



Full length article

Dynamic clustering-based consensus model for large-scale group decision-making considering overlapping communities

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ARTICLE INFO

Keywords:

Large-scale group decision making (LSGDM)
 Consensus reaching model
 Social network
 Community detection

ABSTRACT

Consensus-reaching strategy is crucial in large-scale group decision-making (LSGDM) as it serves as an effective approach to reducing group conflicts. Meanwhile, the common social network relationships in large groups can affect information exchange, thereby influencing the consensus-reaching process (CRP) and decision results. Therefore, how to leverage social network information in LSGDM to obtain an agreed solution has received widespread attention. However, most existing research assumes relative independence between communities in the dimension reduction process of LSGDM and neglects the possibility of different overlaps between them. Moreover, the impact of overlapping communities on CRP has not been adequately explored. Besides, the dynamic variations in clusters and their weights caused by evaluation updates need to be further studied. To address these issues, this paper proposes a dynamic clustering-based consensus-reaching method for LSGDM considering the impact of overlapping communities. First, the LINE-based label propagation algorithm is designed to cluster decision makers (DMs) and detect overlapping communities with social network information. An overlapping community-driven feedback mechanism is then developed to enhance group consensus by utilizing the bridging role of overlapping DMs. During CRP, clusters and their weights are dynamically updated with trust evolution due to the evaluation iteration. Finally, a case study using the Film Trust dataset is conducted to verify the effectiveness of the proposed method. Simulation experiments and comparative analysis demonstrate the capability of our method in modeling practical scenarios and addressing LSGDM problems under social network contexts.

1. Introduction

Group decision-making (GDM) is ubiquitous in our daily lives. It involves a group of decision-makers (DMs) or stakeholders collaboratively assessing a set of alternatives to identify the optimal choice [1]. With the ongoing progression of information technology and social media, increasingly complex decision-making problems necessitate the participation of a greater number of DMs with diverse knowledge backgrounds. Therefore, large-scale group decision-making (LSGDM) methods have become research hotspots. LSGDM involves managing more DMs, addressing more complex decision environments, and coping with substantial divergence of opinions. These methods are extensively applied in various domains, such as emergency management [2], supplier selection [3], and smart city evaluation [4].

To better manage DMs' evaluations in large-scale contexts, clustering algorithms are widely used as dimension-reduction techniques to improve the efficiency of LSGDM. Most existing studies utilize evaluation similarity degree-based clustering methods, such as K-means [5]

and fuzzy C-means [6], to partition the large group of DMs into clusters and treat each cluster as the basic unit in the subsequent process. However, such methods fail to consider the social relationships among DMs. With the popularity of social platforms, communication among DMs has become more convenient, fostering social network relationships [7]. Community detection is a proven effective approach for clustering DMs within social network contexts. However, most LSGDM methods that use community detection algorithms to cluster DMs neglect the potential overlaps between communities [8–10]. In practice, communities are not always isolated, and a DM may belong to multiple communities simultaneously. Thus, it is necessary to consider different types of potential overlaps among communities during the clustering process and investigate their subsequent impact on LSGDM.

Consensus building is essential in LSGDM to coordinate individuals' opinions and mitigate discrepancies among them [11]. Generally, divergence degrees are employed to quantify the group consensus

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degree (GCD). If the obtained GCD does not reach a specific threshold, the feedback mechanism will be activated to enhance the group consensus. The existing consensus-reaching strategies are mainly based on two approaches, i.e., the identification-direction mechanism (linear combination approach) and the optimization model-based methods. In the identification-direction mechanism, the evaluations that contribute less to the group consensus are first identified, and subsequently, the direction rule is formed to navigate the adjustments of the identified evaluations. For the optimization-based methods, typical ones include the minimum cost model [12], minimum adjustment model [13], and maximum consensus improvement model [14]. Consensus-reaching essentially involves dynamic updates in alternative evaluations. However, in social network environments, DMs tend to interact and exchange information with trusted individuals, forming stronger bonds with like-minded peers. Hence, evaluation iterations in CRP can cause dynamic changes in the social network topology, leading to variations in the clustering and weight assignments at individual and cluster levels. Therefore, different aspects of dynamics in CRP that consider overlapping communities need to be further explored.

Motivated by these challenges, this study aims to propose a dynamic clustering-based consensus-reaching method for LSGDM that considers overlapping communities and their impact on CRP. The main contributions of this paper are summarized below.

- (1) The LINE-based label propagation (LNLP) algorithm is designed as an overlapping community detection method to cluster DMs. First, the LINE method is utilized to obtain low-dimensional vector representations of DMs based on the local and global topology of the social network. Second, the node similarity is determined by vector dot product operations. The concept of selection degree is then introduced to enhance traditional label propagation. This involves considering dynamic label information and static network information to select the neighbor DM with the highest selection degree for label propagation. The proposed algorithm improves the stability of overlapping community detection by better preserving network information and overcoming the limitation of randomly selecting neighboring nodes in traditional label propagation.
- (2) The overlapping community-driven feedback mechanism is formulated to improve group consensus. Due to the closer trust relationships among DMs within the same community, those belonging to different communities simultaneously can serve as the bridge to mitigate evaluation conflicts among subgroups [15]. If the GCD falls below a certain threshold, clusters that contribute minimally to group consensus are first identified. Subsequently, cluster-level evaluations are adjusted using evaluations from overlapping DMs to guide the opinion modification. Distinct consensus-reaching strategies are proposed for different typical overlapping scenarios. In this way, intricate overlapping trust relationships can be exploited to facilitate automatic mutual interactions among subgroups to reach group consensus.
- (3) A dynamic clustering-based LSGDM framework that considers overlapping communities is constructed. We elaborate on the evolution of trust caused by evaluation modification in the CRP. On this basis, the dynamic updating on overlapping community detection results and weight allocations influenced by trust evolution is further analyzed. The case analysis using the Film Trust dataset on the film selection problem illustrates the effectiveness and superiority of the proposed method in addressing LSGDM problems in social network environments.

The rest of this paper is organized as follows. Section 2 presents the problem description of LSGDM and reviews the existing literature on clustering and consensus-reaching methods in LSGDM. Section 3 elaborates on the dynamic clustering-based consensus-reaching method with overlapping communities. Section 4 verifies the effectiveness of the proposed approach with a LSGDM problem based on the Film Trust

dataset. Discussions are given in Section 5 to demonstrate the practicality and advantages of the proposed method. Section 6 presents the conclusion and future research directions.

2. Literature review

Since the dimension reduction process and consensus-reaching strategies are pivotal aspects of LSGDM research, this section briefly reviews the relevant studies and analyzes their limitations.

2.1. Clustering methods in LSGDM

In LSGDM, dividing large groups into smaller and more manageable clusters (subgroups) is a crucial step to alleviate dimensional complexity and improve decision efficiency. Based on different perspectives of clustering DMs, current clustering methods for LSGDM can be broadly categorized into two types: Similarity-based clustering and social-network-based clustering.

In similarity-based clustering methods, many studies have extended the traditional K-means algorithm to cluster large groups of DMs, such as the extended comparative linguistic expressions with symbolic translation (ELICIT)-based K-means method [5] and the modification cost-based K-means method [16]. Additionally, fuzzy C-means clustering was employed to classify DMs, such as the cohesion-driving fuzzy C-means clustering method [17]. Compared to K-means methods, fuzzy C-means demonstrate greater robustness against noise and outliers during clustering. However, both K-means and fuzzy C-means methods require the pre-specification of the number of clusters (i.e., K), which can be challenging to determine. The initialization issue may also lead these algorithms to local optima. Moreover, these similarity-based methods do not account for the social relationships among DMs in a social network environment and their impact on clustering. Social network structures can significantly influence the clustering process by introducing factors such as trust, influence, and communication patterns among DMs. Ignoring these relationships can result in suboptimal clustering outcomes that do not reflect the true dynamics within the group.

To address this limitation, social network-based clustering methods have been employed to partition the large group of DMs based on their social relationships. For example, a trust-based Cop-K means clustering algorithm was proposed to overcome the deficiencies of traditional similarity-based clustering methods, which often result in low-trust decision-maker groupings [18]. Additionally, community detection methods were explored to identify subgroups within the network, providing meaningful subgroupings that reflect real-world interactions [19]. For instance, the Louvain algorithm has been widely used to cluster DMs based on their social relationships [9,20]. Considering structural holes and their impact on the CRP, the Leiden algorithm was utilized to segment DMs [21]. However, these methods are not applicable for overlapping community detection. In practical scenarios, a DM may simultaneously belong to multiple communities. Therefore, the Lancichinetti–Fortunato method (LFM) [22] and the community overlap propagation algorithm (COPRA) [23] were explored to reveal the overlapping trust relationships among DMs and classify them into subgroups. However, the LFM randomly selects a node not belonging to any community as the initial seed and employs a fitness function to detect overlapping communities. Although this approach is straightforward and applicable to large-scale complex networks, this optimization-based overlapping community detection method may result in situations where some nodes do not belong to any communities. The COPRA randomly selects neighbor nodes during the label propagation process, which may result in inconsistencies in the overlapping community detection outcome. Therefore, novel overlapping community detection algorithms need to be proposed to cluster DMs more reasonably and effectively in LSGDM problems considering trust relationships.

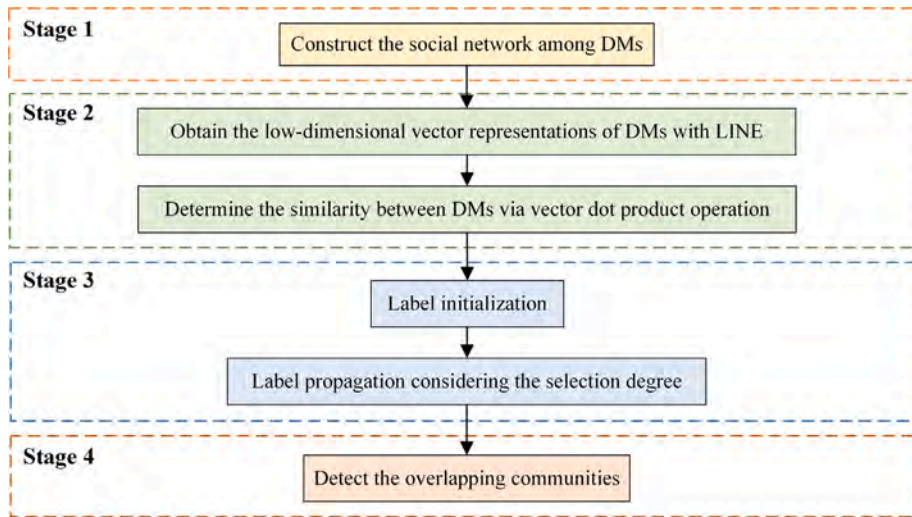


Fig. 1. The flowchart of the LINE-based label propagation algorithm for overlapping community detection.

2.2. Consensus-reaching methods for LSGDM

Due to the diverse backgrounds and professional expertise of stakeholders and experts involved in LSGDM, their opinions on alternatives may vary substantially. Therefore, enhancing group consensus for effective decision implementation has become a research hotspot. The existing consensus-reaching strategies fall into two main perspectives: one is based on optimization models, and the other is based on the identification-direction mechanism.

For optimization-based consensus-reaching methods, different optimization models are employed to enhance the efficiency of the CRP. The most widely used are the minimum adjustment/cost model and its extensions, such as the minimum adjustment model considering limited compromise behavior [24], the two-stage Gini coefficient-based minimum adjustment model [25], the personalized individual semantics-based minimum adjustment model [26], and the polymerization degree-based adjustable minimum cost model [27]. However, these consensus models neglect the impact of social relationships among DMs on the CRP. To overcome this shortcoming, some scholars have proposed models such as the social network-driven bi-level minimum cost consensus model from the perspectives of the Stackelberg game structure [21]. Although optimization-based methods are efficient, the adjusted evaluations obtained are directly output by the optimal model without considering the tendency of DMs during opinion updates, and thus cannot better simulate the opinion iteration process in actual consensus-reaching.

Compared to optimization-based methods, the identification-direction mechanism can guide the CRP of identified evaluations based on specific rules. In existing social network-based modification mechanisms, trust relationships among DMs are often used as the principle to guide opinion updates. For example, the social network DeGroot model is utilized to achieve opinion dynamics [3]. In [28], the modification suggestion for each identified DM is generated by referencing the evaluations of the individuals they trust at a specific ratio. However, such methods fail to consider the impact of potential overlaps between subgroups on the CRP and decision outcomes. Although two studies have considered overlapping communities when clustering a large group of DMs [22,23], they did not comprehensively analyze different types of overlaps and their corresponding consensus-reaching strategies. In [23], the global collective evaluation and community preferences are used as recommendation references for the identified DMs to adjust their opinions. While this idea is easy to understand and widely applied, it fails to leverage the bridging role of overlapping DMs to mitigate opinion conflicts in the CRP. Besides, opinion

modifications are performed on the individual level, rather than on the cluster (subgroup) level. This can increase the complexity of CRP and reduce the efficiency of evaluation iteration, particularly when handling a larger scale of DMs. In [22], evaluations of the overlapping DMs are explored to guide opinion modification within the associated communities. However, this approach only analyzed one overlapping pattern, neglecting other potential overlapping scenarios. In practice, consensus-reaching strategies should be tailored to different overlapping patterns. Additionally, the evaluations of overlapping DMs were not incorporated when calculating the group evaluation and the GCD, leading to information loss and biased results. Moreover, trust evolution and the subsequent dynamic clustering in CRP, considering different types of overlapping communities, have not been adequately analyzed. Therefore, consensus-reaching strategies that can fully leverage the bridging role of overlapping DMs, and simultaneously consider different aspects of dynamics to facilitate CRP require deeper exploration.

3. Dynamic clustering-based consensus model for LSGDM with overlapping communities

This section elaborates on a dynamic clustering-based consensus-reaching method that considers overlapping communities for LSGDM. First, Section 3.1 presents the problem description of LSGDM. Section 3.2 introduces the LINE-based label propagation algorithm to detect overlapping communities among the large group of DMs. In Section 3.3, the overlapping community-driven feedback mechanism for CRP is developed in detail. Lastly, the framework and procedure of the proposed method are presented in Section 3.4.

3.1. LSGDM problem

For an LSGDM problem within social network settings, assume that r ($r \geq 20$) DMs $E = \{e_k | k = 1, 2, \dots, r\}$ are invited to evaluate m ($m \geq 2$) alternatives $A = \{a_i | i = 1, 2, \dots, m\}$ regarding n criteria $C = \{c_j | j = 1, 2, \dots, n\}$. The weight of criteria can be denoted as w_j ($j = 1, 2, \dots, n$), where $w_j \in (0, 1)$ and $\sum_{j=1}^n w_j = 1$. The evaluation that DM e_k provides towards alternative a_i with respect to criterion c_j can be expressed as x_{ij}^k . A higher expected value of x_{ij}^k indicates the better performance of alternative a_i under criterion c_j . According to the characteristics of different LSGDM problems and data, qualitative and quantitative evaluation expression structures (such as crisp numbers, fuzzy numbers, and linguistic variables) are employed to describe DMs' judgments. The decision information matrix of DM e_k is shown in Eq. (1).

$$X^k = \begin{bmatrix} x_{11}^k \\ x_{21}^k \\ \vdots \\ x_{m1}^k \end{bmatrix}_{m \times n} = \begin{matrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{matrix} \begin{bmatrix} c_1 & c_2 & \cdots & c_n \\ x_{11}^k & x_{12}^k & \cdots & x_{1n}^k \\ x_{21}^k & x_{22}^k & \cdots & x_{2n}^k \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1}^k & x_{m2}^k & \cdots & x_{mn}^k \end{bmatrix} \quad (k = 1, 2, \dots, r) \quad (1)$$

The directed graph $G = (E, L)$ is utilized to depict the social network among r DMs, where E denotes the group of DMs and L represents the set of trust relations between individuals. $G = (E, L)$ can also be expressed with the adjacency matrix $T = [t_{kl}]_{r \times r}$, where t_{kl} ($0 \leq t_{kl} \leq 1$) indicates the degree of trust that e_k has in e_l . The group of DMs can be partitioned into z overlapping communities $OC = \{OC_v | v = 1, 2, \dots, z\}$. This study aims to improve the group consensus and identify the optimal alternative for LSGDM problems according to the decision information and trust relationships among DMs.

3.2. Overlapping community detection with the LINE-based label propagation algorithm

Selecting a suitable clustering method to coordinate DMs can greatly enhance the efficiency of CRP in LSGDM. Since individuals tend to form subgroups with people they are closely connected, employing community detection algorithms to cluster DMs based on their trust relationships is more reasonable and acceptable in social network contexts. However, most existing LSGDM methods that utilize community detection algorithms for clustering neglect the potential overlaps between communities. The few methods that consider overlapping communities also have certain limitations, as discussed in Section 2.1. Therefore, this subsection proposes the LINE-based label propagation algorithm to enhance the effective partitioning of the large group of DMs considering overlapping scenarios. The flowchart of the proposed overlapping community detection method can be summarized in Fig. 1.

3.2.1. Determine the similarity between DMs in embedding space

Given the social network among DMs, the overlapping community detection problem can be defined as follows.

Definition 1 (Overlapping Community Detection). Let $G = (E, L)$ denotes a social network, where E represents the large group of DMs (nodes) and L denotes the set of trust relationships (edges). $T = [t_{kl}]_{r \times r}$ represents the adjacency matrix of G , which can be either a binary matrix or a weighted matrix. The goal of overlapping community detection is to cluster all DMs into z communities, which is shown as follows.

$$(G, T) \xrightarrow{F} OC \quad (2)$$

where F denotes the overlapping community detection method. $OC = \{OC_1, OC_2, \dots, OC_z\}$ represents the set of detected communities that satisfies $\mathfrak{R}(OC_1) \cup \mathfrak{R}(OC_2) \cup \dots \cup \mathfrak{R}(OC_z) = E$ and $\exists \mathfrak{R}(OC_i) \cap \mathfrak{R}(OC_j) \neq \emptyset$, where $\mathfrak{R}(OC_i)$ denotes the individuals in OC_i .

First, we utilize the network embedding technique to determine low-dimensional vector representations of DMs in the social network. Network embedding is a technique that transforms network data into a low-dimensional vector space, preserving the structural information of the network. This transformation allows for applying various algorithms to network data, making it easier to analyze and interpret complex network structures. The embedding space refers to this low-dimensional representation where each node in the network is mapped to a point in the vector space, capturing both local and global topological properties. This approach has several advantages, for example, it helps reduce data dimensions, thereby better capturing similarities and relationships between nodes. Besides, representing nodes with low-dimensional vectors with network embedding methods facilitates network computation tasks, such as node classification and clustering.

Multiple network embedding techniques were proposed, including representative ones such as DeepWalk, SDNE, and LINE [29]. However, the DeepWalk method only applies to unweighted networks, and SDNE involves many parameters that must be configured. Therefore, the LINE method, which can better handle weighted networks and capture the first and second-order proximities in the network is employed to obtain low-dimensional vector representations of DMs in this study.

LINE is an effective network embedding technique that optimizes a well-designed objective function considering local and global network structure [29]. It mainly comprises two essential aspects: (i) Preserve the first and second-order proximities. The first-order proximity between DMs reflects the local structure of the social network. As a complement, the global structure of the network is captured by the second-order proximity, which is obtained based on the shared neighborhood of the DMs rather than the strength of edges. (ii) Optimization via edge sampling. The objective function that preserves local and global network structure is optimized to derive the vector representations of DMs. The edge sampling algorithm is employed to overcome the deficiency of stochastic gradient descent.

Subsequently, based on the vector representations of DMs obtained with the LINE method, their similarity in the embedding space can be calculated through dot product operation, as defined in Eq. (3).

Definition 2 ([29] Similarity between Nodes). For two DMs e_k and e_l in the social network $G = (E, L)$, the similarity between them in the embedding space can be determined by the dot production of their low dimensional vector representations, shown as follows.

$$Sim(e_k, e_l) = \eta(e_k) \cdot \eta(e_l) \quad (3)$$

where $\eta(e_k)$ and $\eta(e_l)$ denote the vector representations of e_k and e_l obtained with the LINE method, respectively.

3.2.2. The LINE-based label propagation algorithm

The label propagation algorithm (LPA) is a widely used community detection method because of its simplicity and flexibility [22]. It operates by iteratively updating nodes' labels according to their neighbors' labels. However, traditional LPA randomly selects neighbor nodes in some cases during label propagation, which reduces the stability of the clustering result. Therefore, we propose the LINE-based label propagation (LNLP) algorithm that considers both static network information and dynamic label information to implement the label propagation. The adjacency matrix that represents the relationships of nodes (DMs in this study) in the LNLP algorithm can be weighted and is not restricted to a binary matrix. It is worth noting that the LINE method can better handle weighted networks and capture both first-order and second-order proximities. The weights are embedded in the adjacency matrix, representing the strength of the relationships among the nodes. These weights are part of the input information and are used by the LINE method to compute the low-dimensional vectors for the DMs.

The LNLP starts with label initialization, where a unique label is allocated to each node. A label typically contains two components, i.e., the community identifier and the belonging coefficient of the label. The belonging coefficient can be used to construct the label similarity index to reflect the dynamic label information in the network, which is given as follows.

Definition 3 ([30] Belonging Coefficient). For any DM e_k in the social network, the strength of e_k belonging to the community OC can be reflected by the belonging coefficient of the label.

$$\xi_{(i)}(e_k, OC) = \frac{\sum_{e_l \in N(e_k)} \xi_{(i-1)}(e_l, OC)}{|N(e_k)|} \quad (4)$$

where $N(e_k)$ denotes the neighbors of e_k and i represents the i th iteration.

Definition 4 ([30] Label Similarity). Let e_k and e_l represent any two DMs in the social network with $(e_k, e_l) \in L$, the label similarity between them $\rho(e_k, e_l)$ can be defined as the sum of the products of belonging coefficients when the community identifiers in e_k and e_l 's labels are identical.

$$\rho(e_k, e_l) = \sum_{(OC_{e_k} \in S_{e_k}, OC_{e_l} \in S_{e_l}) \wedge OC_{e_k} = OC_{e_l}} \xi(e_k, OC_{e_k}) \times \xi(e_l, OC_{e_l}) \quad (5)$$

where $\xi(\cdot)$ denotes the belonging coefficient given in Definition 3. S_{e_k} and S_{e_l} denote the label sets of e_k and e_l , respectively.

In addition, the local clustering coefficient [31] and the local structural similarity [32] of DMs can be calculated to preserve the network static information, which is given as follows.

Definition 5 ([31] Local Clustering Coefficient). The local clustering coefficient indicates the proportion of edges established among the neighbors of e_k to the potential edges of nodes in $N(e_k)$, which can be computed as:

$$\chi(e_k) = \frac{\Phi(e_k)}{|N(e_k)| \times |N(e_k) - 1|} \quad (6)$$

where $N(e_k)$ and $\Phi(e_k)$ denote the neighbors of e_k and the set of edges connecting the neighbors of e_k , respectively.

Definition 6 ([32] Local Structural Similarity). Let e_k and e_l denote any two DMs in the social network, where $(e_k, e_l) \in L$. The local structural similarity between e_k and e_l can be determined as follows.

$$\tau(e_k, e_l) = \frac{|\Theta(e_k) \cap \Theta(e_l)|}{|\Theta(e_k) \cup \Theta(e_l)|} \quad (7)$$

where $\Theta(e_k)$ represents the set of e_k and its neighbors.

By considering both dynamic label information and static network structure information, we introduce the selection degree of e_k to e_l for improving the traditional label propagation process. In this way, the label of DMs can be updated by selecting the label from the neighbor with the maximum selection degree.

Definition 7 (Selection Degree). The selection degree of e_k to e_l reflects the probability that the neighbor e_l is chosen to exchange its label with e_k , which can be calculated as follows.

$$SD(e_k, e_l) = \frac{\exp(\widehat{Sim}(e_k, e_l) + \rho(e_k, e_l) + \tau(e_k, e_l) + \chi(e_k))}{\sum_{\bar{e}_l \in N(e_k)} \exp(\widehat{Sim}(e_k, \bar{e}_l) + \rho(e_k, \bar{e}_l) + \tau(e_k, \bar{e}_l) + \chi(e_k))} \quad (8)$$

where $\widehat{Sim}(e_k, e_l)$ denotes the standardized similarity between e_k and e_l in the embedding space, with $\widehat{Sim}(e_k, e_l) = \frac{Sim(e_k, e_l) - \min_{1 \leq k \leq r} \{Sim(e_k, e_l)\}}{\max_{1 \leq k \leq r} \{Sim(e_k, e_l)\} - \min_{1 \leq k \leq r} \{Sim(e_k, e_l)\}}$.

Based on the proposed LINE-based label propagation algorithm, each DM iteratively updates its label with neighbors' labels based on the selection degree. The iteration process stops until the type and quantity of labels remain unchanged before and after iteration. DMs with the same label can be classified to the same community.

3.3. Dynamic clustering-based consensus-reaching model with overlapping communities

After clustering the large group of DMs into communities, the consensus-reaching strategy is proposed to enhance the group consensus for an agreeable decision outcome. First, the consensus measurements on the evaluation, cluster, and group levels are presented. Then, the overlapping community-driven feedback mechanism is developed to construct the dynamic clustering-based consensus-reaching model to enhance group consensus.

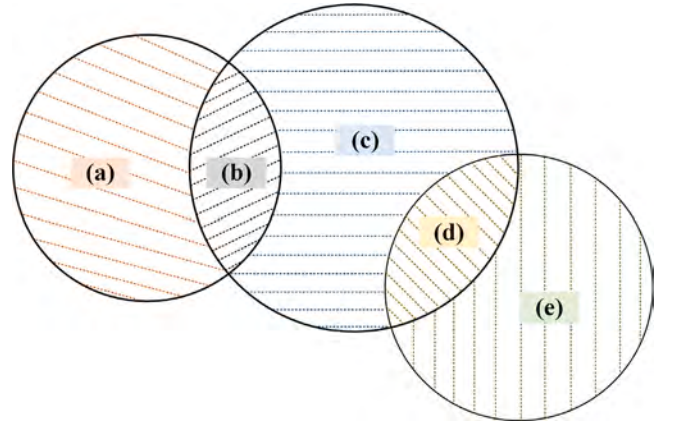


Fig. 2. The schematic illustration of the cluster in this study.

3.3.1. Consensus measurements

Consensus measurements are utilized to quantify the agreement level inside the group of DMs. The group consensus can be measured in two manners: one is by the discrepancy between the individual and collective evaluations, and the other is by the deviation among individual assessments. Here, we adopt the second perspective.

In the context of LSGDM in this study, clusters (instead of DMs) are regarded as basic processing units after dimension reduction to enhance the efficiency of CRP. Fig. 2 is presented to better illustrate the scope of the cluster in this study considering overlapping communities. In Fig. 2, three communities with two overlaps (i.e., (b) and (d)) are detected; we regard (a), (b), (c), (d), and (e) as clusters. In other words, there are two types of clusters in this study: the first type of cluster contains DMs that belong to only one community, such as (a), (c), and (e); the second type of cluster consists of DMs that belong to more than one community simultaneously, such as (b) and (d). It is worth noting that no overlap exists between any two clusters.

According to individual evaluations of alternatives, the evaluation of clusters can be aggregated as follows.

Definition 8 (Cluster Evaluation). Let $CL = \{Cl_p | p = 1, 2, \dots, f\}$ denotes the set of clusters after overlapping community detection, the evaluation of cluster Cl_p on alternative a_i regarding criterion c_j can be determined as follows.

$$x_{ij}^{Cl_p} = \sum_{e_k \in Cl_p} \frac{HC(e_k)}{\sum_{e_k \in Cl_p} HC(e_k)} x_{ij}^k \quad (9)$$

where x_{ij}^k represents e_k 's evaluation on a_i regarding c_j . $HC(e_k)$ represents the hybrid centrality of e_k , reflecting DM's importance from both local and global network topological structure [7]. For brevity, the calculation for hybrid centrality is not elaborated here; please refer to [7] for details.

Based on the above analysis, the consensus measures at the evaluation, cluster, and group levels are defined as follows.

Definition 9 (Consensus at the Evaluation Level [13]). Let $x_{ij}^{Cl_p}$ and $x_{ij}^{Cl_g}$ denote the evaluations of clusters Cl_p and Cl_g on alternative a_i regarding criterion c_j , respectively. The consensus degree at the evaluation level can be calculated as:

$$CD_{ij}^{Cl_p} = \frac{1}{f-1} \sum_{g=1, g \neq p}^f \left(1 - d(x_{ij}^{Cl_p}, x_{ij}^{Cl_g})\right) \quad (10)$$

where $d(x_{ij}^{Cl_p}, x_{ij}^{Cl_g}) = |x_{ij}^{Cl_p} - x_{ij}^{Cl_g}|$ represents the distance between $x_{ij}^{Cl_p}$ and $x_{ij}^{Cl_g}$ in this study, though other distance measures could be used.

Definition 10 (Consensus at the Cluster Level [13]). The consensus degree of cluster Cl_p ($p = 1, 2, \dots, f$) on the set of alternatives with respect to the set of criteria can be determined as:

$$CD^{Cl_p} = \frac{1}{mn} \sum_{i=1}^m \sum_{j=1}^n CD_{ij}^{Cl_p} \quad (11)$$

where $0 < CD^{Cl_p} < 1$.

Definition 11 (Consensus at the Group Level [13]). Based on the consensus degree of each cluster, the group consensus degree can be obtained as follows.

$$GCD = \frac{1}{f} \sum_{p=1}^f CD^{Cl_p} \quad (12)$$

where $0 < GCD < 1$. A higher GCD suggests a stronger agreement among DMs in the large group.

The calculated GCD needs to be compared with a preset consensus threshold γ to determine whether the current group consensus level is satisfactory. If $GCD \geq \gamma$, the alternative selection process will be carried out based on the present group evaluation; otherwise, the overlapping community-driven feedback mechanism will be implemented to enhance group consensus. The setting of the threshold value γ depends on the specific circumstances. If the decision committee requires a higher level of consistency in the decision outcomes, then the value of γ will be larger, resulting in greater modifications to initial evaluations and potential costs incurred.

3.3.2. The overlapping community-driven feedback mechanism for consensus-reaching process

First, the non-overlapped clusters that contribute less to a satisfactory GCD are identified as follows.

(1) The non-overlapped clusters with insufficient consensus degrees are identified as:

$$CPS = \{p \mid CD^{Cl_p} < \gamma\} \quad (13)$$

(2) For clusters in the set of CPS, those evaluations with consensus degrees below the preset threshold are selected as:

$$CVS = \{(i, j, p) \mid p \in CPS \wedge CD_{ij}^{Cl_p} < \gamma\} \quad (14)$$

Let x_{ij}^k ($e_k \in Cl_p$) represent the evaluations of DMs that need to be adjusted in the identified clusters Cl_p ($Cl_p \in CVS$), the direction rule is subsequently elaborated to guide the evaluation modification in the CRP under social network contexts.

Since DMs in the same community share closer trust relationships, the modification suggestions from DMs within the same community are generally more acceptable than group opinions. According to the concept of brokerage in social network theory, brokers are individuals who connect otherwise disconnected communities, facilitating the flow of information across boundaries [15]. These brokers play a crucial role in integrating diverse viewpoints and resolving conflicts by leveraging their unique positions at the intersection of different social circles. Their ability to navigate and integrate diverse perspectives enhances the overall consensus-building process within overlapping communities. Therefore, the overlapping DMs that simultaneously belong to different communities can serve as a bridge to guide consensus-reaching interactions among different clusters in the large group. In this study, the opinions of overlapping DMs are aggregated and utilized as recommendations to construct the overlapping community-driven feedback mechanism to improve group consensus. Several typical scenarios are described as follows, and the consensus-reaching strategy under other situations can be extended accordingly.

(1) The identified cluster is situated within a community that overlaps with only one community, as shown in Fig. 3.

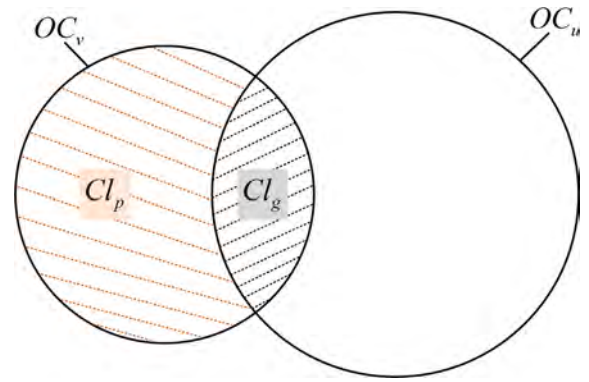


Fig. 3. The community that the identified cluster Cl_p belongs to overlaps with only one other community OC_u .

In this case, the evaluation of cluster Cl_g can be utilized as the modification reference for DMs e_k ($e_k \in Cl_p$) in the identified cluster Cl_p to adjust their opinions. The modified evaluation of x_{ij}^k in cluster Cl_p can be obtained as follows.

$$\overline{x_{ij}^k} = \phi x_{ij}^k + (1 - \phi) x_{ij}^{Cl_g} \quad (15)$$

where x_{ij}^k and $x_{ij}^{Cl_g}$ represent the evaluation of e_k and Cl_g on a_i regarding c_j , respectively. The adjustment coefficient ϕ ($0 < \phi < 1$) reflects the extent to which the original evaluation is retained, and $(1 - \phi)$ reflects the degree of reference to Cl_g . In this study, $\phi = CD^{Cl_p}$. This indicates that the lower the consensus level of Cl_p , the greater its reference to Cl_g in evaluation modification.

(2) The community the identified cluster belongs to overlaps with the other two communities. One potential scenario is illustrated in Fig. 4, and the proposed evaluation modification strategy can be extended to other similar cases.

In this scenario, the evaluations of clusters Cl_g and Cl_f are used to guide the opinion modifications of DMs in the identified cluster Cl_p . To integrate the evaluations of Cl_g and Cl_f , the weight determination method for clusters is defined first.

Definition 12 (The Weight of Clusters). The weight of cluster w_{Cl_p} ($p = 1, 2, \dots, f$) can be determined from the following three aspects: (i) The size of the cluster SI^{Cl_p} , which can be obtained with the number of DMs in the cluster $Num(Cl_p)$. (ii) Intra-cluster cohesion CO^{Cl_p} , which represents the consistency degree among the opinions of DMs within the cluster. (iii) Inter-cluster consensus CD^{Cl_p} , which reflects the agreement level of the cluster evaluation in the large group. The weight of the cluster is directly proportional to these three indicators, which can be calculated as follows.

$$w_{Cl_p} = \alpha \cdot SI^{Cl_p} + \beta \cdot CO^{Cl_p} + \mu \cdot CD^{Cl_p} \quad (16)$$

where $0 < w_{Cl_p} < 1$ and $\sum_{p=1}^f w_{Cl_p} = 1$. $SI^{Cl_p} = \frac{Num(Cl_p)}{r}$, where r denotes the total number of DMs in the large group. $CO^{Cl_p} = \frac{1}{mn(Num(Cl_p)-1)} \sum_{i=1}^m \sum_{j=1}^n \sum_{e_k, e_l \in Cl_p, k \neq l} (1 - d(x_{ij}^k, x_{ij}^l))$, where x_{ij}^k and x_{ij}^l denote the evaluations of e_k and e_l in Cl_p , respectively. CD^{Cl_p} can be determined by Eq. (11). If the variability within a certain indicator is larger, its impact on the cluster weights should be greater. Hence, this indicator should be assigned a larger weight. Based on this principle of maximum deviation, the adjustment parameters can be obtained as: $\alpha = \frac{\sum_{p=1}^f (\max(SI^{Cl_p}) - SI^{Cl_p})}{Sum(dis)}$, $\beta = \frac{\sum_{p=1}^f (\max(CO^{Cl_p}) - CO^{Cl_p})}{Sum(dis)}$, $\mu = \frac{\sum_{p=1}^f (\max(CD^{Cl_p}) - CD^{Cl_p})}{Sum(dis)}$, where $Sum(dis) = \sum_{p=1}^f (\max(SI^{Cl_p}) - SI^{Cl_p}) + \sum_{p=1}^f (\max(CO^{Cl_p}) - CO^{Cl_p}) + \sum_{p=1}^f (\max(CD^{Cl_p}) - CD^{Cl_p})$.

Based on the weight and evaluations of the overlapping clusters, the recommendation advice for cluster Cl_p can be constructed as follows.

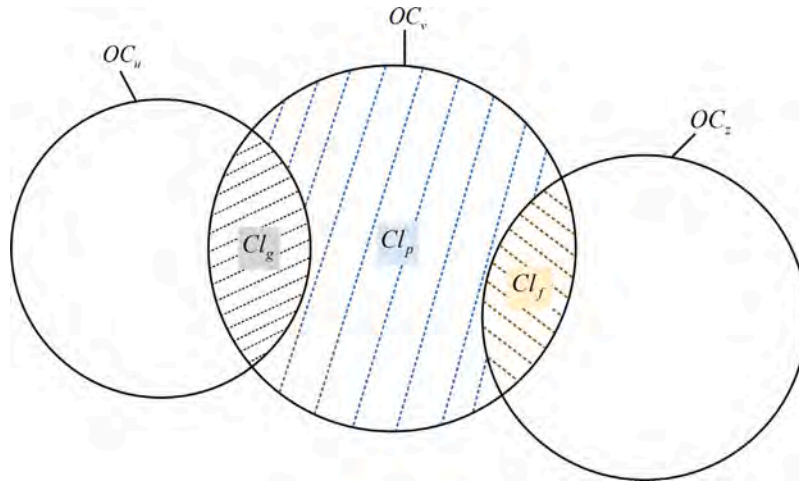


Fig. 4. The community that the identified cluster Cl_p belongs to overlaps with the other two communities OC_u and OC_z .

$$\overline{x_{ij}^{Ref(Cl_p)}} = \frac{w_{Cl_g}}{w_{Cl_g} + w_{Cl_f}} x_{ij}^{Cl_g} + \frac{w_{Cl_f}}{w_{Cl_g} + w_{Cl_f}} x_{ij}^{Cl_f} \quad (17)$$

where $x_{ij}^{Cl_g}$ and $x_{ij}^{Cl_f}$ represent the evaluations of Cl_g and Cl_f , respectively. w_{Cl_g} and w_{Cl_f} represent the weights of $x_{ij}^{Cl_g}$ and $x_{ij}^{Cl_f}$, respectively.

With the modification reference, the modified evaluation of DMs in Cl_p can be determined as:

$$\overline{x_{ij}^k} = \phi x_{ij}^k + (1 - \phi) \overline{x_{ij}^{Ref(Cl_p)}} \quad (18)$$

where x_{ij}^k denotes the original evaluation of $e_k (e_k \in Cl_p)$ on a_i regarding c_j . $\overline{x_{ij}^{Ref(Cl_p)}}$ represents the modification reference for x_{ij}^k . $\phi (0 < \phi < 1)$ is the adjustment coefficient with $\phi = CD^{Cl_p}$. $(1 - \phi)$ reflects the reference degree to the recommendation advice $\overline{x_{ij}^{Ref(Cl_p)}}$. The rationale for using the consensus degree as a feedback coefficient is that a lower consensus indicates a lower level of consensus among DMs. Thus, greater adjustments are required for such evaluations to enhance the overall group consensus. This approach is intuitive and avoids the subjectivity of directly assigning values during the CRP.

(3) The community to which the identified cluster belongs overlaps with the other three communities. Fig. 5 depicts one possible scenario, and the modification strategy can be extrapolated for similar situations accordingly.

Under this circumstance, evaluations from $Cl_g, Cl_q, Cl_h,$ and Cl_f are all utilized to form the modification reference for evaluations in Cl_p . Based on the assessments of $Cl_g, Cl_q, Cl_h, Cl_f,$ and their corresponding weights, the modification reference for Cl_p can be formulated as:

$$\overline{x_{ij}^{Ref(Cl_p)}} = \frac{w_{Cl_g} x_{ij}^{Cl_g} + w_{Cl_q} x_{ij}^{Cl_q} + w_{Cl_h} x_{ij}^{Cl_h} + w_{Cl_f} x_{ij}^{Cl_f}}{w_{Cl_g} + w_{Cl_q} + w_{Cl_h} + w_{Cl_f}} \quad (19)$$

where $x_{ij}^{Cl_g}, x_{ij}^{Cl_q}, x_{ij}^{Cl_h},$ and $x_{ij}^{Cl_f}$ represent the evaluation of Cl_g, Cl_q, Cl_h and Cl_f , respectively. $w_{Cl_g}, w_{Cl_q}, w_{Cl_h},$ and w_{Cl_f} denote the weights of the respective overlapping clusters.

Similarly, the modified evaluations of DMs in cluster Cl_p can be calculated as the linear combination of the original assessment and the recommendation advice:

$$\overline{x_{ij}^k} = \phi x_{ij}^k + (1 - \phi) \overline{x_{ij}^{Ref(Cl_p)}} \quad (20)$$

where x_{ij}^k and $\overline{x_{ij}^{Ref(Cl_p)}}$ represent the original assessment of e_k and its modification reference, respectively. $\phi (0 < \phi < 1)$ is the adjustment coefficient with $\phi = CD^{Cl_p}$. $(1 - \phi)$ indicates the degree to which the recommendation advice $\overline{x_{ij}^{Ref(Cl_p)}}$ is referenced. Using the consensus

degree as the feedback coefficient is based on the idea that evaluations with lower consensus require more significant adjustments to improve the overall group consensus.

(4) In addition to the aforementioned cases, it is also possible that the identified cluster does not overlap with any other community, i.e., the identified cluster itself is a community. One possible scenario is shown in Fig. 6.

In this situation, the evaluations in the identified cluster (community) are adjusted towards the evaluations of the most important (highest-weighted) cluster in the social network. Let Cl_f denotes the cluster with the highest weight; the modified evaluations of DMs in Cl_f can be determined as:

$$\overline{x_{ij}^k} = \phi x_{ij}^k + (1 - \phi) x_{ij}^{Cl_f} \quad (21)$$

where x_{ij}^k is the original evaluation of $e_k (e_k \in Cl_p)$ on a_i regarding c_j . $x_{ij}^{Cl_f}$ denotes the modification reference for x_{ij}^k . $\phi (0 < \phi < 1)$ is the adjustment coefficient with $\phi = CD^{Cl_p}$, which indicates that the evaluation with a lower level of consensus degree requires greater adjustment.

After the first round of iteration, the GCD is recalculated according to the modified evaluations of identified clusters and the original assessments of remaining clusters. If the updated GCD falls below the threshold γ , the overlapping community-driven consensus-reaching strategy is initiated to enhance group consensus. A maximum iteration parameter N_{max} is set, and if the GCD fails to meet the threshold requirement within the given number of iterations, the CRP will be terminated. As individuals tend to form closer trust relationships with those who share similar judgments, a higher level of opinion similarity can result in strengthened trust relationships.

Therefore, after the evaluation modification in the CRP, the trust relationships between DMs need to be evolved by updating their trust degrees. These dynamic changes in the topology of the social network can lead to changes in the adjacency matrix. This updated weighted matrix is then used to re-initiate the LINE method for a new round of overlapping community detection based on the proposed LNL algorithm, resulting in dynamic changes in the identified clusters. In this dynamic clustering-based consensus model, the formula for the trust strength evolution between DMs due to evaluation modifications is given as follows.

Definition 13 (Trust Evolution). Let t_{kl} denotes the original trust degree from e_k to e_l . After evaluation modifications in the CRP, trust degrees need to be updated according to the original trust values and the variation of opinion similarity between e_k and e_l . The updated trust

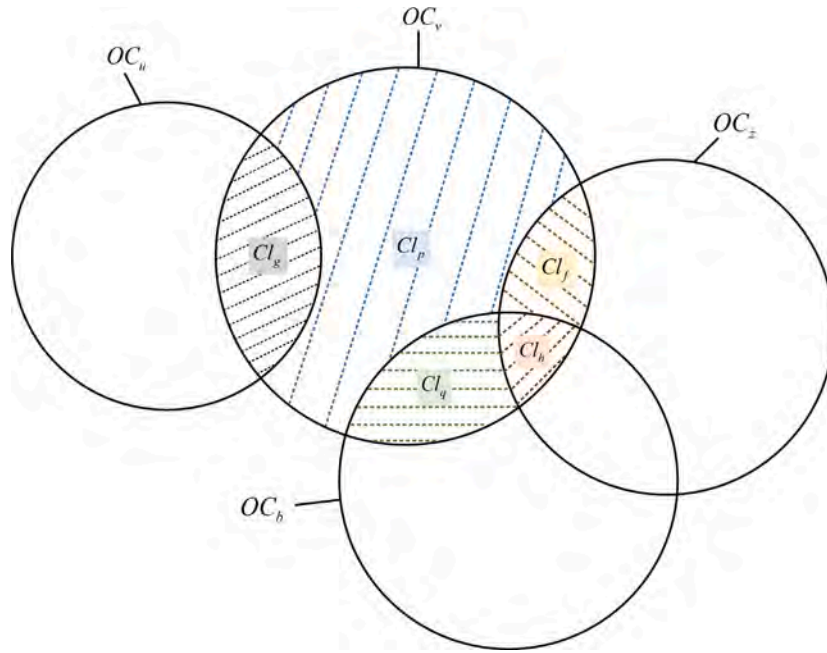


Fig. 5. The identified cluster is in community OC_v , which overlaps with other three communities OC_u , OC_z , and OC_b .

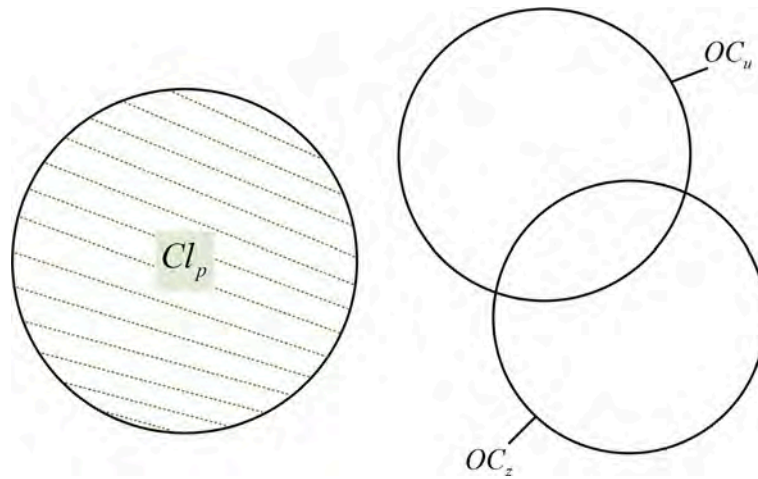


Fig. 6. The identified cluster Cl_p itself is a community without overlapping with others.

level from e_k to e_l ($k \neq l$) can be determined as follows.

$$\widetilde{t}_{kl} = \begin{cases} t_{kl} + \left(\frac{1}{mn} \left(\sum_{i=1}^m \sum_{j=1}^n (1 - d(\overline{x_{ij}^k}, \overline{x_{ij}^l})) - \sum_{i=1}^m \sum_{j=1}^n (1 - d(x_{ij}^k, x_{ij}^l)) \right) \right)^\sigma & \begin{cases} \sum_{i=1}^m \sum_{j=1}^n (1 - d(\overline{x_{ij}^k}, \overline{x_{ij}^l})) \\ \geq \sum_{i=1}^m \sum_{j=1}^n (1 - d(x_{ij}^k, x_{ij}^l)) \end{cases} \\ t_{kl} - \left| \frac{1}{mn} \left(\sum_{i=1}^m \sum_{j=1}^n (1 - d(\overline{x_{ij}^k}, \overline{x_{ij}^l})) - \sum_{i=1}^m \sum_{j=1}^n (1 - d(x_{ij}^k, x_{ij}^l)) \right) \right|^\sigma & \begin{cases} \sum_{i=1}^m \sum_{j=1}^n (1 - d(\overline{x_{ij}^k}, \overline{x_{ij}^l})) \\ < \sum_{i=1}^m \sum_{j=1}^n (1 - d(x_{ij}^k, x_{ij}^l)) \end{cases} \end{cases} \quad (22)$$

where x_{ij}^k and $\overline{x_{ij}^k}$ represent the original and modified evaluation of DM e_k on alternative a_i regarding criterion c_j , respectively. $(1 - d(x_{ij}^k, x_{ij}^l))$ reflects the degree of similarity between x_{ij}^k and x_{ij}^l . $f = (\cdot)^\sigma$ is a monotonically increasing power function. The value of parameter σ ranges between 0 and 1 considering the law of diminishing marginal utility. Based on the relevant literature [33], the empirical value of σ is set to 0.5 in this study.

To facilitate the analysis of the social network among DMs, the updated trust level \widetilde{t}_{kl} is normalized to trust degree \overline{t}_{kl} , as shown below.

$$\overline{t}_{kl} = \frac{\widetilde{t}_{kl} - \min_{k,l=1,2,\dots,r} \{\widetilde{t}_{kl}\}}{\max_{k,l=1,2,\dots,r} \{\widetilde{t}_{kl}\} - \min_{k,l=1,2,\dots,r} \{\widetilde{t}_{kl}\}} \quad (23)$$

Based on the evolved trust degrees, the large group of DMs is re-clustered after each round of evaluation updates. Subsequently, the overlapping community-driven feedback mechanism is re-applied to enhance group consensus. The above CRP terminates when the updated group consensus satisfies $GCD \geq \gamma$. The ranking value of alternatives

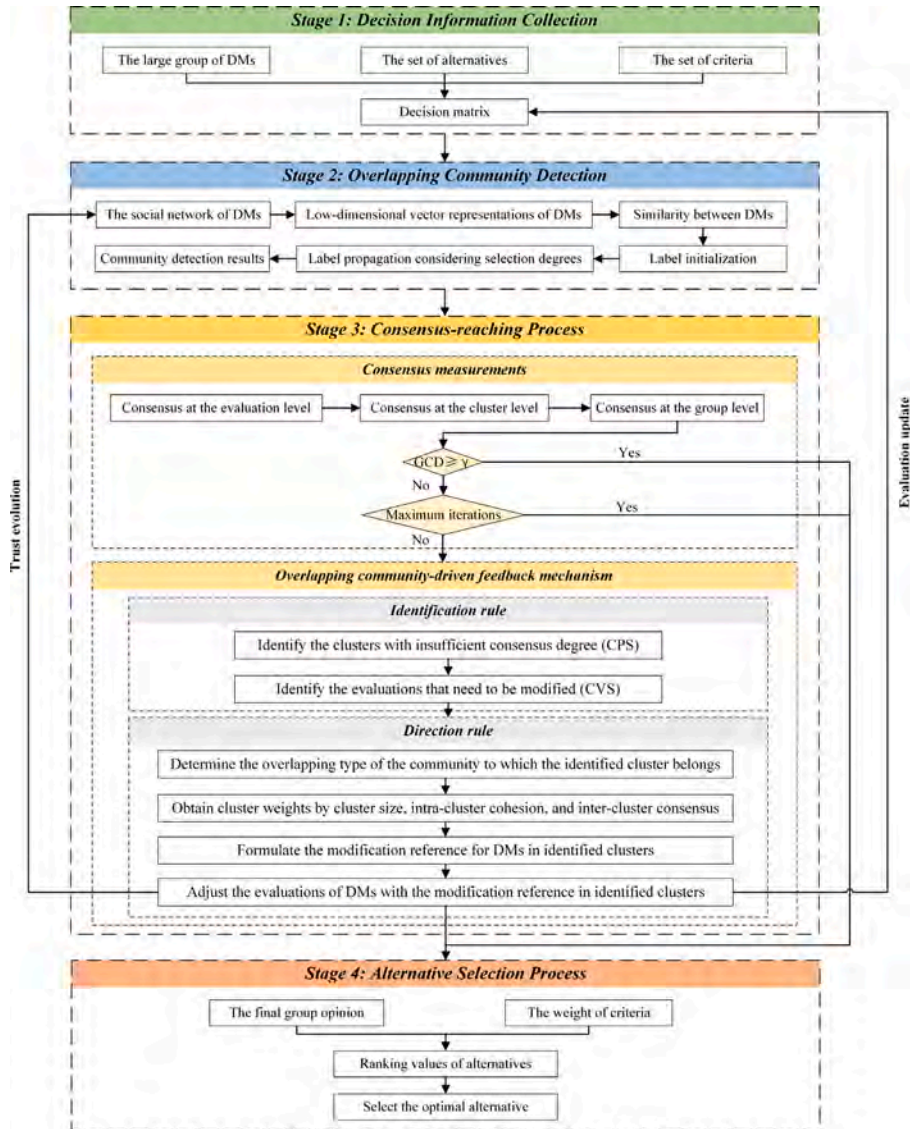


Fig. 7. The framework of the proposed large-scale group decision-making method under social networks.

Table 1
Decision information of e_k ($k = 1, 2, \dots, 15$) on alternatives.

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	e_{11}	e_{12}	e_{13}	e_{14}	e_{15}
a_1	0.5	3	3.5	1	2	2	4	3.5	1	3.5	2	1.5	3	3.5	4
a_2	3	1	4	2.5	2	3	1	1.5	3	2.5	1.5	3	0.5	2	4
a_3	2	1	3	1.5	4	3	3	2.5	1.5	4	0.5	3	3.5	4	3
a_4	1.5	3	4	1	3	2.5	3	2.5	1.5	2	4	1	3.5	4	2.5

a_i ($i = 1, 2, \dots, m$) are determined as follows. The alternative with the maximum ranking value will be identified as the optimal solution.

$$EV(a_i) = \sum_{j=1}^n \sum_{p=1}^f w_j w_{Cl_p} x_{ij}^{Cl_p} \quad (24)$$

where w_j and w_{Cl_p} denote the weight of criterion c_j and cluster Cl_p , respectively. $x_{ij}^{Cl_p}$ represents the consensual evaluation of cluster Cl_p ,

3.4. The framework of the proposed LSGDM method

The framework of the proposed LSGDM method under social networks is illustrated in Fig. 7, and the algorithm of the proposed method is summarized as follows.

Algorithm 1 Dynamic Clustering-based Consensus-reaching Method for LSGDM

Require: A set of potential alternatives $A = \{a_i | i = 1, 2, \dots, m\}$, a set of criteria $C = \{c_j | j = 1, 2, \dots, n\}$, the large group of DMs $E = \{e_k | k = 1, 2, \dots, r\}$, the adjacency matrix $T = [t_{kl}]_{r \times r}$, the preset consensus threshold γ , and maximum iterations N_{\max} .

Ensure: The optimal alternative.

- 1: Evaluate alternatives on each criterion and construct the decision matrix $X^k = [x_{ij}^k]_{m \times n}$.
 - 2: Determine the low-dimensional vector representations of DMs $\eta(e_k)$ with the LINE method.
 - 3: Calculate the similarity between DMs $Sim(e_k, e_l)$ in the embedding space by vector dot product.
 - 4: Assign each node a unique label with a community identifier and belonging coefficient.
 - 5: **repeat**
 - 6: Calculate the selection degree $SD(e_k, e_l)$ of DMs using dynamic label information and static network structure.
 - 7: Update labels by selecting the label with the maximum selection degree from its neighbors.
 - 8: **until** The type and quantity of labels remain unchanged.
 - 9: Classify DMs with the same label into the same community $OC = \{OC_v | v = 1, 2, \dots, z\}$.
 - 10: Quantify the consensus at evaluation level $CD_{ij}^{Cl_p}$, cluster level CD^{Cl_p} , and group level GCD .
 - 11: **if** $GCD \geq \gamma$ **then**
 - 12: Proceed to alternative selection.
 - 13: **else**
 - 14: **if** The iteration count does not exceed N_{\max} **then**
 - 15: Identify clusters and evaluations with consensus degrees below the preset threshold.
 - 16: Determine the overlapping pattern of the community where the identified cluster is situated.
 - 17: Calculate the cluster weight w_{Cl_p} from size, intra-cluster cohesion, and inter-cluster consensus.
 - 18: Formulate recommendation advice $x_{ij}^{Ref(Cl_p)}$ and modify DMs' evaluations based on overlapping patterns.
 - 19: Update the decision information and trust degrees.
 - 20: **end if**
 - 21: **end if**
 - 22: Aggregate the consensual evaluations of clusters to calculate the final group evaluation and ranking value $EV(a_i)$ of each alternative with criteria weights w_j .
 - 23: Select the optimal alternative with the maximum ranking value.
-

4. Case analysis

In this section, a case analysis of a film selection problem using the Film Trust dataset is presented to verify the effectiveness of our LSGDM method in social network environments.

The Film Trust dataset was originally collected by Guo et al. in their research on recommender systems [34]. It contains 35 497 ratings from 1508 users on 2071 films and 1853 directed trust relationships between users. Due to incomplete evaluations of films given by the users in this dataset, this case study selects 853 DMs who have provided complete evaluations for four films (film 592, film 1305, film 1645, and film 1978) in the collection to construct the LSGDM problem. It is important to note that our study does not attempt to provide comprehensive insights into all 2071 films or serve as a definitive market indicator.

Instead, we focus on validating methodologies for managing incomplete decision information in social network-based LSGDM scenarios. LSGDM methods capable of handling incomplete decision information under social networks will be investigated in future research. Besides, Our research does not aim to adjust evaluations for the recommendation system. Instead, we used the dataset related to the recommendation system to validate our proposed LSGDM method in a social network environment. This study focuses primarily on the CRP considering overlapping communities, which involves trust relationships among decision-makers and decision evaluations.

Therefore, to validate the proposed LSGDM method, 853 users with complete evaluations on four films are selected to construct the large group of DMs, denoted as $E = \{e_k | k = 1, 2, \dots, 853\}$. The original trust relationships among them can be utilized to construct the social network $G = (E, L)$ and a binary adjacency matrix $T = [t_{kl}]_{853 \times 853}$. Four films with complete evaluations from this group of DMs are used to establish the set of alternatives, denoted as $A = \{a_i | i = 1, 2, 3, 4\}$. The corresponding 3412 pieces of film ratings (utility values ranging from 0 to 4) are collected as the original decision evaluations. The problem is to determine the consensual group opinion towards the performance of each film in set A and select the best one. Due to the limited space, decision information from the first 15 DMs is presented in Table 1. The trust relationships among the group of DMs are depicted in Fig. 8.

Based on the proposed LINE-based label propagation algorithm, the community detection results can be obtained. The group of 853 DMs is partitioned into 64 communities, which are partially shown in Table 2. The DMs involved in the overlapping communities are illustrated in Fig. 9, and the specific DMs that locate in the overlapping regions are given in Table 3. For example, communities OC_5 and OC_{10} overlap, and the DMs in the overlapping region are e_{666} , e_{812} , and e_{849} . Considering the overlapping regions, the group of DMs can be divided into 69 clusters, denoted as $CL = \{Cl_p | p = 1, 2, \dots, 69\}$. Cl_1 to Cl_{64} correspond to the independent parts of the 64 communities (i.e., the portions that have no overlap with others). Cl_{65} , Cl_{66} , Cl_{67} , and Cl_{68} represent the overlapping regions of OC_5 with OC_{10} , OC_{15} , OC_{31} , and OC_{62} , respectively. Cl_{69} denotes the overlapping region of OC_{20} with OC_{29} .

Based on individual evaluations on alternatives, the cluster evaluations can be determined by Eq. (9). Then, the consensus on evaluation, cluster, and group levels can be obtained via Eqs. (10)–(12). The initial consensus degrees on the cluster level are presented in Table 4.

According to the consensus degrees of 69 clusters, the current group consensus degree is calculated as $GCD = 0.697$. To guarantee the ultimate decision outcome is acceptable to most DMs, a consensus threshold $\gamma = 0.80$ is set. Given that the current group consensus falls below the threshold, the overlapping community-driven consensus-reaching strategy needs to be carried out to enhance group consensus. The maximum number of iterations is set as $N_{\max} = 30$. Following the identification rule, the evaluations that need to be modified are presented in Table 5.

In the proposed consensus-reaching method, the evaluations in the identified clusters are modified towards the overlapping DMs that belong to different communities simultaneously. First, we need to determine the overlapping pattern of the community where the identified cluster is located. Then, the modified evaluations can be calculated based on different consensus-reaching strategies elaborated in Section 3.3.2.

Take the modification of the identified evaluations on alternative a_2 in cluster Cl_5 as an example; the overlapping pattern of the community to which Cl_5 belongs falls into the second category described in Section 3.3.2. According to the direction rule given by Eqs. (17)–(18), the modified evaluations in Cl_5 can be obtained as follows.

$$\bar{x}_2^k = 0.635x_2^k + 0.365 \left(\frac{w_{Cl_{65}}x_2^{Cl_{65}} + w_{Cl_{66}}x_2^{Cl_{66}} + w_{Cl_{67}}x_2^{Cl_{67}} + w_{Cl_{68}}x_2^{Cl_{68}}}{w_{Cl_{65}} + w_{Cl_{66}} + w_{Cl_{67}} + w_{Cl_{68}}} \right) \quad (25)$$

where x_2^k denotes the original evaluation of e_k ($e_k \in Cl_5$) on a_2 .

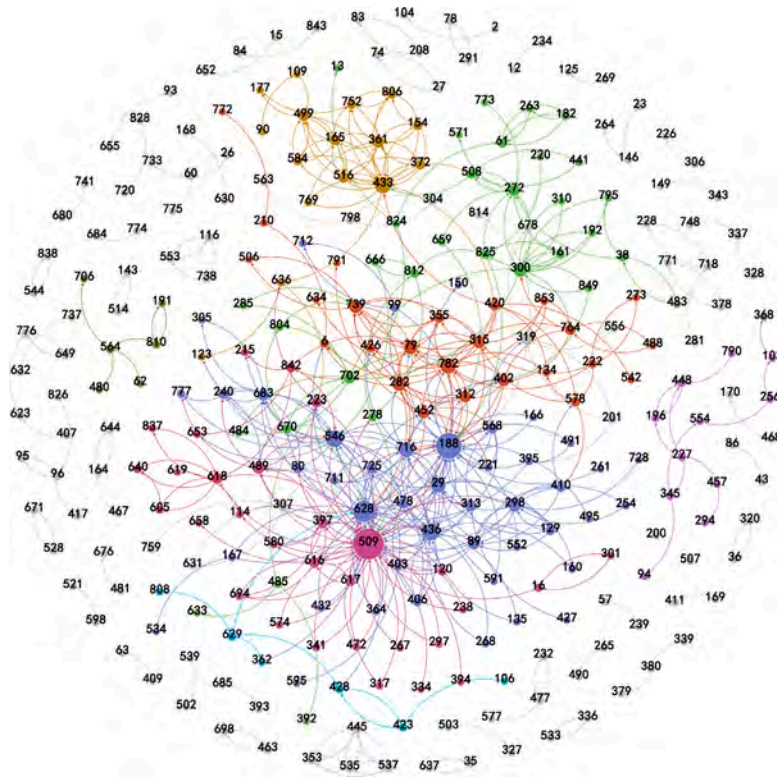


Fig. 8. The social network among the group of DMs.

Table 2
Community detection results.

Comm	Nodes	Comm	Nodes	Comm	Nodes	Comm	Nodes
1	e_2, e_{104}	17	e_{43}, e_{86}	33	$e_{228}, e_{281}, e_{718}$	49	$e_{392}, e_{485}, e_{633}$
2	e_{12}, e_{234}	18	e_{90}, e_{109}	34	$e_{232}, e_{477}, e_{577}$	50	e_{265}, e_{490}
3	e_{15}, e_{652}	19	$e_{95}, e_{96}, e_{417}, \dots$	35	e_{125}, e_{269}	51	e_{327}, e_{503}
4	e_{16}, e_{301}	20	$e_{103}, e_{256}, e_{345}, \dots$	36	$e_{304}, e_{563}, e_{798}$	52	$e_{605}, e_{618}, e_{640}, \dots$
5	$e_{29}, e_{80}, e_{89}, \dots$	21	$e_{154}, e_{165}, e_{177}, \dots$	37	e_{226}, e_{306}	53	e_{521}, e_{598}
6	$e_{25}, e_{60}, e_{733}, \dots$	22	$e_{116}, e_{553}, e_{738}$	38	$e_{328}, e_{337}, e_{749}, \dots$	54	e_{528}, e_{671}
7	e_{27}, e_{74}	23	e_{143}, e_{514}	39	e_{336}, e_{533}	55	e_{502}, e_{539}
8	e_{35}, e_{637}	24	e_{23}, e_{146}, e_{264}	40	$e_{353}, e_{445}, e_{535}, \dots$	56	e_{544}, e_{838}
9	e_{36}, e_{320}	25	e_{149}, e_{343}	41	e_{378}, e_{771}	57	e_{632}, e_{737}
10	$e_{38}, e_{161}, e_{192}, \dots$	26	$e_{164}, e_{467}, e_{644}$	42	$e_{339}, e_{379}, e_{380}$	58	e_{93}, e_{655}
11	e_{57}, e_{239}	27	e_{168}, e_{630}	43	$e_{407}, e_{623}, e_{649}, \dots$	59	e_{393}, e_{685}
12	$e_{13}, e_{61}, e_{182}, \dots$	28	$e_{62}, e_{191}, e_{480}, \dots$	44	e_{169}, e_{411}	60	e_{680}, e_{741}
13	e_{63}, e_{409}	29	$e_{94}, e_{196}, e_{227}, \dots$	45	e_{106}, e_{423}	61	e_{684}, e_{774}
14	e_{78}, e_{291}	30	e_{200}, e_{507}	46	e_{463}, e_{698}	62	e_{629}, e_{808}
15	$e_6, e_{79}, e_{134}, \dots$	31	$e_{201}, e_{319}, e_{556}, \dots$	47	e_{170}, e_{468}	63	e_{720}, e_{828}
16	e_{83}, e_{208}	32	e_{210}, e_{772}	48	e_{481}, e_{676}	64	e_{84}, e_{843}

Table 3
The DMs that specifically locate in the overlapping regions.

Overlapping communities	DMs
OC_5, OC_{10}	$e_{653}, e_{658}, e_{666}, e_{694}, e_{711}, e_{728}, e_{812}, e_{853}, e_{849}$
OC_5, OC_{15}	$e_{426}, e_{534}, e_{546}, e_{574}, e_{591}, e_{616}, e_{670}, e_{683}, e_{842}$
OC_5, OC_{31}	$e_{201}, e_{297}, e_{568}, e_{580}, e_{595}, e_{628}$
OC_5, OC_{62}	e_{629}, e_{808}
OC_{20}, OC_{29}	$e_{345}, e_{448}, e_{457}, e_{554}$

Other identified evaluations can be modified in a similar manner. Based on the adjusted evaluations, the trust degrees among DMs can be updated by Eq. (22). Take the trust relationship from e_{616} to e_{509} as an example, it is updated according to the original trust value from e_{616} to e_{509} and the variation of opinion similarity between e_{616} and e_{509} . Therefore, the updated trust level from e_{616} to e_{509} can be determined by

Eq. (26), shown as follows. The remaining trust degrees can be updated similarly.

$$\widehat{t_{616,509}} = t_{616,509} + \left(\frac{1}{4} \left(\sum_{i=1}^4 (1 - d(\overline{x_i^{616}}, \overline{x_i^{509}})) - \sum_{i=1}^4 (1 - d(x_i^{616}, x_i^{509})) \right) \right)^{0.5} \quad (26)$$

Subsequently, consensus degrees on the cluster level are recalculated by Eq. (11), as shown in Table 6. The group consensus degree is recalculated as $GCD^{(1)} = 0.702$ by Eq. (12). We can observe that the group consensus has been enhanced after one round of modification, but it has not reached the preset threshold. Therefore, it is necessary to continue the above process until the GCD meets the threshold requirement. It is worth noting that, before the next iteration of the CRP process begins, the updated trust degrees must be used to form a new adjacency matrix, upon which the proposed LNL algorithm is

Table 4
The initial consensus degrees on the cluster level.

p	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
CD^{Cl_p}	0.814	0.809	0.806	0.463	0.635	0.668	0.581	0.832	0.412	0.582	0.857	0.801	0.597	0.643	0.830
p	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
CD^{Cl_p}	0.683	0.805	0.846	0.572	0.419	0.487	0.859	0.630	0.818	0.619	0.548	0.811	0.836	0.849	0.865
p	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
CD^{Cl_p}	0.805	0.828	0.843	0.801	0.812	0.540	0.824	0.832	0.563	0.687	0.522	0.841	0.527	0.429	0.532
p	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
CD^{Cl_p}	0.824	0.606	0.567	0.848	0.618	0.806	0.830	0.612	0.558	0.815	0.808	0.813	0.569	0.607	0.480
p	61	62	63	64	65	66	67	68	69						
CD^{Cl_p}	0.826	0.833	0.616	0.820	0.398	0.847	0.675	0.571	0.806						

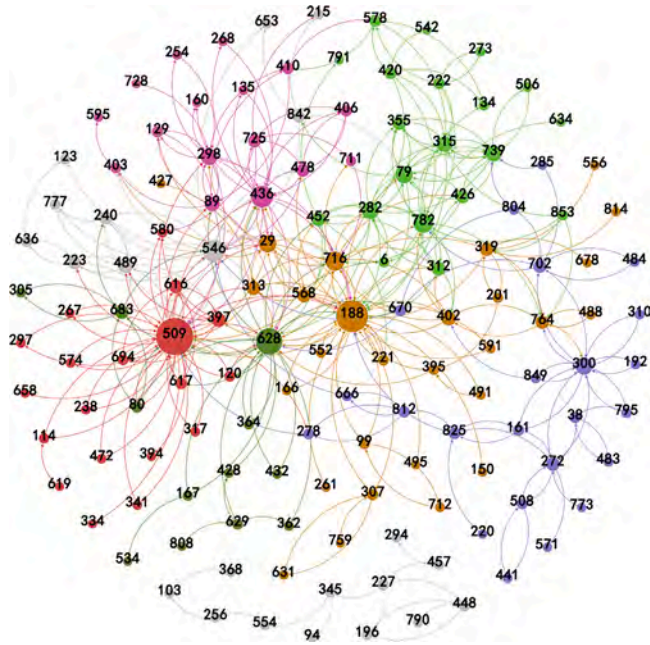


Fig. 9. The DMs involved in the detected overlapping communities.

Table 5
The identified evaluations and clusters that need to be modified.

$CPS(p)$	$CVS((i,p))$	$CPS(p)$	$CVS((i,p))$
4	(1,4)	36	(1,36), (3,36)
5	(2,5), (3,5), (4,5)	39	(2,39), (3,39)
6	(1,6), (3,6)	40	(1,40)
7	(2,7)	41	(2,41), (3,41), (4,41)
9	(3,9), (4,9)	43	(1,43)
10	(1,10), (2,10), (3,10)	44	(2,44)
13	(2,13)	45	(1,45), (4,45)
14	(3,14), (4,14),	47	(1,47), (2,47), (3,47)
15	(1,15), (2,15), (4,15)	48	(4,48)
19	(2,19)	50	(2,50), (3,50)
20	(1,20), (2,20)	53	(1,53), (2,53)
21	(2,21), (3,21), (4,21)	54	(3,54)
23	(1,23), (2,23)	58	(1,58)
25	(2,25), (3,25)	59	(2,59)
26	(1,26), (4,26)	60	(3,60), (4,60)
29	(1,29), (2,29), (3,29)	62	(1,62), (3,62), (4,62)
30	(3,30)	65	(4,65)
31	(1,31), (2,31), (4,31)	67	(3,67)

CPS denotes the set of identified clusters with inadequate consensus degrees. CVS denotes the set of evaluations whose consensus degrees are below the preset threshold in the corresponding clusters.

reapplied for overlapping community detection. The new overlapping community detection results will then guide the next round of the consensus-reaching until the group consensus level reaches the preset threshold or the number of iterations reaches the maximum set limit. After 8 iterations, the GCD reaches 0.806, surpassing the threshold of

0.80. For brevity, the detailed process is omitted. The improvement in group consensus is illustrated in Fig. 10. The final consensual group opinion on the performance of alternatives is (1.828, 2.483, 3.851, 2.672). The ranking of the films can be obtained as $a_3 > a_4 > a_2 > a_1$. Therefore, a_3 can be selected as the best film.

5. Discussions

In this section, sensitivity analysis, validity analysis, and comparative analysis are conducted to demonstrate the effectiveness and superiority of the proposed LSGDM method.

5.1. Sensitivity analysis

To verify the stability of this method, sensitivity analyses on the weight of clusters and the group consensus degree are carried out.

First, the perturbation method is employed to analyze how the variation of cluster weights within $\pm 20\%$ affects the ranking of alternatives. The original weight of cluster Cl_p is w_{Cl_p} , the weight after perturbation is denoted as $\widetilde{w}_{Cl_p} = \delta w_{Cl_p}$, where δ is the perturbation coefficient and $\widetilde{w}_{Cl_g} \in (0, 1)$. Since the weight of clusters conform to $\sum_{p=1}^f w_{Cl_p} = 1$, if the weight of Cl_p is disturbed, the weight of other clusters will be influenced, which is represented as $\widetilde{w}_{Cl_g} = \theta w_{Cl_g}$ ($g \neq p, g = 1, 2, \dots, f$).

Since $\widetilde{w}_{Cl_p} + \sum_{g=1, g \neq p}^f \widetilde{w}_{Cl_g} = 1$, thus $\theta = \frac{1 - \delta w_{Cl_p}}{1 - w_{Cl_p}}$. When δ varies, the

criteria weight adjusts accordingly. Therefore, 100 perturbation simulations are conducted on the criteria weight. Fig. 11 shows the ranking results of alternatives. On one hand, a_3 and a_1 remain insensitive to the variations of cluster weights, where a_3 is always the optimal alternative and a_1 is always the worst choice. On the other hand, a_2 and a_4 are more sensitive to changes in cluster weight, where a_4 ranked second for 86 times and third for 14 times. This illustrates the stability and robustness of the proposed method in differentiating the performance of alternatives.

Then, the impact of variations in group consensus threshold γ on alternative ranking is explored. Table 7 presents the ranking values of different alternatives as γ varies from 0.75 to 0.85 in increments of 0.1. When γ ranges from 0.75 to 0.76, the alternative ranking remains unchanged, being $a_4 > a_3 > a_1 > a_2$. When γ is set to 0.77 and 0.78, the alternative ranking changes to $a_4 > a_2 > a_1 > a_3$, with a_4 being the optimal solution in both cases. As the group consensus threshold continues to increase, ranging from 0.79 to 0.85, the optimal and worst alternatives become a_3 and a_1 , respectively. This highlights the importance of the CRP in obtaining an optimal choice with a higher agreement level among the group.

5.2. Validity analysis

To validate the effectiveness of the proposed LSGDM method, this study employs the validation approach introduced by Wang et al. [35]. This approach relies on the following principle: A reliable decision-making approach should adhere to the transitivity principle. When

Table 6
The updated consensus degrees on the cluster level after the first iteration.

p	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$CD^{Cl_p(1)}$	0.814	0.809	0.811	0.483	0.639	0.674	0.601	0.837	0.445	0.595	0.857	0.801	0.599	0.657	0.835
p	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
$CD^{Cl_p(1)}$	0.691	0.811	0.850	0.588	0.432	0.495	0.858	0.647	0.821	0.629	0.557	0.812	0.839	0.852	0.865
p	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
$CD^{Cl_p(1)}$	0.807	0.832	0.846	0.806	0.815	0.544	0.827	0.834	0.551	0.683	0.526	0.845	0.529	0.431	0.537
p	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
$CD^{Cl_p(1)}$	0.827	0.608	0.562	0.850	0.611	0.809	0.837	0.612	0.554	0.818	0.809	0.811	0.565	0.602	0.476
p	61	62	63	64	65	66	67	68	69						
$CD^{Cl_p(1)}$	0.831	0.835	0.616	0.826	0.411	0.85	0.677	0.584	0.813						

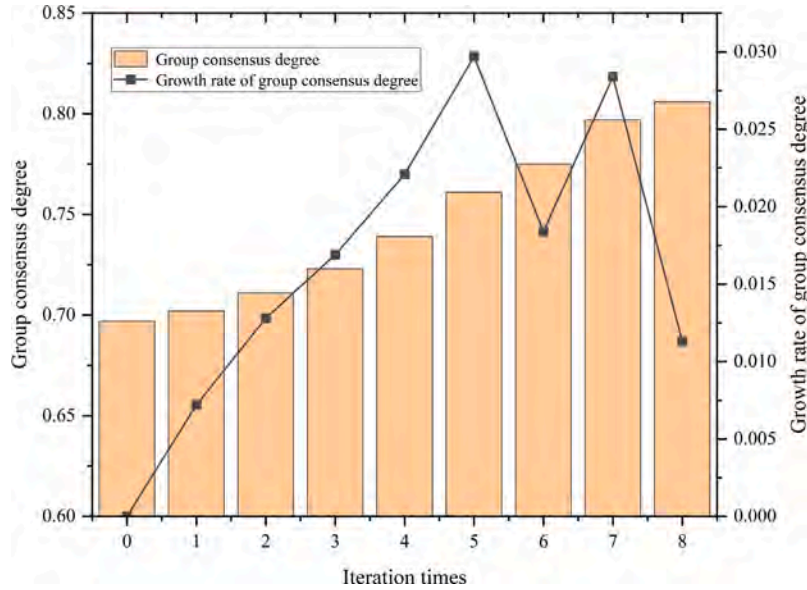


Fig. 10. The improvement of group consensus degree during the consensus-reaching process.

Table 7
The ranking values of alternatives with the variation of γ .

γ	Ranking values of alternatives	Alternative ranking
0.75	$EV(a_1) = 2.534, EV(a_2) = 1.651, EV(a_3) = 2.914, EV(a_4) = 3.346.$	$a_4 > a_3 > a_1 > a_2$
0.76	$EV(a_1) = 2.659, EV(a_2) = 1.483, EV(a_3) = 2.704, EV(a_4) = 3.658.$	$a_4 > a_3 > a_1 > a_2$
0.77	$EV(a_1) = 2.180, EV(a_2) = 2.714, EV(a_3) = 1.366, EV(a_4) = 3.523.$	$a_4 > a_2 > a_1 > a_3$
0.78	$EV(a_1) = 2.526, EV(a_2) = 2.694, EV(a_3) = 1.637, EV(a_4) = 3.605.$	$a_4 > a_2 > a_1 > a_3$
0.79	$EV(a_1) = 1.364, EV(a_2) = 2.174, EV(a_3) = 3.535, EV(a_4) = 2.256.$	$a_3 > a_4 > a_2 > a_1$
0.80	$EV(a_1) = 1.828, EV(a_2) = 2.483, EV(a_3) = 3.851, EV(a_4) = 2.672.$	$a_3 > a_4 > a_2 > a_1$
0.81	$EV(a_1) = 1.631, EV(a_2) = 2.362, EV(a_3) = 3.646, EV(a_4) = 2.476.$	$a_3 > a_4 > a_2 > a_1$
0.82	$EV(a_1) = 1.741, EV(a_2) = 2.483, EV(a_3) = 3.702, EV(a_4) = 2.823.$	$a_3 > a_4 > a_2 > a_1$
0.83	$EV(a_1) = 1.589, EV(a_2) = 2.264, EV(a_3) = 3.739, EV(a_4) = 2.706.$	$a_3 > a_4 > a_2 > a_1$
0.84	$EV(a_1) = 1.357, EV(a_2) = 2.965, EV(a_3) = 3.630, EV(a_4) = 2.624.$	$a_3 > a_2 > a_4 > a_1$
0.85	$EV(a_1) = 1.364, EV(a_2) = 2.462, EV(a_3) = 3.704, EV(a_4) = 2.813.$	$a_3 > a_2 > a_4 > a_1$

decomposing the GDM problem into multiple sub-problems and employing the same method to solve them, the alternative ranking should consistently align with those of the original problem.

Based on the above principle, the original set of alternatives in Section 4 is decomposed into four subsets, given in Table 8. Employing the LSGDM method proposed in this study, the rankings of alternatives within subsets can be determined. Table 8 shows that the alternative priority in these subsets aligns with the priority of the original alternative set, which demonstrates the validity of the proposed method.

5.3. Comparative analysis

The overlapping community detection and consensus-reaching process constitute the LSGDM method proposed in this study. Therefore, in this subsection, comparative analyses of the clustering algorithm and

consensus-reaching strategy are conducted from both qualitative and quantitative perspectives to illustrate the advantages of the proposed method.

5.3.1. comparative analysis on the clustering algorithm

First, we qualitatively compare the proposed LNL algorithm with several typical clustering methods used in the existing LSGDM methods, which are shown in Table 9. Methods in [2,3,23,24] utilize the similarity of evaluations, differences between alternative rankings, or the characteristic of behaviors to partition the group of DMs, which neglect the social relationship among them and their impact on the clustering and CRP. In [9,10,36], the Louvain algorithm is utilized to divide the group of DMs into multiple communities based on their trust relationships. In [37], the fast-unfolding algorithm detects the communities. However, traditional community detection algorithms like the Louvain algorithm fail to consider different potential overlaps

Table 8
The alternative ranking in four subsets.

Subsets	Ranking values of alternatives	Alternative ranking
$\{a_1, a_3, a_4\}$	$EV(a_1) = 0.784, EV(a_3) = 2.159, EV(a_4) = 1.582.$	$a_3 > a_4 > a_1$
$\{a_1, a_2, a_4\}$	$EV(a_1) = 0.916, EV(a_2) = 1.275, EV(a_4) = 2.671.$	$a_4 > a_2 > a_1$
$\{a_1, a_2, a_3\}$	$EV(a_1) = 0.690, EV(a_2) = 1.623, EV(a_3) = 2.174.$	$a_3 > a_2 > a_1$
$\{a_2, a_3, a_4\}$	$EV(a_2) = 0.835, EV(a_3) = 2.619, EV(a_4) = 1.425.$	$a_3 > a_4 > a_2$

Table 9
The qualitative comparison of clustering algorithms in LSGDM methods.

Methods	Clustering method	Cluster overlapping analysis
[2]	Clustered by modeling the characteristic of herding behavior	Not considered
[3]	Maximizing subgroup combined cohesion degree-based clustering method	Not considered
[23]	The similarity of individual evaluations and alternatives' rankings	Not considered
[24]	Improved density peak clustering method	Not considered
[37]	Fast unfolding algorithm	Not considered
[9]	Louvain algorithm	Not considered
[10]	Louvain algorithm	Not considered
[36]	Louvain algorithm	Not considered
[21]	Lancichinetti-Fortunato method (LFM)	Considered
[22]	Community overlap propagation algorithm (COPRA)	Considered
This paper	LINE-based label propagation algorithm	Considered

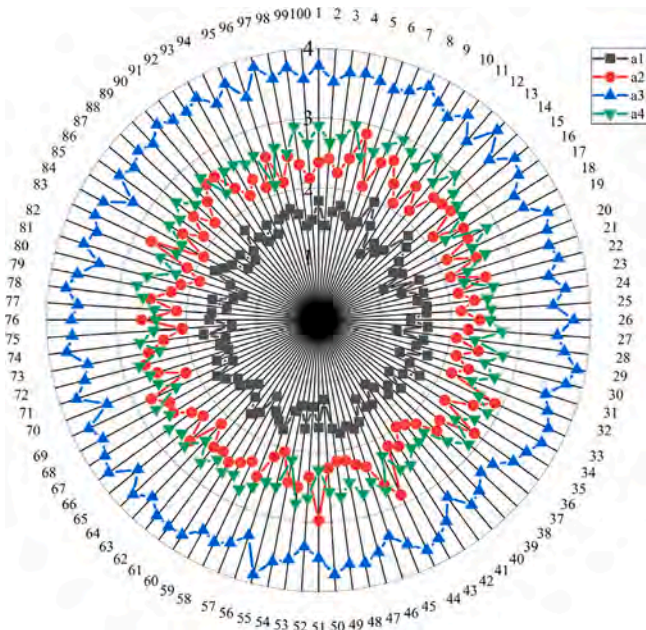


Fig. 11. The ranking values of alternatives under disturbed cluster weights.

among communities. In practice, communities may not necessarily be mutually exclusive, and the influence of DMs in overlapping regions on the CRP deserves further analysis. Although two studies utilize overlapping community detection methods to cluster the large group of DMs, i.e., LFM [21] and COPRA [22], both methods have certain limitations. The LFM may lead to nodes not belonging to any community in the community detection results. The CORPA method selects neighbor nodes randomly in the label propagation process, potentially causing inconsistencies in overlapping community detection outcomes.

In this study, we propose the LNL algorithm to perform overlapping community detection. Compared with other approaches, our method preserves higher-order proximities of DMs in the social network considering local and global network structure. Besides, static network and dynamic label information are utilized to carry out the label propagation process, in which the neighbor node with the highest selection degree is chosen to propagate the label.

Table 10
The statistics of four real-world datasets.

Dataset	Nodes	Edges	Average degree	Average clustering coefficient
Karate	34	78	4.588	0.571
Football	115	613	10.661	0.403
Jazz	198	2742	27.697	0.617
YouTube	5000	4689	1.876	0.208

Table 11
The statistics of four synthetic datasets.

Parameter	LFR1	LFR2	LFR3	LFR4
Nodes	500	500	500	1000–15 000
Average degree	25	25	25	25
Maximum degree	50	50	50	50
Internal connection coefficient	0.1–0.5	0.4	0.4	0.3
Minimum community size	10	10	10	10
Maximum community size	50	50	50	50
Number of overlapping nodes	50	50–300	50	50
Number of memberships of the overlapping nodes	2	2	2–6	2

For quantitative comparison, real-world datasets and synthetic datasets are utilized to carry out the experimental analysis. The real-world datasets include the Karate club dataset [38], the football dataset [39], the Jazz dataset [40], and the YouTube dataset [41]. The statistics are shown in Table 10. Since the Lancichinetti–Fortunato–Radicchi (LFR) network [42] is considered the standard network for community detection, it is employed to validate the performance of the algorithms. The statistics of the synthetic datasets are given in Table 11.

The Modularity and normalized mutual information (NMI) metrics are utilized to evaluate the performance of the LNL algorithm. Modularity measures the strength of the division of a network into modules (communities) by comparing the density of edges inside communities to the density of edges between communities. The NMI metric is used to measure the similarity between two clustering results. The calculation formulas for both metrics are presented in Definitions 14 and 15, respectively.

Definition 14 ([43]). Let $NC(k)$ and $NC(l)$ represent the numbers of communities to which nodes k and l belong, respectively. $DC(k)$ and $DC(l)$ denote the degree centrality of node k and l , respectively. $\delta(OC(k), OC(l))$ is employed to check whether node k and node l

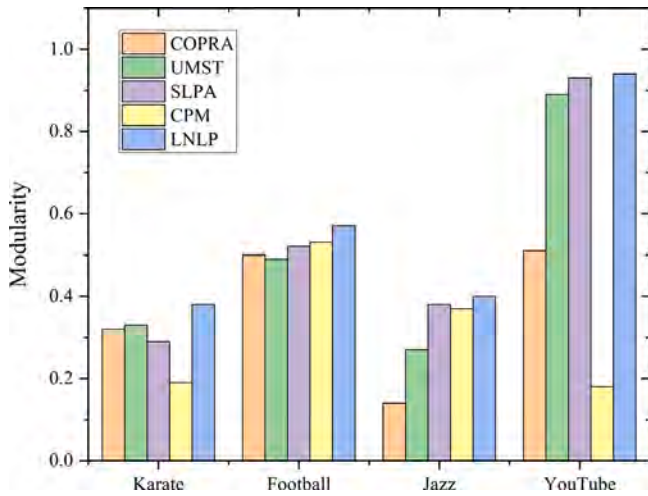


Fig. 12. The modularity of COPRA, UMST, SLPA, CPM, and LNLN on four real-world networks.

are in the same community. The adjacency matrix of the network is represented by $T = [t_{kl}]_{r \times r}$. The modularity of the network can be determined as follows.

$$Q = \frac{1}{2|L|} \sum_{kl} \frac{\delta(OC(k), OC(l))}{NC(k)NC(l)} \left(t_{kl} - \frac{DC(k)DC(l)}{2|L|} \right) \quad (27)$$

where $|L|$ denotes the total number of edges in the network.

Definition 15 ([44]). Let U and V represent the real community partition and algorithm partition. $H(U)$ and $H(V)$ denotes the entropy of U and V , respectively. $H(U|V)$ indicates the conditional entropy between U and V . $H(V|U)$ denotes the conditional entropy between V and U . The NMI between the two community partitions is defined as follows.

$$NMI(U, V) = 1 - \frac{1}{2} \left(\frac{H(U|V)}{H(U)} + \frac{H(V|U)}{H(V)} \right) \quad (28)$$

where $NMI(U, V) \in [0, 1]$.

To validate the effectiveness of the proposed method, we quantitatively compared it with four typical overlapping community detection methods: COPRA [23], Union of All Maximum Spanning Trees (UMST) [45], Speaker-listener Label Propagation Algorithm (SLPA) [46], and Critical Path Methods (CPM) [47]. All experiments were repeated 25 times, and the average values were taken as the final results. The modularity values of COPRA, UMST, SLPA, CPM, and LNLN on four real-world networks are shown in Fig. 12.

Generally, higher modularity values indicate better performance in community detection. From the results in Fig. 12, it can be observed that the proposed LNLN algorithm achieves higher modularity values than the other four methods across four real-world networks, suggesting its superior ability to identify community structures. The main reason is that the proposed LNLN method selects the neighbors with the highest selection degree in the label propagation process. In contrast, COPRA and SLPA involve a stochastic process in the propagation of labels, which can lead to inconsistent results across different runs, requiring multiple iterations to obtain stable results. The CPM method performs poorly on the YouTube network because CPM requires mining all four clique graphs within the dataset (in this experiment, the parameter k for CPM is set to 4). When the number of four-clique graphs is small, the CPM method struggles to identify meaningful communities, leading to lower modularity values. Therefore, CPM's focus on a specific clique size might not effectively capture the broader community structure, especially in heterogeneous networks where community sizes and densities vary widely. While

the UMST method demonstrates stable performance across different datasets, this approach focuses on the most prominent connections, potentially overlooking subtler relationships and community overlaps. By considering both dynamic label information and static network structure, the proposed method detects overlapping communities more effectively.

Fig. 13 presents the NMI values obtained using COPRA, UMST, CPM, SLPA, and the proposed LNLN method on the four LFR synthetic datasets shown in Table 10. In Fig. 13(a), as the internal connection coefficient increases, the NMI values of COPRA, UMST, CPM, SLPA, and LNLN methods all show a decreasing trend. Notably, the NMI values of the SLPA and COPRA methods significantly drop when the internal connection coefficient exceeds 0.3. This is because the network structure becomes more complex, and the label propagation processes of COPRA and SLPA cannot distinguish between noisy and normal neighbors. This randomness can lead to inconsistent community detection results, especially as the complexity of internal connections increases. Overall, the proposed LNLN method performs well, demonstrating better stability and robustness in detecting overlapping communities. In Fig. 13(b), as the number of overlapping nodes increases, the NMI values of all methods decrease, with the NMI values of COPRA and SLPA showing a more significant decline. That is due to their limitations in effectively differentiating between overlapping nodes and those belonging to a single community. In contrast, the UMST and LNLN method exhibits more stable performance. In Fig. 13(c), the LNLN method also demonstrates good performance. This is because, during the label propagation process, the LNLN method does not randomly select neighbor nodes but instead selects nodes with the highest selection degree for label exchange. The CPM achieves the highest NMI value when the number of memberships of the overlapping nodes is 4, due to fewer outliers in the network. The NMI values of COPRA and SLPA methods also show a significant decline as the number of memberships of the overlapping nodes increases, indicating their poorer detection capabilities for overlapping communities in high-overlapping diversity scenarios. In Fig. 13(d), compared to other methods, the LNLN method also demonstrates better performance as the network size increases. Therefore, the proposed LNLN algorithm is more reasonable and effective in clustering nodes, facilitating the overlapping community-driven CRP for LSGDM problems.

5.3.2. comparative analysis on the consensus-reaching strategy

The comparative analysis of typical LSGDM methods in terms of cluster weighting, consensus-reaching process, and dynamic analysis is shown in Table 12.

When determining the cluster weight, the cluster size (i.e., the number of DMs within the cluster) is a factor considered by all typical methods listed in Table 12. In addition to size, method in [36] incorporates the social trust relationships within the cluster; method in [23] considers the confidence degree and trust degree of the subgroup; method in [3] includes the combined cohesion degree of the cluster; method in [24] considers the silhouette coefficient and subgroup evaluation deviation in cluster weighting. Methods in [2,9,10] determine the cluster's weight based on the size of the cluster and the weights of DMs within the cluster. In [22], DMs within the cluster are regarded as internal experts, and their quantity, together with the trust they receive from external experts, are utilized to calculate the cluster weight. In [48], the number of DMs, consensus, and trust level of the subgroups are considered to determine the cluster weight.

Compared to the above methods, the cluster weighting approach proposed in this paper not only considers the size of the clusters but also incorporates the intra-cluster cohesion and inter-cluster consensus within the entire group. Clusters with greater intra-cluster cohesion and inter-cluster consensus will be assigned higher importance. Therefore, our cluster weighting method is more comprehensive and follows intuition more closely.

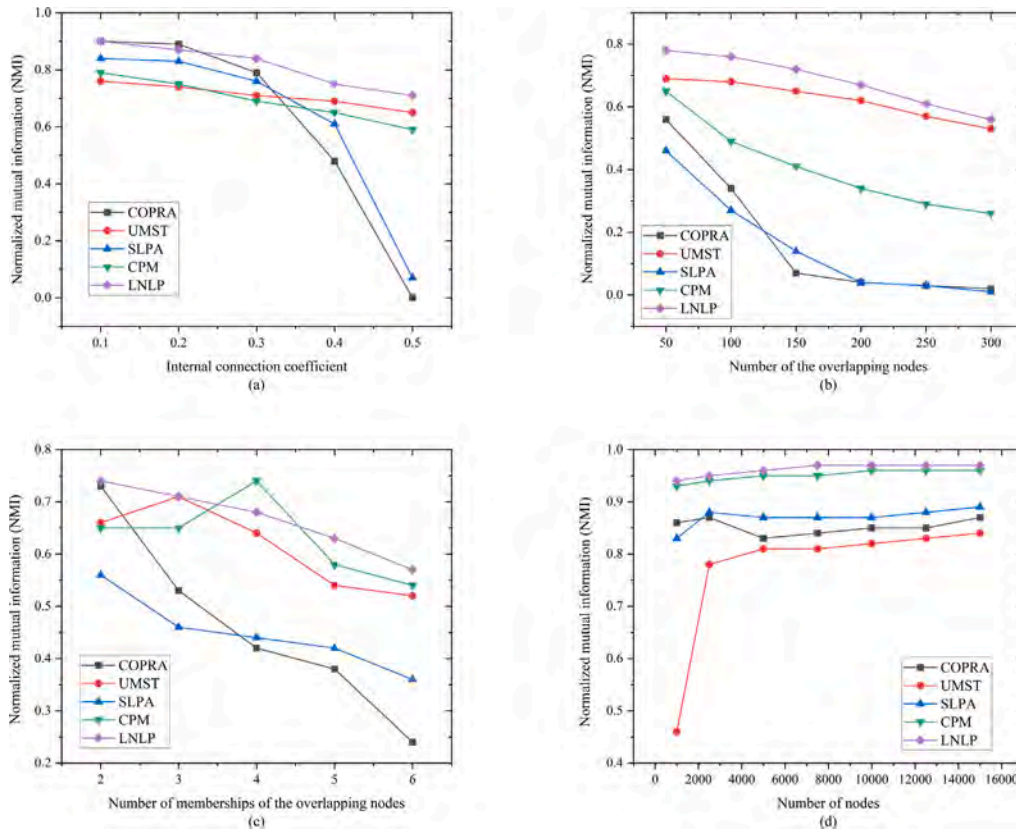


Fig. 13. The NMI values obtained using COPRA, UMST, CPM, SLPA, and LNLN method on the four LFR synthetic datasets.

Table 12
The qualitative comparison of various typical LSGDM methods.

Methods	Cluster weighting	Consensus-reaching strategy	Dynamic elements
[23]	The number of DMs in the subgroup, the confidence degree, and the trust degree of the subgroup	Minimum adjustment model-based consensus-reaching method	Evaluation dynamics, trust dynamics, and behavior dynamics
[3]	The number of DMs in the subgroup and the combined cohesion degree of the subgroup	The social network DeGroot model-based consensus-reaching method for the identified individuals	Evaluation dynamics
[2]	The weight and number of DMs in the cluster	Minimum adjustment model-based consensus-reaching method	Evaluation dynamics
[24]	The silhouette coefficient, the size of subgroup, and subgroup evaluation deviation	Two-stage Gini coefficient-based optimization model	Evaluation dynamics
[37]	The size of the cluster	The identified evaluations are modified towards the DM with the highest consensus level	Evaluation dynamics
[10]	The weight and number of DMs in the cluster	The evaluations of the identified DMs are adjusted towards the DMs they trust and the group opinion	Evaluation dynamics
[9]	The weight and number of DMs in the cluster	Minimum cost model-based consensus-reaching method	Evaluation dynamics
[36]	The size and social trust of the subgroups	Minimum cost maximum satisfaction consensus model	Evaluation dynamics
[21]	The size of the subgroup	The overlapping community guided consensus-reaching method for the identified subgroup pairs	Evaluation dynamics
[22]	The number of internal DMs in the community and the trust weights internal DMs received from external DMs	Group and subgroup evaluations guided consensus-reaching method for the identified individuals	Evaluation dynamics
[48]	The number of DMs, consensus level, and trust level in the subgroup	The overlapping community guided consensus-reaching method for the identified subgroups	Evaluation dynamics, trust dynamics, and cluster dynamics
[49]	Not discussed	Decayed trust propagation-induced feedback mechanism	Evaluation dynamics
This paper	The size of the cluster, intra-cluster cohesion, and inter-cluster consensus	The overlapping community-driven feedback mechanism considering different overlapping patterns for the identified clusters	Evaluation dynamics, trust dynamics, and cluster dynamics

The consensus-reaching strategy summarized in Table 12 can be primarily categorized into two categories. The first category involves optimization-based techniques, i.e., the minimum cost consensus model [9], minimum adjustment consensus model [2,23], and bi-objective consensus models [36]. The second type is based on the identification and direction rule-based principle [10,21,22,37,48,49]. While optimization-based approaches are more efficient, they fail to fully leverage the social relationships when guiding DMs in opinion modification. For instance, in the minimum adjustment consensus model, the identified evaluations are updated by minimizing the adjustment needed to achieve a satisfactory level of consensus, which neglects that DMs' evaluations can be easily influenced by other individuals in the social network.

Regarding the identification and direction rule-based methods, the identified evaluations are modified towards the DM with the highest consensus level [37], which did not consider the social relationships among DMs and the tendency of individuals to refer to those they trust. In [10], the evaluations of the identified DMs are adjusted towards the DMs they trust and the group opinion. In [3], the social network DeGroot model is employed to dynamically update the evaluations below the consensus threshold. However, these methods ignore the potential overlapping among clusters and their impact on the CRP. Although [22] considers the intersection of communities, in its consensus-reaching strategy, group and subgroup opinions are utilized as modification references to update the identified evaluations, which fails to utilize the bridging role of the overlapping DMs across different communities. In addition, the consensus-reaching strategy is implemented on an individual basis rather than a cluster basis, which reduces the efficiency of consensus improvement, particularly when dealing with a larger scale of DMs.

Although methods in [21,48,49] guide the consensus-reaching process by leveraging the bridging role of overlapping DMs, the overlapping community-driven feedback mechanism proposed in this study possesses several advantages compared to [21,48,49]: (i) In [21], the group evaluation is solely aggregated from independent portions of subgroups, excluding evaluations from overlapping DMs. This omission may lead to information loss, diminishing the accuracy of the results. However, our approach includes assessments from overlapping DMs when determining the group evaluation, which improves the accuracy of the result without redundancy in the computation process. (ii) Method [21] also neglects the assessments of overlapping DMs when computing the group consensus level, whereas our approach incorporates the evaluations from the overlapping DMs when calculating GCD, thereby enhancing the rationality of the results. (iii) The consensus-reaching strategy in [21] only identifies one pair of subgroups with the minimum consensus level in each iteration and modifies their evaluations pairwise. In contrast, our approach identifies the set of evaluations falling below the consensus threshold and concurrently initiates changes in each iteration, which can enhance the efficiency of the CRP. (iv) Consensus-reaching strategies in [21,48,49] only consider one or two overlapping patterns between communities. In [21], it only considers Pattern 2 outlined in Section 3.3.2 of this paper. This pattern involves identified subgroup pairs that overlap with the same community and are mutually independent. If other overlapping patterns exist within the identified clusters, the modification strategy in [21] becomes inapplicable. Methods in [48,49] analyze Pattern 1 and Pattern 2 outlined in Section 3.3.2, and the case study involves only 20–40 DMs, thus overlooking other potential overlapping scenarios that could affect the consensus-reaching process. The method in [49] focuses more on how trust propagates through these communities to improve consensus. Therefore, the construction of modification references for various overlapping scenarios is not sufficiently addressed. Conversely, the proposed consensus-reaching feedback mechanism encompasses four typical overlapping scenarios and allows the modification strategy to be extended to other overlapping circumstances. Therefore, the

proposed overlapping community-driven consensus-reaching method is more practical and generalizable.

As for analyzing dynamic elements, all methods listed in Table 12 involve evaluation dynamics, as the CRP inherently implies dynamic updates in alternative judgments. Additionally, in practical scenarios, individuals tend to form closer trust relationships with those who share similar opinions. The iterative updates of opinions lead to dynamic changes in social network relationships. Except for considering opinion dynamics, [23] also analyzes trust dynamics and behavior dynamics. However, these approaches do not delve into the dynamic variations in clustering. In our proposed method and [48], the trust evolution resulting from evaluation dynamics and the dynamic clustering induced by changes in trust strength are both incorporated to establish the dynamic clustering-based LSGDM framework. Therefore, these two methods are more effective and flexible compared to other methods.

Following qualitative comparisons, three typical methods are selected for quantitative comparisons. Given that diverse evaluation expression structures are employed by different methods, crisp numbers are utilized to describe DMs' evaluations and apply the same example to determine the ranking of alternatives, which is presented in Table 13.

Table 13 indicates variations in the ranking values of alternatives obtained by different methods. However, the optimal and worst-ranking alternatives (i.e., a_3 and a_1) remain consistent, illustrating the effectiveness of the proposed approach. In GDM methods, discrimination degrees can reflect the ability of different methods to distinguish the performances of alternatives. The higher the discriminability of a decision-making method, the easier it is to identify the optimal choice. The discrimination degree Dis^ϵ of alternatives a_i and the total discrimination degree $TDis^\epsilon$ of a decision-making method ϵ can be computed as follows:

$$Dis^\epsilon(a_i) = \frac{\max_i (EV^\epsilon(a_i)) - EV^\epsilon(a_i)}{\max_i (EV^\epsilon(a_i))} \quad (29)$$

$$TDis^\epsilon = \sum_{i=1}^4 Dis^\epsilon(a_i) \quad (30)$$

where $EV^\epsilon(a_i)$ denotes the ranking value of a_i in method ϵ .

According to Eqs. (29)–(30), the discrimination degree of alternatives in different methods and the total discrimination degree (TDis) of these approaches are shown in Fig. 14. Compared to other approaches, our method demonstrates a stronger ability to differentiate various alternatives. The key reasons for this stronger differentiation include three aspects: First, by using the LINE method for low-dimensional vector representations and subsequent selection degree-based label propagation, our approach effectively captures similarities and relationships between decision-makers. Second, the dynamic clustering of decision-makers based on updated labels and selection degrees enhances the alignment of individual evaluations within communities. Third, our method incorporates an overlapping community-driven consensus-reaching process. This process ensures that decision-makers consider diverse perspectives and adjust their evaluations accordingly, leading to more distinct and well-formed consensus results. These aspects interact to give the constructed dynamic clustering-based LSGDM framework a stronger ability to differentiate among alternatives.

The comparison of consensus iteration rounds for different methods under various consensus thresholds is shown in Table 14. As the consensus threshold increases from 0.7 to 0.9, the number of iterations required for the consensus model gradually increases. At the same consensus threshold, the consensus models in [3,10] require more iterations. This is because these two models implement the consensus-reaching strategy on an individual basis rather than a cluster basis, reducing the efficiency of reaching consensus.

In contrast, [22] and the proposed consensus model require fewer iterations to achieve the same consensus threshold. This is because both methods conduct consensus-reaching on a cluster basis after performing dimension reduction on large-scale groups. Additionally, these methods

Table 13
The alternative ranking determined by different methods.

Alternative	[3]		[10]		[22]		This paper	
	Ranking value	Ranking	Ranking value	Ranking	Ranking value	Ranking	Ranking value	Ranking
a_1	0.235	4	0.518	4	0.201	4	0.538	4
a_2	0.530	2	0.834	3	0.591	2	1.294	3
a_3	0.736	1	1.745	1	0.836	1	3.756	1
a_4	0.269	3	1.552	2	0.307	3	2.633	2

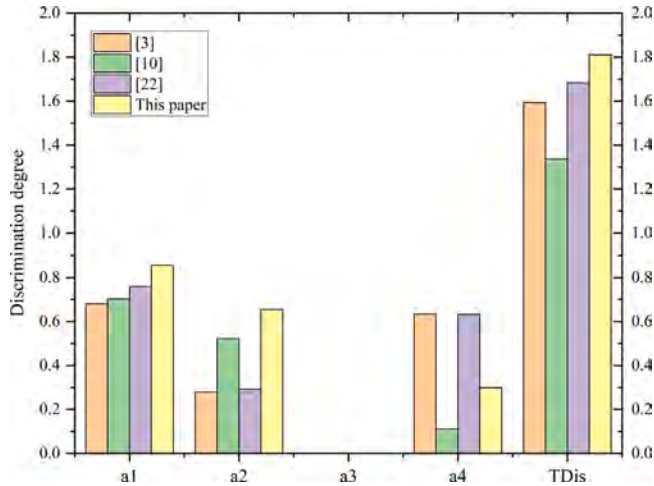


Fig. 14. Discrimination degrees of alternatives and the total discrimination degrees of different methods.

Table 14
Consensus iteration rounds for different methods under various consensus thresholds γ .

Method	$\gamma = 0.7$	$\gamma = 0.75$	$\gamma = 0.8$	$\gamma = 0.85$	$\gamma = 0.9$
[3]	3	8	13	17	25
[10]	2	9	14	18	24
[22]	2	7	11	16	23
This paper	1	5	8	13	19

explore the bridging role of overlapping communities to guide opinion modification. Compared to [22], the proposed method is more efficient because it considers different overlapping scenarios and proposes corresponding consensus-reaching strategies. Therefore, the proposed method can handle more situations, whereas the consensus model in [22] can only deal with one type of overlapping community, limiting the efficiency of opinion modification and consensus improvement.

5.4. Theoretical and practical implications

From the theoretical point of view, this study extends previous theoretical frameworks of LSGDM by overcoming two limitations: (i) the underexplored impact of various overlapping community structures on group consensus-reaching and (ii) the dynamic nature of trust evolution in social network environments. Different from the existing work, this study introduces a more comprehensive understanding of how social networks affect consensus-building by considering different overlapping scenarios in the clustering process. The proposed LINE-based label propagation algorithm enriches the clustering models by considering both local and global network topology in community detection. Additionally, this research provides new insights into the dynamic interplay between trust evolution and CRP, further refining theoretical models of decision-making in large, socially connected groups.

From a practical perspective, this study provides an applicable method to real-world decision-making processes. The dynamic

clustering-based framework, along with the overlapping community-driven feedback mechanism, equips DMs with an effective tool to improve group consensus. This has applications in various domains such as emergency management, public policy formulation, and corporate decision-making, where the ability to adaptively manage group opinions is crucial. The use of overlapping DMs as bridges between communities provides an innovative strategy to reduce conflicts and improve consensus, which can lead to more widely accepted decisions in practical scenarios. The case study using the Film Trust dataset further demonstrates how this method can be adapted to social network-based environments, ensuring its feasibility in a range of LSGDM problems. Moreover, the dynamic nature of the method ensures that it can accommodate ongoing changes in trust and opinion during the decision-making process, making it applicable to real-time scenarios.

6. Conclusion

In LSGDM problems, it is necessary to cluster the large group of DMs into more manageable communities (subgroups) to reduce dimensional complexity. The subsequent consensus-reaching process is essential for mitigating group divergences, which helps generate a decision outcome better accepted by the group. Nevertheless, most existing studies neglect the potential overlaps between different communities. Furthermore, the influence of different overlapping patterns on consensus improvement has not been fully explored. Therefore, this paper proposes a dynamic clustering-based LSGDM framework, where different types of overlapping communities are employed to drive the consensus-reaching process. Through theoretical analysis and method validation, the following conclusions can be drawn:

- (1) The LINE-based label propagation algorithm is developed to cluster the large group of DMs considering overlapping communities. Specifically, the concept of selection degree is introduced to improve the traditional label propagation algorithm with both static network information and dynamic label information, which can enhance the stability of the overlapping community detection result.
- (2) The overlapping community-driven consensus-reaching strategy is proposed to reduce group conflicts. Consensus measurements on evaluation, cluster, and group levels are defined to identify the clusters and evaluations that require adjustments. Then, the evaluations from overlapping DMs are utilized as modification references to construct consensus-reaching strategies that can adapt to various overlapping scenarios, providing a more realistic modeling of practical situations.
- (3) A dynamic clustering-based LSGDM framework is established, where evaluation iteration, trust evolution, and cluster dynamics are analyzed. We validate the effectiveness and rationality of our method through the application to a film selection problem using the Film Trust dataset. Finally, the qualitative and quantitative comparisons with recent typical studies demonstrate the advantages of our method and its applicability in handling other LSGDM problems in social network contexts.

However, this study has some limitations. Firstly, the case study selected only the group of DMs who provided complete evaluations for the films to establish the set of alternatives, without considering

those with missing evaluations. Incomplete information often arises in LSGDM problems. While several studies have proposed estimating missing information through trust relationships, these relationships may become sparse in scenarios with a larger scale of DMs, as seen in scale-free networks. Therefore, how to complete missing information in LSGDM problems deserves further research. Secondly, in this study, the evaluations of DMs are collected from the Film Trust dataset and described using crisp numbers. However, in practice, DMs tend to express their opinions using natural language. Although scholars have proposed linguistic expression structures to characterize the uncertainty in DMs' judgments, their opinions are still constrained within a specific form of expression. Therefore, how to employ natural language processing techniques to directly handle reviews given by DMs and extract their evaluations needs to be further explored. Lastly, this study, like most LSGDM methods, is based on the principle of majority, in which minority opinions are either adjusted or assigned lower weights in aggregation. However, there may be situations where the truth lies in the hands of a few. Therefore, it is worth exploring methods to effectively identify and leverage constructive minority opinions to obtain decision results.

CRedit authorship contribution statement

Zhen Hua: Writing – original draft, Visualization, Validation, Software, Methodology, Formal analysis, Data curation, Conceptualization. **Xiangjie Gou:** Writing – original draft, Visualization, Methodology, Formal analysis, Data curation. **Luis Martínez:** Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Methodology, Formal analysis, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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