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Decision support

Consensus methods with Nash and Kalai–Smorodinsky bargaining game for large-scale group decision-making

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ABSTRACT

With the significant advancements in communication technology, group decision-making (GDM) can now be implemented online, allowing a large number of decision-makers (DMs) to participate concurrently. However, current methods for large-scale group decision-making (LSGDM) are primarily suitable for 20 to 50 DMs, and their effectiveness in scenarios involving thousands or even tens of thousands of participants has yet to be fully validated. Furthermore, as the number of participants increases, the evaluation information becomes increasingly diverse and complex. At the same time, the social networks associated with the DMs typically become sparse, making information sharing and consensus building more challenging. In light of these challenges, we develop two new methods based on cooperative games to effectively address the challenges in super LSGDM. First, we propose a two-stage semi-supervised fuzzy C-means (FCM) clustering method with trust constraints, which aims to address the issue of sparsity in relationships within large-scale social networks. This method utilizes trust relationships as reliable resources and prior knowledge to guide and supervise the clustering process. On this basis, we discuss three scenarios from the perspective of cooperative games: (i) subgroup optimal consensus adjustments in non-cooperative situations, (ii) group optimal consensus adjustments in cooperative situations, and (iii) subgroup optimal consensus adjustments in cooperative situations. Subsequently, we view the consensus adjustment allocation as a cost cooperative game problem and propose two new LSGDM consensus methods based on Nash Bargaining (NB) and Kalai–Smorodinsky Bargaining (KSB). Finally, experiments on real datasets demonstrate the superiority and reliability of our proposed LSGDM methods.

1. Introduction

In recent years, rapid advancements in blockchain and communication technologies have greatly facilitated the widespread involvement of decision-makers (DMs) in day-to-day incident management, which is now becoming the norm (Gupta, Modgil, Bhattacharyya, & Bose, 2022). Traditional group decision-making (GDM) theories usually involve no more than seven DMs and are showing their limitations when faced with more complex modern problems (Chiclana, Herrera-Viedma, Herrera, & Alonso, 2007; Goers & Horton, 2023; Panda, Modak, Basu, & Goyal, 2015). Consequently, large-scale GDM (LSGDM) has emerged as a prominent research topic, attracting widespread attention from scholars (Liu, Shen, Zhang, Chen, & Wang, 2015; Palomares, Martínez, & Herrera, 2013; Rodríguez, Labella, Tré, & Martínez, 2018; Tong & Zhu, 2023; Xu, Du, & Chen, 2015). Ding et al. (2020) formally defined LSGDM, describing it as a decision-making process that involves at least twenty DMs. A core issue in LSGDM research is how DMs from various domains with different knowledge backgrounds can achieve a consensus (Urena, Chiclana, Melancon, & Herrera-Viedma, 2019). In this regard, numerous researchers have proposed various LSGDM methods, including those based on social networks (Lesser, Naamani-Dery, Kalech, & Elovici, 2017), information granularity (Zhang, Dong, & Pedrycz, 2022), online reviews (Chen, Liu, Chin, Pedrycz, Tsui, & Skibniewski, 2021; Guo, Zhan, Kou, & Martínez, 2024), and behavioural decision-making (Chao, Kou, Peng, & Herrera-Viedma, 2021; Shen, Ma, Zhang, & Zhan, 2024). However, most of these studies involve groups of only around 20 to 50 DMs. A recent survey by García-Zamora, Labella, Ding, Rodríguez, and Martínez (2022) criticized this limitation, emphasizing the need to test models with much larger groups, involving

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thousands or even tens of thousands of DMs in LSGDM. Thus, despite significant advancements in the research of LSGDM, several critical challenges remain to be addressed.

To effectively reduce the complexity of LSGDM problems, clustering techniques are widely utilized to divide numerous DMs into more manageable subgroups. This technique simplifies the decision-making process by organizing DMs into several manageable decision units (Meng, Tang, & An, 2023; Shen, Ma, & Zhan, 2023; Tang & Liao, 2021). Traditionally, clustering methods have primarily relied on factors such as opinion similarity, alternative ranking, preference consistency, and conflict (Shen et al., 2024). However, with the advancement of e-democracy and social network analysis (SNA), social relations have emerged as a reliable and effective clustering resource, which are extensively applied in addressing LSGDM challenges. On the other hand, since DMs tend to adopt recommendations from those they trust, the introduction of social networks not only helps to reduce conflicts but also improves the acceptance of advice (Liu, Zhou, Ding, Palomares, & Herrera, 2019). For example, Liu, Jiao, Shen, Chen, Wu, and Chen (2022) proposed a dual-path consensus feedback model based on a hybrid trust network, which improves the handling of dynamic trust relationships and the control of adjustment cost. Additionally, Li et al. (2023) developed a two-stage consensus model that integrates dynamic social networks and employs spectral clustering algorithms to enhance the efficiency of coordination among DMs. Despite these advantages, it is important to note that this method generally works best in smaller, closely-knit groups. In contrast, in some large-scale citizen participation projects in public decision-making, there are fewer direct interactions between individuals due to the large number and dispersal of participants, resulting in more sparse social networks. In this case, it is difficult for traditional community detection algorithms to achieve effective dimensionality reduction due to network sparsity, computational difficulty, and constraints of cost and human resources. Therefore, the application of these algorithms faces significant challenges in the context of both large and sparse networks.

After dividing a large-scale DM group into several subgroups, these subgroups are treated as independent decision-making units, thereby simplifying LSGDM problems into a more manageable traditional GDM framework (Wan, Xu, & Han, 2024). Subsequently, a consensus reaching process (CRP) is employed to coordinate the opinions among subgroups to reduce divergences to an acceptable level for the group. When the consensus criteria are not met, most LSGDM methods employ a feedback iteration mechanism. However, Meng, Wang, Pedrycz, and Tan (2024) have identified several shortcomings in this mechanism, including insufficient quantification, excessive adjustments, high time costs, and the absence of fair and rational explanations. In response to these issues, some scholars have extended the application of optimal model methods in GDM to LSGDM. For instance, Rodríguez, Labella, Nuñez-Cacho, Molina-Moreno, and Martínez (2022) developed a comprehensive minimum cost consensus model (MCCM) suitable for the circular economy sector, while Zhang, Dong, Zhang, and Pedrycz (2020) explored the interactions between coordinators and DMs and constructed a consensus model with maximum return adjustment and minimum cost feedback. Compared to the feedback iteration mechanism, the optimized model mechanism demonstrates better performance addressing the aforementioned issues, particularly alleviating the first three deficiencies. Nevertheless, this mechanism also has limitations, including insufficient consideration of individual DMs' willingness to adjust and the distribution questions of consensus adjustments. Intuitively, as reported by Meng, Gong, and Pedrycz (2023), DMs often prefer to preserve their own interests and are reluctant adjust their initial opinions, tending to minimize any unnecessary changes.

In summary, while some studies have addressed questions such as minimum consensus adjustments (Dong, Xu, Li, & Feng, 2010), selfish behaviours (Meng, Gong, & Pedrycz, 2023), and fair adjustment (Du, Liu, & Liu, 2022), the interactions among various DMs remain insufficiently explored and require further investigation. Clearly, these decision-making paradigms do not meet the complex needs of LSGDM. Consequently, this study addresses this gap by exploring consensus adjustment questions for DMs across various scenarios. Specifically, this study aims to answer the following research questions:

(1) How can information from large-scale sparse social networks be leveraged to address the dimensionality reduction problem?
(2) What strategies can be implemented to ensure the acceptability of final decision outcomes?
(3) Is cooperation universally beneficial for participants, or there are conditions under which it may fail?
(4) How can a fair consensus adjustment mechanism be designed to allocate resources fairly among participants?

To address these issues, this study proposes two new consensus methods for LSGDM based on optimization models and cooperative game theory. Firstly, we introduce a two-stage semi-supervised fuzzy C-means (FCM) clustering method with trust constraints, which effectively tackles the problem of dimensionality reduction failure caused by sparse and incomplete social network data. Next, we explore consensus adjustment strategies under decentralized and centralized decision-making from the perspective of cooperative games, comparing the optimal consensus adjustment costs in cooperative and non-cooperative scenarios. The results indicate that in certain independent decision-making contexts, participants may achieve consensus with lower adjustment costs, suggesting that cooperation may not always be the optimal choice. To this end, we transform the consensus adjustment allocation into a cost cooperative game problem, designing a strategy that minimizes the total cost while ensuring a fair distribution of costs among all participants. Overall, this study contributes significantly to the existing theory and demonstrates its potential for practical applications. Specifically, the main innovations of this study are outlined below:

(1) We introduce a two-stage trust-constrained semi-supervised learning mechanism that effectively addresses the questions of sparse and incomplete social network data in LSGDM. Notably, this method not only improves the quality of dimensionality reduction but also enables flexible control over the role of trust information in clustering through the adjustment of regularization parameters.

(2) We discuss optimal consensus adjustment strategies across three different scenarios. In these scenarios, the global minimum adjustment cost reflects collective interests, while the minimum adjustments cost under non-cooperative and cooperative conditions correspond to the threat points and ideal points in cooperative negotiations, respectively.

(3) We establish a two-type MCCM that fuses non-cooperative and cooperative strategies, and the mechanism effectively balances the interests of cooperative and non-cooperative parties. This method provides a vital innovation for the coordination of complex multi-party interests, and has the potential to be widely applied to other areas involving multi-party competition and cooperation.

(4) We develop two new LSGDM consensus methods based on Nash bargaining (NB) and Kalai–Smorodinsky bargaining (KSB) theories. These methods optimize the process by which DMs achieve their consensuses in both decentralized and centralized environments, thereby enhancing decision quality and fairness.

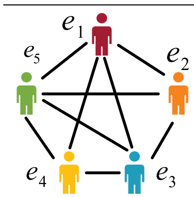
To improve readability, Table 1 lists the definitions corresponding to a number of abbreviations and symbols.

The main structure of this study is exhibited below: Section 2 first introduces the basic concepts of SNA, LSGDM and FCM to lay the theoretical foundations for this study. Section 3 describes in detail the proposed two-stage semi-supervised FCM clustering method with trust constrains. Building on these foundations, Section 4 explores optimal consensus adjustment strategies for subgroups and groups in various situations from the perspective of cooperative games. Section 5 then reveals two LSGDM consensus methods based on cooperative games under fairness concern considerations. Section 6 demonstrates

Table 1
Meaning of some abbreviations and symbols.

Abbreviation	Meaning	Symbol	Meaning
GDM	Group decision-making	SG	The set of all subgroups
LSGDM	Large-scale group decision-making	S_1	The set of consensus subgroups
DM	Decision-maker	S_2	The set of non-consensus subgroups
SNA	Social network analysis	Δ_h	Minimum adjustment cost for the subgroup S_h
FCM	Fuzzy C-means	WC	The set of subgroups willing to cooperate in S_2
NB	Nash bargaining	NC	The set of non-cooperative subgroups in S_2
KSB	Kalai–Smorodinsky bargaining	TP_h	Threat point of the subgroup S_h
CRP	Consensus reaching process	IP_h	Ideal point of the subgroup S_h
MCCM	Minimum cost consensus model	TC^*	Group minimum adjustment cost

Table 2
The different representation schemes for social network.

Graph theory	Algebraic relation	Sociometric matrix
	e_1Re_2, e_1Re_3, e_1Re_4; e_1Re_5, e_2Re_3, e_2Re_5; e_3Re_4, e_3Re_5, e_4Re_5;	$\begin{bmatrix} - & 1 & 1 & 1 & 1 \\ 1 & - & 1 & 0 & 1 \\ 1 & 1 & - & 1 & 1 \\ 1 & 0 & 1 & - & 1 \\ 1 & 1 & 1 & 1 & - \end{bmatrix}$

the practical application effects of these methods through a real case, and sensitivity analyses of key parameters are conducted. Section 7 verifies the applicability and superiority of the new methods through a comparative analysis and discussion. Section 8 elaborates on the theoretical and practical significance of this study based on in-depth analysis of the research results. At last, Section 9 summarizes the major contributions of this study and explores potential directions for future research.

2. SNA, LSGDM and FCM

In this section, we exhibit several key concepts related to SNA, LSGDM and FCM.

2.1. Social network analysis (SNA)

A social network typically comprises DMs and their social relationships, with a graphical representation constructed by synthesizing all DMs relationships in the network. In particular, social networks based on similar views or trust relationships are frequently utilized in LSGDM scenarios. Previous research has indicated that social relationships significantly influence the decision-making process, as DMs may be influenced by close friends to adjust their preferences (Li, Kou, Li, & Peng, 2022).

Definition 2.1 (*Wu & Chiclana, 2014*)**.** A social network can be represented by a graph $G = (E, L)$, where $E = \{e_1, e_2, \dots, e_m\}$ is the set of nodes and $L = \{l_{kh} \mid k, h = 1, 2, \dots, m\}$ denotes the set of edges of relationships between nodes.

Additionally, a social network can be depicted using a matrix $T = (t_{ij})_{m\times m}$, where an element of 1 in the matrix indicates the presence of a connection between nodes and 0 indicates no connection. Table 2 shows three distinct representations of social networks: graph form, algebraic relations, and matrix representation.

In social networks, centrally located nodes often represent active or core members of the community, as these nodes are directly connected to many other nodes. Degree centrality index, as one of the measures of a node's importance in a network, reflects the number of direct connections a node has with other nodes.

Definition 2.2 (*Wu & Chiclana, 2014*)**.** Suppose that $T = (t_{ij})_{m\times m}$ is the adjacency matrix associated with the social network. Then, the degree centrality index DC_k of the node e_k can be computed as follows:

$$DC_k = \frac{1}{m-1}\sum_{k=1,k\neq h}^{m} t_{kh}, \tag{2.1}$$

where m is the total number of nodes in the graph, and t_{kh} denotes the connection weight between the node e_k and the node e_h.

Next, we define $DC^h_{\sigma(p)}$ to denote the pth largest degree centrality index in the community C_h. This notation helps to clearly indicate the position and importance of specific nodes when discussing the internal structure of the community.

2.2. Large-scale group decision-making (LSGDM)

In the LSGDM problem, it is assumed that $E = \{e_1, e_2, \dots, e_m\}$ represents the set of DMs, who typically provide individual evaluation information about alternatives in a clear numerical form, represented by $O = \{o_1, o_2, \dots, o_m\}$. The weights of the DMs are denoted by $W = \{w_1, w_2, \dots, w_m\}$, where $w_k \geq 0$ and $\sum_{k=1}^{m} w_k = 1$.

To reduce the complexity of LSGDM, clustering is often employed to downscale DMs (Xu et al., 2015). Its primary aim is to segregate DMs into subgroups so that those with similar opinions are clustered together. This method simplifies the decision problem by treating each subgroup as a distinct decision-making unit. Suppose the large-scale DMs are divided into k subgroups. The evaluation value of the hth subgroup, denoted as r_h, is obtained by averaging the evaluations of its members. Then, the consensus level CL_h of the subgroup C_h can be calculated as follows:

$$CL_h = 1 - |r_h - g|, \tag{2.2}$$

where g represents the group opinion of the weighted average of all subgroups.

Next, the group consensus level GCL can be calculated as follows:

$$GCL = \frac{1}{k}\sum_{h=1}^{k} CL_h. \tag{2.3}$$

Remark 2.1. Notably, Eqs. (2.2)–(2.3) simplify the computational process and increase efficiency by dividing large-scale DMs into subgroups. By managing subgroups, the entire decision-making process can be observed and controlled at a higher level, enhancing the flexibility and relevance of decision-making.

2.3. Fuzzy C-means (FCM)

As an unsupervised clustering method, FCM is regarded as a powerful mining tool for exploring data structures in machine learning (Cannon, Dave, & Bezdek, 1986). Fundamentally, it operates as a segmentation algorithm that employs a flexible fuzzy segmentation method. By incorporating fuzzy theory into cluster analysis, the FCM algorithm uses the membership function to determine the classification of data objects, allowing it to maintain the flexibility of the algorithm while improving the accuracy of the classification.

To summarize, the core objective of the FCM algorithm is to solve an optimization problem that aims to compute a partitioning matrix which reflects the membership degree of the samples to the different clusters. Formally, in the scenario of a given dataset $X = \{x_1, x_2, \dots, x_m\}$ and with the total number of clusters predetermined to be k, the FCM algorithm achieves its core objective by defining the following specific objective function:

$$\begin{aligned} &\mathbf{min} \quad J(U,C) = \sum_{i=1}^{m} \sum_{j=1}^{k} u_{ij}^{m^*} \; d_{ij}^2 \\ &\text{s.t.} \begin{cases} 0 \le u_{ij} \le 1, & \forall\ i = 1,2,\dots,m, j = 1,2,\dots,k, \\ \sum_{j=1}^{k} u_{ij} = 1, & \forall i = 1,2,\dots,m, \end{cases} \end{aligned} \tag{2.4}$$

where m is the number of data points, k is the number of clusters, u_{ij} denotes the membership degree of data point x_i to the cluster centre c_j, and m^* is the fuzzification parameter usually taken as 2. Additionally, $d_{ij} = \|x_i - c_j\|$ represents the Euclidean distance from the data point x_i to the cluster centre c_j.

By applying the constraints of membership and optimizing the objective function using the Lagrange multiplier method to solve for and set the partial derivatives of the membership and cluster centres to zero, the update formula for the membership is derived as follows:

$$u_{ij} = \frac{1}{\sum_{l=1}^{k} \left(\frac{d_{ij}}{d_{il}}\right)^{\frac{2}{m^*-1}}}. \tag{2.5}$$

Analogously, the update formula for the clustering centre c_i is as follows:

$$c_i = \frac{\sum_{j=1}^{m} u_{ij}^{m^*} \; x_j}{\sum_{j=1}^{m} u_{ij}^{m^*}}. \tag{2.6}$$

The algorithm terminates under either of the following conditions: (a) the decrease in the objective function becomes insignificant, or the magnitude of updates to the cluster centres falls below a predetermined threshold; (b) the maximum number of iterations is reached. These mechanisms ensure the algorithm halts either upon convergence to a stable solution or when reaching the computational limit.

3. Two-stage semi-supervised FCM clustering method with trust constraints

Based on previous discussions, the inherent sparsity of large-scale social networks may affect the effectiveness of traditional community detection methods. To address this challenge, we propose a two-stage semi-supervised FCM clustering method. The first stage aims to identify potential groups within the social network, while the second stage further refines the group partitioning using a semi-supervised FCM clustering method under trust constraints. The motivation and specific steps of this method will be detailed below.

3.1. Main motivations

As García-Zamora et al. (2022) have indicated, advancements in digital technology now allow addressing super LSGDM questions involving thousands of DMs. Consequently, research into integrating LSGDM methodologies with popular technologies is of significant importance. It must be emphasized that reliance solely on theoretical models and case analyses involving only 20–50 DMs is insufficient to demonstrate the efficacy of proposed methods in practical applications. This is because if a model cannot maintain good performance across various scales and complex environments, its real-world adoption becomes highly unlikely. In summary, although unsupervised and supervised clustering methods have achieved significant research outcomes, the challenges of managing LSGDM with a large number of DMs remain inadequately addressed, including:

(1) The existing clustering and community detection algorithms often overlook the phenomenon of cliques and known labels that may exist prior to clustering.
(2) Human and cost constraints in large-scale social networks hinder a comprehensive understanding of DMs' social connections, thereby making it challenging for traditional community detection algorithms to accurately identify tight groups in sparse networks.
(3) When extending decision-making to social network environments, trust relationships as reliable resources and a priori knowledge for clustering and CRP, their roles in guiding and supervising clustering have not been fully investigated.

In light of the aforementioned questions, this section introduces a two-stage semi-supervised FCM clustering method with trust constraints, aimed at improving the effectiveness of classification management and maximizing the utilization of decision-making information. In the following, we implement the proposed method in two stages as follows:

- **Stage 1:** Community detection for sparse social networks using Louvain algorithm to identify and tag groups with strong social connections. (When the labels are known, there is a flexible option to move directly to **Stage 2**)
- **Stage 2:** To apply semi-supervised FCM algorithm to use the data labelled in the **Stage 1** as supervised information, by adjusting the objective function of the FCM algorithm so that it can achieve supervision in the iterative process of clustering.

The execution process of the proposed two-stage clustering method is exhibited in Fig. 1.

3.2. Stage 1: Identification of potential groups in social networks

In the first stage of this section, the objective is to identify potential groups within social networks, particularly when social connections between DMs are sparse or difficult to fully capture. To achieve this, the study employed the Louvain algorithm for community detection, which has demonstrated significant efficiency and accuracy in identifying tightly-knit groups in large-scale networks. Notably, in certain practical scenarios, the labels of these groups may already be known, allowing a direct transition to the second stage.

Initially, we assume that m DMs are classified into r different communities based on their trust relationships (Waltman & Van Eck, 2013), with the clustering results denoted by $C = \{C_1, C_2, \dots, C_r\}$. In the context of sparse social network structures, the clustering process may cover only a few or a single DM, leading to suboptimal clustering results when the expected dimensionality reduction effect is not achieved. Consequently, this study opts to retain those small groups with tight membership and significant size, treating them as known labels. To effectively identify key DMs in the community, this study introduces a degree centrality index based on SNA techniques to identify nodes that may correspond to known labels.

During the potential group identification, our objective is to identify members of the social network who meet specific conditions. We suppose that a potential group should meet the following two criteria:

(1) The community C_h in which the small group B_h is located has a relatively large number of people, where $B_h \subseteq C_h$.

(2) The probability of being a central node should be higher for the members of the small group B_h than for the other members within the same community C_h.

Assume that B_h represents a potential small group, it must satisfy the following conditions:

$$B_h = \{e_j \mid e_j \in C_h \wedge |C_h| \ge \lceil m/r \rceil \wedge DC_j \ge DC_{\sigma(p)}^h\ \}, \tag{3.1}$$

where $2 \le p \le \lceil m/r \rceil$, $|C_h|$ is the number of DMs within C_h and $\lceil\ \rceil$ is a ceiling function, $DC_{\sigma(p)}^h$ denotes the pth largest degree centrality index in the community C_h. m is the total number of decision makers and r is the number of initial clusters.

By identifying these eligible cliques and labelling them as members belonging to the same class (i.e., the known labels), a solid foundation is laid for the subsequent semi-supervised clustering process.

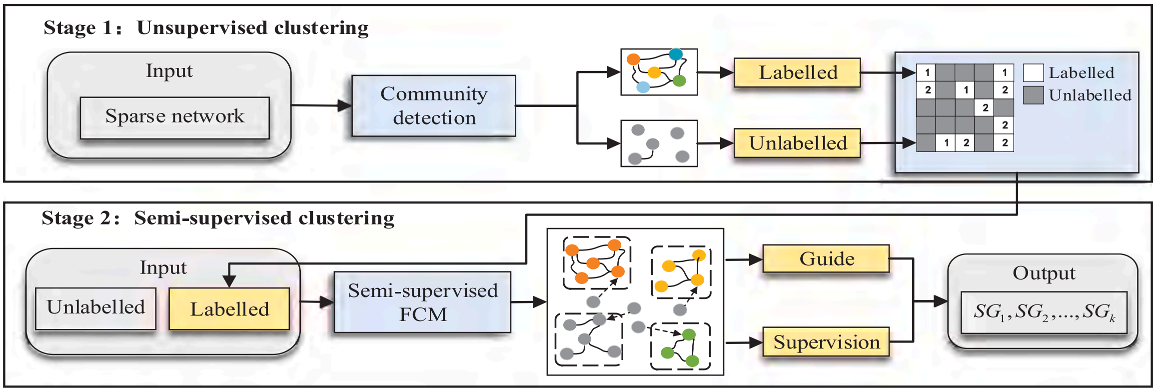


Fig. 1. The execution process of the proposed two-stage clustering method.

3.3. *Stage 2: Semi-supervised FCM clustering method*

Over the past decades, FCM clustering algorithms have seen widespread use in various fields due to their simplicity and effectiveness (Shen, Pedrycz, Chen, Wang, & Gacek, 2019). However, traditional FCM algorithms often fail to fully utilize the available prior knowledge in certain application scenarios (Pedrycz, 1985). Therefore, this study introduces a semi-supervised FCM algorithm that utilizes a small number of category labels as supervisory information, which is integrated into the objective function of the FCM algorithm to achieve the supervisory function in the clustering process. At this point, the objective function of this semi-supervised FCM algorithm is defined as follows:

$$\min \; J(U,C) = \sum_{i=1}^{m}\sum_{j=1}^{k} u_{ij}^{m^*} d_{ij}^2 + \alpha \sum_{i=1}^{m}\sum_{j=1}^{k} (u_{ij} - f_{ij}a_j)^{m^*} d_{ij}^2$$
$$\text{s.t.} \begin{cases} 0 \le u_{ij} \le 1, & \forall\, i = 1,2,\ldots,m, \quad j = 1,2,\ldots,k, \\ 0 \le f_{ij} \le 1, & \forall\, i = 1,2,\ldots,m, \quad j = 1,2,\ldots,k, \\ \sum_{j=1}^{k} u_{ij} = 1, & \forall\, i = 1,2,\ldots,m, \\ a_j \in \{0,1\}, & \forall\, j = 1,2,\ldots,k, \end{cases} \tag{3.2}$$

where α is a regularization parameter used to balance unsupervised and supervised information. It is proportional to the ratio of the total number of samples m to the number of labelled samples q. That is, the value of the parameter α should be adjusted according to the proportion m/q (Pedrycz & Waletzky, 1997). $A = (a_j)_{1\times m}$ represents a labelling indicator, where the value for the known label x_i is $a_i = 1$, and $a_i = 0$ otherwise, f_{ij} represents the membership degree of a labelled sample x_i for c_j.

Using the Lagrange multiplier method, the iterative expression for the membership matrix is given by:

$$u_{ij} = \frac{1}{1+\alpha}\left\{ \frac{1 + \alpha\left(1 - a_j \sum_{h=1}^{k} f_{hj}\right)}{\sum_{h=1}^{k} \frac{d_{ij}^2}{d_{hj}^2}} + \alpha f_{ij} a_j \right\}. \tag{3.3}$$

Analogously, the update formula for the clustering centre c_i is defined as:

$$c_i = \frac{\sum_{j=1}^{m} u_{ij}^{m^*} x_j + \alpha \sum_{j=1}^{m} \left(u_{ij} - f_{ij} b_j\right)^{m^*} x_j}{\sum_{j=1}^{m} u_{ij}^{m^*} + \alpha \sum_{j=1}^{m} \left(u_{ij} - f_{ij} b_j\right)^{m^*}}. \tag{3.4}$$

The optimal solution for J is obtained by iteratively computing and applying Eqs. (3.3) and (3.4). A more detailed description of this clustering process is available in Algorithm 1.

Remark 3.1. It should be noted that this method is flexible and the choice of parameters should be set according to specific decision-making needs. By reasonably adjusting the parameters α and p, it is possible to make full use of the labelled data while maintaining a good generalization ability to the unlabelled data.

Algorithm 1: Two-stage semi-supervised clustering algorithm

Input: The initial evaluation value $O = \{o_1, o_2, \cdots, o_m\}$, the adjacency matrix $T = (t_{kh})_{m\times m}$, the number of clusters k and the parameters α, p.
Output: The final clustering result: $SG = \{SG_1, SG_2, \cdots, SG_k\}$.
Step 1: Initialize each node i as a separate community C_i;
Step 2: For each node i, consider moving it to the community where all its neighbouring nodes j are located and calculate the change in the degree of modularity ΔQ ;
Step 3: This move is performed if moving the node i to another community increases the degree of modularity, i.e., $\Delta Q > 0$;
Step 4: Repeat **Steps 2-3** until the movement of all nodes no longer increases the modularity;
Step 5: Based on the current division of communities, a new network is established in which each community becomes a new node;
Step 6: Repeat **Steps 2-4** on the new network until the modularity no longer increases;
Step 7: Obtain the initial community division result: $C = \{C_1, C_2, \cdots, C_r\}$;
Step 8: The initial partitioning result C is further filtered according to the two conditions in Eq. (3.1) to identify potential labelled groups, and then the labelling indicator $A = (a_j)_{1\times m}$ for the labelled samples and the membership matrix $F = (f_{ij})_{k\times q}$ are derived;
Step 9: The data with label information is used for initial division and then the result obtained is used as the initial clustering centre;
Step 10: Calculate the new membership matrix U and the new clustering centre according to Eqs. (3.3) and (3.4);
Step 11: Calculate the objective function according to Eq. (3.2). If the difference between the two times before and after is less than the threshold or the maximum number of iterations is reached, then the algorithm ends; otherwise, go to **Step 10**;
Step 12: Classification results are obtained based on the final membership matrix;
Step 13: End.

4. Optimal consensus adjustment strategy: A cooperative game perspective

In the previous section, we have elucidated the solution to the classification problem involving large-scale DM. Next, we will discuss the cost allocation problem of consensus adjustment in CRP. Currently, although consensus methods based on feedback iterative mechanisms and minimum cost are widely adopted, they both exhibit different limitations (Meng et al., 2024). For this reason, we propose a new solution in this section from the perspective of cooperative games.

As mentioned earlier, some of the optimal model adjustments are obtained in light of the assumption that participants would accept them unconditionally. However, it is important to note that the adjustments recommended by these models may be difficult to accept when participants have more advantageous options available. Therefore, a more convincing and reliable strategy should take into account the possible selfish or non-cooperative behaviour of all participants. Based on this, the following three hypotheses are presented before proposing a new method:

1. Participants tend to reject further cooperation and negotiation when cooperation fails to provide significant benefits.
2. Although participants generally exhibit selfishness, they do not want their selfish behaviours to be a hindrance to consensus formation.
3. If the consensus adjustment programme is fair and reasonable, all participants will accept it.

4.1. Subgroup optimal consensus adjustment in non-cooperative situations

Inspired by previous work (Ben-Arieh & Easton, 2007; Labella, Liu, Rodríguez, & Martínez, 2020; Meng, Pedrycz, & Tang, 2022), this section explores the optimal consensus adjustment problem in the CRP from the perspective of subgroup independent decision-making in non-cooperative situations. Assume that S_1 and S_2 represent the set of subgroups that has reached a consensus and the set of subgroups that has not reached a consensus, respectively, where $S_1 \cap S_2 = \emptyset$ and $S_1 \cup S_2 = SG$. For the subgroup $SG_h \in S_2$ that does not reach a consensus, we assume that this subgroup achieves a consensus by minimizing the adjustment cost while keeping the initial opinions of all other DMs unchanged. To quantify the minimum adjustment cost required for each subgroup under independent decision-making, we construct the following model:

$$\textbf{Model 1}: \quad \begin{array}{ll} \min & TP_h = c_h\left|r_h - \bar{r}_h\right| \\ \text{s.t.} & \begin{cases} \overline{CL}_h = 1 - \left|\bar{r}_h - \bar{g}\right| \geq \theta, & (a) \\ \bar{g} = \lambda_h \bar{r}_h + \sum_{l=1, l\neq h}^{m} \lambda_l r_l, & (b) \\ 0 \leq \bar{r}_h \leq 1, & (c) \end{cases} \end{array}$$

where r_h and $\bar{r}_h$ denote the original and adjusted opinions of the subgroup SG_h, respectively, λ_h and c_h denote its weight and unit adjustment cost. $\overline{CL}_h$ denotes the consensus level of the subgroup SG_h after the opinion adjustment, $\bar{g}$ is the adjusted group opinion, and θ is the consensus threshold. The predefined consensus threshold should be adjusted according to the specific problem, such as using 0.9 for critical decisions and 0.8 for urgent situations (Tang, Zhou, Liao, Xu, Fujita, & Herrera, 2019).

In the above model, the constraint (a) ensures that the subgroups are able to meet the desired consensus level after adjustment. The constraint (b) details the method of aggregating group opinions, and the constraint (c) limits the scope of opinion adjustment to ensure that the adjusted opinions are within a reasonable range. In short, **Model 1** determines the minimum adjustment cost that each subgroup needs to incur in order to be consistent with the initial opinions of the other subgroups, provided that the consensus threshold is reached.

From the perspective of cooperative game theory, **Model 1** can be viewed as a strategy for optimizing the interests of subgroups, revealing the minimum gains that each subgroup can achieve in the absence of cooperation. Specifically, the threat point represents the adjustment cost required for each subgroup to reach a consensus under conditions of complete independence and no cooperation or compromise. The threat point is a core concept in game theory, defined as the lowest acceptable outcome that participants in a negotiation can independently choose, and has a significant impact on the negotiation process and outcome (Zhang, Xiao, Bu, Yu, Niyato, & Han, 2018). Furthermore, for the subgroup $SG_h \in S_1$ that has already reached a consensus, it can be easily deduced that the adjustment cost indicated by **Model 1** is 0. This indicates that these subgroups in S_1 have maximized their own interests based on the achieving consensus.

Theorem 4.1. *Model 1 has a unique globally optimal solution.*

Proof. Firstly, as the constraints (b) and (c) are both linear constraints and the absolute value constraint (a) can be equivalently transformed to a linear constraint, the feasible domain of Model 1 forms a convex set. Additionally, the objective function of Model 1 is strictly convex, establishing it as a convex programming model. Moreover, it is apparent that $\bar{r}_h = g$ constitutes a solution of Model 1, demonstrating that the feasible set of Model 1 is non-empty. Simultaneously, the objective function confirms that the lower bound of Model 1 is zero. Therefore, according to the principles of convex optimization, it is deduced that there exists a unique globally optimal solution for Model 1. □

4.2. Group optimal consensus adjustment in cooperative situations

In a cooperative situation, information from all participants is aggregated to a coordinator or decision-making body, which makes decisions in a unified manner. The advantage of this mechanism is that it enables global optimization, coordinates the interests of all parties and avoids wasted resources. The following model allows us to calculate the minimum total cost required to achieve group consensus when all subgroups participate in communication and cooperation:

$$\textbf{Model 2}: \quad \begin{array}{ll} \min & TC = \sum_{SG_h \in S_2} c_h\left|r_h - \bar{r}_h\right| \\ \text{s.t.} & \begin{cases} \overline{GCL} = \sum_{h=1}^{k} \overline{CL}_h / k \geq \theta, & (a) \\ \overline{CL}_h = 1 - \left|\bar{r}_h - \bar{g}\right|, SG_h \in S_2, & (b) \\ \overline{CL}_l = 1 - \left|r_l - \bar{g}\right|, SG_l \in S_1, & (c) \\ \bar{g} = \sum_{SG_h \in S_2} \lambda_h \bar{r}_h + \sum_{SG_l \in S_1} \lambda_l r_l, & (d) \\ 0 \leq \bar{r}_h \leq 1, SG_h \in S_2, & (e) \end{cases} \end{array}$$

where $\bar{r}_h$ is the decision variable, constraint (a) is used to calculate the group consensus level, constraints (b) and (c) are used to calculate the subgroup consensus level in the sets S_1 and S_2 after opinion adjustment, and the rest of the constraints are the same as in Model 1.

Under a centralized decision-making mechanism, all subgroups can share information, resources, and viewpoints through cooperation, thereby achieving optimal decisions under a common goal. By calculating the minimum total consensus adjustment cost, we can achieve overall optimization. From the perspective of cooperative game theory, the objective of **Model 2** is to maximize collective benefits. To be specific, all subgroups cooperate to minimize the total adjustment cost required for consensus.

Theorem 4.2. *Model 2 has a global optimal solution.*

Proof. Similar to the proof of Theorem 4.1, it can be easily shown that the feasible domain of Model 2 forms a convex set. Furthermore, it is evident that the objective function of Model 2 is convex, thereby identifying it as a convex planning model. Additionally, it can be clearly observed that $\bar{r}_h = g, \ \forall\ SG_h \in S_2$ serves as a solution of Model 2, demonstrating that the feasible set of Model 2 is non-empty. Therefore, based on the theory of convex optimization, we conclude that there exists a globally optimal solution for Model 2. □

It should be noted that the aforementioned model addresses the issue of consensus adjustment from the perspective of group optimization, which assumes that participants are willing to cooperate. However, if the negotiation results make the situation worse for some participants compared to acting independently, the assumption of sustained cooperation becomes difficult to maintain, indicating that cooperation is not always beneficial for all participants. Theoretically, solutions that fall below the baseline should not be considered feasible negotiation outcomes, as rational participants would prefer no agreement over cooperation that is disadvantageous to them. This view is consistent with the proposed hypothesis 1 that all participants need to gain at least as much by cooperating as by acting independently.

Based on this, we propose an improved version of the MCCM to address this challenge. Specifically, by setting the minimum adjustment cost in **Model 1** as a cost upper bound, it ensures that cooperation not only promotes the realization of collective goals, but also safeguards the actual benefits of each participant. The model is constructed as follows:

$$\textbf{Model 3}: \quad \min \ TC = \sum_{SG_h \in S_2} c_h \left| r_h - \bar{r}_h \right| \quad \text{s.t.} \begin{cases} \overline{GCL} = \sum_{h=1}^{k} \overline{CL}_h / k \geq \theta, & (a) \\ \overline{CL}_h = 1 - \left| \bar{r}_h - \bar{g} \right|, SG_h \in S_2, & (b) \\ \overline{CL}_l = 1 - \left| r_l - \bar{g} \right|, SG_l \in S_1, & (c) \\ \bar{g} = \sum_{SG_h \in S_2} \lambda_h \bar{r}_h + \sum_{SG_l \in S_1} \lambda_l r_l, & (d) \\ 0 \leq \bar{r}_h \leq 1, SG_h \in S_2, & (e) \\ c_h \left| r_h - \bar{r}_h \right| \leq TP_h, SG_h \in S_2, & (f) \end{cases}$$

where $\bar{r}_h$ is this decision variable and TP_h is the optimal objective function value for Model 1, and the rest of the constraints are the same as in Model 2.

By introducing the constraint condition (f), **Model 3** designs a novel cooperation mechanism. It aims to ensure that while pursuing collective optimization goals, the interests of each participant are effectively safeguarded, thereby addressing the issue in traditional models where participants' interests are often overlooked. Additionally, constraint condition (f) is the key premise to ensure that each party is willing to cooperate, serving as the motivation for cooperation. However, we also note that under the constraint of a high consensus threshold, **Model 3** may encounter infeasible solutions. This means that cooperation does not always benefit all participants. According to hypothesis 1, these participants will refuse to cooperate and choose selfish solutions. Based on this, we introduce the following definition:

Definition 4.1. Suppose that the set of non-consensus subgroups is denoted as S_2, where the set of subgroups willing to cooperate is WC, the non-cooperative set is NC and $NC \cup WC = S_2$. Also, for the subgroup $SG_h \in NC$, the following condition must be satisfied for any solution set of Model 2: $c_h \left| r_h - \bar{r}_h \right| > TP_h$. Similarly, for the subgroup $SG_h \in WC$, the following condition must be satisfied for any solution set of Model 2: $c_h \left| r_h - \bar{r}_h \right| \leq TP_h$.

According to Definition 4.1, if no consensus adjustment allocation under cooperative conditions can make any subgroup better off compared to making decisions independently, these subgroups will reject cooperation. At this point, the motivation for refusing to cooperate is the individual's dissatisfaction with his or her consensus adjustment. This implies that cooperation cannot benefit the subgroups within the set NC, leading to **Model 3** having no solution. With this in mind, we construct a cooperative and non-cooperative two-type MCCM as follows:

$$\textbf{Model 4}: \quad \min \ TC^* = \sum_{SG_h \in WC} c_h \left| r_h - \bar{r}_h \right| \quad \text{s.t.} \begin{cases} \overline{GCL} = \sum_{h=1}^{k} \overline{CL}_h / k \geq \theta, & (a) \\ \overline{CL}_h = 1 - \left| \bar{r}_h - \bar{g} \right|, SG_h \in S_2, & (b) \\ \overline{CL}_l = 1 - \left| r_l - \bar{g} \right|, SG_l \in S_1, & (c) \\ \bar{g} = \sum_{SG_l \in S_1} \lambda_l r_l + \sum_{SG_h \in S_2} \lambda_h \bar{r}_h, & (d) \\ 0 \leq \bar{r}_h \leq 1, SG_h \in S_2, & (e) \\ c_h \left| r_h - \bar{r}_h \right| \leq TP_h, SG_h \in WC, & (f) \end{cases}$$

where $\bar{r}_h$ denote the adjusted opinions of the subgroups in the sets S_2, constraint (f) is relaxed only for subgroups favouring cooperation, and the rest of the constraints are the same as in Model 3.

Theorem 4.3. *Suppose that TC^* is the optimal objective function value for Model 4, and TP_h is the optimal objective function value for Model 1, then for any subgroup $SG_h \in WC$, there is $TP_h \geq c_h |r_h - \bar{r}_h|$.*

Proof. First, based on the constraints (e) and (f), it can be demonstrated that the objective function is bounded. Second, according to Theorem 4.2 and Definition 4.1, we can prove the existence of at least one feasible solution. Specifically, Theorem 4.2 indicates that the constraints (a) to (e) can be satisfied, while Definition 4.1 further ensures that the constraint (f) holds. In summary, Model 4 has at least one solution. Therefore, Theorem 4.3 is evidently valid. □

According to Theorem 4.3, it is easy to obtain the following Corollary 4.1.

Corollary 4.1. *Suppose that TC^* is the optimal objective function value for Model 4, and TP_h is the optimal objective function value of Model 1, for the subgroup $SG_h \in WC$, then $\sum_{SG_h \in WC} TP_h \geq TC^*$.*

Thus, Theorem 4.3 and Corollary 4.1 confirm that consensus can be reached at a lower cost via cooperation for some subgroups. However, it is interesting to note that not all of the subgroups cooperate to bring benefits, which responds to the third question raised earlier. It also lays the foundation for cooperation in the subsequent construction of the bargaining model. Given that there may be multiple optimal solutions to **Model 4**, it becomes particularly important to ensure that all subgroups are treated fairly in the allocation process. Specifically, varying assessments of potential benefits by subgroups may lead to perceptions that the distribution in the minimum cost model does not align with their expectations, thereby fostering a sense of unfairness. Therefore, it is particularly critical to choose a strategy that pursues overall optimization while balancing cost-effectiveness and participants' sense of fairness.

To clearly understand the distinctions between **Model 1** and **Model 4** and the cost differences required for different subgroups to reach consensus, a simple example will be provided below.

Example 4.1. Suppose that the initial opinions of the five DMs are $O = \{0.1, 0.2, 0.4, 0.5, 0.8\}$, and they have the same importance and unit cost with a consensus threshold of 0.9. Under this setting, we use Models 1 and 4 to derive the optimal adjusted opinions for different scenarios, respectively, and the specific results are shown in Table 3.

Table 3 presents the results of Example 4.1 for **Models 1** and **4**. The data shows that the adjustment cost for **Model 4** is significantly lower than that for **Model 1**, indicating that **Model 4** is more cost-effective in achieving consensus. Moreover, **Model 4** has multiple optimal solutions, all corresponding to the same objective function value. However, there are significant differences in the adjustment costs for the DMs e_1 and e_2 across different solutions. Therefore, from a fairness perspective, adjusting based solely on the results of **Model 4** may lead to conflicts. This emphasizes the need to ensure fairness in cost allocation in addition to the pursuit of cost minimization when designing consensus mechanisms.

Table 3
The results for **Models 1** and **4** in Example 4.1.

DM	e_1	e_2	e_3	e_4	e_5	Cost
Initial opinion O	0.1000	0.2000	0.4000	0.5000	0.8000	—
Model 1: $\bar{O}$	0.3500	0.3250	0.4000	0.5000	0.4250	0.7500
Model 4: $\bar{O}_1$	0.2959	0.2541	0.4000	0.5000	0.5500	**0.5000**
Model 4: $\bar{O}_2$	0.2750	0.2759	0.4000	0.5000	0.5500	**0.5000**
Model 4: $\bar{O}_3$	0.3451	0.2049	0.4000	0.5000	0.5500	**0.5000**

4.3. Subgroup optimal consensus adjustment in cooperative situations

Although **Models 1** and **4** achieve consensus cost minimization from the perspectives of subgroup independent adjustment and subgroup collaborative adjustment, respectively, both mechanisms exhibit certain limitations. Specifically, **Model 1** focuses on maximizing the benefits of subgroups, which may result in an increase in overall adjustment costs. On the other hand, while **Model 4** reduces total costs, it overlooks the fairness of cost distribution. In light of these considerations, this study proposes treating consensus adjustment as a cost cooperative game problem, aiming to identify a strategy that minimizes total costs while ensuring equitable cost distribution among all participants.

As previously discussed, considering the potential benefits to participants before reaching an agreement is crucial for achieving optimal cost allocation. As illustrated by Example 4.1, participants tend to reach a group consensus at the lowest possible cost, which constitutes a desirable outcome for agreement. Thus, when addressing a practical problem that necessitates simultaneous consideration of the proportion of benefits allocated to each participant, considering the threat point versus the ideal point becomes essential, even though it has often been overlooked in previous studies. Based on this understanding, the next model will explore how subgroups achieve optimal adjustment in cooperative situations. To this end, we construct the following programming model to compute the optimal consensus adjustment cost for subgroups in cooperative situations:

$$
\textbf{Model 5}: \quad \begin{aligned} &\min \quad IP_h = c_h\left|r_h - \bar{r}_h\right| \\ &\text{s.t.} \begin{cases} \overline{GCL} = \sum_{h=1}^{k} \overline{CL}_h / k \geq \theta, & (a) \\ \overline{CL}_h = 1 - \left|\bar{r}_h - \bar{g}\right|, SG_h \in S_2, & (b) \\ \overline{CL}_l = 1 - \left|r_l - \bar{g}\right|, SG_h \in S_1, & (c) \\ \bar{g} = \sum_{SG_l \in S_1} \lambda_l r_l + \sum_{SG_h \in S_2} \lambda_h \bar{r}_h, & (d) \\ 0 \leq \bar{r}_h \leq 1, SG_h \in S_2, & (e) \\ c_h\left|r_h - \bar{r}_h\right| \leq TP_h, SG_h \in WC, & (f) \\ TC^* = \sum_{SG_h \in WC} c_h\left|r_h - \bar{r}_h\right|, & (g) \end{cases} \end{aligned}
$$

where $\bar{r}_h$ is the decision variable and TC^* is the optimal objective function value for Model 4, and the rest of the constraints are the same as in Model 4.

Theorem 4.4. *Suppose that TC^* is the optimal objective function value for Model 4, and IP_h is the optimal objective function value for Model 5. For any set of solutions in Model 4, for any subgroup $SG_h \in WC$, satisfy $IP_h \leq c_h|r_h - \bar{r}_h|$.*

In light of Theorem 4.4, it is easy to obtain the following Corollary 4.2.

Corollary 4.2. *Suppose that TC^* is the optimal objective function value for Model 4, and IP_h is the optimal objective function value for Model 5, there exists $\sum_{SG_h \in WC} IP_h \leq TC^*$.*

The findings of Corollaries 4.1 and 4.2 suggest that participants are entitled to maximize their own interests in both noncooperative and cooperative contexts. Consequently, the results of Corollaries 4.1 and 4.2 establish upper and lower bounds for the interests of the participants. In other words, when considering fair distribution, the minimum and maximum interests of each participant must be taken into account.

To further illustrate this point, Table 4 gives the results of Model 5 on Example 4.1. The results show that the optimal and general solutions derived from Model 5 are significantly different between DMs e_1 and e_2, which further emphasizes the importance of the distribution of benefits.

5. Two bargaining consensus models based on cooperative game under fairness concern

As mentioned previously, this study examines the optimal consensus adjustment strategies implemented by subgroups in cooperative and non-cooperative frameworks, with the aim of providing a reasonable allocation method for multiple solutions of the **Model 4**. In this regard, this study formally introduces two negotiation models: the NB model and the KSB model, and proposes two new LSGDM consensus adjustment methods based on these two models.

5.1. Concepts of NB and KSB

The development of bargaining models has been crucial in explaining and predicting how parties acquire the final consensus through negotiation within contemporary research fields of economics and management (Livne, 1989). Particularly, within the multi-party bargaining framework, the traditional NB model and the KSB model have been adapted to accommodate the complex dynamics of multiple participants (Luo, Zhou, & Lev, 2022; Monroy, Rubiales, & Mármol, 2017). The aim of this study is to formalize the introduction of these models into the LSGDM problems and to explore their application to consensus adjustment allocation. The model attempts to find an equilibrium point that maximizes the product of the parties' negotiated outputs and their relative gains relative to the threat point. In a multi-party negotiation situation, the NB model can be extended to the following general form:

Definition 5.1 (*Nash, 1950*). Assume that a multi-party negotiation includes n participants and the utility of each participant i is represented by U_i. In the absence of any agreement, each participant can guarantee a minimum utility of D_i, known as the noncooperative point or threat point. The multi-party NB model aims to find a utility vector $U = (U_1, U_2, \ldots, U_n)^T$ that maximizes the product of all participants' utility gains relative to their threat point. In light of the axiomatic definition of the NB solution (NBS), the Nash solution is the solution to the following model:

$$
U^* = \max_{U_i > D_i} \prod_{i=1}^{n} \left(U_i - D_i\right). \tag{5.1}
$$

The NBS has been broadly applied in areas such as business negotiations, environmental governance and supply chain management, due to its tendency to produce solutions that are both easy to interpret and analyse. Nash characterized his solution by four axioms, inclusive of independence of irrelevant alternatives (IIA). This axiom caused much controversy in this context, as pointed out by Luce and Raiffa (1957), page 120. In response to these critiques, scholars have proposed alternative bargaining solutions that replace IIA with monotonicity (as in the Kalai–Smorodinsky solution), although other alternative solutions

Table 4
The results for **Model 5** in Example 4.1.

DM	e_1	e_2	e_3	e_4	e_5	Cost
Initial opinion O	0.1000	0.2000	0.4000	0.5000	0.8000	—
Model 5: $\bar{O}_1$	**0.1507**	0.3993	0.4000	0.5000	0.5500	0.5000
Model 5: $\bar{O}_2$	0.3500	**0.2000**	0.4000	0.5000	0.5500	0.5000
Model 5: $\bar{O}_3$	0.2750	0.2750	0.4000	0.5000	**0.5500**	0.5000

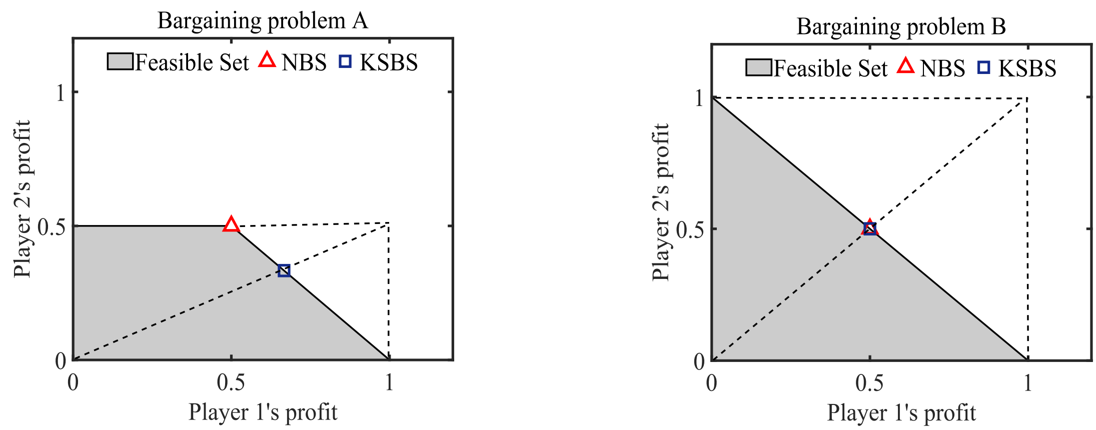


Fig. 2. Demonstrations on NBS and KSBS.

retained IIA (one remarkable example is the egalitarian solution). In particular, the solution proposed by Kalai and Smorodinsky (Kalai & Smorodinsky, 1975) has been applied in a number of fields, but has not yet been adopted in LSGDM.

Definition 5.2 (*Kalai & Smorodinsky, 1975*)**.** Assume that a multi-party negotiation includes n participants and the utility of each participant i is represented by U_i. In the absence of any agreement, each participant can guarantee a minimum utility of D_i, known as the noncooperative point or threat point. In addition, the maximum possible utility that party i can obtain with cooperation is defined as M_i. The multi-party KSB model aims to find a utility vector $U = (U_1, U_2, \ldots, U_n)^T$ that satisfies the equality of proportions between the utility gains of all participants, which is expressed as:

$$\frac{U_1 - D_1}{M_1 - D_1} = \cdots = \frac{U_n - D_n}{M_n - D_n}. \tag{5.2}$$

By ensuring that the utility gain ratio of each party is equal, the multi-party KSB model provides an effective mechanism to address potential inequality issues. This model is particularly suitable for complex negotiations involving numerous stakeholders. Its application facilitates the formulation of more cooperative and equitable solutions in both bilateral and multilateral negotiations, thereby increasing the likelihood of reaching agreements acceptable to all parties involved. Next, we will illustrate the differences in outcomes between these two models using an example from the economic literature.

Example 5.1. Consider two bargaining problems A and B whose profit distribution sets are convex packets $A = \{(0,0),(0,0.5),(0.5,0.5),(1,0)\}$ and $B = \{(0,0),(0,1),(1,0)\}$, as shown in Fig. 2.

Clearly, Player 2 has better prospects in Problem B than in Problem A. However, applying the NBS yields a negotiation result of (0.5,0.5) in both cases. In short, Player 2 obtains higher profits in the KSB solutions (KSBS) compared to Problem A in Problem B due to better prospects. This suggests that compared to the NBS, the KSBS provides a more realistic solution by considering the best prospects of each party. Ideally, the negotiated solution should be based on a geometric configuration of all possible trade prospects to ensure a fair distribution of benefits.

5.2. Consensus adjustment game model based on NB

As previously discussed, we have examined the non-uniqueness issue in consensus adjustments within **Model 4**. Therefore, selecting a fair consensus adjustment strategy naturally becomes the core problem of this study. To address this, we transform the consensus adjustment problem into a NB problem and utilize NB theory to develop a consensus adjustment allocation model. This theory possesses several ideal properties, including independence of irrelevant alternatives, invariance to affine transformations, symmetry, and Pareto efficiency (Feng, Li, & Shanthikumar, 2022), and is widely accepted as a profit distribution scheme. Inspired by the work of Meng et al. (2022), this study adopts the cost reductions brought about by cooperation relative to independent action as equal as possible as a fairness criterion for the NB cooperation game. In other words, it is ensured that the cost reductions of consensus adjustment for each subgroup relative to the threat point should be as equal as possible. In this way, it can be ensured that all parties make the same compromises or concessions in cooperation, thus increasing the acceptance and recognition of the adjustment scheme. This method aims to prevent a single participant or a few individuals from bearing too much of the adjustment burden, and to avoid the resulting dissatisfaction and possible failure of consensus. Therefore, we construct the following NB consensus model based on the cooperative game:

$$\textbf{Model 6}: \quad \Omega = \max_{TP_h \geq \Delta_h} \prod\nolimits_{SG_h \in WC} \left(TP_h - \Delta_h\right)$$
$$\text{s.t.} \begin{cases} \Delta_h = c_h \left| r_h - \bar{r}_h \right|, & (a) \\ TC^* = \sum_{SG_h \in WC} \Delta_h, & (b) \\ \text{The constraints in the } \textbf{Model 4}, & \text{(c)} \end{cases}$$

where $\bar{r}_h$ and Δ_h are the decision variables, TP_h is the optimal objective function value of Model 1 for the subgroup SG_h and TC^* is the optimal objective function value for Model 4.

As noted above, **Model 6** employs the well-known Nash product as the objective function, defining the negotiation breakdown point as the minimum cost required for independent adjustment by each subgroup. Furthermore, we assume that all subgroups in **Model 6** have equal bargaining power, which is only applicable to symmetric NB problems (Binmore, Rubinstein, & Wolinsky, 1986). In fact, considering the heterogeneity among subgroups, it is necessary to revise this assumption. To address this, we introduce the parameter δ_h to relax the original assumption. Based on this adjustment and accounting for differences in bargaining power among subgroups, we can extend the model to the following general form:

$$\textbf{Model 7}: \quad \Omega = \max_{TP_h \geq \Delta_h} \prod_{SG_h \in WC} \left(TP_h - \Delta_h\right)^{\delta_h}$$
$$\text{s.t.} \begin{cases} \Delta_h = c_h \left|r_h - \bar{r}_h\right|, & (a) \\ TC^* = \sum_{SG_h \in WC} \Delta_h, & (b) \\ \sum_{SG_h \in WC} \delta_h = 1, & (c) \\ \text{The constraints in the } \textbf{Model 4}, & (d) \end{cases}$$

where $\bar{r}_h$ and Δ_h are the decision variables, $\delta_h \geq 0$, and the rest of the constraints are the same as in Model 6.

In practical implementation, each δ_h can be adjusted according to specific circumstances to accurately reflect the actual bargaining power of each negotiation participant. If all participants have equal bargaining power, **Model 7** naturally simplifies to **Model 6**. This indicates that **Model 6** is a special case of **Model 7** in the absence of differences in bargaining power. The asymmetric NB model provides a more flexible and realistic framework for multi-party negotiations involving different bargaining powers. Therefore, considering the initial consensus level and the size of the subgroups, a method to calculate the bargaining power of subgroup is given as follows:

$$\tau_h = (1 + |SG_h|)^{\beta \cdot CL_h}, \tag{5.3}$$

where $\beta > 0$ is the parameter used to modulate the effect of consensus level CL_h, $|SG_h|$ is the number of DMs within SG_h. Additionally, the experiment by Rodríguez et al. (2018) suggests $\beta = 0.3$, and $\delta_h = \tau_h / \sum_{SG_h \in WC} \tau_h$.

5.3. Consensus adjustment game model based on KSB

In this section, we employ the KSB model, which aims to achieve fairness through cooperative negotiation. The KSB model is designed based on the principle of fair allocation, which ensures that all participants receive the same proportional utility gain relative to their threat and desirable points. Unlike the NB model, the KSB model seeks a cooperative solution in which all participants receive proportionate fair gains. Specifically, we use the minimum consensus adjustment cost for each participant in the independent decision-making situation as the base point (threat point) and the minimum adjustment cost in the cooperative situation as the target point (ideal point). These two points clarify each participant's position in the utility space and its potential negotiation range. Subsequently, we construct a utility possibility boundary representing all achievable combinations of utilities through negotiation. On this boundary, the KSB model identifies a specific point that ensures that the utility gains of all participants remain at the same proportional difference between their ideal and threat points. Following the above definition, the consensus adjustment model based on KSB can be defined as follows:

$$\textbf{Model 8}: \quad \frac{TP_h - \Delta_h}{TP_h - IP_h} = Z, \ \forall \ SG_h \in WC,$$

where Δ_h is the decision variable, Z is the intermediate variable, TP_h, TC^* and IP_h are the optimal objective function values for Models 1, 4 and 5, respectively.

In other words, the core of the KSB model is to ensure that the benefits of cooperation are distributed fairly among all participants, rather than merely maximizing the total gains. Given that the feasible set in **Model 4** is limited and discrete, and the requirements of **Model 8** are overly stringent, it is not always possible to find a suitable solution. To address this issue, we propose a compromise KSB model by constructing the following model:

$$\textbf{Model 9}: \quad \Theta = \max \prod_{SG_h \in WC} \left(\frac{TP_h - \Delta_h}{TP_h - IP_h}\right)$$
$$\text{s.t.} \begin{cases} \Delta_h = c_h \left|r_h - \bar{r}_h\right|, & (a) \\ TC^* = \sum_{SG_h \in WC} \Delta_h, & (b) \\ \text{The constraints in } \textbf{Model 4}, & (c) \end{cases}$$

where $\bar{r}_h$ and Δ_h are the decision variables, TP_h, TC^* and IP_h are the optimal objective function values for Models 1, 4 and 5, respectively, and $TP_h \geq \Delta_h \geq IP_h$.

These two bargaining models construct a theoretical framework aimed at analysing and explaining how negotiation can maximize collective benefits and safeguard individual fairness while minimizing costs in the process of reaching consensus. The NB model emphasizes the worst-returns scenario, where the parties reach a consensus through a compromise to the benefit of all the participants. The KSB model not only takes into account the worst-returns scenarios of the participants, but also considers the best deal scenario. In conclusion, these models offer valuable guidance for understanding and optimizing the CRP, helping to maximize cost-effectiveness and fairness.

Considering the case that there may be multiple optimal solutions for **Models 7** and **9**, we provide the following method to select the optimal solution from them:

$$\textbf{Model 10}: \quad \Gamma = \mathop{\mathbf{Arg\,max}} \min_{SG_h \in SG} \left(\overline{CL}_h\right)$$
$$\text{s.t.} \begin{cases} \Delta_h^* = c_h \left|r_h - \bar{r}_h\right|, & (a) \\ TC^* = \sum_{SG_h \in WC} \Delta_h^*, & (b) \\ \text{The constraints in } \textbf{Model 4}, & (c) \end{cases}$$

where $\bar{r}_h$ is the decision variable, Δ_h^* is the set of optimal solutions obtained from Models 7 or 9.

5.4. Two new methods for LSGDM

In this section, two new LSGDM methods are proposed, namely one is the consensus adjustment mechanisms based on NB and another is based on KSB. Fig. 3 shows the specific flow, and Algorithms 2 and 3 describe their computation process in detail.

Algorithm 2: The first new LSGDM method: the consensus model based on NB

Input: The initial evaluation value $O = \{o_1, o_2, \cdots, o_m\}$, the adjacency matrix $T = (t_{kh})_{m \times m}$, the unit cost vector $c = \{c_1, c_2, \cdots, c_m\}$, the number of clusters k, the parameters α, p and the consensus threshold θ.
Output: Output the final evaluation value of the alternatives.
Step 1: The DMs are clustered into k subgroups using Algorithm 1, denoted as $SG = \{SG_1, SG_2, ..., SG_k\}$;
Step 2: Calculate the initial group consensus level GCL through Eqs. (2.2)–(2.3);
Step 3: If the consensus requirement is met, go to **Step 5**; Otherwise, turn to the next step;
Step 4: The sets S_1 and S_2 are determined based on the initial subgroup consensus level, and then the optimal adjustment TP_h for the subgroup in the noncooperative case and the optimal adjustment TC^* for the group in the cooperative case are determined by Model 1 and Model 4;
Step 5: The bargaining power δ_h of each subgroup SG_h is calculated according to Eq. (5.3), and the optimal consensus adjustment allocation is obtained by choosing to solve using either Model 6 or Model 7;
Step 6: Output final consensus adjustment opinion;
Step 7: End.

In this study, we investigate the application of the NB model and the KSB model to LSGDM, focusing on the problem of consensus adjustment allocation among multiple participants. Each of these two models has unique advantages and applicability conditions in different decision-making environments. The NB model emphasizes the minimum utility

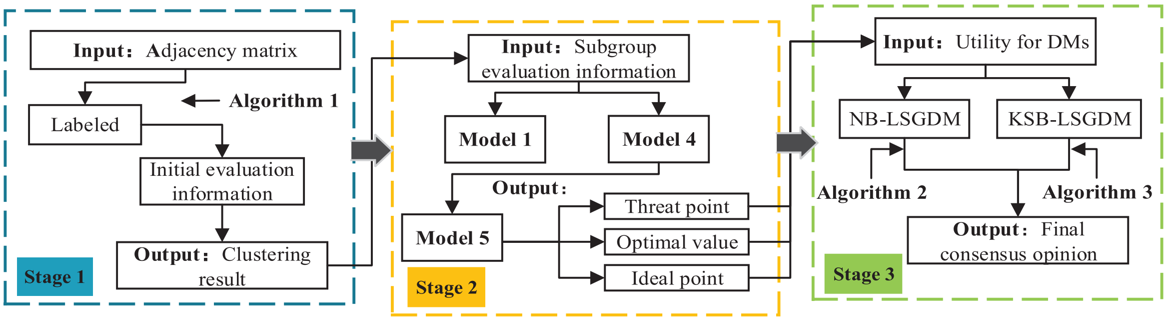


Fig. 3. The workflow of the constructed LSGDM methods.

Algorithm 3: The second new LSGDM method: the consensus model based on KSB

Input: The initial evaluation value $O = \{o_1, o_2, \cdots, o_m\}$, the adjacency matrix $T = (t_{kh})_{m\times m}$, the unit cost vector $c = \{c_1, c_2, \cdots, c_m\}$, the number of clusters k, the parameters α, p and the consensus threshold θ.
Output: Output the final evaluation value of the alternatives.
Step 1: The same as **Steps 1-3** in Algorithm 2;
Step 2: Based on the group optimal adjustment TC^* of Model 4, the optimal adjustment IP_h of each subgroup in the cooperative scenario is determined according to Model 5;
Step 3: First use Model 8 to determine whether a solution exists, if so move to **Step 5**; otherwise, go to the next step;
Step 4: Using Model 9 to find a compromise KSBS;
Step 5: Output final consensus adjustment opinion;
Step 6: End.

value of each party in the event of a negotiation failure and ensures that each participant receives the maximum benefit from its threat point. The model is able to effectively deal with power and resource asymmetries among participants, allowing each party to obtain reasonable utility gains even under unequal conditions. In contrast, the KSB model provides a more equitable solution based on simultaneous consideration of initial conditions and future prospects. The model works to achieve absolute fairness in the decision-making process by integrating threat points and ideal points. However, in complex and multi-participant environments, the KSB model faces significant challenges that make finding ideal solutions difficult. As a result, although the KSB model theoretically provides fairer solutions, its practical application is limited. To address these challenges, this study proposes a compromise KSB model that aims to alleviate its high demands on solutions. Although this compromise may be inferior to the NB model in some cases, it provides a new solution in complex environments.

In conclusion, both the NB and KSB models have unique advantages and applicable conditions in addressing the complexities of LSGDM. The NB model ensures that the benefits from cooperative relatively independent actions are fairly distributed and excels in dealing with unequal bargaining power. On the other hand, the KSB model, by considering potential benefit spaces, is more suitable for decision-making environments that require a high degree of fairness. Therefore, depending on the specific decision-making needs, these two models can be flexibly applied to achieve optimal outcomes.

6. Case study and parameter analysis

In this section, we apply the two proposed LSGDM models to real-world decision-making scenarios to evaluate their applicability. Additionally, we analyse the key parameters related to the proposed methods to further validate their effectiveness and practicality.

6.1. A numerical example

This section illustrates a practical application of the proposed method using a dataset (Cantador, Brusilovsky, & Kuflik, 2011) from the Last.fm social music website.[1] This dataset encompasses users' listening frequencies to artists and social network information derived from these interactions. The social network is constructed based on the friend relationships on the Last.fm social music platform, which are considered as trust relationships between users. Specifically, we assume that trust exists between users if they are friends on the platform. Thus, a friend relationship corresponds to a trust value of 1, while the absence of a friend relationship corresponds to a trust value of 0. During the SNA of the artist recommendation index, user biases may influence the analysis results. For this case study, we select the listening data and social network information of 100 users related to artist ID 62 to evaluate the model's effectiveness. Fig. 4(a) displays the initial social network layout of these 100 users. The parameters for the experiment are set as follows: $m = 100$, $\alpha = 2$, $p = 5$, $k = 6$, with a unit cost c_h of 1 for all subgroups, all DMs and subgroups are treated as equally important.

Given the data distribution characteristics in the dataset, this study employs a mapping strategy that converts the frequency of a user's listens to an artist into a unit interval of [0,1]. This mapping strategy serves as the foundation for developing the recommendation index, ensuring uniformity and fairness in the evaluation process. The specific equation used for this mapping is defined as follows:

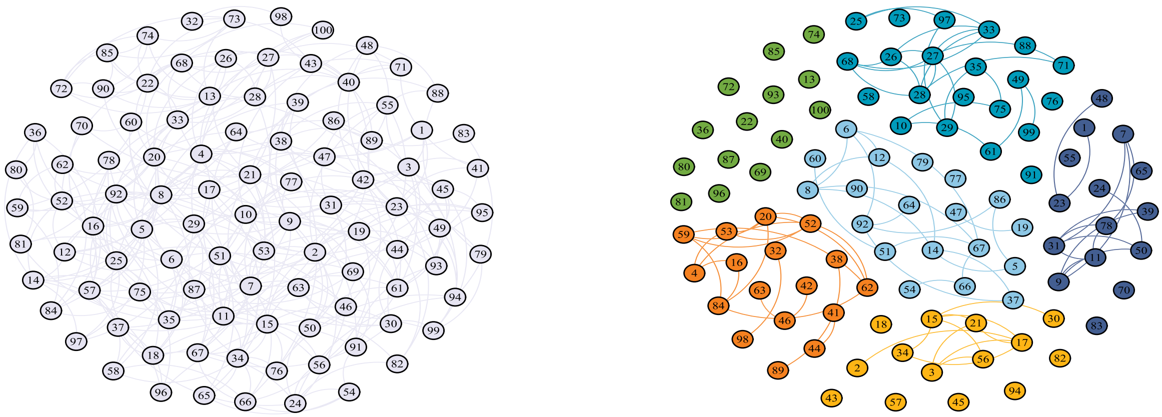
$$o_h = \frac{log_{10}(L_h)}{log_{10}(\max(L_1, L_2, \ldots, L_m))}, \tag{6.1}$$

where L_h represents the number of listens, o_h represents the original opinion of the DM e_h, and m represents the total number of DMs.

To obtain the final score for the artist, we use the first LSGDM method proposed to solve this problem, which are implemented as follows:

- **Step 1:** By processing the listening data of 100 users for artist ID 62 through Eq. (6.1), we acquire an initial evaluation vector O of DMs, and the specific results are presented in Table 5. Then, Table 6 shows the initial community division results obtained by the Louvain algorithm. Obviously, the subgroups that satisfy the size requirement condition ($|C_h| \geq \lceil 100/14 \rceil$) are: C_1, C_3, C_5, C_7, C_9, C_{11}, C_{13}. According to Eq. (2.1), we can calculate and rank the degree centrality indices of the DMs in the above subgroups. Next, According to Eq. (3.1), the potential small groups identified as meeting the criteria are: $B_1 = \{e_5, e_6, e_8, e_{12}, e_{14}\}$, $B_2 = \{e_3, e_{15}, e_{17}, e_{21}, e_{34}\}$, $B_3 = \{e_{10}, e_{42}, e_{45}, e_{49}, e_{61}\}$, $B_4 = \{e_7, e_9, e_{11}, e_{31}, e_{78}\}$, $B_5 = \{e_{26}, e_{27}, e_{28}, e_{33}, e_{68}\}$, $B_6 = \{e_{20}, e_{52}, e_{53}, e_{59}, e_{62}\}$, $B_7 = \{e_{13}, e_{22}, e_{72}, e_{74}, e_{85}\}$, which corresponds to the bolded DMs in Table 6. By using potential small groups as known

[1] https://www.last.fm



(a) Initial social network of 100 DMs (b) Six subgroups after clustering

Fig. 4. The two-stage semi-supervised clustering process.

labels, in the second stage we can obtain the final clustering results using the semi-supervised FCM algorithm as shown in Table 7 and Fig. 4(b). It is important to note that some nodes within the community, although aggregated together due to similarity of opinions, are not directly connected due to lack of trust relationships. Meanwhile, the connections between communities are hidden in order to clearly show the community structure.

- **Step 2:** Let the consensus threshold $\theta = 0.95$, the initial group consensus level is calculated according to Eqs. (2.2)–(2.3), and since $GCL = 0.8608 < \theta$, the consensus feedback adjustment process is entered. In light of the consensus level of each subgroup, it is determined that $WC = \{SG_2, SG_3, SG_4, SG_5, SG_6\}$, and then the optimal adjustment cost TP_h for the subgroup in the non-cooperative case and the optimal adjustment cost TC^* for the group in the cooperative case are determined by **Models 1** and **4**: $TP_2 = 0.1118, TP_3 = 0.2077, TP_4 = 0.2255, TP_5 = 0.0195, TP_6 = 0.0939, TC^* = 0.5350$.

- **Step 3:** The optimal consensus adjustment for each subgroup in the case of reaching a cooperation can be calculated according to **Model 5**: $IP_2 = 0, IP_3 = 0.0844, IP_4 = 0.1025, IP_5 = 0, IP_6 = 0$. Then, according to Eq. (5.3), the negotiating power of the subgroup can be calculated as: $\delta_2 = 0.2163, \delta_3 = 0.1917, \delta_4 = 0.1815, \delta_5 = 0.2087, \delta_6 = 0.2018$. Based on the above results, the optimal consensus adjustment allocation is obtained by using **Model 7**: $\Delta_2 = 0.0842, \Delta_3 = 0.1815, \Delta_4 = 0.2019, \Delta_5 = 0.0001, \Delta_6 = 0.0673$. Output final consensus opinion: $\bar{r} = \{0.6004, 0.5779, 0.6783, 0.6007, 0.7030, 0.6977\}$.

6.2. The sensitivity analyses for the clustering parameter p

In this study, we designed a two-stage trust constrained semi-supervised clustering algorithm that utilizes a small amount of labelled data to guide and supervise the clustering process. Additionally, this method introduces an adjustable parameter p to control the amount of supervision, thereby regulating the influence of prior information on the clustering constraints. In order to further investigate the specific effects of different DMs sizes and the parameter p on the clustering effect, we do the following experiment to give some references for the choice of p. The parameters of this experiment are set as follows: $m \in \{20, 50, 100\}$, $p \in (\{2, 5\}, \{3, 6\}, \{4, 7\})$, $k = 5$, $\alpha = 2$, $t = 100$. The accuracy is calculated by comparing the actual clustering results with the known labels, and the distribution of these results is shown in Fig. 5.

The experimental results indicate that as the value of p increases, clustering accuracy significantly improves. A smaller p value implies prioritizing DMs with higher degree centrality as labelled data. These DMs, who typically have higher connectivity within the network, are usually positioned at the core of the group. This ensures their importance and representativeness in the group, thereby enhancing clustering accuracy. However, when p values are higher, accuracy may decline due to the redistribution caused by differences in DMs' opinions. In summary, a lower p value ensures that closely connected DMs within the community are prioritized for initial clustering, while a higher p value may allow for more labelled data, where unstable clustering can be corrected by unknown labels. Therefore, it is reasonable to choose an appropriate p value based on the accuracy and reliability of the labels, and opting for a lower p value is more cautious when information is uncertain.

6.3. The sensitivity analyses for the clustering parameter α

In this section, we further analyse another important clustering parameter α, and investigate the effect of different parameter values of α on clustering. The parameters of this experiment are set as follows: $m \in \{20, 50, 100\}$, $\alpha \in \{0.3, 0.6, 1\}$, $k = 5$, $p = 5$, $t = 100$. Similar to the previous experiment, the results of the experiment with different parameters are given in Fig. 6.

As instructed by the objective function, the parameter α controls the weight of the label information in the clustering process, thus influencing the updating of the cluster centres and the adjustment of the affiliation matrix. In general, a smaller value of α implies a lower weighting of the label information, which may lead to the clustering results being more influenced by the distribution of the data itself, while a larger value of α enhances the role of the label information in the clustering and helps to steer the clustering process to be more in line with the a priori distribution of the labels. In addition, as shown in Fig. 6, the clustering accuracy exhibits different trends with increasing a values for different numbers of DMs m. This indicates that there are differences in the optimal values of the regularization parameter α on datasets of different sizes.

Based on the experimental results, a higher value of α can enhance the importance of supervised information, ensuring that labelled samples are correctly classified. However, as the sample size increases, the clustering accuracy gradually decreases, indicating that clustering accuracy is closely related to the number of labelled samples and the sample size. Typically, the parameter α is proportional to the ratio of the sample size to the number of labelled samples. Based on this empirical observation, when we aim to prevent labelled samples from being reassigned and to effectively supervise the clustering process, a larger

Table 5
Initial opinions of the DMs.

DM	o_k	DM	o_k	DM	o_k	DM	o_k	DM	o_k
e_1	0.7705	e_{21}	0.6346	e_{41}	0.9137	e_{61}	0.4686	e_{81}	0.4197
e_2	0.6897	e_{22}	0.1068	e_{42}	0.8277	e_{62}	0.9695	e_{82}	0.7068
e_3	0.6905	e_{23}	0.7912	e_{43}	0.7064	e_{63}	0.8446	e_{83}	0.8102
e_4	0.8427	e_{24}	0.7825	e_{44}	0.8263	e_{64}	0.5879	e_{84}	0.9709
e_5	0.8163	e_{25}	0.5027	e_{45}	0.6874	e_{65}	0.7432	e_{85}	0.5815
e_6	0.5348	e_{26}	0.5471	e_{46}	0.8813	e_{66}	0.6057	e_{86}	0.5891
e_7	0.8779	e_{27}	0.5131	e_{47}	0.6078	e_{67}	0.6408	e_{87}	0.2416
e_8	0.4561	e_{28}	0.4342	e_{48}	0.7682	e_{68}	0.5432	e_{88}	0.5266
e_9	0.6545	e_{29}	0.5198	e_{49}	0.5131	e_{69}	0.3399	e_{89}	0.9290
e_{10}	0.5471	e_{30}	0.7298	e_{50}	0.7470	e_{70}	0.8033	e_{90}	0.6174
e_{11}	0.7136	e_{31}	0.8388	e_{51}	0.5492	e_{71}	0.4477	e_{91}	0.4909
e_{12}	0.7092	e_{32}	0.8382	e_{52}	0.8084	e_{72}	0.3562	e_{92}	0.5641
e_{13}	0.6388	e_{33}	0.4477	e_{53}	0.5516	e_{73}	0.4732	e_{93}	0.3701
e_{14}	0.4769	e_{34}	0.6698	e_{54}	0.5961	e_{74}	0.4116	e_{94}	0.6864
e_{15}	0.5805	e_{35}	0.4579	e_{55}	0.7972	e_{75}	0.4732	e_{95}	0.5151
e_{16}	0.8353	e_{36}	0.4028	e_{56}	0.6755	e_{76}	0.4506	e_{96}	0.4417
e_{17}	1.0000	e_{37}	0.6274	e_{57}	0.7295	e_{77}	0.6271	e_{97}	0.5131
e_{18}	0.6550	e_{38}	0.8596	e_{58}	0.5283	e_{78}	0.6326	e_{98}	0.9269
e_{19}	0.5994	e_{39}	0.7442	e_{59}	0.9497	e_{79}	0.5678	e_{99}	0.4525
e_{20}	0.8417	e_{40}	0.4427	e_{60}	0.6335	e_{80}	0.4184	e_{100}	0.4116

Table 6
The first stage initial community division results.

Subgroup	Member	Subgroup	Member
C_1	$\boldsymbol{e_5}$, $\boldsymbol{e_6}$, $\boldsymbol{e_8}$, $\boldsymbol{e_{12}}$, $\boldsymbol{e_{14}}$, e_{16}, e_{18}, e_{25}, e_{29}, e_{35}, e_{36}, e_{37}, e_{92}, e_{96}, e_{97}	C_2	e_{24}, e_{82}
C_3	$\boldsymbol{e_3}$, $\boldsymbol{e_{15}}$, $\boldsymbol{e_{17}}$, $\boldsymbol{e_{21}}$, $\boldsymbol{e_{34}}$, e_{41}, e_{44}, e_{54}, e_{56}, e_{58}, e_{65}, e_{66}, e_{67}, e_{76}	C_4	e_{79}, e_{88}
C_5	$\boldsymbol{e_{10}}$, e_{30}, $\boldsymbol{e_{42}}$, $\boldsymbol{e_{45}}$, e_{46}, e_{47}, $\boldsymbol{e_{49}}$, e_{55}, $\boldsymbol{e_{61}}$, e_{63}, e_{75}, e_{99}	C_6	e_{51}, e_{60}, e_{70}
C_7	$\boldsymbol{e_7}$, $\boldsymbol{e_9}$, $\boldsymbol{e_{11}}$, $\boldsymbol{e_{31}}$, e_{39}, e_{50}, e_{64}, e_{69}, e_{77}, $\boldsymbol{e_{78}}$, e_{81}, e_{87}	C_8	e_{83}, e_{86}, e_{94}
C_9	e_4, $\boldsymbol{e_{26}}$, $\boldsymbol{e_{27}}$, $\boldsymbol{e_{28}}$, $\boldsymbol{e_{33}}$, e_{40}, $\boldsymbol{e_{68}}$, e_{71}, e_{100}	C_{10}	e_{32}, e_{43}, e_{98}
C_{11}	$\boldsymbol{e_{20}}$, $\boldsymbol{e_{52}}$, $\boldsymbol{e_{53}}$, e_{57}, $\boldsymbol{e_{59}}$, $\boldsymbol{e_{62}}$, e_{80}, e_{84}	C_{12}	e_1, e_{23}, e_{38}, e_{48}, e_{89}
C_{13}	$\boldsymbol{e_{13}}$, $\boldsymbol{e_{22}}$, $\boldsymbol{e_{72}}$, e_{73}, $\boldsymbol{e_{74}}$, $\boldsymbol{e_{85}}$, e_{90}	C_{14}	e_2, e_{19}, e_{91}, e_{93}, e_{95}

Table 7
Final clustering results and relevant indicators.

Subgroup	Member	CL_h	r_h
SG_1	e_5, e_6, e_8, e_{12}, e_{14}, e_{19}, e_{37}, e_{47}, e_{51}, e_{54}, e_{60}, e_{64}, e_{66}, e_{67}, e_{77}, e_{79}, e_{86}, e_{90}, e_{92}	0.9636	0.6004
SG_2	e_{10}, e_{25}, e_{26}, e_{27}, e_{28}, e_{29}, e_{33}, e_{35}, e_{49}, e_{58}, e_{61}, e_{68}, e_{71}, e_{73}, e_{75}, e_{76}, e_{88}, e_{91}, e_{95}, e_{97}, e_{99}	0.8568	0.4936
SG_3	e_4, e_{16}, e_{20}, e_{32}, e_{38}, e_{41}, e_{42}, e_{44}, e_{46}, e_{52}, e_{53}, e_{59}, e_{62}, e_{63}, e_{84}, e_{89}, e_{98}	0.7769	0.8598
SG_4	e_{13}, e_{22}, e_{36}, e_{40}, e_{69}, e_{72}, e_{74}, e_{80}, e_{81}, e_{85}, e_{87}, e_{93}, e_{96}, e_{100}	0.7621	0.3988
SG_5	e_2, e_3, e_{15}, e_{17}, e_{18}, e_{21}, e_{30}, e_{34}, e_{43}, e_{45}, e_{56}, e_{57}, e_{82}, e_{94}	0.9338	0.7030
SG_6	e_1, e_7, e_9, e_{11}, e_{23}, e_{24}, e_{31}, e_{39}, e_{48}, e_{50}, e_{55}, e_{65}, e_{70}, e_{78}, e_{83}	0.8718	0.7650

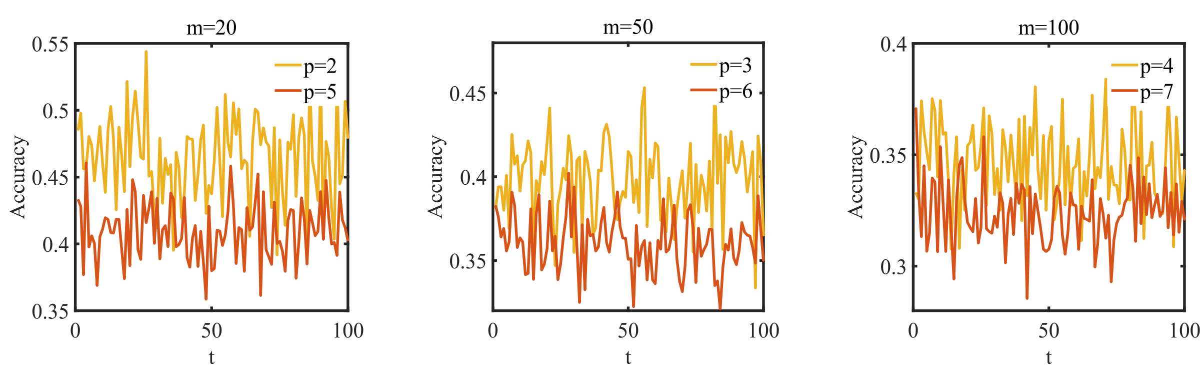


Fig. 5. Sensitivity experiments for the parameter p.

α value can be selected. Conversely, if we intend for labelled samples to serve merely as a guide and allow for reassignment, a smaller α value can be chosen. Therefore, for datasets of different scales and characteristics, determining the value of through pre-experiments before implementation is a critical step in improving clustering performance.

6.4. The sensitivity analyses for the consensus threshold θ

The decision results in the case study are derived for a consensus threshold of 0.95. However, when different consensus thresholds are set, the final consensus opinion and the cost required will be different. In this experiment, we aim to analyse the possible decision costs associated with different consensus thresholds, as a reference to the choice of θ. The experimental parameters are set as follows: $m \in \{20, 50, 100\}$, $\theta \in \{0.85, 0.90, 0.95\}$, $k = 5$, $p = 5$, $t = 100$. The experimental results are reflected in Fig. 7.

As can be easily observed in Fig. 7, the total cost shows an increasing trend as the consensus threshold θ increases for DMs sizes m of 20, 50 and 100 respectively. This means that the cost required to reach a higher degree of consensus increases with the consensus threshold,

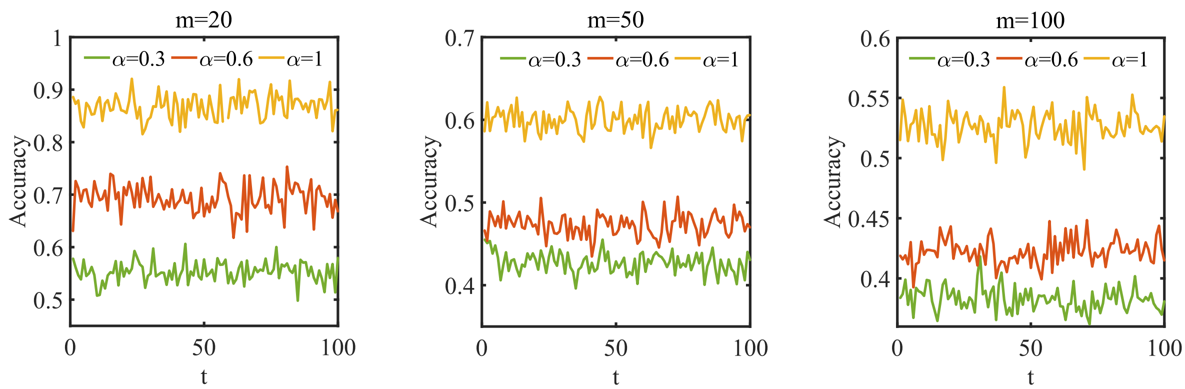


Fig. 6. Sensitivity experiments for the parameter α.

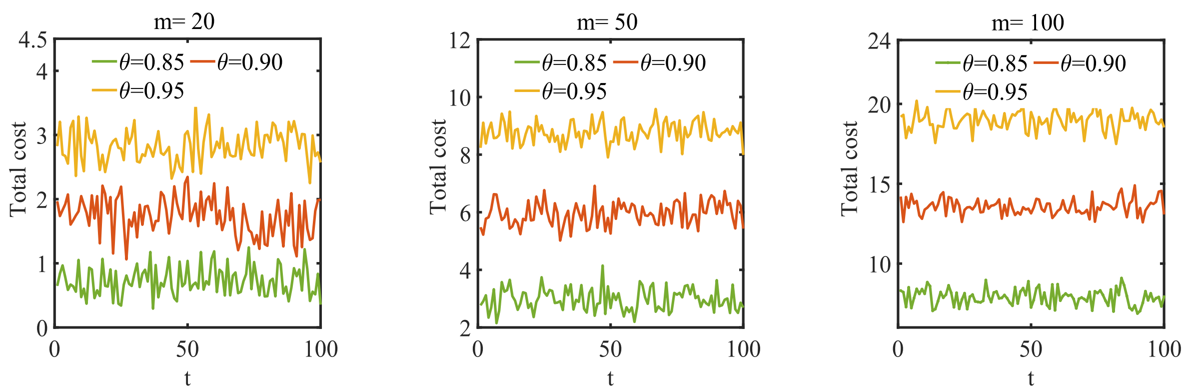


Fig. 7. Sensitivity experiments for the parameter θ.

when the number of DMs is fixed. Therefore, depending on the urgency of the problem and the available resources, a reasonable consensus threshold is critical to controlling the cost of decision-making. In occasions where resources are limited or rapid decision-making is required, consideration may be given to selecting a lower consensus threshold to reduce costs; conversely, in important and adequately resourced situations, it may be more appropriate to select a higher consensus threshold to ensure the quality of decision-making.

7. Comparative study and simulation analysis

In this section, the performance and adaptability of two new LSGDM methods for clustering and consensus are evaluated through detailed experiments and analyses. The results show that these methods have unique advantages and potential applications in solving LSGDM problems.

7.1. Comparison of clustering methods

In what follows, to exhibit the merits and effectiveness of the established clustering method more clearly, we compare the established method with the traditional FCM clustering method (Cannon et al., 1986). Specifically, the accuracy of 100 simulation experiments is used as the performance measure by setting different regularization parameters $\alpha \in \{0.2, 0.5, 3\}$. These different parameters are chosen to reflect the low, medium and high influence of the regularization term in the proposed clustering method.

As shown in Fig. 8, at the low value of α, the regularization term acts as a kind of bootstrap, assisting in adjusting the affiliation of the data points without directly dominating the formation of clusters. This guiding role allows the algorithm to slightly incorporate a priori knowledge while maintaining a certain level of adaptivity and exploration. In contrast, when the α value is high, the regularization term exerts a direct supervisory effect on the clustering results, compulsorily pushing the data points towards a predetermined clustering structure, which somewhat reduces the algorithm's adaptability but improves the accuracy of conforming to a priori knowledge.

In practical large-scale networks, sparsity is a common feature, meaning the number of existing edges is much smaller than the maximum possible number of edges. For example, a May 2011 survey of the Facebook friend-relationship network showed that it contained 721 million active users and 68.7 billion edges, with a sparsity of about 0.3×10^{-7}, suggesting that real-life social network environments are very sparse (Striga & Podobnik, 2018). In light of this, we propose a two-stage trust-constrained semi-supervised FCM method. To verify the effectiveness of this method in dealing with sparse social networks, we design a series of experiments to compare the performance of different methods in sparse networks. Table 8 summarizes the performance of each method under different sparsity conditions by comparing our method with three common clustering methods.

Based on the experimental results, the main advantages of the clustering method proposed in this study can be summarized as follows:

(1) The proposed method is able to stabilize the aggregation of DMs into fewer and more consistent clusters under different sparsity conditions, avoiding the generation of clusters of isolated DMs that lead to failure in dimensionality reduction. In contrast, two classical community detection methods perform poorly in these aspects, which proves the reliability and stability of the method in complex networks.

(2) The proposed method significantly improves the quality of clustering results by considering both evaluation information and trust information. By fully utilizing trust relationships as a reliable resource and a priori knowledge, our method provides a more comprehensive picture of DMs' relationships and opinions.

(3) This clustering method exhibits high flexibility, allowing for a balance between supervisory information and clustering results, thereby achieving higher quality outcomes. Larger parameters can

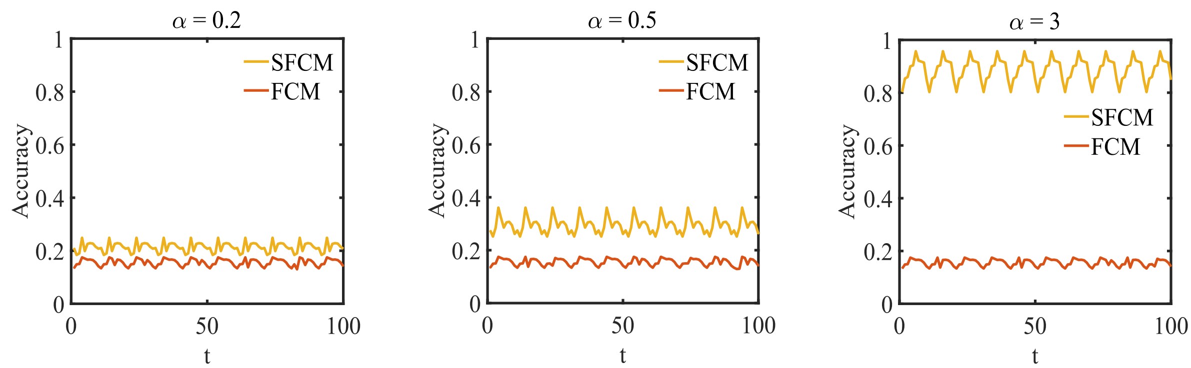


Fig. 8. Performance analysis of the proposed clustering method.

Table 8
Performance comparison under different sparsity conditions.

Number of edges	Clustering method	Evaluation information	Trust information	Number of clusters	Number of clusters with less than 2 DMs	Average number of edges within clusters
3000	Louvain	×	✓	373	25	1.7507
	Label propagation	×	✓	26	25	114.9615
	FCM	✓	×	5	0	126.2000
	The proposed	✓	✓	5	0	127.4000
2000	Louvain	×	✓	415	70	1.4410
	Label propagation	×	✓	74	70	25.1081
	FCM	✓	×	5	0	83.0000
	The proposed	✓	✓	5	0	84.4000
1000	Louvain	×	✓	529	206	0.8922
	Label propagation	×	✓	280	206	2.7071
	FCM	✓	×	5	0	44.4000
	The proposed	✓	✓	5	0	46.4000

[1] "×" denotes the factors not considered by the method, "✓" denotes the factors considered by the method.

utilize modular optimization to identify community structures, while smaller parameters are better suited for capturing the overall structure.

In summary, the designed clustering algorithm is ideal and effective to extend the application range of LSGDM. Certainly, this algorithm has some limitations. Therefore, future research can gather the following directions: (1) Exploring the development of clustering algorithms based on other types of prior knowledge (e.g., pairwise constraints) to enhance the adaptability and generality of the algorithm. (2) Investigating how to adaptively determine several parameters in the proposed clustering method is another meaningful work.

7.2. Simulation experiments of the proposed two new methods

Many methods have been proposed to address the LSGDM problem, with cost and fairness being crucial evaluation metrics. Since the two new methods proposed in this study are both based on the minimum cost model, this section focuses on comparing their performance in terms of fairness. First, we analyse the relationship between the NBS and the threat point. Subsequently, we compare the KSBS with the threat point and the ideal point. The experimental parameters were set as: $m = 100$, $\alpha = 0.6$, $k = 7$, $p = 3$, $\theta = 0.95$, and 30 repeated simulations were conducted. The results are shown in Figs. 9 and 10. This comparison helps to reveal the advantages of the two proposed LSGDM methods in terms of fairness.

In this experiment, we select three of the subgroups involved in the negotiation for visualization and analysis. Firstly, it can be observed that the negotiated solutions obtained in the cooperative condition are lower than the costs required for each subgroup to make decisions independently, which is considered to be the basis for the cooperation. Furthermore, by looking at the three subgraphs in Fig. 9, it can be noted that there is a consistency in the gap between the threat points and the NBS. This suggests that the model developed successfully achieves the goal of fair negotiation, with all the cooperative subgroups making similar concessions relative to their respective threat points. Obviously, the model can also achieve fair and effective negotiation in some asymmetric negotiations.

Unlike the NB model, the KSB model focuses more on the fairness and symmetry of the negotiation outcome. As shown in Fig. 10, the model emphasizes on finding a balance between the threat point and the ideal point to ensure that all parties obtain relatively satisfactory utility. Therefore, it is particularly suitable for negotiation scenarios where roles are equal and proportional gains are pursued. For example, in wage negotiations, the KSB model ensures that workers and employers are relatively satisfied with the wage level by evaluating the potential benefits of workers joining or leaving the firm, leading to a fairer outcome.

7.3. Performance analysis of the proposed two new methods

With the advancement of social media and e-democracy, more and more DMs are able to participate in the decision-making process, especially when dealing with LSGDM questions, where broad participation becomes a necessity. Therefore, it is important for the current research to take note of this by proposing LSGDM methods suitable for dealing with problems involving a large number of DMs instead of being limited to only 11 DMs or 20 DMs in the traditional definition.

By virtue of the above considerations, in order to confirm the flexibility of the proposed new method, we next conduct simulation experiments with 100, 500 and 2000, 10000 DMs, respectively. The number of clusters is assumed to be set to $k = 6$, the consensus threshold is preset to 0.95, and the initial evaluation information and adjacency matrix are randomly generated according to the size of DMs. The final results are presented in Table 9, which gives the time and cost derived for different number of DMs and the final evaluation opinions.

As shown in Table 9, as the number of DMs involved in the decision-making process increases, the consensus model proposed in this study is

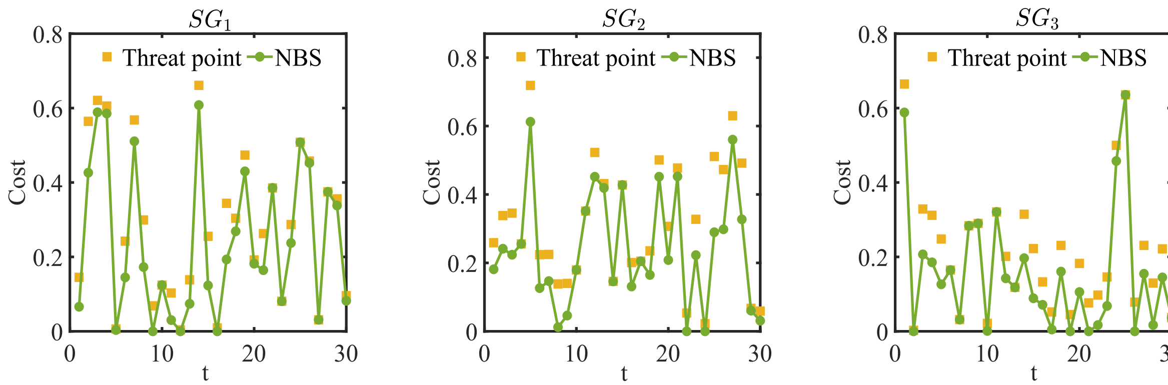


Fig. 9. Simulation experiments of the first new method.

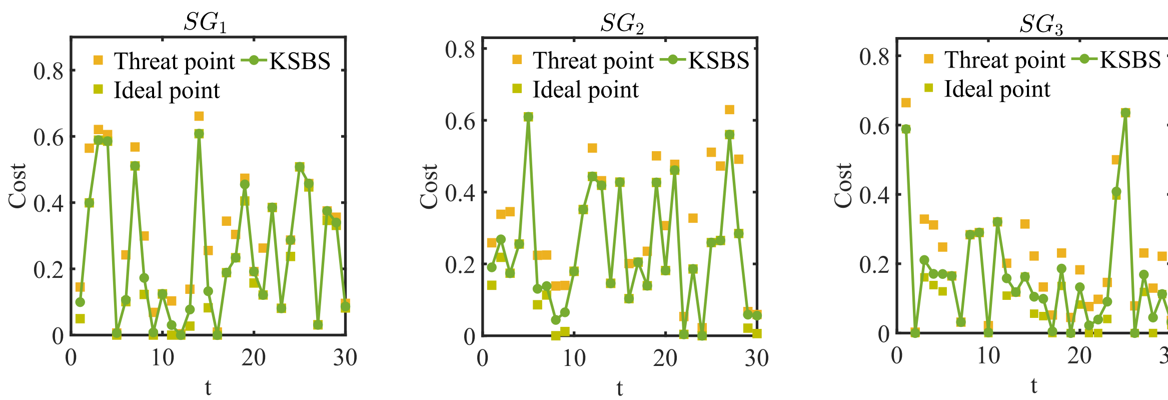


Fig. 10. Simulation experiments of the second new method.

Table 9
Simulation analysis results for processing DMs of different sizes.

Simulation experiments	Consensus model	Time cost	Final opinion	Cost
Simulation experiments for LSGDM with 100 DMs	NB-LSGDM KSB-LSGDM	7.6515 s 8.0158 s	$(0.6206, 0.5452, 0.6041, 0.5797, 0.4288, 0.5304)^T$ $(0.6206, 0.5452, 0.6041, 0.6965, 0.5457, 0.5304)^T$	16.0013 16.0013
Simulation experiments for LSGDM with 500 DMs	NB-LSGDM KSB-LSGDM	19 min 41 s 21 min 28 s	$(0.4669, 0.3804, 0.5239, 0.5544, 0.4188, 0.4876)^T$ $(0.4765, 0.3573, 0.5109, 0.5406, 0.4487, 0.5310)^T$	96.4938 96.4938
Simulation experiments for LSGDM with 2000 DMs	NB-LSGDM KSB-LSGDM	8 h and 21 min 8 h and 49 min	$(0.4325, 0.4175, 0.4929, 0.5626, 0.5282, 0.4337)^T$ $(0.4908, 0.4560, 0.6284, 0.5426, 0.5105, 0.4347)^T$	395.0093 395.0093
Simulation experiments for LSGDM with 10000 DMs	NB-LSGDM KSB-LSGDM	23 h and 54 min 24 h and 12 min	$(0.5035, 0.5363, 0.5707, 0.4298, 0.4430, 0.4377)^T$ $(0.5927, 0.5187, 0.5519, 0.4661, 0.5013, 0.3960)^T$	2000.0886 2000.0886

able to achieve a significant level of consensus in a limited time with a low adjustment cost. This result highlights the efficiency and reliability of the newly proposed LSGDM method dealing with a large number of DMs involved, proving its applicability scalable to tens of thousands of experts. In particular, it is noted that the simulation experiments are performed via MATLAB R2022a on a computer configured with a 12th Gen Intel(R) Core(TM) i5-12500 processor operating at 3.00 GHz.

7.4. Summary and discussion

In this section, we summarize and compare the advantages and characteristics of different consensus methods. As shown in Table 10, this study makes a comparison with some of the latest studies. Furthermore, to clearly delineate our study from the existing literature, the relevant studies have been categorized into three types: LSGDM consensus methods based on feedback iteration mechanisms (FIM-LSGDM), LSGDM consensus methods employing optimization strategies (OS-LSGDM), and LSGDM consensus methods from a game theory perspective (GT-LSGDM). Specifically, the research by Chao et al. (2021), Li et al. (2022), Guo, Zhang, Gong, Kou, and Xu (2024), and Shen et al. (2024) falls under FIM-LSGDM. For OS-LSGDM, the methods developed by Zhang et al. (2022), Rodríguez et al. (2022), Qin, Wang, and Liang (2023), and Zhao, Guo, Xu, and Wu (2024) are included. Lastly, the GT-LSGDM category includes the works of Tang, Liao, and Wu (2023), Meng, Tang, and An (2023), Meng et al. (2024), as well as the contributions of this study, which focus on a game theory to LSGDM. Through this analysis, we derive the following results:

(1) In terms of clustering studies: In this study, we propose a two-stage trust-constrained semi-supervised clustering learning method that effectively utilizes trust information to assist clustering. By introducing a regularization term, we overcome the limitations of traditional community detection algorithms and enhance their applicability in real-world scenarios. To the best of our knowledge, this is the first time that a semi-supervised clustering algorithm considering sparse social networks has been applied in LSGDM. This represents a significant advancement in the field of LSGDM clustering. Notably, comparative analysis demonstrates substantial performance improvements, making this method more suitable for addressing complex LSGDM problems.

(2) In terms of consensus design: As previously mentioned, both FIM-LSGDM and OS-LSGDM exhibit their respective strengths and

Table 10
Comparison of different LSGDM methods.

Types	References	Year of publication	Known labels	Sparse social network	Minimum cost	Fairness concern	Non-cooperation behaviour	Super large scale DMs
FIM-LSGDM	Chao et al.	2021	×	×	×	×	✓	×
	Li et al.	2022	×	×	×	×	×	×
	Guo, Zhang, et al.	2024	×	×	✓	✓	×	×
	Shen et al.	2024	×	×	×	×	×	×
OS-LSGDM	Zhang et al.	2022	×	×	✓	✓	×	×
	Rodríguez et al.	2022	×	×	✓	×	×	×
	Qin et al.	2023	×	×	✓	×	×	×
	Zhao et al.	2024	×	×	✓	×	×	×
GT-LSGDM	Tang et al.	2023	×	×	✓	✓	×	×
	Meng, Gong, and Pedrycz	2023	×	×	✓	✓	×	✓
	Meng et al.	2024	×	×	✓	✓	×	×
	This paper	–	✓	✓	✓	✓	✓	✓

[1] "×" denotes the factors not considered by the method, "✓" denotes the factors considered by the method.

weaknesses in addressing LSGDM problems. To overcome the limitations of traditional strategies, we propose two bargaining methods based on cooperative game theory in this study. Specifically, this study focuses on addressing the cost-related drawbacks of the FIM-LSGDM method and improving the fairness shortcomings of the OS-LSGDM method. In particular, we developed a two-type MCCM that integrates both cooperative and non-cooperative elements, taking into account the potential selfish behaviour of participants. By incorporating participants' self-interest tendencies and potential non-cooperative behaviour, this method ensures the effectiveness and reliability of achieving consensus.

(3) In terms of methods testing: To demonstrate the exact implementation of the proposed methods, contemporary research tends to be limited to case studies involving 20–50 DMs. This limited scope is insufficient to demonstrate the effectiveness of the method in larger and more complex situations. Therefore, we consider it important to conduct additional simulations or tests after the case studies to validate the method's ability to manage the challenges of Super LSGDM. Unfortunately, these necessary experiments are rare in the current literature. Additionally, the constructed method is time-efficient compared to the study by Meng, Tang, and An (2023). The time savings increase as the number of DMs increases. In essence, this study introduces an efficient and reliable method for LSGDM that is efficient, fair and cost-effective.

The LSGDM problem in social network environments is complex because it involves a large number of DMs, frequent interactive behaviours and significant cost requirements. In this environment, the structural information of social networks and human behaviour have important impacts on the clustering process, but this area has not been sufficiently researched. With this in mind, this study introduces the concepts of NB and KSB to construct two novel LSGDM methods based on cooperative games. These methods combine the dynamic characteristics of social networks, non-cooperative behaviours, and game theory principles. In summary, the proposed methods demonstrate their scientific validity and practicality in real LSGDM environments. Nevertheless, there are some limitations to be acknowledged. Specifically, we do not consider the effect of trust relationship on the CRP, which may limit the performance of the proposed method to some extent, thus affecting the actual consensus adjustment and selection.

8. Summary of managerial implications

In this study, we explore how to design fair consensus adjustment strategies based on individual and group interests in different scenarios, and analyse the effects of non-cooperative and cooperative behaviours on individual decision-making. Based on the previous analyses, this section further discusses the academic value and practical implications of these two LSGDM methods.

(1) With the rapid development of information technology, GDM has been able to be conducted on online platforms, allowing a large number of DMS to participate simultaneously. However, a common feature of large-scale networks is sparsity, meaning that the actual number of connections in the network is far less than the maximum possible number of connections. To address this challenge, this study proposes a two-stage trust-constrained semi-supervised learning mechanism that significantly optimizes information utilization in LSGDM, especially for dealing with sparse and incomplete data in social networks. This mechanism has wide applicability in practical applications. For example, in recommendation systems, the mechanism can make use of sparse trust relationships and interaction data between users to generate more personalized and accurate recommendations, improving user satisfaction and system efficiency.

(2) In modern society, as the public's demand for high-quality collective decision-making increases, LSGDM methods have gradually become a research hotspot. In studies related to LSGDM, the integration of SNA, machine learning methods, and optimization algorithms has been extensively investigated. In this study, we introduce two new negotiation methods to achieve fairness and cost minimization in CRP from a cooperative game theory perspective. Notably, these concepts have not been fully investigated in LSGDM, although they have been widely used in other domains. For example, in the process of public policy making, these methods can effectively find the balance point of multi-party interests, thus increasing public participation and satisfaction.

(3) From the perspective of implementer, this study is based on the principle of global minimum cost, aiming to achieve consensus through the optimization of adjustment cost distribution. By comparing the adjustment costs in independent and centralized decision-making environments, we have developed a non-cooperative and cooperative two-type MCCM, which facilitates more efficient cooperation and consensus. Therefore, this study not only provides effective tools for optimizing resource allocation and improving decision-making efficiency but also helps implementers maintain flexibility and adaptability in dynamic decision-making environments.

(4) From the perspective of the participants, this study aims to explore a strategy that ensures fair cost allocation based on the principle of maximum fairness. By considering the participants' negotiation ideal points and threat points, we have developed two new LSGDM negotiation methods. These methods not only enhance fairness in the decision-making process, but also effectively reduce conflicts arising from uneven cost distribution. In practical negotiations, this can prevent excessive bias towards one party's interests, thereby contributing to more stable and enduring agreements.

9. Conclusions

The proliferation of the internet has not only propelled a paradigm shift in decision-making but also heightened the demands for fairness and efficiency in these processes. With the increase in the number of participants, LSGDM methods have demonstrated unique advantages in addressing complex decision questions, thus becoming an important research topic in the field of decision-making. In this study, we have introduced two novel LSGDM methods, whose superiority and reliability have been validated on real datasets from a music platform. The main contributions of this study are as follows:

(1) To address the potential issue of network sparsity caused by the participation of a large number of DMs in social networks, we have developed a two-stage semi-supervised FCM clustering method with trust constraints. This method effectively alleviates the impact of data sparsity on clustering outcomes and improves the validity of the clustering results.

(2) From the perspective of cooperative game theory, we have analysed the optimal consensus adjustment strategies for participants under both cooperative and non-cooperative conditions. In doing so, we have also accounted for potential selfish behaviours to ensure the acceptability of the decision outcomes.

(3) The research results indicate that cooperation does not always bring the expected benefits to participants. Therefore, by analysing the outcomes in various scenarios, we can predict the actions that participants are likely to take, providing valuable insights for the decision-making process.

(4) To address the issue of fair distribution in global optimal consensus adjustment, we have developed two LSGDM consensus methods based on bargaining. These methods ensure fair decision adjustments in situations involving multiple participants.

Future work will focus on further expanding the application and theoretical exploration of the current methods. We plan to conduct in-depth research in the following areas:

(1) We will explore various social relationships within social networks, such as cooperative, competitive, and power relationships, and study game mechanisms based on these relationships to better understand their role in decision-making processes.

(2) We will focus on analysing the optimal strategy choices of participants in non-cooperative games and examine the realization of Nash equilibrium, further enhancing the theoretical foundation of decision models.

(3) We plan to apply these methods to more complex real-world scenarios, such as emergency decision-making, the green economy, and competitive supply chains, to validate their broad applicability and practical value in diverse decision-making environments.

CRediT authorship contribution statement

Yufeng Shen: Writing – original draft, Methodology, Investigation. **Xueling Ma:** Writing – original draft, Methodology, Investigation. **Gang Kou:** Writing – review & editing, Methodology. **Rosa M. Rodríguez:** Writing – review & editing, Methodology. **Jianming Zhan:** Supervision, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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