A consensus model for large scale using hesitant information

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Nowadays due to the technological development, large-scale group decision making problems (LSGDM) are common and they often need to obtain accepted solutions for all experts involved in the problem. To do so, a consensus reaching process (CRP) is applied. A challenge in CRP for LSGDM is to overcome scalability problems. This paper presents a new consensus model to deal with LSGDM that is able to reduce the time cost of the CRP.

Keywords: Large-scale; consensus reaching process; scalability; hesitant fuzzy sets.

1. Introduction

Many real-world problems affect society and might require agreed decisions. In such cases, it is necessary to apply a CRP.[1] However, most of the results obtained in this area are focused on GDM dealing with a few number of experts, but due to the current demands in the society, it is necessary to propose CRPs to deal with LSGDM. Taking into account the main challenges of classical CRPs for LSGDM problems,[2] this contribution proposes a CRP model for LSGDM to overcome the scalability problem related to time cost. The proposal includes the following novelties:

- It detects subgroups of experts according to their preferences and computes the relevance for each subgroup considering its size and cohesion.

- Most of the CRPs aggregate experts' preferences in the early stages of the process, the aggregation might result in a loss of information. To avoid such situations, this proposal will model experts subgroup preferences by hesitant fuzzy sets (HFS).[3]
- It defines a new feedback process that guides the CRP according to the consensus degree achieved.

The contribution is structured as follows: Section 2 revises some concepts. Section 3 presents the new consensus model to deal with LSGDM problems using HFS. Finally, section 4 points out some conclusions.

2. Preliminaries

2.1. *Large scale group decision making*

The concept of LSGDM is quite similar to GDM, but differs because in the former the number of experts who express their assessments is much greater than in the latter.

LSGDM has used solution schemes similar to GDM, such as the selection process. However, this process does not always guarantee that the selected decision is accepted by all experts involved in the problem.[1] A solution to obtain an agreed decision is to apply a CRP which implies that experts modify their preferences making them closer to each other.[4]

In LSGDM, it is also necessary to apply CRPs. Thus, some proposals have been introduced.[4,5] Nevertheless, these proposals aggregate expert's preferences in early stages of the decision process, which implies a loss of information and do not consider different level of agreement across the CRP that can provoke high time cost.

2.2. *Hesitant information*

HFSs[6] model the uncertainty provoked by the doubt that might occur when an expert wants to assign the membership degree of an element to a set.

Definition 2.1.[3] Let X be a reference set, a HFS on X is a function $\mathfrak{h}$ that returns a subset of values in [0,1]: $\mathfrak{h} : X \to \wp([0,1])$.

This definition is completed with the mathematical representation, $A = \{\langle x, h_A(x)\rangle : x \in X\}$, where $h_A(x)$ is called Hesitant Fuzzy Element (HFE) and is a set of some values in [0,1].

Definition 2.2.[7] Let X be a reference set, a hesitant fuzzy preference relation (HFPR) on X is represented by a matrix $H = (h_{ij})_{n\times n} \subset X \times X$, where $h_{ij} = \{\gamma_{ij}^s | s = 1, 2, \ldots, \#h_{ij}\}$($\#h_{ij}$ is the number of elements in h_{ij}) is a HFE that indicates the membership degrees that denote to which extent x_i is preferred to x_j. Additionally,

$$\gamma_{ij}^{\sigma(s)} + \gamma_{ji}^{\sigma'(s)} = 1, \quad h_{ii} = \{0.5\}, \quad \#h_{ij} = \#h_{ji}, \quad i, j = \{1, 2, \ldots, n\}$$
$$\gamma_{ij}^{\sigma(s)} < \gamma_{ij}^{\sigma(s+1)}, \quad \gamma_{ji}^{\sigma'(s+1)} < \gamma_{ji}^{\sigma'(s)},$$

where $\{\sigma(1), \ldots, \sigma(\#h_{ij})\}$ is a permutation of $\{1, \ldots, \#h_{ij}\}$, such that, $\gamma_{ij}^{\sigma(s)}$ is the s^{th} smallest element in h_{ij}.

HFS computations sometimes require that the HFEs involved have the same number of elements, to solve this issue, a β-normalization is applied.[8] Let h_j be the shortest one, $h_j^- = min\{\gamma | \gamma \in h_j\}$ and $h_j^+ = max\{\gamma | \gamma \in h_j\}$, the value to be added a number of times such that its length becomes equal to the largest one, is obtained by $\gamma' = \eta h_j^+ + (1 - \eta) h_j^-$, $\eta (0 \leq \eta \leq 1)$.

3. A New Consensus Model for LSGDM

This section presents a novel consensus model for LSGDM that is able to deal with the scalability challenge of a CRP. It consists of 6 main phases.

3.1. *Gathering preferences*

Each expert $e_r \in E$, provides his/her opinions on X by means of a fuzzy preference relation (FPR), $P^r = (p_{ij}^r)_{n \times n}$, that is reciprocal $p_{ij}^r + p_{ji}^r = 1$, $i, j \in 1, \ldots, n$.

3.2. *Framework configuration*

In a LSGDM problem there are a set of possible alternatives $X = \{x_1, \ldots, x_n\}$ and a large number of experts $E = \{e_1, \ldots, e_m\}$ involved in the problem, being $m >> n$. Three parameters are established in the CRP: (i) a consensus threshold, $\vartheta \in [0, 1]$, (ii) a parameter used in the adaptive feedback process, $\delta \in [0, 1]$, $\delta < \vartheta$ and (iii) the maximum number of rounds, Max_round.

3.3. *Managing subgroups*

3.3.1. *Subgroups identification*

This phase detects subgroups of experts according to their similar preferences by using a k-means based algorithm.

(1) Initially, there is a cluster for each alternative, $C = \{C_1, \ldots, C_n\}$.

(2) A centroid c^l is computed for each cluster.
(3) The distance between each FPR, P^r, and the centroid c^l, $l \in \{1, \ldots, n\}$, is calculated by a distance measure, e.g. the Euclidean distance.
(4) The preference relation P^r is assigned to the cluster for which, the distance between P^r and the centroid, is minimum.
$C^{l(t)} = \{P^r : d(P^r, c^{l(t)}) \leq d(P^r, c^{z(t)}), \forall 1 \leq z \leq n\}$
(5) New centroids are computed, $c_{ij}^{l(t+1)} = \frac{1}{|C^{l(t)}|} \sum_{P^r \in C^{l(t)}} p_{ij}^r$, $i, j \in \{1, \ldots, n\}$, where $|C^{l(t)}|$ is the number of preference relations that belong to the cluster C^l during iteration t.
(6) Repeat steps (3)-(5) until the assignments to the clusters do not change.

3.3.2. *Managing subgroups hesitation*

To keep as much information as possible in the CRP, unlike of oversimplifying the preference modelling with aggregation procedures, our proposal considers that the different experts' preferences elicited in the subgroup, despite of being similar, show a kind of hesitation in the group. Let $G^l = \{e_1^l, \ldots, e_r^l\}$ be the subgroup of experts belonging to cluster, C^l, with FPRs, $P^{l1} = (p_{ij}^{l1})_{n \times n}, \ldots, P^{lr} = (p_{ij}^{lr})_{n \times n}$. A HFPR, $HP^l = (h_{ij}^l)_{n \times n}$, $l \in \{1, \ldots, n\}$, that fuses all experts' preferences in G^l, is built such that, $h_{ij}^l = \{p_{ij}^{lk} | k = 1, 2, \ldots, |G^l|\}$ where $|G^l|$ is the cardinality of G^l.

3.3.3. *Weighting subgroups*

The relevance of a subgroup is based on its size (i.e., number of experts in the subgroup) and its cohesion (i.e. the level of togetherness among experts).

A geometric approach to compute the *cohesion* is defined:

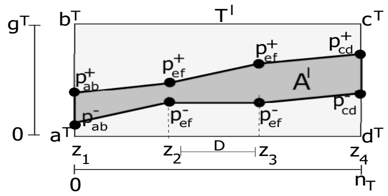

Fig. 1. Graphical representation of the cohesion of a subgroup.

(1) Let T^l be the total area of the rectangle consisting of the points a^T, b^T, c^T and d^T (see Fig. 1), i.e., $T^l = g^T \times n^T$.
(2) Let $I = \bigcup_{i,j \in n, i \neq j} \{(i,j)\}$ be a set with all the possible combinations over the set of alternatives $X = \{x_1, \ldots, x_n\}$. The minimum and maximum assessments for each pair of alternatives taking into account all the preferences of the subgroup G^l are obtained as follows.
$\gamma_{ij}^- = min\{\gamma_{ij}^1, \gamma_{ij}^2, \ldots, \gamma_{ij}^s\},\ \forall (i,j) \in I$
$\gamma_{ij}^+ = max\{\gamma_{ij}^1, \gamma_{ij}^2, \ldots, \gamma_{ij}^s\},\ \forall (i,j) \in I$
The first and last pair of alternatives considered on the X-axis are,
$\gamma_{ab}^- = min_{i,j \in I}\{\gamma_{ij}^-\},\ (a,b) \in I$
$\gamma_{cd}^+ = max_{i,j \in I}\{\gamma_{ij}^-\},\ (c,d) \in I$
A function f is defined to obtain the indexes of the pairs of alternatives, $f : \{z_1, z_2, \ldots, z_{n(n-1)}\} \rightarrow I$.
The area A^l, between the maximum and minimum assessments ordered on the X-axis by the minimum is computed by,

$$A^l = \left[\sum_{i,j \in I} (\gamma_{ij}^+ - \gamma_{ij}^-) - \frac{(\gamma_{ab}^+ - \gamma_{ab}^-) + (\gamma_{cd}^+ - \gamma_{cd}^-)}{2} \right] \cdot D \qquad (1)$$

where D is the distance between z_i and z_{i+1}, which in our case is 1.
(3) Finally, the cohesion is given by, $cohesion(G^l) = 1 - \frac{A^l}{T^l}$.

The value of the *size* is obtained from the subgroup identification and is modelled through a fuzzy membership function μ_{size} as shown in Figure 2.

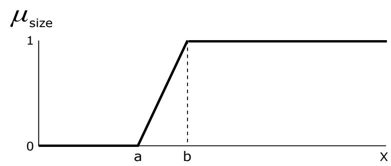

Fig. 2. Membership Function.

The values (size and cohesion) are fused by the convex combination.

Definition 3.1. Let $Y_{G^l} = \{y_1, y_2\}$ be the values obtained for the cohesion and size, $y_1, y_2 \in [0,1]$, of the subgroup G^l which are aggregated. The convex combination of Y_{G^l} is given by,

$$\varphi(Y_{G^l}) = y_1 \cdot \alpha_1 + y_2 \cdot \alpha_2 \qquad (2)$$

being $\alpha = \{\alpha_1, \alpha_2\}$ a weighting vector, $\alpha_i \in [0,1], i = \{1,2\}$ and $\sum_i \alpha_i = 1$.

3.4. *Computing the consensus degree*

The consensus degree among experts is computed in 3 steps.

(1) For each pair of subgroups G^l and G^k, a similarity matrix $SM^{lk} = (sm^{lk}_{ij})_{n\times n}$ is computed, $sm^{lk}_{ij} = 1 - d(h^l_{ij}, h^k_{ij})$, where d is a distance measure for HFEs.[9]
(2) A consensus matrix, $CM = (cm_{ij})_{n\times n}$, is obtained by aggregating the similarity matrices by the arithmetic mean, $cm_{ij} = \frac{\sum_{l=1}^{n-1}\sum_{k=l+1}^{n} sm^{lk}_{ij}}{n(n-1)/2}$.
(3) The consensus degree is computed by, $cr = \frac{\sum_{i=1}^{n}\sum_{j=1, i\neq j}^{n} cm_{ij}}{n(n-1)}$.

3.5. *Consensus control*

The consensus degree cr is compared with the consensus threshold $\vartheta \in [0,1]$. If $cr \geq \vartheta$, then the consensus process ends, otherwise more discussion rounds are necessary. The maximum number of discussion rounds is controlled by the parameter Max_round.

3.6. *Adaptive feedback process*

The rules for the advice generation depend on the consensus level achieved, cr, that determines whether the consensus level is "high" or "low". The feedback process consists of 3 steps.

- Obtain a collective matrix, HP^C, by aggregating the preferences represented by normalized HFPRs $\{\overline{HP}^1, \ldots, \overline{HP}^n\}$ (see Def. 2.2).
- Compute the proximity between each subgroup $\{\overline{HP}^1, \ldots, \overline{HP}^n\}$, and the collective matrix HP^C, by using a similarity measure for HFSs,[9] $pr^l = sim(HP^C, \overline{HP}^l) = 1 - d(HP^C, \overline{HP}^l)$.
- Adapt the feedback process according to reached consensus degree cr.

Group feedback process: If the consensus degree $cr < \delta$, this means that the consensus is "low". Then, all experts of the furthest subgroups are recommended to modify their preferences. This is done as follows.

(1) If $pr^l \leq \overline{pr}$, then the subgroup G^l is selected, $\overline{pr} = \frac{1}{n}\sum_{l=1}^{n} pr^l$.
(2) If $pr^l_{ij} \leq \overline{pr}_{ij}$, then the pair of alternatives (x_i, x_j) is selected, $\overline{pr}_{ij} = \frac{1}{n}\sum_{l=1}^{n} pr^l_{ij}$ and $pr^l_{ij} = 1 - d(h^C_{ij}, h^l_{ij})$

If $(v^l_{ij}) < (v^C_{ij})$, then all experts who belong to the subgroup G^l should increase their preferences over the pair of alternatives (x_i, x_j) and if $(v^l_{ij}) > (v^C_{ij})$, they should decrease them.

Let v_{ij}^{l} and v_{ij}^{C} be calculated as follows, $v_{ij}^{l} = \frac{1}{\#h}\sum_{s=1}^{\#h}\gamma_{ij}^{l,s}$ and $v_{ij}^{C} = \frac{1}{\#h}\sum_{s=1}^{\#h}\gamma_{ij}^{C,s}$.

Individual feedback process: If the consensus degree, $\delta \leq cr < \vartheta$, then the consensus level is "high".

(1) If $pr^{l} \leq \overline{pr}$, then the subgroup G^{l} is selected.
(2) If $pr_{ij}^{l} \leq \overline{pr}_{ij}$, then the pair of alternatives (x_i, x_j) is selected.
(3) If $pr_{ij}^{lr} \leq pr_{ij}^{l} = \{(r)|(1 - |v_{ij}^{C} - \gamma_{ij}^{lr}|) \leq pr_{ij}^{l}\}$, the expert e_r is selected.

- If $(\gamma_{ij}^{lr}) < (v_{ij}^{C})$, then the expert e_r who belongs to the subgroup G^{l} should increase his/her preference over the pair of alternatives (x_i, x_j) and if $(\gamma_{ij}^{lr}) > (v_{ij}^{C})$, then he should decrease it.
- If $(\gamma_{ij}^{lr}) = (v_{ij}^{C})$, then it is not necessary to make changes.

4. Conclusions

LSGDM problems are common and agreed decisions are more appreciated, thus CRPs are necessary. Because current approaches are time consuming, scalability problems are a challenge. Therefore, a new CRP model dealing with LSGDM is proposed to overcome these problems.

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