A consistency-driven approach to set personalized numerical scales for hesitant fuzzy linguistic preference relations

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***Abstract*—In decision making dealing with computing with words, the importance of the statement that words mean different things for different people has been highlighted. In this paper, we focus on personalizing numerical scales of linguistic terms in decision making with hesitant fuzzy linguistic preference relations (HFLPRs). First, an average consistency measure for HFLPRs is provided, and then an optimization-based model to personalize individual semantics via numerical scales is presented, aiming at maximizing the average consistency of HFLPRs. Numerical examples are used to illustrate the proposal.**

***Index Terms*—computing with words; hesitant information; numerical scale; average consistency; optimization**

I. Introduction

In real-world decision making, solving a decision problem with linguistic information implies the need of Computing with Words (CW) [8, 17, 28]. CW is a methodology in which the objects of computation are words and propositions drawn from a natural language that arises to emulate human behavior [29, 30]. Different linguistic models have been proposed. In particular, Herrera and Martínez [9, 16] proposed the 2-tuple linguistic representation model. The 2-tuple linguistic model has been successfully used in a wide range of applications (e.g., [15, 18, 19, 20]). In recent years, different models based on linguistic 2-tuples have been developed, such as the proportional 2-tuple linguistic representation model [26], the model based on a linguistic hierarchy [5, 10], and numerical scale model [1, 3, 4, 6]. Moreover, complexity and time pressure of decision making problems nowadays make decision makers need more elaborated expressions than a simple linguistic label [25]. Hence, to overcome this limitation, Rodríguez et al. [23, 24] introduced the concept of a Hesitant Fuzzy Linguistic Term Set (HFLTS) to serve as the basis of increasing the flexibility of the elicitation of linguistic information by means of linguistic expressions and also presented a deep study on hesitant fuzzy sets.

In decision making dealing with CW, there is a fact that words mean different things for different people [11, 21, 22]. For example, when reviewing an article, two referees may think the reviewed article is interesting, but the term interesting often has different numerical meaning for both referees. The existing studies developed the use of type-2 fuzzy sets [21] and the use of multi-granular linguistic models [7, 12]. Although these two methods that deal with multiple meanings of words are quite useful, they do not represent yet the specific semantics of each individual. To overcome this problem, Li et al. [14] proposed a new approach to personalize individual semantics by means of numerical scales [4, 6] with the 2-tuple linguistic model [9].

Therefore, keeping the previous fact in mind, in decision making problems with Hesitant Fuzzy Linguistic Preference Relations (HFLPRs), for different decision makers, the hesitant linguistic term sets may have different numerical meanings. Hence, to handle the different numerical meaning of the hesitant linguistic term sets for different decision makers, in this paper we continue the study of personalized individual semantics by means of numerical scales and propose a consistency-driven optimization model to set personalized numerical scales for linguistic terms with HFLPRs. First, an average consistency measure for the HFLPR is proposed, which reflects the average consistency degree of all linguistic preference relations associated to the HFLPR. Then, an optimization model to obtain the personalized numerical scales based on the average consistency measure is provided. Finally, numerical examples to illustrate the use of the proposed optimization model are presented.

The rest of this paper is arranged as follows. In Section 2, we present the basic knowledge regarding the 2-tuple linguistic model, numerical scale model and HFLTSs. In Section 3, a consistency-driven optimization-based model is proposed to set personalized numerical scales with HFLPRs. In Section 4, numerical examples are provided to illustrate the use of the proposed model. Section 5 then concludes this paper with final remarks.

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II. Preliminaries

This section introduces some basic concepts about the 2-tuple linguistic model, the numerical scale model and HFLTSs.

A. The 2-tuple linguistic model

The 2-tuple linguistic representation model, presented by Herrera and Martínez [9], represents the linguistic information by a 2-tuple $(s_i, \alpha) \in \overline{S} = S \times [-0.5, 0.5)$, where $s_i \in S$ and $\alpha \in [-0.5, 0.5)$.

Definition 1: [9] Let $S = \{s_0, s_1, ..., s_g\}$ be a linguistic term set and $\beta \in [0, g]$ be a value representing the result of a symbolic aggregation operation. The 2-tuple that expresses the equivalent information to β is then obtained as:

$$\Delta : [0, g] \rightarrow \overline{S},$$

being

$$\Delta(\beta) = (s_i, \alpha), \;\; with \left\{ \begin{array}{l} s_i, \;\; i = round(\beta) \\ \alpha = \beta - i, \;\; \alpha \in [-0.5, 0.5) \end{array} \right.$$

Function Δ, it is a one to one mapping whose inverse function $\Delta^{-1} : \bar{S} \rightarrow [0, g]$ is defined as $\Delta^{-1}(s_i, \alpha) = i + \alpha$. When $\alpha = 0$ in (s_i, α) is then called simple term.

In [9] it was also defined a computational model for linguistic 2-tuples in which a 2-tuple comparison operator, a 2-tuple negation operator and several 2-tuple linguistic aggregation operators were introduced (see [9, 16]).

B. Numerical scale model

The concept of the numerical scale was introduced by Dong et al. [1] for transforming linguistic terms into real numbers:

Definition 2: [1] Let $S = \{s_0, s_1, ..., s_g\}$ be a linguistic term set, and R be the set of real numbers. The function: $NS : S \rightarrow R$ is defined as a numerical scale of S, and $NS(s_i)$ is called the numerical index of s_i. If the function is strictly monotone increasing, then NS is called an ordered numerical scale.

Definition 3: [1] Let S be defined as before. The numerical scale NS for (s_i, α), is defined by

$$NS(s_i, \alpha) = \left\{ \begin{array}{l} NS(s_i) + \alpha \times (NS(s_{i+1}) - NS(s_i))\, \alpha \geq 0 \\ NS(s_i) + \alpha \times (NS(s_i) - NS(s_{i-1}))\, \alpha < 0 \end{array} \right.$$

In particular, the numerical scale model provides a connection framework [6] among the Herrera and Martínez model [9], the Wang and Hao model [27] and the model based on a linguistic hierarchy [10].

C. Hesitant fuzzy linguistic term sets

The concept of HFLTS is introduced by Rodríguez et al. [23], as follows.

Definition 4: [23] Let $S = \{s_0, s_1, ..., s_g\}$ be a linguistic term set. A HFLTS, H_S, is an ordered finite subset of consecutive linguistic terms of S.

Definition 5: [23] Let H_S be a HFLTS of S. Let $H_S^- = \min_{s_i \in H_S}(s_i)$, $H_S^+ = \max_{s_i \in H_S}(s_i)$ and $env(H_S) = [H_S^-, H_S^+]$. Then, H_S^-, H_S^+ and $env(H_S)$ are called the lower bound, the upper bound and the envelope of H_S.

Based on the use of HFLTSs, the concept of HFLPR is provided.

Definition 6: [26] Let M_S be a set of HFLTSs based on S. A HFLPR based on S is presented by a matrix $H = (H_{ij})_{n \times n}$, where $H_{ij} \in M_S$ and $Neg(H_{ij}) = H_{ji}$.

III. An approach to set personalized numerical scales in decision making with HFLPRs

In this section, we propose an approach to personalize numerical scales of linguistic terms in decision making with HFLPRs. First, an average consistency measure for HFLPRs based on numerical scale model is provided, and then a consistency-driven optimization-based model to set personalized numerical scales for linguistic terms is presented.

A. Average consistency measure of HFLPRs

Let $S = \{s_0, s_1, ..., s_g\}$ be a linguistic term set. Let $H = (H_{ij})_{n \times n}$ be a HFLPR based on S, where $H_{ij} = \{H_{ij}^k \,| k = 1, ..., \#H_{ij}\}$, and $\#H_{ij}$ is the number of linguistic terms in H_{ij}.

Definition 7: Let $H = (H_{ij})_{n \times n}$ be defined as before. $L = (l_{ij})_{n \times n}$ is a linguistic preference relation associated to H, if $l_{ij} = H_{ij}^k$ $(k = 1, ..., \#H_{ij})$ and $l_{ij} = Neg(l_{ji})$.

We denote N_H as the set of the linguistic preference relations associated to H.

Let NS be an ordered numerical scale on S, and in this paper we set the range of NS in the interval [0,1]. Let $V = (V_{ij})_{n \times n}$, in which $V_{ij} = \{V_{ij}^k \,| k = 1, ..., \#V_{ij}\} = \{NS(H_{ij}^k) \,| k = 1, ..., \# H_{ij}\}$, be the hesitant fuzzy preference relation transformed by NS, associated with H. Clearly, based on the numerical scale NS, the HFLPR H can be transformed into the corresponding hesitant fuzzy preference relation V. Similarly, the linguistic preference relations associated to H can be also transformed into fuzzy preference relations.

Here, we propose a method to measure the average consistency index (ACI) of HFLPRs based on numerical scales NS.

Additive transitivity is often used to character the consistency of linguistic preference relations [2, 13]. Following the additive transitivity, the consistency index (CI) of a linguistic preference relation L based on the numerical scales NS is defined as,

$$CI(L) =$$

$$1 - \tfrac{2}{3n(n-1)(n-2)} \sum_{i,j,z=1}^{n} |NS(l_{ij}) + NS(l_{jz}) - NS(l_{iz}) - 0.5|$$

with $NS(l_{ij}) \in [0, 1]$.

Definition 8: Let H be a HFLPR. The value of $ACI(H)$ is determined by the average consistency degree of all linguistic preference relations associated to the HFLPR, i.e.,

$$ACI(H) = \frac{1}{\#N_H} \times \sum_{L \in N_H} CI(L)$$

where $\#N_H$ is the number of linguistic preference relations in H, i.e., $\#N_H = \prod_{i=1}^{n} \prod_{j=i+1}^{n} \#H_{ij}$.

Let L^h $(h = 1, 2, ..., \#N_H)$ be the linguistic preference relations associated to H, i.e., $L^h \in N_H$. We provide Algorithm 1 to show the procedure to obtain the ACI.

Algorithm 1. The procedure to obtain the ACI of a HFLPR based on numerical scales NS

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1. Input the HFLPR H.
2. For each linguistic preference relation associated to H, $L^h \, (h = 1, 2, ..., \#N_H)$
   do
   calculate the consistency degree of L^h,
   $CI(L^h) =$
   $1 - \frac{2}{3n(n-1)(n-2)} \sum_{i,j,z=1}^{n} \left| NS(l_{ij}^h) + NS(l_{jz}^h) - NS(l_{iz}^h) - 0.5 \right|$
   End for
3. Calculate the average consistency degree of H,
   $ACI(H) = \frac{1}{\#N_H} \times \sum_{h=1}^{\#N_H} CI(L^h)$
4. Output $ACI(H)$.

B. Consistency-driven optimization-based model to personalize numerical scales

In the following, we construct an optimization-based model to set personalized numerical scales for linguistic terms with HFLPRs based on the average consistency measure (see Fig.1).

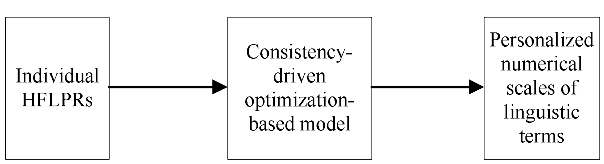

Fig.1 The framework to set personalized numerical scales

As mentioned before, using a numerical scale NS it is possible to transform a HFLPR $H = (H_{ij})_{n \times n}$ into a hesitant fuzzy preference relation $V = (V_{ij})_{n \times n}$. Hence, H and V represent the same preference of decision maker. So V should be consistent if H is consistent. From this reason, the following premise is provided:

Premise 1: [4] If HFLPRs provided by individuals are consistent, then the hesitant fuzzy preference relations, transformed by the established numerical scale, should be as much as consistent as possible.

Based on Premise 1, in order to guarantee the HFLPR H is as consistent as possible, the objective function is to maximize the ACI of HFLPR H, i.e.,

$$\max \; ACI(H) \tag{1}$$

where

$$ACI(H) = \frac{1}{\#N_H} \sum_{h=1, L^h \in N_H}^{\#N_H} \left(1 - \frac{2 \sum_{i,j,z=1}^{n} \left| NS(l_{ij}^h) + NS(l_{jz}^h) - NS(l_{iz}^h) - 0.5 \right|}{3n(n-1)(n-2)}\right)$$

In this paper, we set the range of numerical scales for linguistic terms as follows,

$$NS(s_i) \begin{cases} = 0 & i = 0 \\ \in [(i-1)/g, (i+1)/g] & i = 1, 2, ..., g-1 \\ = 1 & i = g \end{cases} \tag{2}$$

Besides, NS must be ordered. We introduce a constraint value $\lambda \in (0, 1)$ to restrict the distance between $NS(s_i)$ and $NS(s_{i+1})$, i.e.,

$$NS(s_{i+1}) - NS(s_i) \geq \lambda \tag{3}$$

Thus, the consistency-driven optimization model P to obtain personalized numerical scales for linguistic terms with HFLPR H is constructed as follows,

$$\begin{cases} \max \quad ACI(H) \\ s.t. \; ACI(H) = \\ \frac{1}{\#N_H} \sum_{h=1, L^h \in N_H}^{\#N_H} \left(1 - \frac{2 \sum_{i,j,z=1}^{n} \left| NS(l_{ij}^h) + NS(l_{jz}^h) - NS(l_{iz}^h) - 0.5 \right|}{3n(n-1)(n-2)}\right) \\ \quad NS(s_0) = 0 \\ \quad NS(s_i) \in [(i-1)/g, (i+1)/g] \quad i = 1, ..., g-1 \\ \quad NS(s_g) = 1 \\ \quad NS(s_{i+1}) - NS(s_i) \geq \lambda \qquad i = 0, 1, ..., g-1 \end{cases}$$

By solving this model using the software package Lingo, we obtain the personalized numerical scales for each term in S, i.e., $NS(s_0)$, $NS(s_1)$, ..., $NS(s_g)$, and also obtain the optimal ACI of H.

Solving model P with different HFLPRs, the obtained personalized numerical scales for linguistic terms may be different. They reflect individual differences in understanding the meaning of linguistic terms.

IV. Numerical analysis

In this section, we provide numerical examples to illustrate the use of the consistency-driven optimization-based model to set personalized numerical scales for linguistic terms with HFLPRs.

Example 1: Suppose that there are five alternatives $X = \{x_1, x_2, x_3, x_4, x_5\}$. The decision maker provides his/her preferences over the alternatives using the following linguistic term set,

$$V^1 = \begin{pmatrix} \{0.5\} & \{0.4, 0.5\} & \{0.55, 0.75\} & \{0.25, 0.35\} & \{0, 0.25\} \\ \{0.5, 0.55\} & \{0.5\} & \{0.75, 0.8\} & \{0.35, 0.4\} & \{0, 0.25, 0.35\} \\ \{0.35, 0.4\} & \{0.25, 0.35\} & \{0.5\} & \{0.55, 0.75\} & \{0.5, 0.55, 0.75\} \\ \{0.75, 0.8\} & \{0.55, 0.75\} & \{0.35, 0.4\} & \{0.5\} & \{0.5, 0.55, 0.75\} \\ \{0.55, 0.75\} & \{0.75, 0.8, 1\} & \{0.35, 0.4, 0.5\} & \{0.35, 0.4, 0.5\} & \{0.5\} \end{pmatrix}$$

$$V^2 = \begin{pmatrix} \{0.5\} & \{0.45\} & \{0.5\} & \{0.5\} & \{0, 0.25, 0.375\} \\ \{0.55\} & \{0.5\} & \{0.55\} & \{0.625, 0.75\} & \{0, 0.25, 0.375\} \\ \{0.5\} & \{0.45\} & \{0.5\} & \{0.55\} & \{0.5\} \\ \{0.5\} & \{0.25, 0.375\} & \{0.45\} & \{0.5\} & \{0.75, 1\} \\ \{0.625, 0.75, 1\} & \{0.625, 0.75, 1\} & \{0.5\} & \{0, 0.25\} & \{0.5\} \end{pmatrix}$$

$$\begin{aligned} S = \{ & s_0 = extremely\ poor, s_1 = very\ poor, s_2 = poor, \\ & s_3 = slightly\ poor, s_4 = fair,\ \ s_5 = slightly\ good\ \ , \\ & s_6 = good, s_7 = very\quad good\ \ , s_8 = extremely\ good\} \end{aligned}$$

Consider the following HFLPR provided by the decision maker,

$$H^1 = \begin{pmatrix} \{s_4\} & \{s_3, s_4\} & \{s_5, s_6\} & \{s_1, s_2\} & \{s_0, s_1\} \\ \{s_4, s_5\} & \{s_4\} & \{s_6, s_7\} & \{s_2, s_3\} & \{s_0, s_1, s_2\} \\ \{s_2, s_3\} & \{s_1, s_2\} & \{s_4\} & \{s_5, s_6\} & \{s_4, s_5, s_6\} \\ \{s_6, s_7\} & \{s_5, s_6\} & \{s_2, s_3\} & \{s_4\} & \{s_4, s_5, s_6\} \\ \{s_7, s_8\} & \{s_6, s_7, s_8\} & \{s_2, s_3, s_4\} & \{s_2, s_3, s_4\} & \{s_4\} \end{pmatrix}$$

Based on Eq. (2), the range of $NS(s_i)$ is set as follows,

$$NS(s_i) \begin{cases} = 0 & i = 0 \\ \in [(i-1)/8, (i+1)/8] & i = 1, 2, ..., 7 \\ = 1 & i = 8 \end{cases}$$

According to Eq. (3), without loss of generality, set the constraint value $\lambda = 0.05$ to restrict the difference between $NS(s_i)$ and $NS(s_{i+1})$, i.e.,

$$NS(s_{i+1}) - NS(s_i) \geq 0.05$$

The consistency-driven optimization-based model to obtain the numerical scale NS is as follows,

$$\begin{cases} \max \quad ACI(H^1) \\ s.t.\ ACI(H^1) = \frac{1}{3456} \times \\ \sum\limits_{h=1, L^h \in N_{H^1}}^{3456} (1 - \frac{1}{15} \sum\limits_{i<j<z}^{n} \left| NS(l_{ij}^h) + NS(l_{jz}^h) - NS(l_{iz}^h) - 0.5 \right|) \\ \quad NS(s_0) = 0 \\ \quad NS(s_i) \in [(i-1)/8, (i+1)/8] \quad i = 1, ..., 7 \\ \quad NS(s_8) = 1 \\ \quad NS(s_{i+1}) - NS(s_i) \geq 0.05 \quad i = 0, 1, ..., 7 \end{cases}$$

By solving the above model using the software package Lingo, we have

$NS(s_0) = 0$, $NS(s_1) = 0.25$, $NS(s_2) = 0.35$, $NS(s_3) = 0.4$, $NS(s_4) = 0.5$, $NS(s_5) = 0.55$, $NS(s_6) = 0.75$, $NS(s_7) = 0.8$ and $NS(s_8) = 1$.

The optimal ACI of the HFLPR H^1 is $ACI(H^1) = 0.783$. Based on the NS of each linguistic term, the transformed hesitant fuzzy preference relation V^1 associated with H^1 is obtained.

Example 2: Let $S = \{s_0, s_1, ..., s_8\}$ be defined as Example 1. Consider the following HFLPR,

$$H^2 = \begin{pmatrix} \{s_4\} & \{s_3\} & \{s_4\} & \{s_4\} & \{s_0, s_1, s_2\} \\ \{s_5\} & \{s_4\} & \{s_5\} & \{s_6, s_7\} & \{s_0, s_1, s_2\} \\ \{s_4\} & \{s_3\} & \{s_4\} & \{s_5\} & \{s_4\} \\ \{s_4\} & \{s_1, s_2\} & \{s_3\} & \{s_4\} & \{s_7, s_8\} \\ \{s_6, s_7, s_8\} & \{s_6, s_7, s_8\} & \{s_4\} & \{s_0, s_1\} & \{s_4\} \end{pmatrix}$$

Same to Example 1, set $\lambda = 0.05$. The optimization-based model to obtain the numerical scale NS is as follows,

$$\begin{cases} \max \quad ACI(H^2) \\ s.t.\ ACI(H^2) = \frac{1}{36} \times \\ \sum\limits_{h=1, L^h \in N_{H^2}}^{36} (1 - \frac{1}{15} \sum\limits_{i<j<z}^{n} \left| NS(l_{ij}^h) + NS(l_{jz}^h) - NS(l_{iz}^h) - 0.5 \right|) \\ \quad NS(s_0) = 0 \\ \quad NS(s_i) \in [(i-1)/8, (i+1)/8] \quad i = 1, ..., 7 \\ \quad NS(s_8) = 1 \\ \quad NS(s_{i+1}) - NS(s_i) \geq 0.05 \quad i = 0, 1, ..., 7 \end{cases}$$

By solving the above model using the software package Lingo, we obtain

$NS(s_0) = 0$, $NS(s_1) = 0.25$, $NS(s_2) = 0.375$, $NS(s_3) = 0.45$, $NS(s_4) = 0.5$, $NS(s_5) = 0.55$, $NS(s_6) = 0.625$, $NS(s_7) = 0.75$ and $NS(s_8) = 1$.

The optimal solution of the ACI of the HFLPR H^2 is $ACI(H^2) = 0.798$. Besides, based on $NS(s_i)$ $(i = 0, 1, ..., 8)$, the transformed hesitant fuzzy preference relation V^2, associated with H^2, is obtained.

From Examples 1 and 2, we conclude that the numerical scales for linguistic terms with different HFLPRs are different, which shows the individual difference in understanding the words.

V. Conclusion

In this paper, we propose a consistency-driven approach to set personalized numerical scales for linguistic terms with HFLPRs. First, we provide an average consistency measure for HFLPRs, which is determined as the average consistency degree of all linguistic preference relations associated to the HFLPR. Then a model based on the average consistency measure to obtain the personalized numerical scales with the aim of maximizing the average consistency of HFLPRs is proposed.

In the future, we plan to study the consistency-driven methodology to set numerical scales with HFLPRs in the GDM based on personalized individual semantics.

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