Dealing with Diversity and Novelty in Group Recommendations using Hesitant Fuzzy Sets

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***Abstract*—Diversity and novelty are appreciated features by users of recommender systems, which alleviate the information overload problem. These features are more important in recommendation to groups because members interests and needs differ from each other or are even in conflict. Various techniques have been used to recommend to groups. However, these techniques apply an aggregation step that imply a loss of information, which negatively affect the recommendation. We aim at avoiding the negative influence of the aggregation step considering the various interests and needs of the group members as the group hesitation, thus, our proposal uses Hesitant Fuzzy Sets to model the group information. A case study is performed to evaluate the proposal, whose results show its performance regarding recommendation diversity, novelty and accuracy.**

I. Introduction

The exponential growth of information available online to users, makes that supporting them in information overload scenarios has become a key task. Recommender Systems (RSs) [1], [2] filter available alternatives regarding users interests or needs, which results in a customized list of suitable alternatives for each user. Among the various techniques for RSs, collaborative filtering (CF) [3], [4] is the most successful one.

The effectiveness of a given RS has been traditionally evaluated using the error when predicting ratings, but users also perceive other recommendation features [5], such as diversity and novelty of the recommendations. Users value diversity because the recommendation covers additional aspects to their taste [6]. Regarding novelty, users may perceive recommended items as more elaborated when they are not correlated with items in their profile [5]. Diversity and novelty are the most fundamental dimensions of a RS beyond accuracy [7], while accuracy should also be considered to deliver good recommendations.

In this paper, we focus on recommendation targeted to groups. There are items that are consumed by groups because of their social features. An example of this situation is some friends who want to eat at a restaurant or to select a movie to watch together. The group recommendation should, therefore, satisfy all group members. This scenario has been widely explored in the literature, which resulted in several approaches for group recommender systems (GRSs) that extend individual RSs through the aggregation of individual information [8], [9]. The aggregation, however, may result in loss of important features of the information, such as distribution or shape. An important challenge for GRSs is to keep such features to provide better recommendations.

Torra et al. [10] extended Fuzzy Sets [11] introducing Hesitant Fuzzy Sets (HFSs), in which the membership function can provide multiple membership values and serves as a way of modelling hesitation within fuzzy sets. In this work, we model the various ratings of the members for a given item as the hesitation of the group about the rating of such item in order to manage the group recommendation.

The proposal, Hesitant Group Recommender Model (HGRM), is the basis for a GRS based on CF whose aim is to keep the maximum information in the recommendation process through the usage of HFSs, which allows to bypass the aggregation process in the group neighbourhood search. This characteristic should allow to increase the recommendation diversity and novelty.

This contribution is structured as follows. Section II shows preliminary concepts about GRSs and HFSs. Section III details the proposal of a HGRM. Section IV shows the findings of the case study and discusses the results. Finally, Section V concludes the paper.

II. Preliminaries

GRSs and HFSs basic concepts, needed to understand our proposal, are reviewed in this section.

A. Group Recommender Systems

GRSs face new challenges that individual RSs do not present [8], [12], being the most important one the variety of members tastes. The most common GRS approach is based on CF [13], [14], which builds the neighbourhood of an active user to recommend items that neighbours have positively experimented in the past. It works with three main sets: users, $U = \{u_1, \ldots, u_q\}$, items, $I = \{i_1, \ldots, i_m\}$ and ratings, $R = \{r_{11}, \ldots, r_{ij}, \ldots, r_{qm}\}$, where r_{ij} is the rating value of user u_i for item i_j. A GRS recommends to a group $G \subset U$ instead of a single user. There are different schemes for CF GRS [12] (see Fig. 1):

1) *Aggregating preferences*: individual preferences are aggregated to obtain group preferences (see Fig. 1 (a)).
2) *Aggregating predictions*: individual predictions that CF obtains are aggregated to recommend (see Fig. 1 (b)).

In these GRSs approaches there is an aggregation process. Common aggregation operators used to build group profiles

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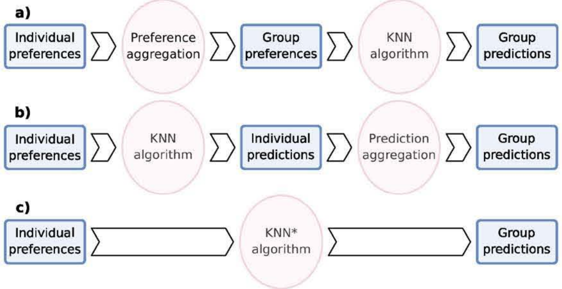


Fig. 1: Three approaches for CF GRS

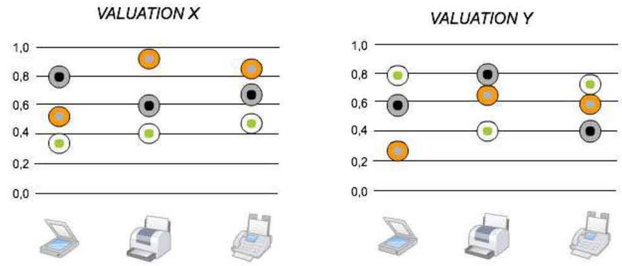


Fig. 2: Correlation of Hesitant Valuations

are [8]: *Mean*, *Root Mean Squared (RMS)*, *Least Misery* and *Average without misery*. Even though aggregations work generally well, there are some cases in which such aggregated group profiles may not be a good representation of members ratings:

1) *Mean* aggregation is a typical operator in GRS that uses the arithmetic mean. Lets assume two situations: (a) user u_1 rates item i_1 with 7 and user u_2 rates 4, and (b) user u_1 rates i_1 with 10 and user u_2 rates 1. In both cases, the group rating for the item i_1 will be 5.5, and then, the GRS can recommend an item i_2 similar to i_1. However, the two situations are quite different. In case (a) both users should be satisfied with i_2, but in situation (b) the user u_2 would be unsatisfied because he/she dislikes i_1 and probably also dislikes i_2, similar to i_1.
2) *Least misery* and *Average without misery* use the minimum operator. It tries to minimize users' misery to satisfy most of them. Only one low rating is enough to penalize an item. This can be a disadvantage, mainly for large groups where it is highly probable that every item has a detractor at least.

These situations happen because aggregation always implies some loss of information from different viewpoints such as distribution, diversity or shape of data. Therefore, an approach that avoids the aggregation can improve results in GRSs.

B. Hesitant Fuzzy Sets for Group Recommendations

In this contribution, Hesitant Fuzzy Sets will be used to model multiple members ratings as the group hesitation regarding an item, which allows us to avoid the aggregation process. Therefore, some basic concepts about HFSs are introduced.

Hesitant Fuzzy Sets (HFSs), introduced by Torra [10], extend Fuzzy Sets [15], [16] giving for each element in the domain a set of membership degrees.

Definition 1. *[10]. Let X be a reference set, a Hesitant Fuzzy Set (HFS) on X is a function h that returns a non-empty subset of [0,1]:*

$$h : X \to \wp([0,1]) \tag{1}$$

This definition is completed by Xia and Xu [17], who proposed the concept of Hesitant Fuzzy Element (HFE) as a subset of [0,1] for a given $x \in X$.

Definition 2. *[17]. Let X be a reference set, a HFS on X can be represented as*

$$E = \{\langle x, h_E(x)\rangle : x \in X\} \tag{2}$$

The set of values $h_E(x)$ denotes the possible membership degrees of the particular element x.

In the context of group recommendations, given an item $x \in X$, each member can rate it with a different value. This situation can be considered as hesitation about ratings of items. Thus, for each item in X we will have a HFE composed of members ratings.

Definition 3. *[18]. Let $E = \{H_1, \ldots, H_n\}$ be a set of n HFS and Θ a function, $\Theta : [0,1]^n \to [0,1]$, we then export Θ on fuzzy sets to HFSs defining*

$$\Theta_E = \bigcup_{\gamma \in H_1(x) \times \cdots \times H_n(x)} \{\Theta(\gamma)\} \tag{3}$$

This is an extension principle that provides a way of extending crisp or fuzzy functions to HFS. In particular, it has been applied to the Pearson's correlation coefficient (PCC), a widely-used function in RSs to calculate similarities between users or items. PCC, noted as ρ, has been extended for HFSs in [19] to the Hesitant Pearson Correlation Coefficient (HPCC), ρ_{HFS}. Figure 2 shows an example. There are two valuations represented by HFSs due to the hesitation that appears when rating each item. The correlation between both valuations can be measured by HPCC.

Definition 4. *Let X and Y be two HFSs on S and $h_X(s_i) \times h_Y(s_i)$ be the collection of all pairs of HFEs,* $\left((h_X(s_i))^{(j)}, (h_Y(s_i))^{(k)}\right)$ *where $j \in \{1, \ldots, l_X(s_i)\}$ and $k \in \{1, \ldots, l_Y(s_i)\}$, being $l_X(s_i)$ and $l_Y(s_i)$ the cardinals of $h_X(s_i)$ and $h_Y(s_i)$ respectively. The set of all pairs HFEs for each $s_i \in S$ is:*

$$R_{HFS} = \cup_{s_i \in S} h_X(s_i) \times h_Y(s_i) \tag{4}$$

The number of pairs of values in R_{HFS} is computed as:

$$|R_{HFS}| = \sum_{i=1}^{n} (l_X(s_i) \times l_Y(s_i)) \quad (5)$$

Definition 5. *[19]. Let X and Y be two HFSs on S, the Hesitant Pearson Correlation Coefficient (HPCC), ρ_{HFS}, is:*

$$\rho_{HFS}(X,Y) = \frac{SSC(h_X, h_Y)}{\sqrt{SS(h_X)}\sqrt{SS(h_Y)}}, \quad (6)$$

where SSC is the covariance of both sets defined as:

$$SSC(h_X, h_Y) = = \sum_{i=1}^{n} \sum_{j=1}^{l_X(s_i)} \sum_{k=1}^{l_Y(s_i)} \left((h_X(s_i))^{(j)} - \overline{h_X} \right) \left((h_Y(s_i))^{(k)} - \overline{h_Y} \right), \quad (7)$$

being $\overline{h_X}$ and $\overline{h_Y}$ the arithmetic mean of the corresponding values of the first and second elements of the pairs, respectively.

$$\overline{h_X} = \frac{1}{|R_{HFS}|} \sum_{i=1}^{n} l_Y(s_i) \left(\sum_{j=1}^{l_X(s_i)} (h_X(s_i))^{(j)} \right) \quad (8)$$

$$\overline{h_Y} = \frac{1}{|R_{HFS}|} \sum_{i=1}^{n} l_X(s_i) \left(\sum_{j=1}^{l_Y(s_i)} (h_Y(s_i))^{(j)} \right) \quad (9)$$

And $SS(h_X)$ and $SS(h_Y)$, correspond to the standard deviation of the respective sets, defined as:

$$SS(h_X) = \sum_{i=1}^{n} l_Y(s_i) \left(\sum_{j=1}^{l_X(s_i)} \left((h_X(s_i))^{(j)} - \overline{h_X} \right)^2 \right) \quad (10)$$

$$SS(h_Y) = \sum_{i=1}^{n} l_X(s_i) \left(\sum_{j=1}^{l_Y(s_i)} \left((h_Y(s_i))^{(j)} - \overline{h_Y} \right)^2 \right) \quad (11)$$

HPCC has been used in the proposed model in order to improve features of recommendations in GRS.

III. Hesitant Group Recommender Model

This section introduces a novel group recommendation model based on CF that uses HFSs to avoid the aggregation processes to keep as much information as possible. Therefore, the proposal follows the scheme shown in Figure 1 (c) because it works with the whole set of ratings using HFSs. This should provide better results regarding diversity and novelty maintaining high accuracy.

The proposed model extends the classical KNN algorithm to deal with groups instead of individual users. To do so, HPCC is used to find the nearest neighbours of the group. This correlation measure computes the similarity between the target group and the neighbours using the whole set of group ratings without aggregation. This proposal consists of two main phases (see Fig. 3):

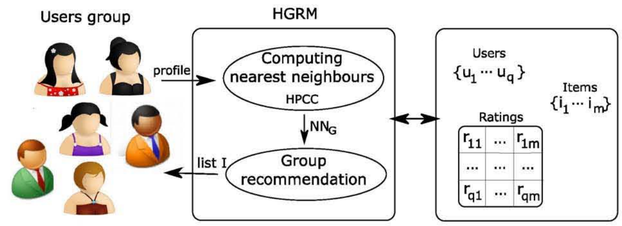


Fig. 3: General scheme of HGRM

TABLE I: Notations.

Symbol	Description
$U = \{u_1, \ldots, u_q\}$	set of users.
$I = \{i_1, \ldots, i_m\}$	set of items.
$G = \{u_1, \ldots, u_n\} \subset U$,	target group, with $n << q$.
$G^c = \{u_{n+1}, \ldots, u_q\} = G \setminus U$	complementary set of G.
$D = [d_{min}, \ldots, d_{max}]$	rating domain
$R = \{r_{11}, \ldots, r_{ij}, \ldots, r_{qm}\}$	set of ratings
$r_{ij} \in D$	rating of user u_i for item i_j.

1) *Computing the nearest neighbours*. The KNN algorithm based on HPCC is used to obtain a set of K nearest neighbours for the target group G.
2) *Group recommendation*. From the ratings of the K nearest neighbours, the model predicts the ratings and recommends the top N items with the highest prediction to the target group.

These phases are explained in further detail below, but first we introduce the notation used for the proposal.

A. Notation

Table I describes the notation used. Additionally, the proposal deals with two ratings profiles: one contains the ratings of group members and the other one contains the ratings of a potential neighbour. Such ratings profiles are represented by means of HFSs:

- Group profile represented by a HFS, X_G, to manage the multiplicity of ratings provided by the users of group G, for each item. Ratings values are normalised to transform them in membership values.

$$X_G = \{\langle i_j, h_X(i_j) \rangle : i_j \in I\}$$
$$h_X : I \to \wp([0,1])$$
$$h_X(i_j) = \left\{ \frac{r_{ij} - d_{min}}{d_{max} - d_{min}}, \text{ for } i = 1, \ldots, n \right\}$$

- User profile, $u_k \in G^c$, represented by a HFS, Y_k, that is composed only by one membership function.

$$Y_k = \{\langle i_j, h_Y(i_j) \rangle : i_j \in I\}$$
$$h_Y : I \to \wp([0,1])$$
$$h_Y(i_j) = \left\{ \frac{r_{kj} - d_{min}}{d_{max} - d_{min}} \right\}$$

Both, $h_X(i_j)$ and $h_Y(i_j)$ are sets, therefore, it is necessary to use superscripts to refer to an element of the set. For example, $(h_X(i_j))^{(z)}$ is the z-th element of the set.

B. Computing the nearest neighbours

Neighbourhoods used in CF are computed by using the KNN algorithm based on HPCC, which calculates the similarity between the group profile G and users in G^c.

Given group G and user $u_k \in G^c$, HPCC obtains the similarity between their profiles represented by two HFSs, X_G and Y_k.

$$\rho_{HFS}(X_G, Y_k) = \frac{SSC(h_X, h_Y)}{\sqrt{SS(h_X)}\sqrt{SS(h_Y)}} \tag{12}$$

where

$$SSC(h_X, h_Y) = $$
$$= \sum_{j=1}^{m}\sum_{z=1}^{n}\left((h_X(i_j))^{(z)} - \overline{h_X}\right)\left((h_Y(i_j))^{(1)} - \overline{h_Y}\right)$$

$$\overline{h_X} = \frac{1}{n \cdot m}\sum_{j=1}^{m}\sum_{z=1}^{n}(h_X(i_j))^{(z)}$$
$$\overline{h_Y} = \frac{1}{m}\sum_{j=1}^{m}(h_Y(i_j))^{(1)}$$

$$SS(h_X) = \sum_{j=1}^{m}\sum_{z=1}^{n}\left((h_X(i_j))^{(z)} - \overline{h_X}\right)^2$$
$$SS(h_Y) = n \cdot \sum_{j=1}^{m}\left((h_Y(i_j))^{(1)} - \overline{\mu_Y}\right)^2$$

The K nearest neighbours to the group G, noted as NN_G, are obtained. Such neighbours are users u_k who maximize $\rho_{HFS}(X_G, Y_k)$. Negative correlations are not considered [20].

C. Group recommendation

Once the nearest neighbours NN_G have been obtained, their ratings are combined to compute the predictions for the target group. In our proposal, it is used the compensated prediction [3], which reduces the influence of user bias when rating.

$$pred(G, i_j) = \overline{r}_G + \frac{\sum_{u_k \in NN_G} \rho_{HFS}(X_G, Y_k) \cdot (r_{kj} - \overline{r}_k)}{\sum_{u_k \in NN_G} \rho_{HFS}(X_G, Y_k)} \tag{13}$$

where $\overline{r}_G$ is the average value of the set of ratings $\{r_{ij}\}$, $u_i \in G$, and $\overline{r}_k$ of the set $\{r_{kj}\}$, for each item i_j.

Finally, an ordered list $\tilde{I}$ with the top N items is provided.

$$\tilde{I} = (<\tilde{i}_1, \tilde{r}_1>, \cdots <\tilde{i}_N, \tilde{r}_N>) \tag{14}$$

being $\tilde{r}_j = pred\left(G, \tilde{i}_j\right)$ and $\tilde{r}_j \geq \tilde{r}_k$ for each $j < k$.

IV. Case Study

A case study was performed to evaluate the proposal, HGRM, and compare it with other well-known GRS approaches regarding accuracy, diversity and novelty. These GRSs features are important to deliver good recommendations and they need to be evaluated together, because if the diversity and novelty improve, the accuracy may decay [6].

A. Experiment description

The proposal was compared with the group recommendation model based on rating aggregation with two aggregation operators (see Fig. 1 (a)): (a) Mean and (b) RMSMean. It has been tested in a standard version and in a no-duplicate version, which removes repeated values when several users provide the same rating for an item. In summary, four techniques are compared:

- *Mean*: Aggregate ratings with Mean aggregation.
- *RMSMean*: Aggregate ratings with RMSMean aggregation.
- *HGRM*: Recommend with neighbourhood based on hesitant similarity.
- *HGRM no-dup.*: Hesitant similarity considers only unique rating values.

B. Data set

The ml-100k dataset [1] was used in the evaluation. It is composed of 1682 items, 943 users and 100k ratings. This dataset uses the traditional one to five rating scale, in which users can provide only entire stars.

To compare the techniques in different GRS scenarios, various group formation techniques can be used [12]:

- *Similar members*: Groups are composed of people with similar features, which can be interests, age, beliefs or education.
- *Dissimilar members*: Groups are composed of people with diversity of features.
- *Random members*: Groups are composed of members without any relation among each other.

Our study is focused on similar members, which is the most usual scenario that a GRS faces. Within this group formation we considered different group sizes ranging from 1 to 500 users. In the figures, only results for groups of size 20, 25, 50, 100, 200, and 500, are shown. Within these experiments, the 5-cross fold validation protocol has been performed 20 times and their results were averaged.

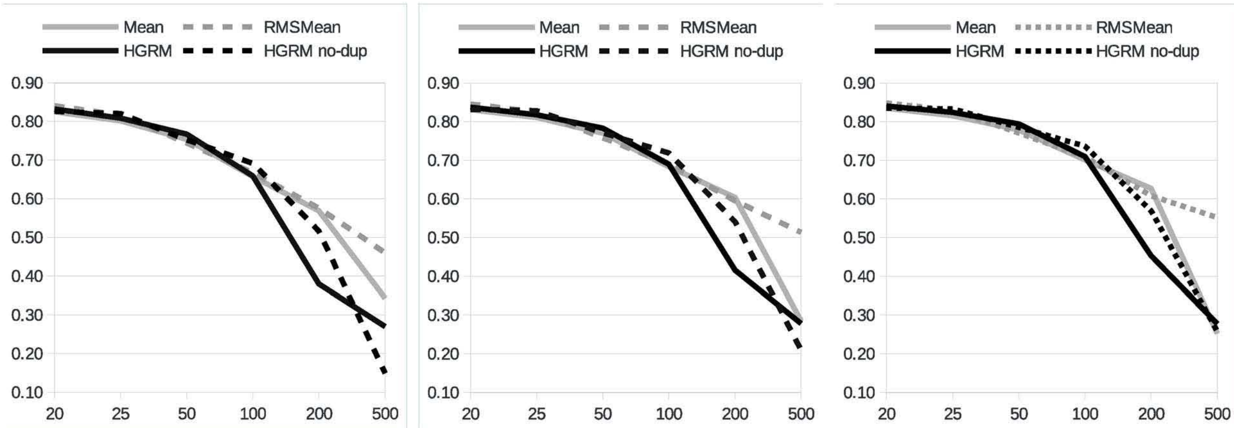
C. Evaluation measures

In this experiment, we evaluate the proposal regarding accuracy, diversity and novelty [6]. Therefore, three evaluation measures are used:

[1] http://grouplens.org/datasets/movielens/

TABLE II: NRMSE

Group size	Mean	RMSMean	HGRM	HGRM no-dup.
20	0.2548	0.2553	0.2555	0.2553
25	0.2550	0.2551	0.2553	0.2552
50	0.2557	0.2556	0.2565	0.2562
100	0.2565	0.2563	0.2567	0.2569
200	0.2580	0.2570	0.2575	0.2587
500	0.2613	0.2636	0.2632	0.2623



(a) ILS@4

(b) ILS@5

(c) ILS@6

Fig. 4: Intra list similarity of the compared approaches stratified by group size.

- Normalized Root Mean Squared Error (NRMSE). Measures how close a prediction $\tilde{r_i}$ is to the true rating r_i to study accuracy through prediction error.

$$NRMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\frac{\tilde{r_i} - r_i}{d_{max} - d_{min}}\right)^2} \quad (15)$$

- Intra List Similarity (ILS) [6]. Measures the similarity of the items recommended, which studies diversity of the recommendation. The less similarity, the more diversity of the recommendation.

$$ILS\left(\tilde{I}\right) = \frac{\sum_{i_j \in \tilde{I}} \sum_{i_k \in \tilde{I}, j \neq k} cosine\left(v_j, v_k\right)}{2} \quad (16)$$

where v_j and v_k are, respectively, characteristic vectors of items i_j and i_k built using the SVD [4] with 20 features.

- User Specific Unexpectedness (USU) [6]. Measures how different the recommendations are as compared to the items rated by the user to study novelty.

$$USU_u(\tilde{I}) = \frac{1}{|\tilde{I}||R_u|} \sum_{i_j \in \tilde{I}} \sum_{i_k \in \tilde{R_u}} sim(i_j, i_k) \quad (17)$$

where $\tilde{I}$ is the set of recommended items and R_u is the set of items rated by the user u.

D. Results

This section shows the results of the experiment and their analyses to evaluate the techniques compared. As stated in the previous section, we evaluated the techniques regarding accuracy, diversity and novelty of the recommendations. Therefore, the results of RMSE are analysed first, then ILS and, finally, the USU results are shown.

Table II shows results for NMRSE. The results have been stratified by group size, which range from 20 to 500 members. NRMSE is an error measure, therefore, the lower value, the better. This table shows that the four approaches differ less than one hundredth for NRMSE, which means that the four techniques have a similar performance regarding rating prediction.

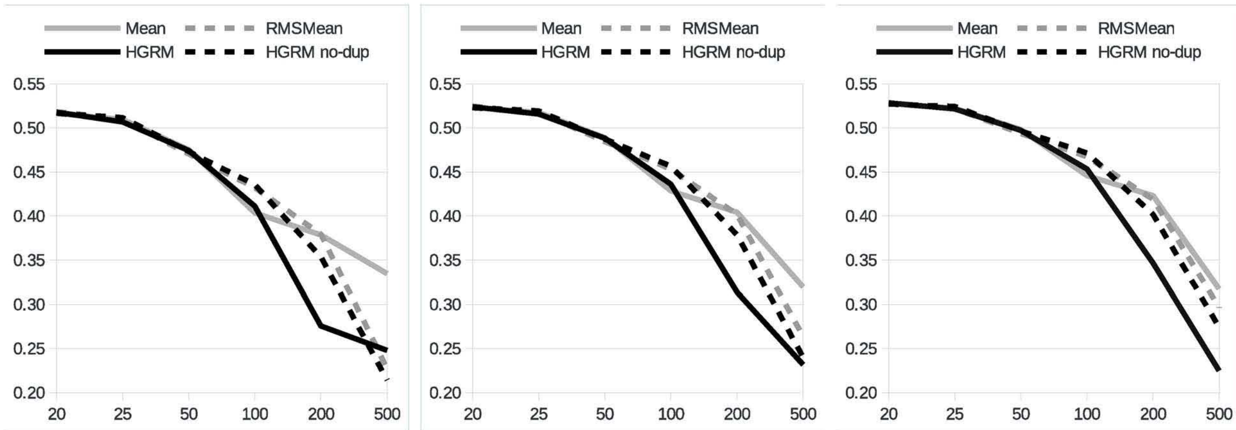
Figure 4 shows results for ILS (Figures 4a, 4b, and 4c show the results for recommendations of 4, 5 and 6 items, respectively). The results have been stratified by group size. ILS measures the similarity of the items in the recommendation, therefore, the lower value, the better. This figure shows that the proposal in both versions improves the results of the compared approaches for large groups. Specifically, for groups of size 200, both HGRM and HGRM no dup. achieve the best results. For groups of size 500 the best approach is the HGRM no-dup. These results confirm that the proposal provides more diverse results when it recommends to large groups while it maintains a similar prediction error.

Figure 5 shows the results for USU (Figures 5a, 5b, and 5c show the results for recommendations of 4, 5 and 6 items, respectively). The results have been stratified by group size. USU measures the novelty of the items recommended using PCC, therefore, the lower value, the more novelty in the recommendation. This figure shows that both versions of the proposal achieve the best results for large groups. Specifically, for groups of size 200 and 500, both HGRM and HGRM no dup. achieve the best results. These results confirm that the proposal provides recommendations with more novelty when it recommends to large groups while it maintains a similar prediction error.

We can conclude that HGRM properly balances the accuracy, diversity and novelty when recommending to large groups as compared with other aggregation based approaches. This fact make them suitable for recommendation in contexts where accuracy and novelty are also considered.

V. Conclusions

This contribution proposes a HGRM, which considers the multiple members ratings over one item as the group hesitation



(a) USU@4 (b) USU@5 (c) USU@6

Fig. 5: User specific unexpectedness of the compared approaches stratified by group size.

wih a HFS. An extension of Pearson's correlation coefficient for HFSs is used to build the neighbourhood of the group avoiding the information loss associated to the aggregation process applied in group recommendation.

Our interest is focused on accuracy, diversity and novelty. The diversity of recommendations is a desired property in GRSs because members may have either different or conflicting interests, therefore, recommending diverse items increases the possibility of all members being satisfied with at least one item. Moreover, novelty is appreciated by users because they perceive the recommendations as more useful.

The proposal has been compared with other techniques in the case study and its results show that the proposal improves diversity and novelty of recommendations in large groups, while it maintains good accuracy results. This makes it suitable for recommendation in this scenario.

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References

[1] G. Adomavicius and A. Tuzhilin, "Toward the next generation of recommender systems: A survey of the state-of-the-art and possible extensions," *IEEE Transactions on Knowledge and Data Engineering*, vol. 17, no. 6, pp. 734–749, June 2005.

[2] F. Ricci, L. Rokach, B. Shapira, and P. Kantor, Eds., *Recommender Systems Handbook*. Springer, 2011.

[3] J. Schafer, D. Frankowski, J. Herlocker, and S. Sen, "Collaborative filtering recommender systems." in *The adaptive web, Berlin: Springer.*, 2007, pp. 291–324.

[4] Y. Koren, R. B. R., and C. Volinsky, "Matrix factorization techniques for recommender systems," *Computer*, vol. 42, no. 8, pp. 30–37, 2009.

[5] D. Kotkov, S. Wang, and J. Veijalainen, "A survey of serendipity in recommender systems," *Knowledge-Based Systems*, vol. 111, pp. 180 – 192, 2016.

[6] M. Kaminskas and D. Bridge, "Diversity, serendipity, novelty, and coverage: A survey and empirical analysis of beyond-accuracy objectives in recommender systems," *ACM Trans. Interact. Intell. Syst.*, vol. 7, no. 1, pp. 2:1–2:42, 12 2016.

[7] P. Castells, N. J. Hurley, and S. Vargas, *Novelty and Diversity in Recommender Systems*. Boston, MA: Springer US, 2015, pp. 881–918.

[8] J. Masthoff, *Group recommender systems: Combining individual models*. Springer, 2011, pp. 677–702.

[9] J. Castro, F. Quesada, I. Palomares, and L. Martínez, "A consensus-driven group recommender system," *International Journal of Intelligent Systems*, vol. 30, no. 8, pp. 887–906, 2015.

[10] V. Torra, "Hesitant fuzzy sets," *International Journal of Intelligent Systems*, vol. 25, no. 6, pp. 529–539, 2010.

[11] L. Zadeh, "Fuzzy sets," *Information Control*, vol. 8, pp. 338–353, 1965.

[12] A. Jameson and B. Smyth, *Recommendation to groups*. Lecture Notes in Computer Science. Springer-Verlag, Berlin Heidelberg New York, 2007, vol. 4321.

[13] J. F. McCarthy and T. D. Anagnost, "Musicfx: An arbiter of group preferences for computer supported collaborative workouts," in *The 1998 ACM Conference on Computer Supported Cooperative Work*, New York, NY, USA, 1998, pp. 363–372.

[14] H. Lieberman, N. W. Van Dyke, and A. S. Vivacqua, "Let's browse: A collaborative web browsing agent," in *The 4th International Conference on Intelligent User Interfaces*, New York, NY, USA, 1999, pp. 65–68.

[15] R. Rodríguez, L. Martínez, V. Torra, Z. Xu, and F. Herrera, "Hesitant fuzzy sets: state of the art and future directions," *International Journal of Intelligent Systems*, vol. 29, no. 6, pp. 495–524, 2014.

[16] R. Rodríguez, B. Bedregal, H. Bustince, Y. Dong, B. Farhadinia, C. Kahraman, L. Martínez, V. Torra, Y. Xu, Z. Xu, and F. Herrera, "A position and perspective analysis of hesitant fuzzy sets on information fusion in decision making towards high quality progress," *Information Fusion*, vol. 29, pp. 89–97, 2016.

[17] N. Xia and Z. Xu, "Hesitant fuzzy information aggregation in decision making," *International Journal of Approximate Reasoning*, vol. 52, pp. 395–407, 2011.

[18] V. Torra and Y. Narukawa, "On hesitant fuzzy sets and decision," *The 18th IEEE International Conference on Fuzzy Systems*, pp. 1378–1382, 2009.

[19] T. González-Arteaga, J. Alcantud, and R. de Andrés Calle, "New correlation coefficients for hesitant fuzzy sets," in *The 16th World Congress of the International Fuzzy Systems Association*, 2015, pp. 427–434.

[20] P. Massa and P. Avesani, "Trust-aware collaborative filtering for recommender systems," in *The International Conference on Cooperative Information Systems, CoopIS*, R. Meersman and Z. Tari, Eds. Springer Berlin Heidelberg, 2004, pp. 492–508.