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RESEARCH ARTICLE

Constructing Composite Indicators Through Extreme Values Reductions-Ordered Weighted Averaging: Human Development Index

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ABSTRACT The Human Development Index is a complex social phenomenon composed of multiple criteria. While many studies are concerned with dealing with outliers in the representation of this composite indicator, the problem of extreme values is overlooked. This study falls within the multi-criteria decision framework aimed at introducing a new approach to constructing composite indicators specifically that addresses the issue of extreme values, called the Extreme Values Reductions-Ordered Weighted Averaging approach. The study findings reveal that popular methods such as Principal Component Analysis, Benefit of the Doubt, Entropy, and Equal Weights ignore outliers and bias the composite indicator scores. The Extreme Value Reduction approach provides an unbiased representation of the Human Development Index, preventing extreme lower and higher values from uncovering weaknesses and unmasking strengths of the decision-making units. The possibility of adjusting the weights of the intermediate values also provides the opportunity to reveal nuances that are difficult to detect by controlling the coefficient of variation of the composite indicator scores.

INDEX TERMS Multicriteria methods, multidimensional problems, composite indicators, ordered weighted averaging, extreme values reductions, human development index.

I. INTRODUCTION

The Human Development Index (HDI) [1], Multidimensional Poverty Index (MPI) [2], Global Competitiveness Index (GCI) [3], and Global Innovation Index (GII) [4] are summary measures used to represent complex and multifaceted realities. These multidimensional indices, or composite indicators, consist of aggregating diverse sub-indicators previously normalized and weighted [5]. They reduce the cognitive stress that the simultaneous reading of multiple sub-indicators exerts on decision-makers, facilitating the understanding and interpreting the multidimensional phenomenon [6].

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The ability to simplify and facilitate the understanding of complex multidimensional phenomena has attracted the attention of researchers in the social and environmental sciences to the literature on composite indicators [7]. Researchers in the multicriteria decision-making field are also very interested in and active in composite indicators literatures [8] and [9].

There are many multicriteria methods for constructing composite indicators. These methods are distinguished essentially by how the sub-indicators are weighted. Some are extremely popular, such as Principal Component Analysis, the Analytical Hierarchy Process, the Benefit of the Doubt, Entropy, and Simple Additive Weighting with Equal Weights [10].

Each of these methods deals with a specific problem in constructing composite indicators. Principal Component Analysis seeks to reduce the informational loss of sub-indicator aggregation by seeking the maximum variance extracted from the input data [11]. The Analytic Hierarchy Process aims to match the weights of the sub-indicators with their conceptual importance [12]. The Benefit of the Doubt seeks to minimize political disputes concerning sub-indicator weights by assigning higher weights to the best-performing sub-indicators in each decision-making unit [13]. Entropy seeks to facilitate the differentiation between decision-making units by assigning greater weights to sub-indicators with greater informational diversity [14]. Finally, Simple Additive Weighting with Equal Weights facilitates the explanation and simplifies the communication of results [5].

Although they offer effective responses to different challenges, these methods have a common flaw: they cannot identify and deal with extreme values. Extreme values are not necessarily outliers but rather values that are far from the mean [15], [16]. While many studies address the problem of outliers in constructing composite indicators [17], [18], [19], the issue of extreme values is often neglected.

Extreme values are problematic in constructing composite indicators as they bias the scores [20]. In most methods, the data scaling transformation functions are based on these extreme values (Max/Min function) or are influenced by them (z-scores function) [21], [22]. In these cases, the composite indicator scores are biased upwards in the presence of lower extreme values and downwards in the presence of higher extreme values [23]. Although scoring bias does not necessarily affect the ranking of decision-making units, it distorts the representation of the multidimensional phenomenon, hiding deficiencies (upward bias) or masking strengths (downward bias).

Extreme values are even more problematic in composite indicators constructed using the Benefit of the Doubt method [24]. The Benefit of the Doubt method assigns higher weights to the sub-indicators with the highest value and lower weights to the sub-indicators with the lowest value [25]. Although this weighting logic benefits all decision-making units, it biases the composite indicator scores maximally upward. In this situation, the negative characteristics of each decision-making unit are undervalued, and the positive characteristics are overvalued. In this way, deficiencies are more strongly hidden in the presence of lower extreme values, and qualities are no longer masked but emphasized in higher extreme values [18], [25].

It is important to remember that extreme values are not necessarily outliers [15], [16]. They have been successfully treated within the multi-criteria decision-making literature through the Ordered Weighted Averaging operator [26], [27], by the so-called Extreme Values Reductions approach [28], [29].

Ordered Weighted Averaging does not assign weights to sub-indicators as in other methods but is based on the values of the sub-indicators of each decision-making unit, ordered from highest to lowest [30]. Applications based on Ordered Weighted Averaging that deal with extreme values take advantage of this data structure and assign higher weights to intermediate values and lower weights to extreme values [29]. This weighting logic allows information to be merged unbiasedly [29].

However, the application of the Extreme Value Reduction approach remains confined to multi-criteria decision-making problems, in which extreme opinions are underweighted while intermediate opinions are overweighted [28], [29]. Therefore, the applicability and effectiveness of the Extreme Values Reduction approach in constructing composite indicators are unknown.

This paper aims to develop the Extreme Values Reductions-Ordered Weighted Averaging approach to address the presence of extreme values in constructing composite indicators. Its main contribution lies in revealing how overvaluations of intermediate values and undervaluations of extreme values impact the scores and ranking of decision-making units in the Human Development Index and contribute to assertive decisions. Additionally, the study creatively compares the Extreme Values Reductions-Ordered Weighted Averaging approach with the Benefit of the Doubt methods [31], [32], Principal Component Analysis [33], [34], Simple Additive Weighting with Equal Weights [35], [36], and Entropy [14], [37]. Although some studies adopt subjective weighting schemes for the Human Development Index [38], the unavailability of the data prevents their inclusion in this comparison.

Based on the information the authors have accessed and reviewed, the application of the Extreme Value Reduction approach in constructing composite indicators makes this study original. Furthermore, the study is groundbreaking in offering an extensive comparison of the Human Development Index constructed by different methods. It also expands the literature on representations of multidimensional phenomena through the Ordered Weighted Average, providing a pioneering representation of the Human Development Index.

The remainder of this research is organized into four sections. Section II introduces composite indicators and Ordered Weighted Averaging and delves into the literature connecting these two frameworks. Section III presents the Human Development Index, its concept, data, and the methods applied in the study. Section IV presents the research results, and Section V presents the conclusions, main findings, limitations, and lines of future research.

II. COMPOSITE INDICATORS AND ORDERED WEIGHTED AVERAGING

Composite indicators result from a set of mathematical operations that allow the representation of multiple criteria (sub-indicators) associated with a multidimensional phenomenon through a one-dimensional measure [25], [39]. In other words, composite indicators result from aggregating previously normalized sub-indicators and weighing them with equal or different weights [20].

The sub-indicator's normalization, weighting, and aggregation operations can be performed differently [40]. The most used normalization functions are max/min and z-scores [23]. The most common aggregation is by the arithmetic mean. However, non-compensatory aggregations such as the geometric mean and Mazziota-Pareto present a desirable property: the non-compensability between poor and above-average sub-indicators [41]. On the one hand, the most popular subjective weighting schemes are the Analytic Hierarchical Process and Budget Allocation. On the other hand, the most popular objective weighting schemes are Equal Weights, Entropy, Principal Component Analysis, and Benefit of the Doubt [5].

At this point, it is important to highlight that the weighting schemes of all the multicriteria methods most used in constructing composite indicators [10] assign weights to the sub-indicators (criteria). In contrast, Ordered Weighted Averaging is a multicriteria method in which weights are not assigned to the criteria but to the ordered values according to each decision-making unit [26], [27]. This peculiar weighting structure offers the decision-maker flexibility in allocating and assigning the subindicator's weights [30], [42].

Yager [26] designed Ordered Weighted Averaging as an aggregation method that assigns weights to input values according to their order [30]. The method has the following properties: boundedness, idempotency, monotonicity, and symmetry [42]. These properties allow obtaining a result that respects the order of the input values equally, making it possible to represent pessimistic and optimistic perspectives or decisions in different intensities through the regulation of the weights [40].

To construct a composite indicator by the Ordered Weighted Averaging, the sub-indicators must be normalized by the following function:

$$\mathrm{DMU}_{nk} = \frac{Sb_k - Sb_{k_{\min}}}{Sb_{k_{\max}} - Sb_{k_{\min}}} \tag{1}$$

where DMU_{nk} is the normalized value of the sub-indicator or criterion k for decision-making unit n, $Sb_{k_{\min}}$ is the lowest value of the sub-indicator k between all decision-making units, and $Sb_{k_{\max}}$ is the highest value of the sub-indicator or criterion k between all decision-making units.

Then, the aggregation of normalized sub-indicators $\mathrm{DMU}_1, \mathrm{DMU}_2, \ldots, \mathrm{DMU}_n$ by the Ordered Weighted Averaging operator of dimension n and function $[0, 1]^n \rightarrow [0, 1]$ is executed from

$$\mathrm{OWA}\left(\mathrm{DMU}_1, \mathrm{DMU}_2, \ldots, \mathrm{DMU}_n\right) = \sum\nolimits_{i=1}^{n} w_i b_i, \tag{2}$$

where b_i is the highest value sub-indicator between $\mathrm{DMU}_1, \mathrm{DMU}_2, \ldots, \mathrm{DMU}_n$ and the weights w_i satisfy the following conditions: $w_i \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$.

Note, therefore, that (1) and (2) consist of performing six elementary operations: 1.) normalizing the data; 2.) transposing the data; 3.) sorting the data from largest to smallest; 4.) defining the weights; 5) multiplying the weight values by their respective weights; 6.) adding the weighted values.

These steps demonstrate how easy it is to implement ordered weighted averaging and highlight the importance of defining their weights.

The definition of weights is a key element of the Ordered Weighted Averaging operator because it allows the implementation of the concept of the decision maker's level of optimism, called the Orness degree [28]. The Orness degree reflects the distribution of the magnitudes of the Ordered Weighted Averaging weights and indicates the level of optimism or pessimism of the decision maker [29]. In (2), the weights must satisfy $w_1 \leq w_2 \leq \ldots \leq w_n$ for an optimistic attitude and $w_1 \geq w_2 \geq \ldots \geq w_n$ for a pessimistic attitude

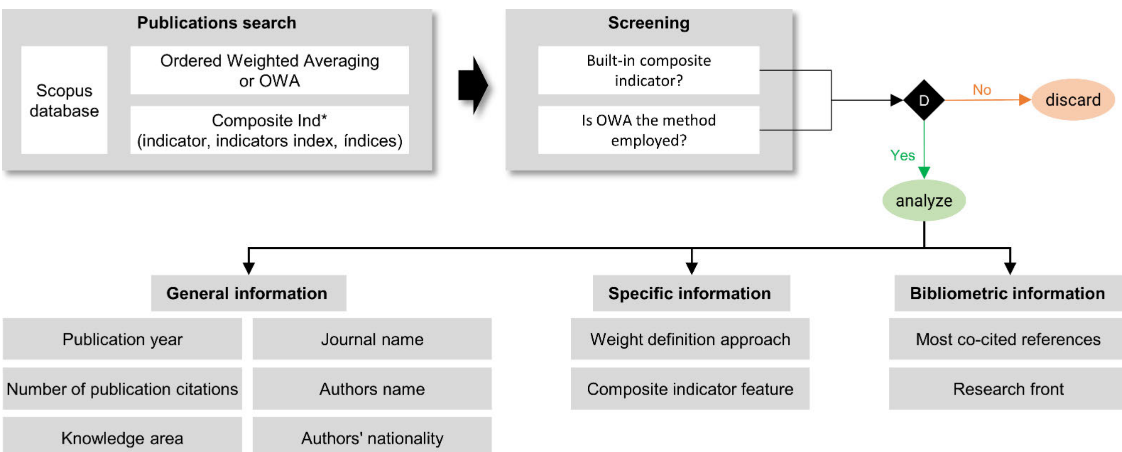

FIGURE 1. Literature review framework on ordered weighted averaging in composite indicators.

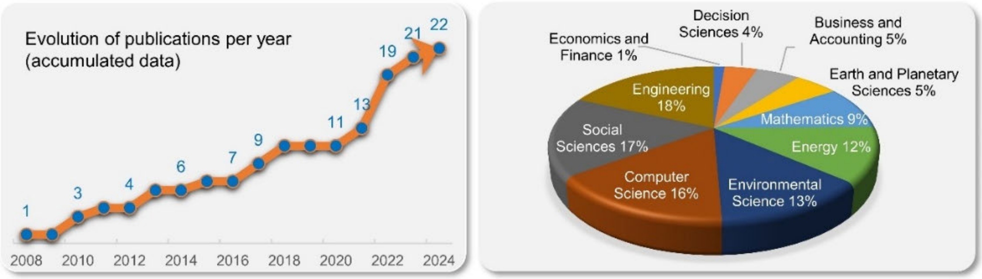

FIGURE 2. Evolution and profile of publications that use ordered weighted averaging to construct composite indicators.

Despite its flexibility and applicability, the following section highlights that Ordered Weighted Averaging is rarely used in constructing composite indicators.

A. ORDERED WEIGHTED AVERAGING-BASED COMPOSITE INDICATORS LITERATURE

The literature review on using Ordered Weighted Averaging to construct composite indicators was carried out following the collection and investigation procedures presented in Figure 1.

Bibliometric information was used to identify research fronts. The research fronts are identifiable through the co-citations of the publications' references through the Vosviewer software [43]. The co-citation analysis allows for the constitution of clusters of publications with topics that are similar to each other and dissimilar among the clusters [44]. The distance between the clusters indicates the strength of the relationship between the publications regarding co-citation, signaling the proximity between the research fronts [43].

B. VOLUME AND PROFILE OF PUBLICATIONS

A total of 212 publications indexed in the Scopus database on May 10, 2024, mention the terms "Ordered Weighted Averaging" OR OWA AND "composite ind*" (where * corresponds to indicators, indicators, indexes, or indices) in the text and NOT in the references. The screening process of these publications reveals that only 22 use Ordered Weighted Averaging and explore composite indicators' theoretical and operational framework.

On the one hand, this number can be considered adequate compared to using composite indicators on a specific topic. The literature review carried out by Asadzadeh et al. [45] identified 17 publications regarding composite indicators for measuring community disaster resilience. Stefana et al. [46] identified 22 publications regarding composite indicators for measuring the quality of working life in Europe. On the other hand, 22 publications that use Ordered Weighted Averaging to construct composite indicators can be considered a low number compared to the literature review of El Gibari et al. [10], which identifies a hundred composite indicators constructed using multicriteria methods.

During the screening process, 16 publications were identified that mentioned the term "composite ind*" in the references. These data suggest that 46% of the 35 publications explicitly mention "Ordered Weighted Averaging" and "composite ind*" construct composite indicators without considering the appropriate theoretical framework. Researchers do not mention constructing a new index or indicator in 12 of these 16 publications. The remaining research [47], [48], [49] construct new indices or indicators from the aggregation of sub-indicators but do not mention the term composite indicator or synonymous terms such as synthesis indicator, overall indicator, or aggregated indicator.

The information in Figures 2, 3, and 4 considers only the 22 publications that explore the concept of composite indicators in the body of the research and employ Ordered Weighted Averaging. These publications occurred between 2008 and 2024, with 68% occurring in the last five years (e.g., [50], [51], [52]. The areas of knowledge explored by Ordered Weighted Averaging in construction composite indicators are Engineering, Social Sciences, and Computer Science, concentrating 18%, 17%, and 16% of the publications, respectively.

Publications that use Ordered Weighted Averaging to construct composite indicators have very different characteristics from publications concerning the literature on composite indicators. El Gibari et al. [10] show that Environmental Sciences, with 48%, and Operations Research and Management Science, with 27%, concentrate most publications constructing composite indicators through multicriteria methods.

C. LEADING AUTHORS AND COUNTRIES

Figure 3 shows the leading authors and countries with publications on composite indicators using Ordered Weighted Averaging. In particular, the two researchers who use Ordered Weighted Averaging and explore the concept of composite indicators in the body of research are not on the list of the top 10 researchers in the field [53]. These two researchers and their respective publications are Ratick and Osleeb [54], [55], [56] and Hermans et al. [57], [58], [59]. The map of the publications' countries of origin shows that researchers from North America (6) and Belgian (4) are the most active. China,

FIGURE 3. Leading authors and countries that most explore composite indicators using ordered weighted averaging.

Brazil, the Republic of China, Hungary, Italy, and Serbia have two publications each.

Among the 22 publications that simultaneously explore the use of Ordered Weighted Averaging and the concept of composite indicators in the body of research, four are cited more than 50 times [60], [61], [62], [63].

The map of the publications is very different from the literature that uses Ordered Weighted Averaging disconnected from the composite indicator's framework. He et al. [53] show that the Republic of China concentrates more than half of the publications exploring composite indicators and using Ordered Weighted Averaging. The United States occupies the third position in the ranking, while Belgium is not part of the group of the ten countries that most publish about Ordered Weighted Averaging.

D. RESEARCH FRONTS (CO-CITATION MAP)

The co-citation map in Figure 4 reveals that the 22 publications that simultaneously explore the concept of composite indicators and use Ordered Weighted Averaging are divided into three research fronts.

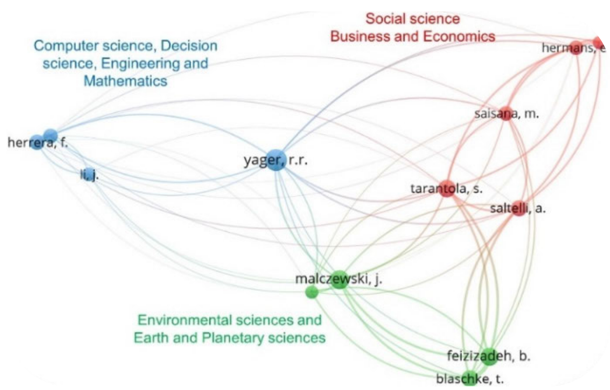

FIGURE 4. Co-citation map of publications that explore the concept of composite indicators and use ordered weighted averaging.

The analysis of the co-citation map indicates that three research fronts are associated with using Ordered Weighted Averaging to construct composite indicators. Publications from the research front that cite Yager and Herrera (e.g., [46], [60]) are grounded in decision-making theory. Publications of this research front are associated with composite indicators from social sciences, business, and economics, such as the costs of doing business index [64] and quality of life index [65].

The use of geographic information systems is the main characteristic of publications that refer to Malczewski [66], [67], Feizizadeh et al. [68], [69], and Feizizadeh and Blaschke [48]. In particular, Ordered Weighted Averaging can be easily operationalized through Geographic Information Systems (GIS) modules such as IDRISI [70], QGIS/Grass [71], [72], and ArcGIS/ArcView [73]. These GIS modules are straightforward, favoring the use of Ordered Weighted Averaging in studies based on satellite images. For this reason, this research front is quite broad, comprising eleven publications. This publication shows the relevance of Ordered Weighted Averaging in research involving geographic data, which are especially significant in environmental, earth, and planetary sciences, including urban and regional geography. On this research front, there are composite indicators associated with heat vulnerability assessment [60] and vulnerability to climate change [54], [74], as well as regional development [75], environmental benefits [76], residential suitability [77], and urban inequality [78].

Publications that refer to Hermans and Tarantola discuss composite indicators' theoretical and operational framework as their main characteristics in a more profound way. Composite indicators constructed in this research are associated with road safety [57], [58], [59], transport planning [62], energy supply [61], [63], and disaster resilience [79].

Only research by Yang et al. [80] was not categorized among the identified lines of research. This research constructs a comprehensive composite indicator of megacity resilience without referencing any of the authors of the co-citation map.

E. QUALITY TESTS AND DEFINITION OF WEIGHTS

Quality tests are found in 58% of the 22 analyzed publications. Linking the indicator to external variables is the most frequent quality test, applied in 42% or eight publications. Robustness analyses (sensitivity or uncertainty) are less frequent and are performed in 16% of publications.

At this point, it is important to highlight that uncertainty analysis has been the most recommended method for testing the quality of the composite indicator [20], [25], [39]. Uncertainty analysis is a measure of the average ranking variation ($\bar{\mathbf{R}}_{\mathbf{v}}$) of decision-making units of composite indicators constructed by different methods, this variation being obtained in the following manner

$$\bar{\mathbf{R}}_{\mathbf{v}} = \frac{1}{m}\sum\nolimits_{r=1}^{m}\left|\mathrm{Rank}\left(CI_n^R\right) - \mathrm{Rank}\left(CI_n^A\right)\right| \quad (3)$$

where $\bar{\mathbf{R}}_{\mathbf{v}}$ is the average ranking variation of all decision-making units, $\mathrm{Rank}\left(CI_n^R\right)$ is the rank of the n-th decision-making unit in the reference composite indicator, and $\mathrm{Rank}\left(CI_n^A\right)$ is the rank of the n-th decision-making unit in the alternative composite indicator.

Linguistic quantifiers and orness degree are the most widespread approaches for defining Ordered Weighted Averaging operator weights and are applied in 37% of the publications each. Entropy measures are applied in 11% of the publications. The remaining 16% of publications do not mention the approach used in defining Ordered Weighted Averaging weights. The choice of linguistic operator allows the representation of the multidimensional phenomenon from different perspectives. For example:

- Max operator: assigns a weight of one to the line with the highest values and a weight of zero to the remaining lines, skewing the composite indicator scores upwards to a maximum extent;
- Min operator: assigns a weight of zero to the lines with the highest values and a weight of one to the line with the lowest values, skewing the composite indicator scores downwards to a maximum extent;
- ''At least J'' criteria operator: assigns a weight of 1/J to the first J lines and a weight of zero to the remaining lines, allowing the composite indicator scores to be skewed upwards to different degrees based on the definition of J.
- ''More than J'' criteria operator: assigns a weight of 1/J to the last J lines and a zero weight to the remaining lines, reflecting a pessimistic view of the decision maker by skewing the composite indicator scores downwards to different degrees.

This flexibility in allocating and defining weights allows the decision-maker to emphasize positive or negative aspects of the decision-making units and capture nuances of the multidimensional phenomenon [30].

III. MATERIALS AND METHODS

A. HUMAN DEVELOPMENT INDEX DATA

The Human Development Index is a composite indicator encompassing the dimensions of a long and healthy life, knowledge, and a decent standard of living [1], [81]. The sub-indicator used to represent the dimension of a long and healthy life is life expectancy at birth. The education dimension combines the sub-indicators mean years of schooling (for adults aged 25 and over) and expected years of education (for children). Finally, the sub-indicator gross national income (GNI) per capita represents the decent standard of living dimension. The Human Development Index's construction method is Simple Additive Weighting by geometric mean with Equal Weights.

This study analyzes the Human Development Index for a small number of countries for the sake of simplicity and comprehensibility. The data retrieved are from 2022 and cover eleven countries in South America: Argentina, Bolivia, Brazil, Chile, Colombia, Ecuador, Guyana, Paraguay, Peru, Uruguay, and Venezuela. This set of countries was chosen because it offers a clearer view of the results through maps.

The data were extracted on February 1, 2025, from the Human Development Reports at https://hdr.undp.org/data-center/human-development-index#/indicies/HDI. The descriptive statistics of the data are presented in Table 1.

TABLE 1. Descriptive statistics of the human development index sub-indicators of south american countries.

	Education	GNI per capita	Life expectancy
Mean	0.40	0.36	0.56
Standard Deviation	0.27	0.28	0.31
Skewness	1.15	1.05	-0.48
Maximum	1.00	1.00	1.00
Minimum	0.06	0.01	0.01

The data with the greatest skewness are education and GNI per capita. Both data show positive skewness, while the Life Expectancy Index shows negative skewness. Positive skewness suggests the presence of lower extreme values, while negative skewness suggests the presence of higher extreme values.

B. COMPOSITE INDICATORS OF THE HUMAN DEVELOPMENT INDEX

Fourteen composite indicators of the Human Development Index were constructed. The arithmetic mean was chosen to allow comparability of results. Table 2 shows that weight vectors w_i and w_n were used. The weight vector w_i is used in Ordered Weighted Averaging and is associated with the values of the dimensions ordered from largest to smallest. The weight vector w_n is used in the Principal Component Analysis, Entropy, and Simple Additive Weighting with Equal Weights methods and is associated with the dimensions. Note that Benefit of the Doubt does not have a weight vector that allows comparison, but a $n \times k$ weight matrix.

IV. RESULTS AND ANALYSIS

From the literature review of the Ordered Weighted Averaging-based composite indicators, it is possible to highlight eight points. First, the literature reviews on composite

TABLE 2. Vector of weights used in composite indicators.

Approach \ Method	Weight vector
Ordered Weighted Averaging	
Max operator	$w_i = [1.00, 0.00, 0.00]$
Min operator	$w_i = [0.00, 0.00, 1.00]$
At least J criterion operator	$w_i = [0.50, 0.50, 0.00]$
More than J criterion operator	$w_i = [0.00, 0.50, 0.50]$
Popular methods of constructing composite indicators	
Principal Component Analysis	$w_n = [0.41, 0.28, 0.31]$
Benefit of the Doubt	$n \times k$ weight matrix
Entropy	$w_n = [0.30, 0.43, 0.28]$
Simple Additive Weighting with Equal Weights	$w_n = [0.33, 0.33, 0.33]$
Extreme Values Reductions-Ordered Weighted Averaging	
Very high reduction	$w_i = [0.05, 0.90, 0.05]$
High reduction	$w_i = [0.10, 0.80, 0.10]$
Medium-high reduction	$w_i = [0.15, 0.70, 0.15]$
Medium-low reduction	$w_i = [0.20, 0.60, 0.20]$
Low reduction	$w_i = [0.25, 0.50, 0.25]$
Very low reduction	$w_i = [0.30, 0.40, 0.30]$

indicators [10], [82] do not include Ordered Weighted Averaging. Second, the Ordered Weighted Averaging allows the construction of composite indicators with non-compensatory aggregation of sub-indicators, with a pessimistic or optimistic emphasis, and considering the heterogeneity of sub-indicator weights according to the decision-making unit [78]. Third, the research front with the most publications comprises the Environmental, Earth, and Planetary Sciences, although many studies have a methodological focus and are associated with multicriteria decision-making literature [30]. Fourth, geography journals concentrate on most of the published studies, mainly due to GIS software that allows you to apply Ordered Weighted Averaging to treat satellite images [66], [67], [68]. Sixth, adjusting the orness degree is the most common approach for defining Ordered Weighted Averaging weights [65]. The most frequent quality test is the link of the composite indicator with external variables [40]. Eighth, the robustness analysis is the quality test most used in the composite indicator's literatures [25] and [39] but rarely used in studies that employ Ordered Weighted Averaging.

Figure 5 illustrates the flexibility offered by Ordered Weighted Averaging in emphasizing positive or negative aspects of the dimensions of the Human Development Index in South American countries. The maps demonstrate this flexibility through the average scores of the Human Development Index, which are higher in the Max and "At least J" operators and lower in the "More than J" and Min operators. The maps also show that uncertainty is more significant in the Max and Min operators as they emphasize the positive and negative aspects of the Human Development Index. The "At least J" operator presents lower uncertainty than the "More than J" operator.

On the one hand, the Max and "At least J" criteria give decision-makers an optimistic representation of the Human Development Index. Governments can benefit from this optimistic representation in identifying countries that develop effective public human development policies, serving as a benchmark for countries with lower Human Development Index. On the other hand, a pessimistic representation of the multidimensional phenomenon, which emphasizes the negative dimensions of the Human Development Index, allows for a more straightforward and precise identification of the countries that should be prioritized in allocating investments.

In general, the maps illustrate the applicability of Ordered Weighted Averaging in constructing composite indicators, highlighting its flexibility in emphasizing positive or negative aspects in different intensities.

Table 3 shows a positive relationship between the weight assigned to intermediate values (0.4 to 0.9) and the ratio of the scores of composite indicator constructed by Ordered Weighted Averaging operators (Max, Min, "At least J" and "More than J") and composite indicator been constructed by the Extreme Value Reduction approach. In other words, the higher the weight assigned to intermediate values, the higher the ratio between the scores of the composite indicators. This positive relationship indicates the presence of higher extreme values.

These results reveal that the Max and "At least J" operators overvalue the upper extreme values, resulting in scores up to 1.86 times higher. Meanwhile, the Min and "More than J" operators completely disregard the upper extreme values, offering a pessimistic-conservative perspective of the Human Development Index.

Table 4 confirms the presence of upper extreme values as it shows a positive relationship between the weight assigned to

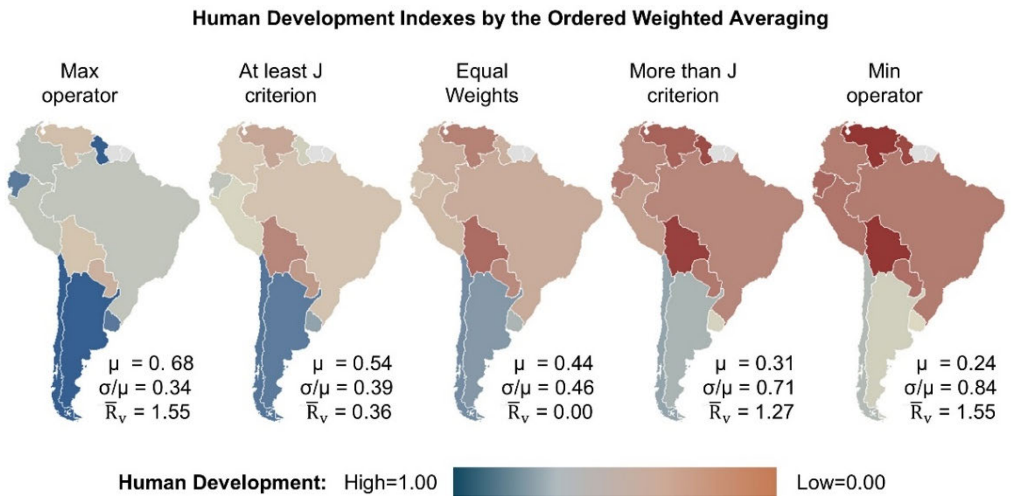

FIGURE 5. Representations of the human development index by ordered weighted averaging.

TABLE 3. Ratio between composite indicator scores (Ordered Weighted Averaging operators by Extreme Value Reduction approach).

	Max	Min	At least J	More than J
Extreme Values =0.30	1.63	0.56	1.24	0.71
Extreme Values =0.25	1.67	0.57	1.28	0.72
Extreme Values =0.20	1.72	0.59	1.31	0.74
Extreme Values =0.15	1.76	0.60	1.34	0.76
Extreme Values =0.10	1.81	0.62	1.38	0.78
Extreme Values =0.05	1.86	0.64	1.42	0.81

intermediate values (0.4 to 0.9) and the ratio of the scores of composite indicator constructed by popular methods: Equal Weights, Principal Component Analysis, Benefit of the Doubt and Entropy. In addition, the scores of the composite indicators Equal Weights, Principal Component Analysis, and Benefit of the Doub are higher than those of the composite indicators Extreme Values Reductions-Ordered Weighted Averaging. Only one of the six composite indicators constructed by Entropy has a lower score than the Extreme Values Reductions-Ordered Weighted Averaging.

TABLE 4. Ratio between composite indicator scores (popular methods by the Extreme Value Reduction approach).

	Equal Weights	Principal Component Analysis	Benefit of the Doubt	Entropy
Extreme Values =0.30	1.02	1.02	1.63	1.02
Extreme Values =0.25	1.04	1.05	1.68	1.04
Extreme Values =0.20	1.07	1.07	1.72	1.07
Extreme Values =0.15	1.10	1.10	1.76	1.10
Extreme Values =0.10	1.13	1.13	1.81	1.13
Extreme Values =0.05	1.16	1.16	1.86	1.16

These results show that popular composite indicator construction methods do not identify and deal with extreme values. Furthermore, the results show that the Benefit of the Doubt biases the composite indicator scores more strongly in the presence of higher extreme values. The ratio between the scores of Benefit of the Doubt and Extreme Values Reductions-Ordered Weighted Averaging is slightly higher than the ratio of the scores of the Max operator.

The results in Table 4 show that the scores of the composite indicator Entropy are closer to those of the Extreme Values Reductions-Ordered Weighted Averaging. This proximity occurs because Entropy assigns a lower weight to the dimension that presents negative asymmetry, life expectancy (see Table 2).

Tables 5 and 6 show the average ranking variation ($\bar{\mathbf{R}}_\mathbf{v}$) of decision-making units of composite indicators constructed by different Extreme Values and methods.

TABLE 5. Uncertainty between composite indicators constructed by the ordered weighted averaging operators and the extreme value reduction approach.

	Max	Min	At least J	More than J
Extreme Values =0.30	1.55	1.18	0.73	0.91
Extreme Values =0.25	1.55	1.18	0.91	0.73
Extreme Values =0.20	1.55	1.18	0.91	0.73
Extreme Values =0.15	1.91	1.36	1.09	0.55
Extreme Values =0.10	1.91	1.36	1.09	0.55
Extreme Values =0.05	2.09	1.18	1.27	0.36

Figure 6 illustrates the Human Development Index constructed by the Principal Component Analysis, Benefit of the Doubt, Entropy, and Equal Weights methods, and the Extreme Value Reduction approach (Extreme Values = 0.30, 0.15, and 0.05). It reveals that the Human Development Index scores are biased upward in the Principal Component Analysis,

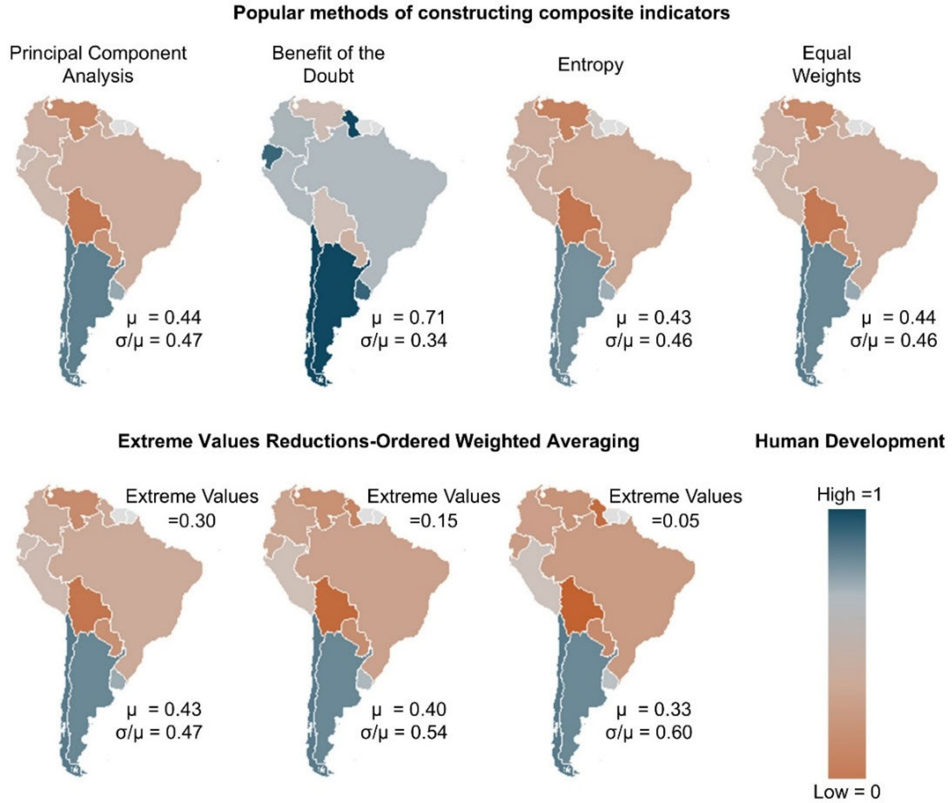

FIGURE 6. Visualization of the effect of extreme values on the human development index constructed by the principal component analysis, benefit of the doubt, entropy and equal weights methods.

TABLE 6. Uncertainty between composite indicators constructed by popular methods and the extreme value reduction approach.

	Equal Weights	Principal Component Analysis	Benefit of the Doubt	Entropy
Extreme Values =0.30	0.36	0.36	1.55	0.73
Extreme Values =0.25	0.55	0.55	1.55	0.73
Extreme Values =0.20	0.55	0.55	1.55	0.73
Extreme Values =0.15	0.91	0.91	1.91	1.09
Extreme Values =0.10	0.91	0.91	1.91	1.09
Extreme Values =0.05	1.09	1.09	2.09	1.27

Benefit of the Doubt, and Equal Weights. The Human Development Index by Entropy is comparable to the very low reduction of the Extreme Value Reduction approach.

The coefficients of variation of the Human Development Index of the Extreme Value Reduction approach are higher than those of popular methods. Higher coefficients of variation indicate more significant variability of the data around the mean, facilitating the process of differentiating decision-making units.

In addition to these findings, the study also highlights that while many studies address the problem of outliers in constructing composite indicators, the problem of extreme values is not. Popular methods such as Principal Component Analysis, Benefit of the Doubt, Entropy, and Equal Weights ignore outliers and bias the composite indicator scores. The Extreme Value Reduction approach provides an unbiased representation of the Human Development Index, preventing extreme lower and higher values from uncovering weaknesses and unmasking strengths of the decision-making units.

Furthermore, the Ordered Weighted Averaging operators offer the decision maker the possibility of highlighting positive and negative aspects of the multidimensional phenomenon through Max and "At least J," and Min and "More than J," respectively. The Extreme Value Reduction approach also allows the decision-maker to emphasize positive or negative aspects of the multidimensional phenomenon. However, while the Ordered Weighted Average emphasizes positive or negative aspects by overvaluing extreme values, the Extreme Value Reduction approach emphasizes positive or negative aspects by adjusting the weights attributed to intermediate values, offering an unbiased representation.

Finally, adjusting the weights of the intermediate values also allows for discovering nuances that are difficult to detect by controlling the coefficient of variation of the composite indicator scores.

V. CONCLUSION

The results of this study highlight the critical role of the Extreme Values Reduction approach in delivering an unbiased representation of the Human Development Index. This approach effectively mitigates the influence of extreme lower and higher values, ensuring that the weaknesses and strengths of decision-making units are accurately represented. The study results reveal that popular methods of constructing composite indicators, such as Principal Component Analysis, Entropy, and Benefit of the Doubt, often overlook the challenges posed by extreme values, leading to skewed interpretations of the multidimensional phenomenon.

In contrast, the proposed approach facilitates a more nuanced understanding of these complexities, allowing for a more straightforward data analysis. Additionally, the Ordered Weighted Averaging operators allow decision-makers to emphasize either positive or negative aspects of the data, thereby enhancing the interpretability and relevance of the results. This flexibility is particularly valuable in identifying areas for improvement and investment, ultimately contributing to more informed policy decisions in human development.

The study was limited to providing evidence on the effects of overvaluations of intermediate values and undervaluations of extreme values on the scores and ranking of decision-making units in the Human Development Index. The study does not indicate which of the composite indicators constructed by the Extreme Values Reduction approach decision-makers should base themselves on when formulating public policies. This limitation suggests the development of methodologies for identifying and selecting composite indicators, which is a promising line of future research. Future research also includes the application of the Extreme Values Reduction approach to other multidimensional phenomena, considering spatial dependence in the definition of the weights of the Ordered Weighted Averaging operator and using aggregation by arithmetic mean. Another avenue of research is using the Extreme Values Reduction approach in constructing composite indicators that combine Data Envelopment Analysis and Ordered Weighted Averaging.

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