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Analyzing the performance of classical consensus models in large scale group decision making: A comparative study

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ABSTRACT

Consensus reaching processes (CRPs) in group decision making (GDM) attempt to reach a mutual agreement among a group of decision makers before making a common decision. Different consensus models have been proposed by different authors in the literature to facilitate CRPs. Classical CRP models focus on achieving an agreement on GDM problems in which few decision makers participate. However, nowadays, societal and technological trends that demand the management of larger scale of decision makers add new requirements to the solution of consensus-based GDM problems. This paper presents a comparative study of different classical CRPs applied to large-scale GDM in order to analyze their performance and find out which are the main challenges that these processes face in large-scale GDM. Such analyses will be developed in a java-based framework (AFRYCA 2.0) simulating different scenarios in large scale GDM.

1. Introduction

Group decision making (GDM) problems, in which multiple individuals/experts with their own attitudes/opinions need to achieve a common solution to a decision problem consisting of several alternatives or possible solutions, have become the focus of a large body of research [1–4]. GDM problems widely exist in diverse application areas that require the joint participation of multiple experts, such as management, engineering, politics and so on [5–7]. In the traditional resolution process of GDM problems [8], the best alternative/alternatives should be chosen after each expert provides his/her own preference over alternatives, disregarding the level of agreement among the preferences of different experts. This often leads to the shortcoming that some experts may not accept the decision result [2], because they might consider that their opinions have not been considered. For this reason, consensus reaching processes (CRPs), in which individuals/experts discuss and modify their preferences in order to reach a collective agreement before making decisions [9], have become an increasingly prominent research topic in GDM problems [10–12].

Classically, GDM problems have been solved by a few number of experts. However, the expansion of technological paradigms, such as e-democracy [6], social networks [13], and marketplace selection for group shopping [14], call for the public attention for the so-called large scale GDM (LGDM) problems, in which a larger number of experts take part in the decision process and responsibility for the decision result. Chen and Liu [15] classified the GDM problems in which the decision makers exceed 20 into LGDM problems. It is noticed that experts have to face a lot of new challenges in terms of the resolution of LGDM problems, such as the higher resources consuming and the time invested for decision making. It requires a higher complexity with respect to the analysis of experts' preferences in LGDM problems, for instance, to detect the conflicts and the closeness amongst experts' opinions, identify the scale of experts that agree/disagree with each other and find coalitions/subgroups of the same or similar interests in the group, etc.

Thorough the study on CRPs over the past few decades, different theoretical consensus models have been proposed [16–22]. On the other hand, in order to provide groups with computer-based decision support systems focused on supporting CRPs, some researches have been done in the development of consensus support systems (CSSs) [20,23–25], based on the implementation of different consensus models.

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Despite the research already conducted on CRPs, there are still some aspects that require improvement. One of them is the demand for managing large groups in such processes. Managing LGDM problems makes more frequent the existence of strong disagreement cases among some experts in the group, therefore the necessity of applying a CRP is higher [26]. As far as we know, most of the existing CRPs are focused on GDM problems with few experts. There is no any depth and systematic study about their performance dealing with LGDM problems yet. Even though, specific proposals for CRPs in LGDM have been introduced [26–28], it seems necessary to make a study about the performance of classical CRPs developed for GDM with few decision makers to evaluate their ability and shortages in the new contexts of LGDM. Consequently, this paper aims at developing a comparative study of different classical CRPs widely used in the literature by using AFRYCA 2.0 [29], a framework which allows to simulate different scenarios for GDM in which decision makers can adapt different behaviors regarding the CRP.

With this study our goal is to answer the following questions:

1. Which is the performance of different types of classical CRPs in the context of LGDM?
   This question is two-fold:
   - The number of experts involved in the GDM can influence the performance of the consensus model, if so, at what extent?
   - A large number of experts make easier to break the collaboration contract to achieve an agreement and non-cooperative behaviors can appear and bias the agreement. Can classical consensus models reach consensus in such LGDM contexts?
2. Is time cost crucial in all classical CRPs to deal with a LGDM problem?
   It also implies a two-fold view:
   - The number of experts involved in the GDM can imply an increasing of time cost in the CRP, can classical consensus models manage the time cost in LGDM?
   - What kind of consensus models deal better with the time cost in LGDM to achieve the agreement?

By a comparative study on the performance of different existing classical consensus models in LGDM problems, the answer of the previous questions could be achieved, and provided some suggestions and necessary conditions that should be added to consensus models in order to manage CRPs in LGDM problems.

This paper is structured as follows: in Section 2, some basics about GDM, LGDM, CRPs and a taxonomy of classical consensus models are reviewed. In Section 3, the framework, AFRYCA 2.0, for the analysis of consensus approaches is briefly introduced. Based on this framework Section 4 introduces and develops a comparative study on performance of different consensus models in LGDM. Section 5 shows new challenges that CRPs should face to deal with LGDM inferred from previous study. Finally, some concluding remarks are provided in Section 6.

2. Background

In this section, GDM problems and several main concepts related to CRPs and a taxonomy about them are reviewed. The notion of large-scale GDM is then revised, as well as some main challenges which experts may encounter during the CRP of LGDM problems.

2.1. Group decision making

GDM is the process of reaching a common judgment or a common solution for a decision making problem, which consists of a set of alternatives or possible solutions, with the participation of multiple individuals. Decision making results made by multiple experts with various types of knowledge and experience are usually supposed to be better compared with those made by only one expert [3].

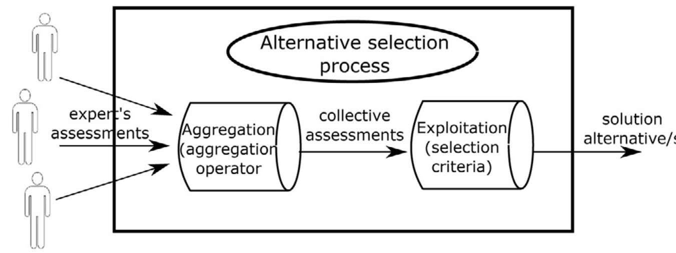

Fig. 1. Selection process for the solution of GDM problems.

A GDM problem can be formally defined as a decision situation in which there are [4]:

1. A group of m individuals/experts, $E = \{e_1, e_2, \ldots, e_m\}$, each one of them has his/her own knowledge and attitude.
2. A decision problem containing n alternatives or possible solutions, which is denoted by $X = \{x_1, x_2, \ldots, x_n\}$.
3. The individuals/experts try to achieve a common solution.

In the common process of a GDM problem, each expert in E expresses his/her preferences over different alternatives in X, by means of a certain kind of preference structure. *Preference Ordering of the Alternatives* [30], *Utility Values/ Utility Function* [31] and *Preference Relation* [32] are some widely used preference representation formats. *Preference Relation* is briefly reviewed below.

For each expert $e_i \in E$, construct a function $\mu_{pi} : X \times X \to D$ where D is the information representation domain and $\mu_{pi}(x_l, x_k) = p^i_{lk}(l, k \in \{1, 2, \ldots, n\})$ denotes the preference degree or intensity of the alternative x_l over x_k in D. Then, these expert's preferences on all alternatives in X can be described as a matrix $P^i = (p^i_{lk})_{n\times n}$. Depending on the information representation domain D, different types of preference relations can be used, such as fuzzy preference relations [4,33], multiplicative preference relations [34] and linguistic preference relations [35–39].

The most commonly used preference structure in GDM approaches is the fuzzy preference relation associated to expert e_i represented by matrix $P^i = (p^i_{lk})_{n\times n}$, where:

1. p^i_{lk} denotes the preference degree associated to expert e_i of alternative x_l to x_k;
2. $D = [0, 1]$, that is, $p^i_{lk} \in [0, 1]$;
3. $p^i_{lk} = 0.5$ indicates indifference between x_l and x_k;
4. $p^i_{lk} > 0.5$ indicates that x_l is preferred over x_k. Especially, $p^i_{lk} = 1$ indicates that x_l is absolutely preferred over x_k ;
5. In order to obtain the consistent preference relations, it is usual to assume the additive reciprocity property, i.e. $p^i_{lk} + p^i_{kl} = 1$ ($\forall l, k \in \{1, \ldots, n\}$).

Regarding GDM solving approaches, there are two common approaches to solve a GDM problem: a direct approach or an indirect approach [8]. In the former approach, the solution can be directly obtained based on the individual preferences of experts, rather than constructing a social opinion first. Meanwhile in the latter approach, a social opinion or a collective preference is computed first, and it is then utilized to achieve a solution for the problem. The classical alternative selection process for reaching a solution to GDM problems contains two phases [40], as shown in Fig. 1: (i) Aggregation phase: by using an aggregation operator, the experts' preferences are combined. (ii) Exploitation phase: by using

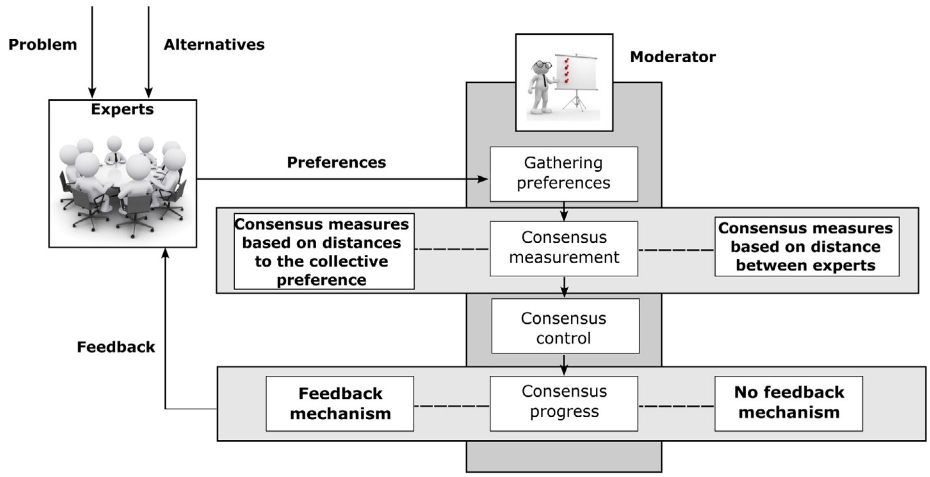

Fig. 2. General CRP scheme.

a selection criterion, an alternative or a subset of alternatives will be obtained as the solution for the problem.

2.2. Consensus in group decision making

If a GDM problem is solved only by the selection process, the existence of agreement amongst experts cannot be guaranteed, which may lead to a solution which cannot be accepted by some experts who feel that their individual opinions have not been taken into consideration [2]. Since a high level of acceptance degree by the whole group could be critical in a number of real-life GDM problems, it is necessary to add a phase so-called "consensus" to the resolution process for GDM problems. A CRP is a dynamic and iterative process consisting of several rounds of discussion, it is designed to reach a compromise before making a decision [2,9]. Reaching consensus implies that experts should modify their initial opinions throughout the CRP in order to bring them closer to the opinions of the rest of the group. The term consensus can be defined to refer to "the mutual agreement produced by consent of all memberships in a group or between several groups" [9]. The concept of consensus has been interpreted from various perspectives, from unanimity to some more flexible interpretations considering different degrees of partial agreement [41]. As one of the most accepted approaches to soften the concept of consensus, the notion of *soft consensus* which is defined as "most of the important individuals agree with almost all of the relevant opinions", was introduced by Kacprzyk et al. based on the concept of "fuzzy majority" [4].

The process of reaching consensus is usually coordinated by a human figure known as moderator. The moderator takes responsibility for supervising and guiding the discussion amongst experts [2,9]. A general CRP scheme followed by a large number of consensus models consists of four main phases (see Fig. 2):

1. Gathering preferences The preferences of each expert are provided and collected in this phase.
2. Consensus measurement The moderator makes use of experts' individual preferences to estimate the current group agreement level by consensus measures. Based on the type of computations and information fusion procedures applied to measure consensus, the existing consensus measures have been classified by Palomares et al. [42] into two categories:
   - Consensus measures based on distances to the collective preference: In this case, firstly a collective preference should be computed by aggregating all individual preferences of experts, then the consensus degrees are obtained by computing the distances between each individual preference and the collective preference [37,43,44].
   - Consensus measures based on distances between experts: In this case, firstly the similarity values between each different pair of experts in the group should be calculated based on the similarity/distance metrics, then the consensus degrees are obtained by aggregating these similarity values [18,45–47].
3. Consensus control The consensus degree obtained previously is compared with a *threshold value* $\boldsymbol{\mu} \in [0, 1]$, which indicates the minimum value of acceptable agreement. If the consensus degree exceeds the threshold value, μ, means that the desired consensus has been achieved, the group moves into the selection process; otherwise, another discussion round should be carried out. It is worth noticing that another threshold value *maxrounds* $\in \mathbb{N}$, which indicates the maximum number of allowed rounds can be introduced in order to prevent a never ending process.
4. Consensus progress A procedure should be adopted to increase the level of agreement throughout the discussion rounds of the CRP. The procedure can also be classified into two categories [42]:
   - Traditionally, such a procedure incorporates a feedback generation process, in which the moderator identifies the farthest assessments from consensus and then advises them to modify their assessments in the direction to increase the consensus degree in the following rounds [9,41]. Each expert has the responsibility to modify his/her own assessments to get close to the collective preference.
   - Some other consensus models employ a procedure without a feedback generation process, by implementing approaches in which the experts' assessments can be updated automatically to increase consensus in the group [44,48,49].

A lot of different consensus approaches have been proposed during the past decades. So far, various criteria have been used to categorize different consensus approaches, such as the reference domain used to compute the soft consensus measures, the coincidence method used to compute the soft consensus measures, the generation method of recommendations supplied to the experts and the kind of measures used to guide the CRP [11]. In this paper it is utilized the categorization introduced in [42] that considers two types of consensus measures and two classes of consensus progress procedures to propose a taxonomy for consensus models, graphically shown in Fig. 3:

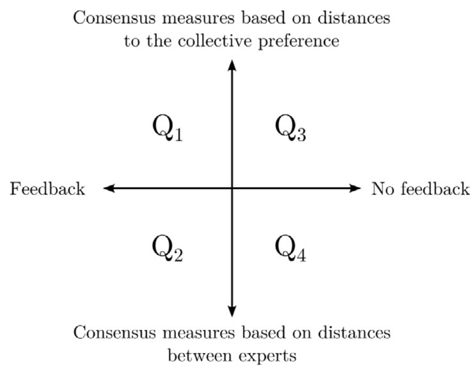

Fig. 3. A taxonomy of approaches for consensus reaching.

Q_1: Consensus models with feedback mechanism and a consensus measure based on computing distances to the collective preference.

Q_2: Consensus models with feedback mechanism and a consensus measure based on computing pairwise similarities.

Q_3: Consensus models without a feedback mechanism and with a consensus measure based on computing distances to the collective preference.

Q_4: Consensus models without a feedback mechanism and with a consensus measure based on computing pairwise similarities.

2.3. Large-scale decision making and its challenge in consensus

Current technological and societal demands have made necessary to make decisions in which a huge amount of participants take part. As a result, LGDM which indicates GDM with a larger number of individuals/experts, attain a greater importance. The presence of a larger number of participants could definitely increase the complexity of a given problem. So far, studies on LGDM concentrate on four categories, i.e., cluster methods in LGDM, CRP in LGDM, LGDM methods, and LGDM support systems [50].

Two main differences between classical GDM and LGDM are: (1) the number of decision makers and the amount of information in the latter case is larger; (2) in LGDM, more time is needed to achieve a final decision, especially when agreement is required.

Some of the challenges that CRPs should face caused by LGDM problems are the following ones:

1. *Non-cooperative behaviors*: Since the amount of decision makers is very large in a LGDM problem, experts cannot cooperate to achieve an agreement. Two typical non-cooperate experts' behaviors in a LGDM problem are described below and noted in this paper as follows;
   - *Refuse behavior* (see Fig. 4): After receiving some suggestions to get closer to the group opinion, the individuals/experts may refuse to change his/her initial preference.
   - *Defense behavior* (see Fig. 5): In this case, the individuals/experts may change his/her initial preference in an opposite direction in order to bias the consensus.

   This paper also refers to the cooperative behavior of experts as *accept behavior*, which indicates the expert will accept the suggestions to get closer to the group.
2. *Subgroup behaviors*: Non-cooperative behavior may be no longer just a personal behavior in LGDM. In other words, when CRPs are carried out in large-scale contexts, there may exist some subgroups of experts who have similar interests and do not want to change their initial positions. They may collaborate to break the collaboration contract [41] at some stage, by refusing to modify their preferences [27], or by moving their preferences on the contrary way in order to bias the final solution for the GDM problem [51]. Hence, it is critical to identify timely and dispose effectively these subgroup non-cooperative behaviors to ensure correct CRPs development.
3. *Minority opinions*: In order to ensure a correct decision result, Xiong et al. [52] spoke highly of the importance of minority opinions in the CRPs and proposed a consensus mechanism to protect such opinions. However, it will be much more difficult to take into account all the minority opinions in a large group situation.
4. *Supervision*: The need for constant human supervision for preferences by either the moderator or experts during the CRP will be much more complex in a LGDM problem [22,26,53,54].

Fig. 4. Refuse behavior.

Fig. 5. Defense behavior.

Other difficulties caused by time cost in a LGDM problem which must be considered in consensus models may be the following ones:

1. Some emergency decision problems ask for a relatively satisfactory result within a short time, which requires effective coordination of the non-cooperative behaviors mentioned above [55]. In LGDM, the existence of non-cooperative behaviors and group non-cooperative behaviors indicate higher time cost in

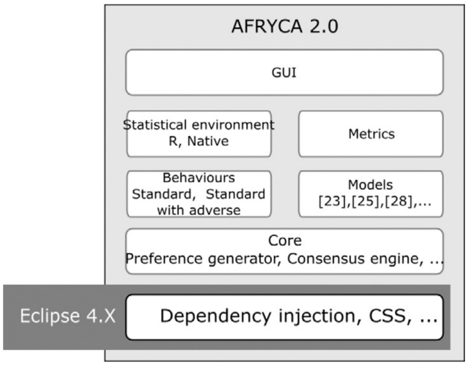

Fig. 6. AFRYCA 2.0 architecture.

CRPs. Then the issue of balancing the relationship between decision quality and time invested emerges.

2. Under a consensus model with feedback mechanism, time cost of supervising and modifying opinions might not only increase the CRP's discussion rounds considerably in LGDM, but also lead to a result that some experts may lose their motivation and interest and then eventually abandon the discussion process [41].
3. The phenomenon that human moderator may tend to consider the opinions of his/her own interest would be more apparent and serious in large-scale decisions, since they need to save time cost. This phenomenon implies that real consensus cannot be reached by the whole group [41]. Although some existing CSSs have took place of human moderator in order to prevent constant supervision by the human moderator [21,22,53], dealing with large-scale CRPs still requires the development of more appropriate architectures that manage the large amount of information efficiently.

3. A framework for the analysis of consensus approaches: AFRYCA 2.0

Our paper aims to analyze the performance of different classical consensus models in LGDM problems and the development of this task is not simple, specially when it is necessary to take into account a large number of experts in the CRP. The necessity of a suitable tool which allows to simulate the performance of the distinct consensus models and the behavior of the experts who take part in the CRP, is clear to achieve our objective. For this reason, this section revises briefly a software so-called, *A Framework for the analYsis of Consensus Approaches* (AFRYCA) [42], that will be used to carry out the simulation of CRPs and the solving process of GDM problems by using different consensus models proposed in the literature. Specifically, the latest version of this software, AFRYCA 2.0 [29], is used to simulate different experts behavior patterns during the CRPs. In technological terms, AFRYCA 2.0 is a component-based application which has been developed by using Eclipse Rich Client Platform (Eclipse RCP) [56], a platform to build and deploy desktop rich client applications easy to maintain and extend. AFRYCA 2.0 [29] uses more than 40 components which are grouped in six types (see Fig. 6):

- *Graphical User Interface (GUI)*: Components which allow to interact with the framework.
- *Statistical environments*: Two statistical environments are included in AFRYCA 2.0, R[1] and a native statistical environment. They are able to carry out Multi-Dimensional Scaling (MDS) of the preferences and the simulation of behavior patterns by means of probability distributions. The statistical environment can be selected during the runtime of the program.
- *Metrics*: Components to analyze several consensus models and the CRPs performance.
- *Behavior patterns*: Components which simulate expert's behavior regarding the advice received. AFRYCA 2.0 includes two behavior patterns: (1) the *standard behavior pattern* (see Fig. 7), which simulates behaviors of experts accept/refuse suggestions; (2) the *standard with adverse behavior pattern* (see Fig. 8), which allows to simulate behaviors of experts accept/refuse/defense recommendations.
- *Models*: Components which implement consensus models proposed in the literature. Each component corresponds to a consensus model and it includes the different phases and parameters considered in such a model. AFRYCA 2.0 implements eight consensus model components [26,28,31,57–61]. Furthermore, to carry out this paper, another consensus model has been included in AFRYCA 2.0 [62].
- *Core*: Components which implement the main features of AFRYCA 2.0 such as, preference generator, consensus engine, etc.

Fig. 7. Optional behaviors of experts in *standard behavior pattern*.

Fig. 8. Optional behaviors of experts in *standard with adverse behavior pattern*.

Therefore AFRYCA components provide different functionalities that can be used for:

[1] https://www.r-project.org/.

Table 1
Behaviors default values.

	Standard	Standard with adverse
p	0.5	0.5
c	–	0.25
μ	0.05	0.05
Size	0.2	0.2

- The performance analyses of consensus models to analyze their advantages and weaknesses.
- Performance analysis of a consensus model under different situations by using its setting configuration.
- Selection of the most suitable consensus model for a specific type of GDM problem through its results reporting.
- Easy comparison of different consensus models by using the graphical interface.

AFRYCA 2.0 allows to carry out experiments with different consensus models implemented in the framework. It is possible to evaluate and compare the performances of different consensus models in LGDM by AFRYCA 2.0, since it provides important information such as initial consensus degree, final consensus degree, ranking of alternatives and final solutions. Furthermore, AFRYCA 2.0 is able to show graphically the state of the experts' preferences for each round by means of a graphical 2-D representation, with MDS [63] (see Fig. 9).

When using AFRYCA 2.0 to simulate the resolution of a GDM problem with a consensus model implemented, the methodology can be divided into 5 steps:

1. *Framework defining*: A specific example of GDM problem should be settled, to be solved by applying the pre-selected consensus model.
2. *Model choosing*: A consensus model is chosen from those included in the framework.
3. *Parameters configuration*: Configure the parameters for the consensus model and behaviors of experts, such as consistency of generated preference relations, consensus thresholds, aggregation operators, etc.
4. *Simulation of the CRP*: Once the consensus model settings are fixed, the CRP should be carried out.
5. *Alternative selection process and analysis of the results*.

In AFRYCA 2.0, two behaviors patterns can be simulated (see Figs. 7 and 8). In the *standard behavior pattern*, the experts are allowed to accept/refuse suggestions. In the *standard with adverse behavior pattern*, the experts are allowed to accept/refuse/defense suggestions. To carry out such behaviors different aspects are taken into account in AFRYCA 2.0:

- In the standard behavior pattern, the probability for experts to accept suggestions has been simulated by a binomial probability distribution, which is configured by a parameter p.
- In the standard with adverse behavior pattern, besides the refuse behavior, the defense behavior has also been taken into consideration. Hence, besides parameter p mentioned above, a new parameter c will be added to configure another binomial probability distribution, which is used to simulate the probability for experts to move into an opposite direction of suggestions.

Although all parameters can be configured in AFRYCA 2.0, this framework has been defined with some default values (see Table 1).

4. Comparative study on the performances of classical CRPs models in LGDM

In this section, a comparative study on the performance of classical CRPs models in LGDM is carried out. First, different representative consensus models with different features are selected for the study. Second, it is necessary to describe the LGDM scenarios in which the comparative study will be developed. Afterwards, the simulation by using AFRYCA 2.0 will be carried out for all models in each scenario defined previously; obtaining different results that will be analyzed for each consensus model in order to find out necessary conditions to reach consensus in LGDM problems. And from such individual analyses a comparative analysis among all models is performed. Eventually, previous study will support us to obtain key characteristics that may be necessary to add to classical CRPs for dealing successfully with LGDM problems. In this way, if it is possible, managers/decision makers will be able to select suitable classical consensus models for LGDM, and even construct some new appropriate consensus models which fit such a type of problems.

4.1. Choosing classical CRPs for study

Due to the multiple proposals introduced in the specialized literature to carry out CRPs in GDM before developing our comparative study it is necessary to choose several classical CRPs to show their performance in LGDM. Therefore for such a selection and according to the taxonomy revised in Fig. 3 from [42], one representative model from each quadrant is selected:

- *Representative model in* Q_1: consensus model with a feedback mechanism and a consensus measure based on computing distances to the collective preference. The model selected was proposed by Herrera-Viedma et al. in [31], it has been selected because:
  - ◯ It follows the soft consensus view [11].
  - ◯ It is the first attempt to use proximity measures taking place of the moderator.
  - ◯ Both consensus measure and proximity measures are based on the comparison of the individual solutions and the collective solution.
  - ◯ The comparison for alternatives is done by comparing the position of the alternatives in each solution, which allows us to know the real consensus situation in each moment during the consensus process.
  - ◯ It allows experts to express their preferences by using different preference structures and then uniform diverse preferences into fuzzy preference relations.

  The Herrera-Viedma et al.'s consensus model needs several parameters, for its implementation in the simulation framework, which are briefly introduced here (see [31] for further detailed descriptions):
  - β: parameter to control the OR-LIKE of the aggregation operator that computes the global consensus degree.
  - Aggregation quantifiers: parameters of the linguistic quantifier used to compute the collective preference by means of the OWA operator.
  - Exploitation quantifiers: parameters of the linguistic quantifier used to compute dominance and non-dominance degrees and conduct preferences of experts into preference orderings.
- *Representative model in* Q_2: consensus models with a feedback mechanism and a consensus measure based on computing pairwise similarities. The model selected is the proposed by Chiclana et al. in [57], because:
  - Initially it was introduced as a framework for integrating individual consistency into a consensus model.

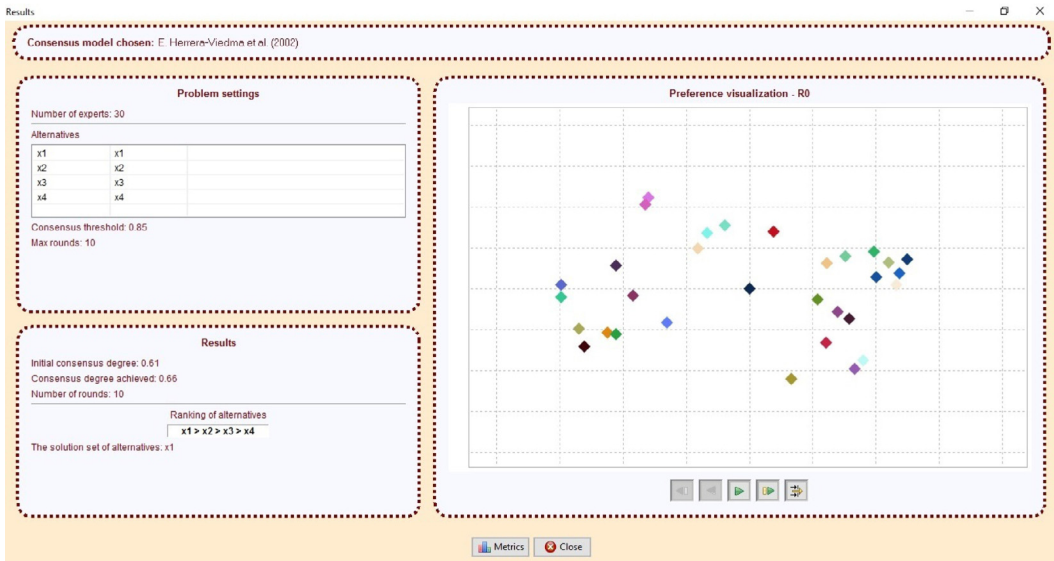

Fig. 9. AFRYCA 2.0 results.

- It has been the basis for further extensions for AHP consensus models introduced by Dong et al. [64].
- Also it has been the basis for a linear optimization model for reaching consensus proposed by Zhang et al. [62].

Similarly to the previous model, Chiclana et al.'s one needs also several parameters for the simulation (further detail about parameters in [57]):

- B: consistency threshold for preferences.
- θ_1, low consensus threshold: if consensus degree is lower than this value, a low consensus preference search is applied.
- θ_2, medium consensus threshold: a medium or high consensus level is applied depending on whether consensus degree is lower or higher than this value, respectively.

• *Representative model in* Q_3: consensus models without a feedback mechanism and with a consensus measure based on computing distances to the collective preference. In this case, two models proposed by Wu et al. in [58] and Xu et al. in [60] are selected because they have similar characteristics, but the former one deals with individual consistency and it is worthy to analyze in this type of consensus models. Therefore, we preferred in this case to study both of them because due to the lack of feedback mechanism their simulation will be easier (see Remark 1):

- Both models considered are simple and straightforward.
- These two consensus models can be easily extended with different features generalizing them.
- In the achievement of a predefined consensus level, each individual preference relation is still ensured to be of acceptable consistency [58].
- Both the individual consistency and the group consensus are stressed in the consensus process introduced in [58].

Some parameters in Wu et al.'s consensus model are necessary for its simulation (please refer to [58] to see detailed descriptions):

- CI: individual consensus threshold.
- β: update coefficient for assessments.
- W_i: experts weights.

The parameters for Xu et al.'s consensus model are shown below (see [60] for further detail):

- CI: individual consensus threshold.
- γ: group consensus threshold.
- W_i: experts weights.

• *Representative model in* Q_4: consensus models without a feedback mechanism and with a consensus measure based on computing pairwise similarities. The model selected in this case is proposed by Zhang et al. in [62], because:

- It optimally preserves the original preference information when constructing individual consistency and reaching consensus.
- This model extends the consistency-driven consensus model of Chiclana et al. to ensure a minimum cost of modifying preferences.
- It can be used not only for conducting the CRP, but also to reach a high level of consistency for each individual preference relation.

The parameters necessary in Zhang et al.'s model for the simulation are (see [62] for detailed descriptions):

- *cl*: consistency level for each preference. The expert's preferences change and each one has to reach this minimal consistency threshold.
- *ccl*: consensus consistency level. The consensus among the different preferences have to reach this minimal consensus threshold.

4.2. LGDM scenarios

Earlier it was pointed out that LGDM problems present several challenges. One of the most important is the different behaviors which appear in the CRP, due to the large numbers of experts involved in it. It is vital to take into account that many experts can present a non-cooperate behavior in real life and, although these experts can refuse the suggestions provided or even go in an opposite direction of the suggestions, they can never been ignored in the evaluation of CRP in LGDM. For this reason, it is necessary to define different *scenarios* which adjust to these challenges by simulating different behaviors. In this way, different simulations as real as possible are proposed. AFRYCA 2.0 first generates consistent fuzzy preference relations, according to [65], for the experts involved in the LGDM and then it will develop the consensus simulation in the following three scenarios (initial preferences are the same for all simulations):

- *Scenario 1*: In this scenario, all experts accept all the recommendations. This kind of scenario is the *ideal* one but not very common in real world problems. It is interesting check how classical consensus models work with favorable conditions.

- *Scenario 2*: In this scenario, 80% of the experts accept all the recommendations. On the other hand 20% of the experts present a defense behavior.
- *Scenario 3*: In this scenario, 70% of the experts accept all the recommendations. On the other hand 20% of the experts refuse the suggestions and the 10% of the experts present a defense behavior.

Remark 1. It is important to highlight that those consensus models without feedback will perform similarly in the three scenarios because they do not consider the experts' opinions after round one and then, the experts' behavior is not meaningful in their performance.

4.3. Results and analysis

This section presents an experimental study to compare the performance of different consensus models in LGDM, during the resolution of GDM problems with a large amount of experts.

Therefore, let us suppose the following LGDM problem: the International Olympic Committee organizes a special committee which is composed of 30 members from all over the world $E=\{e_1, e_2, \ldots, e_{30}\}$, to make a decision on the place where the Olympic Games in 2040 will be held. It is final selection round and there are only four candidate cities: $X=\{x_1$: Paris, x_2: Tokyo, x_3: Madrid, x_4: New York$\}$.

All preferences are expressed as fuzzy preference relations generated by AFRYCA, the corresponding data sets are available in the public access of AFRYCA website.[2] To find a satisfactory solution for this problem, the consensus threshold and the maximum consensus discussions rounds in the CRP are set as $\boldsymbol{\mu}=\mathbf{0.85}$ and **maxround = 30** respectively. *Maxround* has been selected for sake of clarity about consensus models performance, but usually is much smaller. Hence, if the consensus threshold, μ, is not reached after 30 discussion rounds, the simulation stops and then the results at that round are shown indicating that the consensus has not been reached.

This comparative study is carried out on the previous LGDM problem, such that the five consensus models selected in Section 4.1 will be applied to it taking into account the different scenarios of application.

Remark 2. Wu et al.'s model measures consensus with Individual Consensus Indices $ICI(P_i)=d(P_i, P_c)$ for each $e_i \in E$ [58], and Xu et al.'s model measures consensus with Group Consensus Index (GCI) [60]. To facilitate the comparative analysis in this section, benefiting from the idea in [42], the consensus degrees for Wu et al.'s model and Xu et al.'s model are given by $1-max_i\, ICI(P_i)$ and $1-GCI$, respectively.

For each simulation performed, experts behaviors have been configured with the parameter values shown in Table 1. The consensus models have been configured with the parameter values shown in Table 2. Results of the LGDM problem resolution with different consensus models are shown in Tables 3–5 , keeping in mind that the results in Table 5 are not sensitive to experts' behaviors (see Remark 1).

4.3.1. Analysis for each representative model

Here a single analysis for each consensus model according to its performance in the different scenarios for the LGDM problem is developed. Such an analysis consists of a brief explanation of the results obtained with their graphical visualization together an analysis of its performance inferring the main advantages and disadvantages of each model.

- Herrera-Viedma et al.'s model [31]
  - ○ Simulation results:

    This model reaches consensus in the three scenarios evaluated, even when there exists non-cooperative behaviors such as in scenarios 2 and 3 (see Fig. 10). Evidently such non-cooperative behaviors may imply more discussions rounds (Scenario 3 needs 8 discussion rounds, others only 6).

    The ranking of alternatives and the solution set of alternatives in all the scenarios are the same which shows that model is robust and coherent in their consensus process.
  - ○ Analysis:

    It is worth noticing that this model weights the alternatives for computing the consensus measure by using S-OWA OR-LIKE operator [66]. By using a parameter β, that bounds the impact of non-cooperative behaviors to a certain degree.

    That is the reason why the simulation results in Scenarios 1, 2 and 3 have similar performances. However, it should be remarked that experts' consensus degree on each alternative is based on an average operator that does not weight expert's behavior in the CRP process. Hence, the impact of non-cooperative behavior is limited to some extent but not in a general way. If we look carefully at Fig. 10 some experts, in Scenarios 2 and 3, seems to be quite far away from mutual agreement. Therefore, to show the good performance of the model is limited, we carried out a new simulation in which the consensus threshold was fixed as $\mu=0.9$, in such a case the scenario 2 could not reach consensus after *maxrounds* = 30 (see Fig. 11), due to the averaging process is not enough for this situation.

    Based on previous analysis, in order to guarantee a robust and correct performance of this model in LGDM, it is necessary the weighting of the set of alternatives and include some penalization in the computation of the consensus degree to decrease the impact of behaviors in Scenarios 2 and 3.
  - ○ Advantages:

    Benefiting from the simulation results and the analysis, it can be seen that the performance of Herrera-Viedma et al.'s model in this LGDM could be good because:
    - – The existence of refuse and defense behaviors can be managed by using S-OWA OR-LIKE operator but not in all situations;
    - – The decision results tend to be robust in different scenarios.
    - – The number of discussion rounds necessary to reach consensus is relatively small taking into account the LGDM problem.
  - Disadvantages:
    - – As is shown in Fig. 10, although the model reaches the consensus, there are some experts far away from the mutual agreement, which indicates that the final consensus is reached by *ignoring* some experts' opinions.
    - – The weighting of alternative set versus the weighting of experts regarding their behavior can lead to deadlock situations in which agreement is not reaching.
- Chiclana et al.'s model [57]
  - Simulation results:

    Unlike the previous one, this model just reaches the consensus within the *maxrounds* in Scenario 1 with 13 rounds, but not in Scenarios 2 and 3 in which not all experts accept suggestions from feedback process. Additionally, the ranking obtained by the model in different scenarios and solution set are not robust.

[2] http://sinbad2.ujaen.es/afryca/.

Table 2
Consensus models parameters.

Wu et al. [58]	Xu et al. [60]	Zhang et al. [62]	Herrera-Viedma et al. [31]	Chiclana et al. [57]
$\mu = 0.85$ $\beta = 0.8$ $\bar{CI} = 0.15$ $w_i = \frac{1}{30}, i = 1, \ldots, 30$	$\mu = 0.85$ $\gamma = 0.2$ $\bar{CI} = 0.15$ $w_i = \frac{1}{30}, i = 1, \ldots, 30$	$\mu = 0.85$ $cl = 0.95$ $ccl = 0.85$	$\mu = 0.85$ $\beta = 0.8$ *Aggregation quantifier* $= F_{most}$ *Exploitation quantifier* $= F_{as\ many\ as\ possible}$	$\mu = 0.85$ $B = 0.8$ $\theta_1 = 0.7$ $\theta_2 = 0.8$

Table 3
CRP simulations results with Herrera-Viedma et al.'s model [31].

Herrera-Viedma et al. [31]	Initial consensus degree	Final consensus degree	Number of rounds	Ranking	Solution
Scenario 1	0.61	0.87	6	$x_1 \succ x_2 \succ x_3 \succ x_4$	x_1
Scenario 2	0.61	0.86	8	$x_1 \succ x_2 \succ x_3 \succ x_4$	x_1
Scenario 3	0.61	0.85	8	$x_1 \succ x_2 \succ x_3 \succ x_4$	x_1

Table 4
CRP simulations results with Chiclana et al.'s model [57].

Chiclana et al. [57]	Initial consensus degree	Final consensus degree	Number of rounds	Ranking	Solution
Scenario 1	0.603	0.855	13	$x_1 \succ x_4 \succ x_2 \succ x_3$	x_1
Scenario 2	0.603	0.72	–	$x_4 \succ x_2 \succ x_1 \succ x_3$	x_4
Scenario 3	0.603	0.703	–	$x_4 \succ x_2 \succ x_1 \succ x_3$	x_4

Table 5
CRP simulations results with no feedback consensus models.

Models without feedback	Initial consensus degree	Final consensus degree	Number of rounds	Ranking	Solution
Wu et al. [58]	0.568 (0.432)	0.72 (0.28)	–	$x_1 \succ x_2 \succ x_4 \succ x_3$	x_1
Xu et al. [60]	0.303 (0.697)	0.876 (0.124)	4	$x_1 \succ x_2 \succ x_4 \succ x_3$	x_1
Zhang et al. [62]	0.605	0.85	1	$x_1 \succ x_2 \succ x_3 \succ x_4$	x_1

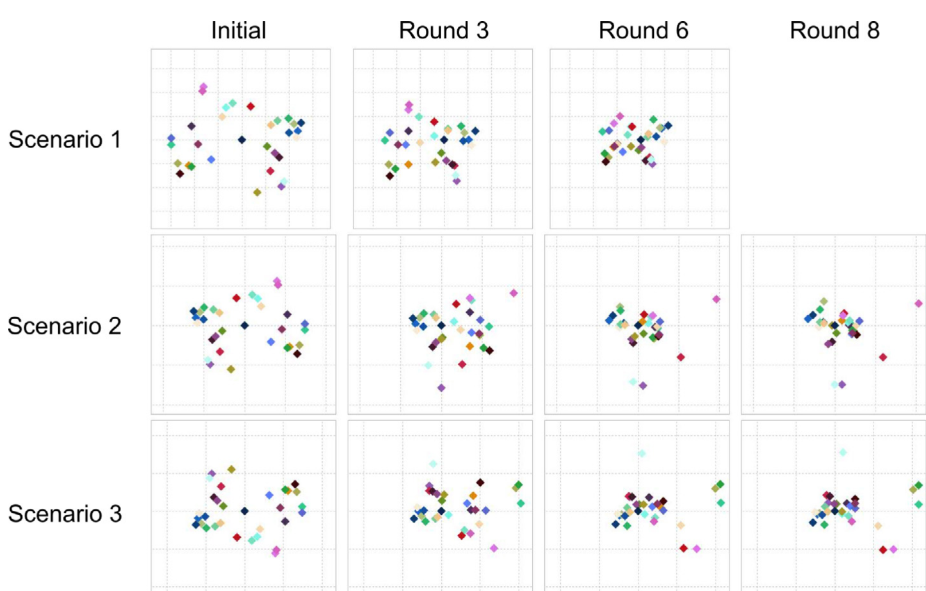

Fig. 10. MDS visualization of CRP using Herrera-Viedma et al.'s model [31].

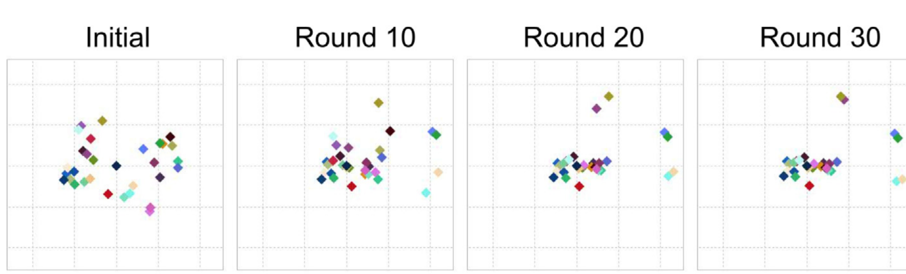

Fig. 11. MDS visualization of CRP using Herrera-Viedma et al.'s model [31] with a consensus threshold 0.9.

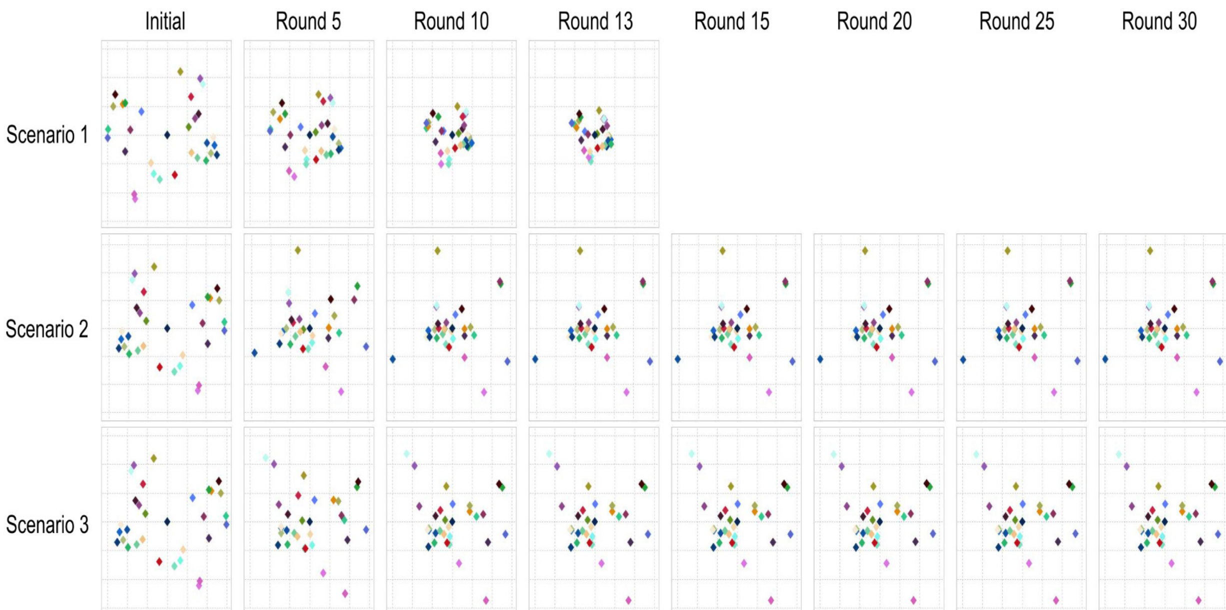

Fig. 12. MDS visualization of CRP using Chiclana et al.'s model [57].

In Fig. 12 can be seen that model cannot effectively manage non-cooperative behaviors of Scenarios 2 and 3.

- Analysis:

  This model deals with consensus at different levels: relation, alternatives and pair of alternatives. The *consensus on the relation* is calculated based on the average of all alternatives, and the *consensus on alternatives* is calculated based on the average of the *consensus on pairs of alternatives*. The weights of experts have been neither determined nor updated based on their behaviors during the CRP when people calculate the consensus. Besides, the proximity degree is calculated in a similar way of the consensus degree based on an average operator, there are not any mean to detect and deal with non-cooperative behaviors during the feedback process, that is the reason why the existence of non-cooperative behaviors leads to deadlock in the consensus process. However, in lots of practical situations of LGDM problems, experts' non-cooperative behaviors cannot be avoided. Hence, the current model need improvements to fit real-world LGDM problems.
- Advantages:
  - In the ideal situation when all experts accept suggestions, the consensus can be successfully reached within several discussion rounds but more than previous model;
  - Determined by the construction of the model which adopts different feedback methods when reaching different consensus degrees, the CRP saves human-being efforts to a certain degree by limiting the rounds for specific experts to change their preferences;
- Disadvantages:
  - The existence of non-cooperative behaviors is not well managed by the model and leads to situations in which the consensus cannot be reached;

- Wu et al.'s model [58]
  - Simulation results:

    Taking into account Remark 1, this model does not consider experts' behaviors because there is not a feedback mechanism in the model. Therefore, the results shown Fig. 13 are the same for the three scenarios, and it can be seen that the model cannot reach the consensus threshold, μ, in any of them within *maxrounds*.
  - Analysis:

    Due to the fact that in this model just one expert's preferences are changed in each round, the consensus process is very slow for LGDM and then a large amount rounds of changing will be needed to reach the consensus threshold by the group.
  - Advantages:
    - Behaviors not affect to the CRP;
    - This model considers not only the group consensus, but also the individual consistency at the same time.
  - Disadvantages:
    - Each round changes only one expert's preferences, which result in a slow process to achieve agreement, especially in large-group problems.
    - Due to the consensus process in this model, it might happen that expert's preferences close to the collective preference should be changed, because the expert is the farthest from the group.
    - This model ignores real experts' preferences because there is not a feedback mechanism that guides experts to express their genuine modified preferences.
- Xu et al.'s model [60]
  - Simulation results:

    Similarly to the previous model, the results in Fig. 14 are valid for all scenarios (see Remark 1). In this case the consensus model reaches the consensus threshold, μ, with just four rounds.
  - Analysis:
    - This consensus model carries out the consensus progress without feedback mechanism but unlike the Wu et al.'s model, in this case the experts' preferences changed in each round are much more than in [58].
    - These changes are carried out based on a group and individual indexes that optimize the distances among experts by means of a quadratic program, which makes the CRP more efficient to reach the consensus threshold.
  - Advantages:
    - Its efficiency to reach consensus within few rounds due to the mathematical programming process.
    - Behaviors not affect to the CRP.
  - Disadvantages:

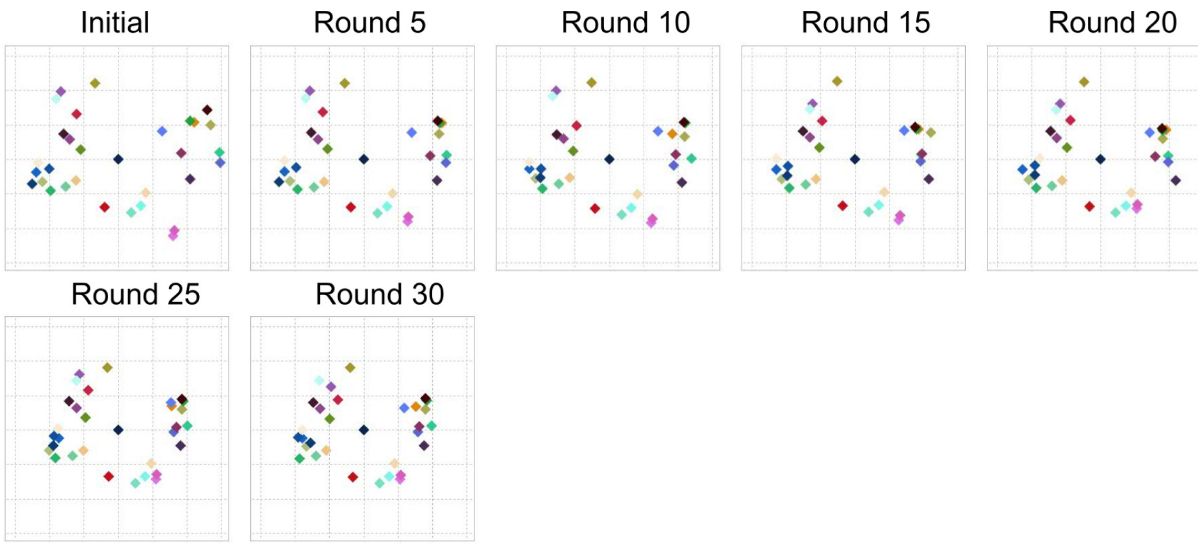

Fig. 13. MDS visualization of CRP using Wu et al.'s model [58].

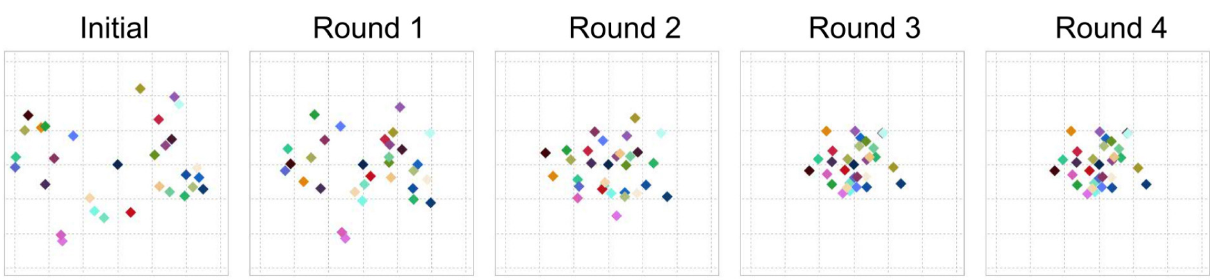

Fig. 14. MDS visualization of CRP using Xu et al. model [60].

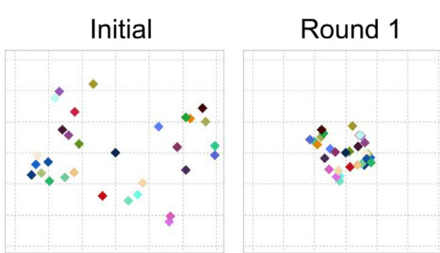

Fig. 15. MDS visualization of CRP using Zhang et al. model [62].

– It does not consider the individual consistency to reach the consensus despite no experts' uncertainty is involved in the revised preferences.
– As a consensus model without feedback, it ignores real experts' preferences.

• Zhang et al.'s model [62]
  • Simulation results:
    This model is the last one with no feedback and again all scenarios obtain the same results graphically shown in Fig. 15. It is remarkable that this model reaches the consensus threshold, μ, just in 1 round, because it just looks for the preferences that achieve an agreement by means of a linear optimization model.
  • Analysis:
    In spite of this simulation the model performs quite well, we should be aware that this model presents an important risk, because the linear optimization model utilized for computing and controlling the consensus process, might be irresolvable and hence other model should be applied to achieve the agreement.
  • Advantages:
    – If the linear optimization consensus model can be solved for the LGDM problem the consensus threshold can certainly be reached within one round.
    – Zhang et al.'s model takes into account not only group consensus but also individual consistency.
  • Disadvantages:
    – Since the restrictions of linear optimization model are very strict, it is hard to determine when a consensus threshold can be reached a priori.
    – Despite some experts' preferences are substantially changed, it ignores real experts' preferences, that it is a common drawback of consensus models without feedback;
    – In Zhang et al.'s model, the time cost is highly dependent on the number of experts, so this model presents an important problem of scalability.

4.3.2. *Comparative analysis*

Taking into account research questions introduced in Section 1 and looking at the previous results as a whole. There are some important issues that should be stressed:

• Even though consensus models without feedback mechanism are not affected by non-cooperative behaviors like the models with feedback, the former ones with their automatic changing strategies highly impact on the expert's preferences changing many of them in each round that can initially seem more suitable for the context of LGDM to reach the consensus threshold, but even these models might not be able to achieve the consensus threshold, μ established in the LGDM, either. They face the scalability problem with more difficulties in LGDM than the latter models because of their mathematical background to carry out the consensus progress. Eventually the large modification of experts' preferences in such a type of problems without considering experts' genuine opinions can lead to decisions not accepted by the large group.

- Due to the fact of unaccepted decisions by experts within the large group, it should be considered the use of consensus models for LGDM despite could be less efficient to achieve the agreement. But in such cases experts can perform refuse/defense behaviors not only just in one round, but also across the whole consensus process. Therefore, the performance of classical consensus models with feedback mechanisms in LGDM with this real-world circumstances in which the consensus model cannot reach an agreement because these experts does not follow the collaboration contract [41] such as it happens in [31,57] although the Herrera-Viedma's model shows a better management of LGDM, even with non-cooperative behaviors, and seems promising to deal with any LGDM problem with just some adjustments.
- Comparing the performances of Herrera-Viedma et al.'s model and Chiclana et al's model in Scenarios 2 and 3, it seems clear that the use of weighting processes for computing consensus fits better LGDM problems, hence it appears the opportunity to revise/penalize weights for experts based on their behaviors during the CRP when calculating the collective preference can improve the performance to ensure the reaching of the consensus threshold in these models.
- Analyzing Figs. 10–12 it is easy to see that models with feedback from experts face the non-cooperative behaviors and when they are able to reach the consensus threshold, several experts are still far away from the mutual agreement and that is the reason that sometimes agreement is not possible to reach. However, when models without feedback reach the consensus threshold the cohesion of the different experts is higher. This issue is quite interesting for further analysis later on.

5. New challenges

As it is revised in Section 2, CRPs needs to deal with several challenges and difficulties when they are applied to LGDM problems that have already been noticed by experts, such as the higher time consuming, the need for more time on constant preferences supervision and the higher complexity with respect to dealing with experts' non-cooperative behaviors. All these challenges have been clearly visualized in the case study. To overcome these challenges, Palomares et al. [27] provided several tools to detect and manage the non-cooperative behaviors in the context of LGDM:

- A fuzzy clustering-based scheme was used to detect non-cooperating individuals or subgroups in their research. In [28] was proposed an extended method to manage participators' behavior in CRP in LGDM, in which, a weighting approach coorperates with uninorm aggregation operator which determines the importance weights of participators according to their overall behavior across the CRP, and thus overcomes the shortage in [27] in which the participators' importance weights cannot be increased again, even though they change their attitudes and decide to adopt more cooperating behaviors.
- Palomares et al. [26] proposed a semi-supervised multi-agent system which reduces time cost of preference supervision and allows experts to revise preferences manually when human supervision is convenient and necessary, which can be regarded as a consensus model with semi-feedback mechanism.

According to our previous study, it is clear that not all the classical consensus models are appropriate for managing LGDM problems. Therefore, although new consensus models are necessary to deal with LGDM, first it should be analyzed if the improvement of classical existing models is a better way to face the challenges of LGDM. Some models can be easily improved to fit the context of LGDM, whereas others can be too much complex and maybe it is better to design other type of specific consensus models for LGDM.

By studying the different performances of the consensus models in the comparative study, it can be observed several key conditions that can be added to consensus models in order to manage LGDM problems in a suitable way. These new conditions can be summarized as below:

1. For consensus models with feedback mechanism:
   - *Weighting measures*: Consensus and proximity measures based on the distance offer an easier and effective way to weight the experts based on their behaviors during the CRPs when calculating the collective preference.
   - *Weighting alternatives*: If alternatives are also weighted when calculating the consensus measure the convergence to consensus threshold could be quicker.
2. For consensus models without feedback mechanism:
   - *Automatic changing scheme*: It should be able to manage multiple experts' opinions at each round otherwise it is not adequate for LGDM problems.
   - *Flexibility*: The conditions of optimization models should be flexible enough to reach consensus when they are used to deal with LGDM problems.

From the results and analyses obtained in the comparative study together the previous conditions, we can figure out several new challenges which should be faced by consensus models within LGDM problems in the future:

1. *Weighting processes*:
   - Within consensus models with feedback mechanism, different weighting mechanisms not only for experts but also for alternatives can provide suitable ways to penalize non-cooperative behaviors in LGDM.
2. *Optimization models*:
   - Within consensus models with feedback mechanism, they should consider seriously time cost for consensus models in LGDM.
   - The restrictions should be decreased or make more flexible in order to adapt them to LGDM, otherwise they become irresolvable.
3. *Hybrid consensus models*:
   - According to our intuition and the visualization of the different models in the previous study, it makes sense to think that in real-world LGDM, it may be useful the use of models without feedback mechanism when there is a cohesive group/subgroup and models with feedback when the group/subgroup is diverse.
4. *Time cost versus experts' willingness*:
   - Setting up consensus models for LGDM considering experts' real willing and keeping or decreasing the time cost at the same time is a promising research topic.

6. Conclusions

The need of solving LGDM under agreement demands CRPs able to deal with these problems. Even though a few new specific proposals of CRPs for LGDM have been done, there have not been carried out so far a study about the performance of classical CRP models designed for GDM problems with a small number of experts within LGDM problem. Therefore, this paper has utilized the consensus simulation framework AFRYCA 2.0 to carry out a comparative study of different types of classical CRPs in different scenarios that are similar to the ones that can be found in real-world LGDM.

From the results obtained, it is clear that the straightforward application of such classical consensus models to LGDM is not always working well, but some models can be easily adapted to deal with LGDM with some improvements that have been pointed out in the analyses provided across the paper.

Finally, some new challenges, that consensus models should cope with in LGDM problems, have been elicited to show their needs if they want to obtain successful results in their performance.

As future research it should be interesting carry out specific analysis of consensus models, such as [12,21,39,48], in addition to the a general study.

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