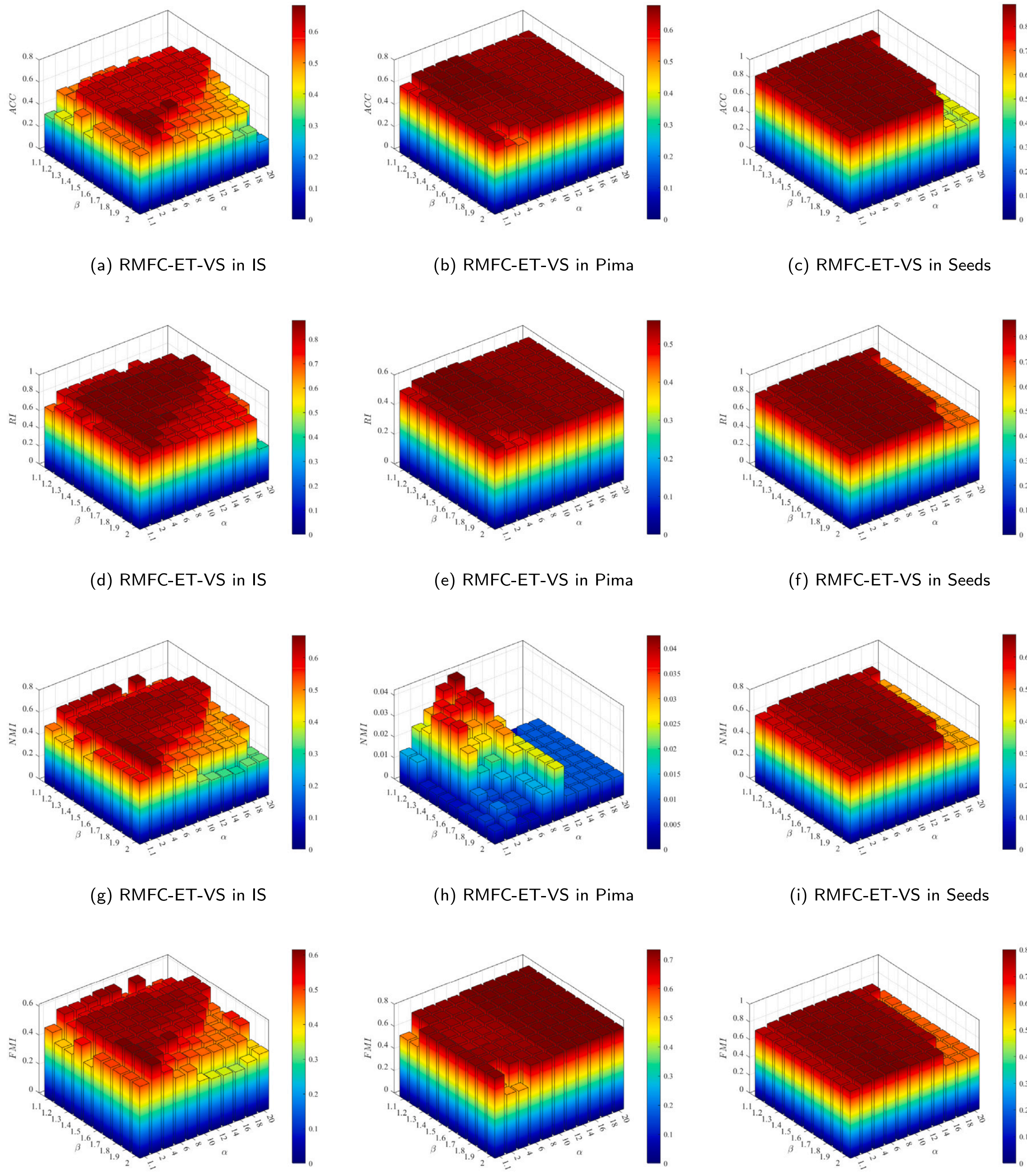
4.5. Ablation study

To thoroughly assess the impact of the exponential transformation of Euclidean distance in our proposed algorithms, we conduct an ablation study. The aim is to compare and analyze the clustering performance of our proposed algorithms against their respective variants, in which the exponential distance transformation is replaced with the standard Euclidean distance. The objective functions for the four variants are defined as follows:

$$\begin{aligned} &J_{MFC-VS}(\mathbf{U},\mathbf{Z},\mathbf{v}) = \sum_{g=1}^{\mathcal{G}}\sum_{i=1}^{\mathcal{N}}\sum_{k=1}^{\mathcal{K}} v_g^\alpha u_{ik}^\beta \left\|\mathbf{x}_i^{(g)}-\mathbf{z}_k^{(g)}\right\|^2 \\ &s.t. \sum_{k=1}^{\mathcal{K}} u_{ik}=1, u_{ik}\geq 0,\ \sum_{g=1}^{\mathcal{G}} v_g = 1, v_g \geq 0 \end{aligned} \tag{49}$$

$$\begin{aligned} &J_{MFC-VP}(\mathbf{U},\mathbf{Z},\mathbf{v}) = \sum_{g=1}^{\mathcal{G}}\sum_{i=1}^{\mathcal{N}}\sum_{k=1}^{\mathcal{K}} v_g u_{ik}^\beta \left\|\mathbf{x}_i^{(g)}-\mathbf{z}_k^{(g)}\right\|^2 \\ &s.t. \sum_{k=1}^{\mathcal{K}} u_{ik}=1, u_{ik}\geq 0,\ \prod_{g=1}^{\mathcal{G}} v_g = 1, v_g > 0 \end{aligned} \tag{50}$$

$$\begin{aligned} &J_{MFC-MS}(\mathbf{U},\mathbf{Z},\mathbf{V}) = \sum_{g=1}^{\mathcal{G}}\sum_{i=1}^{\mathcal{N}}\sum_{k=1}^{\mathcal{K}} v_{kg}^\alpha u_{ik}^\beta \left\|\mathbf{x}_i^{(g)}-\mathbf{z}_k^{(g)}\right\|^2 \\ &s.t. \sum_{k=1}^{\mathcal{K}} u_{ik}=1, u_{ik}\geq 0,\ \sum_{g=1}^{\mathcal{G}} v_{kg} = 1, v_{kg} \geq 0 \end{aligned} \tag{51}$$



(a) RMFC-ET-VS in IS (b) RMFC-ET-VS in Pima (c) RMFC-ET-VS in Seeds

(d) RMFC-ET-VS in IS (e) RMFC-ET-VS in Pima (f) RMFC-ET-VS in Seeds

(g) RMFC-ET-VS in IS (h) RMFC-ET-VS in Pima (i) RMFC-ET-VS in Seeds

(j) RMFC-ET-VS in IS (k) RMFC-ET-VS in Pima (l) RMFC-ET-VS in Seeds

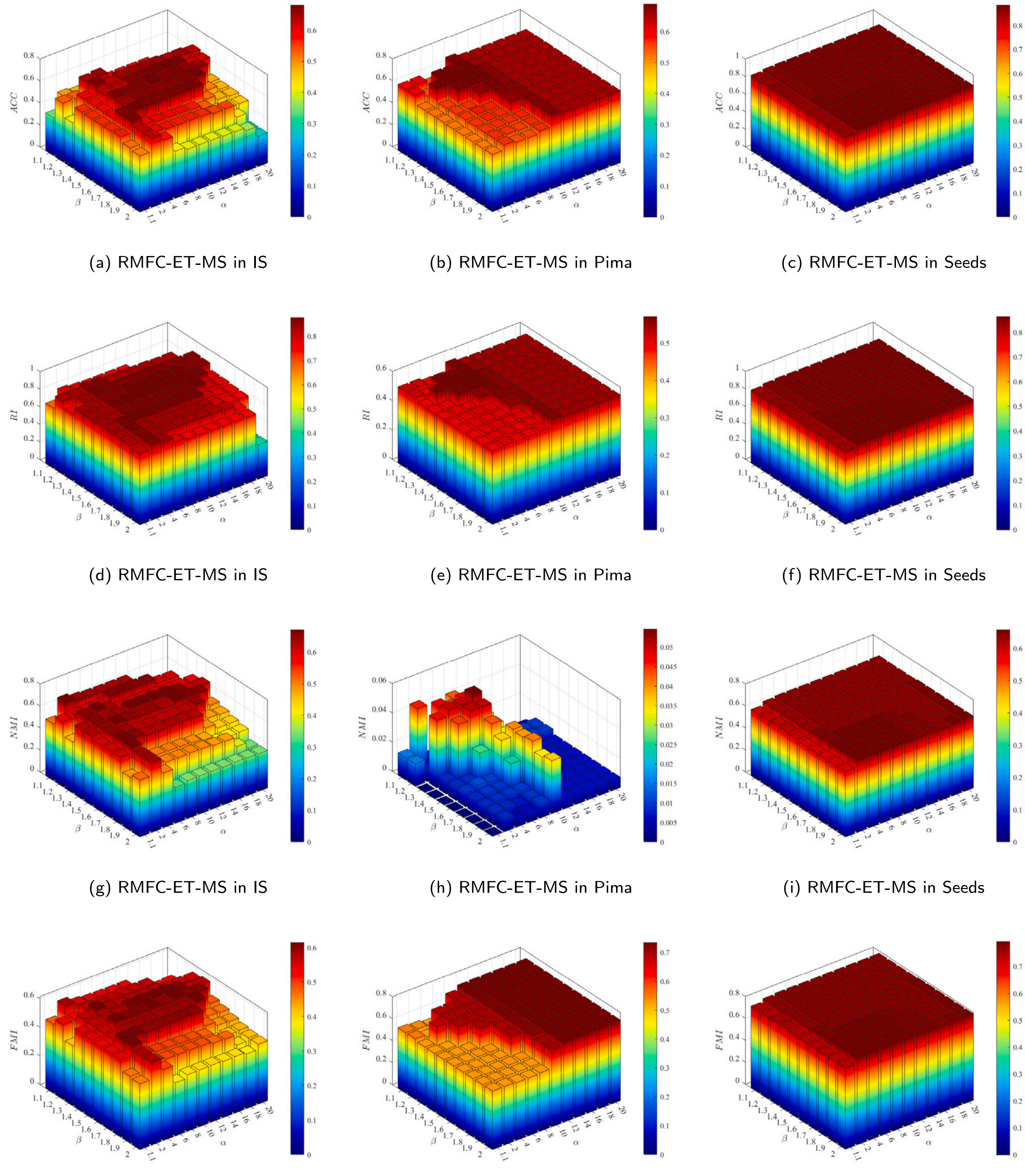
Fig. 3. The performance of RMFC-ET-VS with different α and β in terms of ACC, RI, NMI and FMI.

Table 4
The average rank r_i of each algorithms on all multi-view datasets.

Metrics	Co-FKM	TW-k-means	Co-FCM	WV-Co-FCM	SWVF	TW-Co-k-means	Co-FW-MVFCM	MVASM	OMFC-CS	FedMVFCM	RMFC-ET-VS	RMFC-ET-VP	RMFC-ET-MS	RMFC-ET-MP
ACC	13.4286	8.7857	11.3571	7.2857	8.7143	6.0000	11.1429	10.4286	7.8571	9.8571	2.7857	3.2857	1.7857	2.2857
RI	13.2857	8.9286	11.8571	7.6429	8.3571	6.5000	10.6429	9.9286	7.5000	9.9286	3.0714	3.2143	1.2857	2.8571
NMI	12.5714	8.7143	11.5714	7.5714	7.8571	6.1429	11.2143	10.2143	7.0000	11.2857	2.6429	3.3571	1.5714	3.2857
FMI	13.4286	8.9286	11.4286	7.1429	8.0000	6.7143	10.5000	10.2143	7.7143	10.3571	2.6429	3.5000	1.4286	3.0000

$$J_{MFC-MP}(\mathbf{U},\mathbf{Z},\mathbf{V}) = \sum_{g=1}^{\mathcal{G}}\sum_{i=1}^{\mathcal{N}}\sum_{k=1}^{\mathcal{K}} v_{kg} u_{ik}^{\beta} \left\| \mathbf{x}_i^{(g)} - \mathbf{z}_k^{(g)} \right\|^2$$
$$s.t. \sum_{k=1}^{\mathcal{K}} u_{ik} = 1, u_{ik} \geq 0, \prod_{g=1}^{\mathcal{G}} v_{kg} = 1, v_{kg} > 0 \tag{52}$$

Fig. 7 presents the comparison results of our proposed algorithms and their respective variants on the MSRCv1 and Prop datasets. It is clear that the proposed algorithms outperform their Euclidean distance variants across all four metrics: ACC, RI, NMI and FMI. This performance improvement can be attributed to the exponential transformation of the Euclidean distance integrated into our framework. The exponential transformation can effectively suppress the influence of



(a) RMFC-ET-MS in IS (b) RMFC-ET-MS in Pima (c) RMFC-ET-MS in Seeds

(d) RMFC-ET-MS in IS (e) RMFC-ET-MS in Pima (f) RMFC-ET-MS in Seeds

(g) RMFC-ET-MS in IS (h) RMFC-ET-MS in Pima (i) RMFC-ET-MS in Seeds

(j) RMFC-ET-MS in IS (k) RMFC-ET-MS in Pima (l) RMFC-ET-MS in Seeds

Fig. 4. The performance of RMFC-ET-MS with different α and β in terms of ACC, RI, NMI and FMI.

noise while enhancing the separation between clusters, which enhances the clustering performance, especially in complex multi-view datasets.

5. Conclusions

This paper proposes a robust framework for multi-view fuzzy clustering, introducing four novel algorithms: RMFC-ET-VS, RMFC-ET-VP, RMFC-ET-MS and RMFC-ET-MP. Such algorithms incorporate an exponential transformation of the Euclidean distance to effectively suppress noise and reduce the influence of outliers, thereby significantly enhancing the robustness of the clustering process. Furthermore, we develop view-weighting mechanisms, including vector-based and matrix-based constraints with sum-to-1 and product-to-1, which dynamically adjust the contribution of each view. Extensive experiments on real-world datasets demonstrate that the proposed algorithms outperform the state-of-the-art multi-view clustering algorithms, achieving better clustering accuracy and greater robustness. The combination of robust distance metrics and flexible view-weighting strategies leads to significant improvements in clustering performance. The proposed algorithms provide a significant advancement in multi-view fuzzy clustering. By addressing the challenges of noise sensitivity and inflexible view weighting, our algorithms offer a more robust and adaptive approach to clustering multi-view data, making them suitable for real-world applications such as image recognition, bioinformatics and multimedia analysis where data quality is often variable. Our results demonstrate

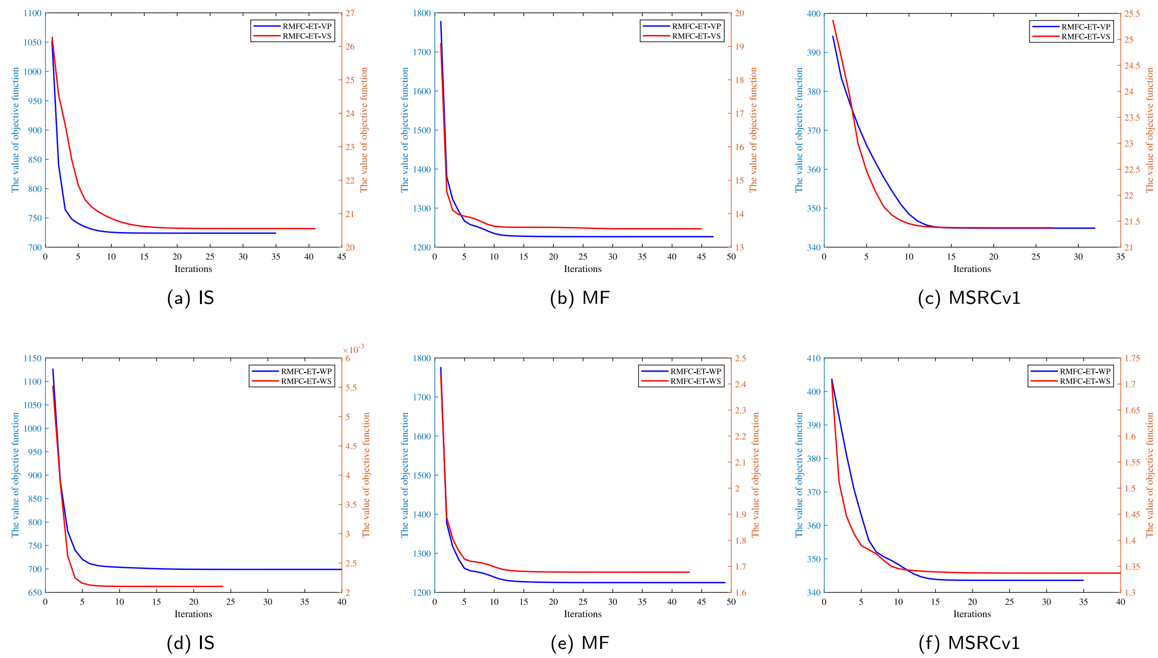


Fig. 5. Convergence analysis of the proposed algorithms on IS, MF and MSRCv1 datasets.

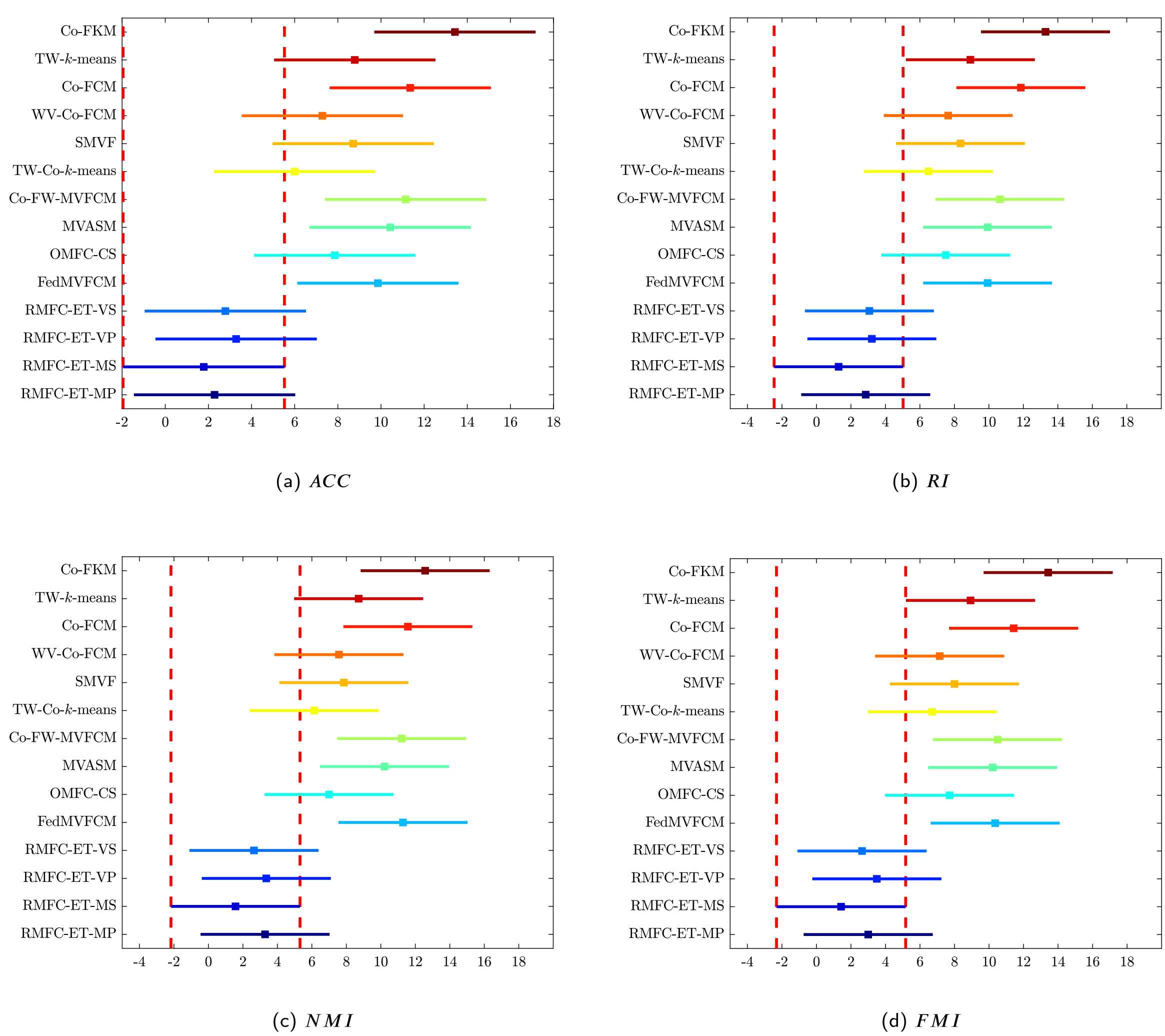
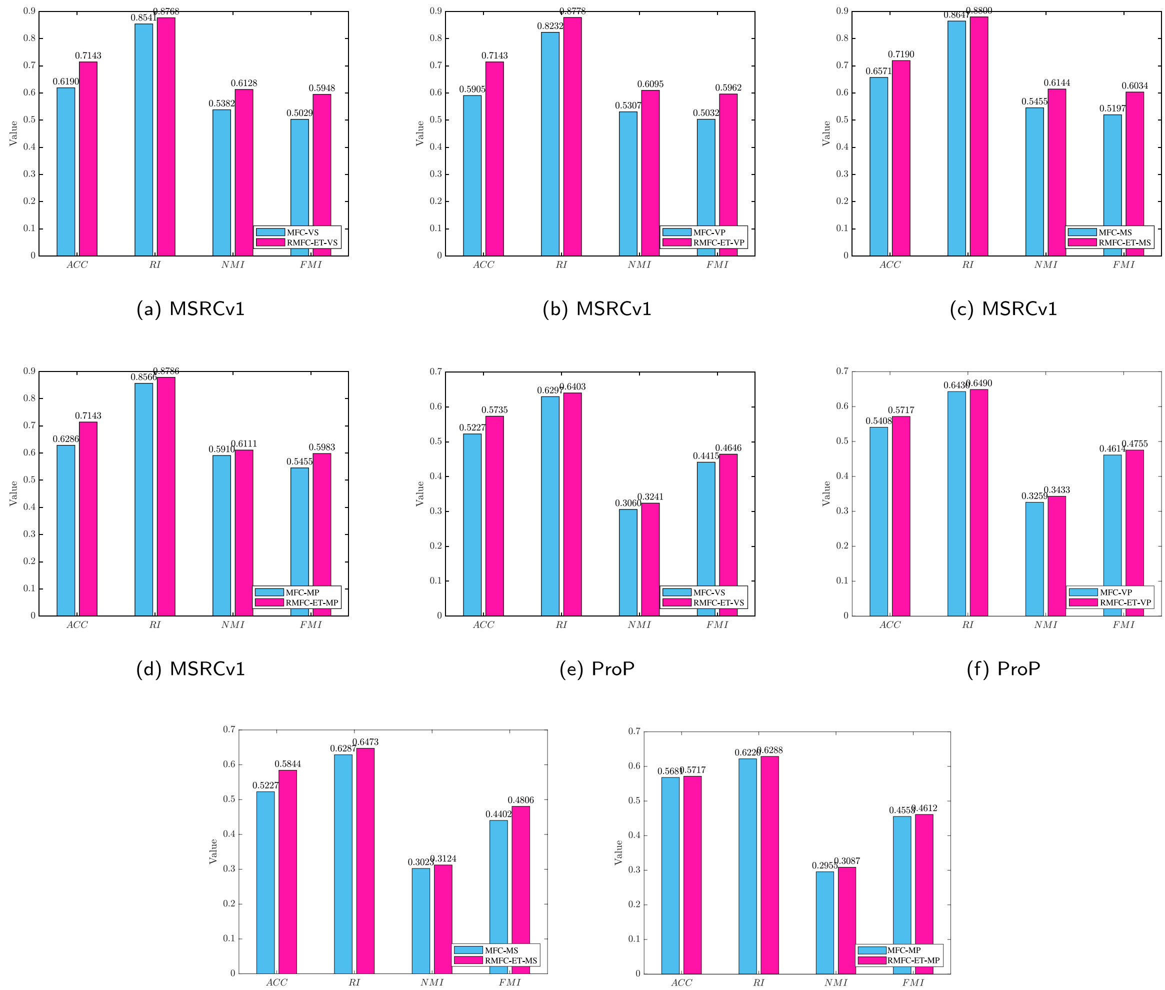


Fig. 6. The Friedman testing diagram in terms of ACC, RI, NMI and FMI.



(a) MSRCv1 (b) MSRCv1 (c) MSRCv1
(d) MSRCv1 (e) ProP (f) ProP
(g) ProP (h) ProP

Fig. 7. Performance comparison of the proposed four algorithms with exponential transformation against their Euclidean distance counterparts on MSRCv1 and ProP datasets.

that these algorithms can handle complex and noisy datasets with varying levels of view relevance, offering a reliable tool for multi-view clustering. Despite the contributions, this paper has several limitations that warrant further investigation. For example, when dealing with high-dimensional datasets, the curse of dimensionality may affect clustering performance. Also, the proposed framework does not consider the need for privacy-preserving. For sensitive fields such as medical care and finance, directly clustering datasets may bring privacy risks. Therefore, future research will focus on exploring more efficient weighting techniques to further learn the importance of views and attributes in clustering. Additionally, we aim to further investigate privacy-preserving aspects of multi-view fuzzy clustering to make these algorithms applicable to sensitive domains.

CRediT authorship contribution statement

Zhe Liu: Writing – original draft, Validation, Software, Methodology, Investigation, Conceptualization. **Haoye Qiu:** Writing – original draft, Validation, Software. **Muhammet Deveci:** Writing – review & editing, Supervision, Resources, Methodology. **Sukumar Letchmunan:** Writing – review & editing, Investigation. **Luis Martínez:** Writing – review & editing, Supervision, Methodology.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The data that has been used is confidential.

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