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RESEARCH ARTICLE

Dealing With Heterogeneous Information in Multi-Criteria Group Decision-Making Problems: A Comprehensive Design Framework

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ABSTRACT This study provides a comprehensive review of the literature on group decision-making with heterogeneous information, aiming to identify the main characteristics in this field, highlight gaps, and suggest new promising approaches and methods to overcome current limitations. The research reveals the main research front directions, key authors, most common application areas, the most frequently used preference formats, and aggregation schemes in existing studies. The research innovation and originality lie in developing five new approaches and methods to overcome the identified gaps and limitations. First, a transparent, comprehensive, and intuitive framework for dealing with heterogeneous information in multi-criteria group decision problems is proposed. Second, a standardization scheme is introduced to define the name of the preference format when it has many names and establish a standard naming structure for all formats. Third, an easy-to-follow framework for categorizing existing and new formats is studied to facilitate an understanding of the structure of each format. Fourth, the new "relational ordered preference" is introduced, a format that increases agility and accuracy in alternative assessments. Fifth, we introduce a pioneering aggregation scheme (consensus-based ordered weighted averaging operator) to maximize the consensus level between individual and collective assessments. An illustrative and a real-world example are also provided. The example of the governance composite indicator demonstrates that relational ordered preference enhances the accuracy of assessments and, consequently, increases the degree of consensus. In turn, the consensus-based approach achieved higher degrees of consensus than extreme value reductions, indicating that preserving more convergent opinions contributes more to consensus than preserving intermediate opinions.

INDEX TERMS Decision making, alternatives and criteria evaluation, consensus levels, format conversion, transformation functions, composite indicators.

I. INTRODUCTION

Group multi-criteria decision-making has as its fundamental characteristic the evaluation of alternatives based on multiple criteria realized by a group of experts [1], [2], [3]. In this context, experts evaluate alternatives using a suitable way of expressing preference that offers them greater psychological comfort. For example, one can view ordering by assigning numerical and linguistic values or by realizing pairwise comparisons of alternatives [3], [4].

On the one hand, the flexibility in choosing the preferred format reduces the cognitive effort required by experts

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when evaluating alternatives, thereby helping to avoid evaluation errors [5]. On the other hand, this flexibility can generate heterogeneous evaluations, preventing their direct aggregation and obtaining a collective assessment [6], [7], which in turn can help reach the desired consensus level [8].

These situations make considering heterogeneous information and achieving consensus in group decision-making a matter of great importance [9]. In particular, two strategies have been frequently employed to deal with heterogeneous information. The first strategy involves applying transformation functions to convert assessments into a homogeneous format and their subsequent aggregation in a collective evaluation [2], [10]. The second strategy consists of applying optimization models to extract the collective assessment that closely matches individual assessments [7], [11]

Many successful applications and recent developments of these strategies solve group decision-making problems with heterogeneous information from different areas [12], [13]. Although quite voluminous, deep, and comprehensive, it is possible to identify some gaps in the literature on group decision-making with heterogeneous information. First, few efforts have been made to systematize and categorize the different preference formats. Second, the current literature lacks information on the most frequently used preferred formats and methods for handling heterogeneous information, as well as their advantages and limitations.

This research narrows these evident gaps by extracting knowledge to build a comprehensive framework for dealing with heterogeneous information in multi-criteria group decision-making problems. Firstly, the proposed framework organizes and standardizes the nomenclatures of the most used preference formats, facilitating their understanding and application by decision-makers. Second, it provides a simple and intuitive categorization of existing and new preference formats. Third, it establishes a new aggregation scheme that ensures that the consensus level established by the decision-maker is achieved.

The covered research results contribute to a greater understanding of the elements that make up each of the preference formats, including their mathematical structure (vector or matrix), means of expression (linguistic or numerical), and estimation process (individual or relational). A deeper understanding of these elements has a significant impact on decision-making, as it enables categorizing preference formats more transparently, comprehensively, and intuitively. The categorization of preference formats based on their mathematical structure, means of expression, and estimation process used in evaluations favors the development of new preference formats, helps decision-makers understand the different ways of evaluating alternatives and criteria, and facilitates categorizing preference formats that have not yet been mapped. In this sense, this research contributes to the development of new approaches for utilizing heterogeneous information in group decision-making. In particular, this study introduces a new preference format that combines precision, speed, and ease of assessment, as well as a mixed aggregation model that utilizes homogenized individual assessments and consensus levels to define the collective assessment.

These contributions are valuable because existing systematizations and categorizations do not adopt a universal structure, consider a limited number of preference formats, and present conflicting categorizations [9], [13]. Furthermore, implementing the proposed aggregation scheme has high application potential, as it prevents solutions with low consensus levels from being used in the decision.

This research offers some important facets of originality. A transparent, comprehensive, and intuitive framework for dealing with heterogeneous information in multi-criteria group decision-making problems. The formulation of a standardized nomenclature of preference formats. A pioneering model for categorizing mapped and unmapped preference formats. A preference format that combines agility and precision in evaluating alternatives. A new method of aggregating individual assessments that maximizes consensus levels between individual and collective assessments through a data-based weighting approach.

This research is organized as follows. Section II presents a focused and comprehensive literature review on group decision-making based on heterogeneous information, providing an overview of the current state of the art and its key features. Section III presents the framework for dealing with heterogeneous information in multi-criteria group decision-making problems. It also introduces the two innovations introduced in the research, namely the related ordered preferences format and the consensus-based ordered weighted averaging operator. Finally, conclusions are presented in Section IV, which cover final considerations, research limitations and identify directions of future investigations.

II. GROUP DECISION-MAKING BASED ON HETEROGENEOUS INFORMATION: A COMPREHENSIVE REVIEW

Figure 1 summarizes the highlights of the search carried out on February 6th, 2024, in the Scopus database for articles with the term "preference format" in the title, abstract, or keyword. The search was configured to retrieve only articles published up to 2023, ensuring temporal comparability of the relevant literature.

Thirty-nine articles were retrieved from this search. Seven articles unrelated to group decision-making based on heterogeneous information were discarded in the screening process. Eighty-eight percent of articles identified as relevant were downloaded. The four articles that were not downloaded were analyzed based on their abstracts [15], [16], [17], [18].

Thus, the analysis of evolution, main knowledge areas, research fronts, and key authors was constructed considering the thirty-two relevant articles. The twenty-eight downloaded and fully studied articles were analyzed in terms of preference formats, methods for dealing with heterogeneous information, and consensus-level measures.

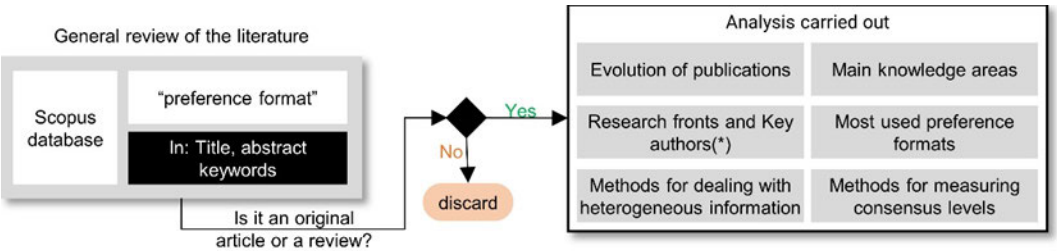


FIGURE 1. Configuration of the literature review on group decision-making using heterogeneous information. Note. Methodology adapted from [14].

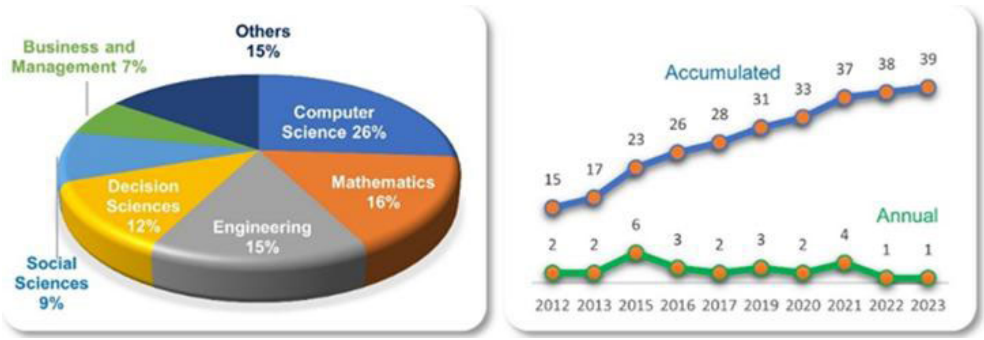


FIGURE 2. Areas of knowledge and evolution of literature related to group decision-making using heterogeneous information.

Firstly, it is important to highlight that the following analyses are based on a limited sample. The filter applied in the literature search prioritizes precision over volume. This is the reason for the high level of relevance of the articles to the literature on group decision-making using heterogeneous information.

A. MAIN KNOWLEDGE AREAS AND EVOLUTION OF PUBLICATIONS

After contextualizing the scope of this literature review, it is interesting to note in Figure 2 that decision science is only the fourth area that publishes the most articles on group decision-making using heterogeneous information. Researchers in computer science, mathematics, and engineering have primarily studied this subject, accounting for 57% of publications.

The graph of the accumulated evolution of the literature offers evidence of a constant growth in publications on group decision-making using heterogeneous information. The annual evolution of the literature suggests a relative decline in interest in the subject. The number of publications on group decision-making using heterogeneous information was most intense between 2015 and 2021. In 2022 and 2023, the number of publications was the lowest in the last ten years.

B. DOMINANT AUTHORS AND DOMINANT RESEARCH FRONTIERS

Figure 3 shows the co-citations analysis of the thirty-two articles on group decision-making using heterogeneous information. This analysis enables us to identify the dominant authors in the literature and their connections, thereby establishing research frontiers in specific specialties [19]. The density of connections and the strength of dominant authors in the cocitation map are also helpful for understanding patterns and trends within each research front [20].

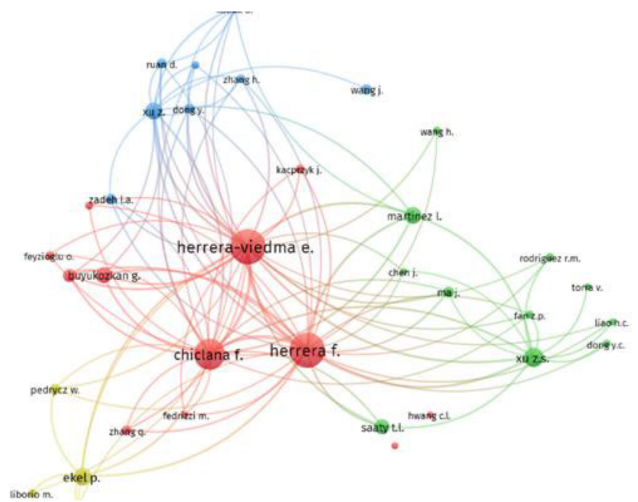


FIGURE 3. Research fronts on group decision-making using heterogeneous information constructed by co-citation analysis in the Vosviewer software [22].

The cocitation map reveals that Herrera, F., Herrera-Viedma, E., and Chiclana are the dominant authors in the literature. Ninety percent of the thirty-two articles reviewed cite Chiclana et al. [22], [23] and Chiclana et al. [24]. These studies occupy a central position in the literature on

TABLE 1. Synthesis of studies by dominant authors in the literature on group decision-making using heterogeneous information.

Reference	Frequency	Synthesis
Chiclana et al. [22]	63%	They establish a general model of preference formats, including orderings, utility values, and fuzzy preference relations. They introduce functions to convert evaluations to fuzzy preference relations format. They show how to aggregate the ordered weighted averaging operator.
Herrera et al. [24]	53%	They construct transformation functions to convert the information in the order of alternatives format and utility functions to the multiplicative preference relation format. The ordered weighted geometric operator aggregates the homogenized information in the multiplicative preference relation format.
Chiclana et al. [23]	38%	They present a transformation function to convert the multiplicative preference relations format to fuzzy preference relations. They study the consistency of this transformation function, which guarantees the informative content of the multiplicative preference relation. They solve a fuzzy decision-making problem that involves information in the formats of order of alternatives, utility value, and multiplicative preference relation, which are converted into fuzzy preference relations. They introduce the ordered weighted geometric operator to aggregate the homogenized information.

group decision-making as they establish the basis for the homogenization and aggregation of information. Table 1 presents a summary of these studies.

Studies by Yager and Zadeh orbit the cocitation map. Their works provide the foundation for developments in group decision-making using heterogeneous information. Herrera et al. [25] present a summary of fuzzy logic and an overview of recent applications of fuzzy sets. In particular, Yager and Zadeh [26] provides the foundations for considering fuzzy logic, which is the basis for several preference formats. Zadeh [27] introduces the ordered weighted averaging aggregation operator, which aggregates information in group decision-making models.

Red front studies specialize in developing methods for aggregating heterogeneous information. In particular, the intuitionistic fuzzy weighted averaging/geometric, intuitionistic fuzzy ordered weighted averaging/geometric, and the intuitionistic fuzzy hybrid averaging/geometric operators [28], [29], [30]. Yellow front studies focus on group decision-making problems in uncertain environments [1], [31], [32]. Finally, blue front studies apply group decision-making to quality function deployment problems [8], [33], [34], [35].

Analyzing the most frequently used keywords helps to gain a deeper understanding of the research fronts. Naturally, the keyword most commonly used in the articles analyzed is "group decision-making." Figure 4 also reveals that the terms "multi-criteria decision-making," "preference formats," and "transformation functions" are used in conjunction. This reinforces the need to convert multi-criteria decision problems involving information in different preference formats into a homogeneous format through transformation functions. Another interesting finding provided by the cloud of keywords used concomitantly is the proximity between the preference formats "fuzzy preference relations," "multiplicative preference relations," "utility values," and "preference orderings." This proximity suggests that these four formats are used simultaneously in the thirty-two articles analyzed.

The most frequent keyword cloud indicates that quality function deployment is the application area with the largest number of articles. Twenty-three percent of the articles reviewed are related to quality function deployment.

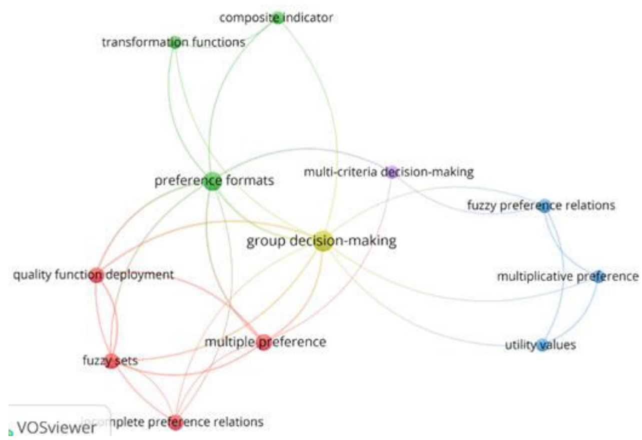


FIGURE 4. Most frequent keywords in publications on group decision-making using heterogeneous information.

A common feature in articles concerning quality function deployment and multi-criteria decision-making problems is the use of the ordered weighted geometric operator. Sixty-seven percent of these articles aggregate homogenized information using ordered weighted geometric and the linguistic operator "at least half" [8], [13], [33], [34], [35], [36], [37], [38]. Unlike other studies, Wang [39] employs optimization modeling to aggregate heterogeneous information without homogenizing the information into a common format. Finally, it was impossible to identify the main characteristics of the two articles related to quality function deployment, which were unavailable for download [16], [17]. However, none of the articles on quality function deployment studied addressed measuring the consensus levels between individual and collective assessments.

The six articles that use ordered weighted averaging to aggregate information homogenized by transformation functions are predominantly aimed at developing new approaches, models, and systems involving fuzzy multi-criteria group decision-making. Among the possibilities for applying these approaches, models, and systems are the student group project assessment [40] and expansion planning for an electrical energy distribution system [6]. A peculiar aspect of articles that use ordered weighted averaging is the diversity of fuzzy linguistic operators used: "max./min." [41], "at least half" [42], "most" [43], and "as many as possible" [44].

TABLE 2. Preference formats most used in the reviewed research.

Preference format	Number	%	Example
Multiplicative preference relations	24	26	Wu and Liao [9]
Order of alternatives	19	20	Zhang et al. [42]
Utility values	18	19	Büyüközkan and Çifçi [35]
Fuzzy preference relations	14	15	Kokshenev et al. [6]
Fuzzy estimates	6	6	Ramalho et al. [41]
Selected subsets	5	5	Zhang et al. [43]
Interval weights	3	3	Wu et al. [11]
Intuitionistic fuzzy preference relations	3	3	Ervural and Kabak [10]
Hesitant fuzzy preference relations	1	1	Zhou and Xu [51]
Ratio bounds	1	1	Wang and Chin [13]

Another four articles employ a simplified approach for aggregating information homogenized by transformation functions [5], [10], [45], [46], [47]. Although they employ a simple aggregation approach, except for Ramalho et al. [45], these articles focus on measuring consensus levels between individual and collective assessments.

Seven articles employ optimization modeling to obtain collective evaluations without transforming the assessments into a single format. Three of these articles measure the consensus levels between individual and collective assessments [7], [11], [48]. Contradictorily, three articles that ignore consensus levels contain the terms' group' or 'multiperson' in their titles [29], [49], [50]. The fourth article that ignores consensus levels is the only one related to quality function deployment [39].

Three articles exhibit characteristics that distinguish them from articles on other research fronts. In short, these articles explore new methodologies for homogenizing information and preference functions. In particular, Zhou and Xu [51] introduce the new preference format asymmetric hesitant fuzzy sigmoid preference relations. Wu and Liao [9] developed an innovative aggregation scheme in which homogeneous information is aggregated before being converted into a single format. Parreiras and Ekel [52] introduced a novel approach based on the membership function for the homogenization of information in nonreciprocal fuzzy preference relations.

C. MOST COMMONLY USED PREFERENCE FORMATS

Firstly, it is worth noting that the analyzed articles simultaneously employ an average of 3.2 preference formats. The four most frequently used formats are found at least once in ninety-three percent of the twenty-eight articles entirely analyzed. Only the articles by Xu [53] and Wu et al. [11] did not use these preference formats. The multiplicative preference relations format and order of alternatives were used in seventy-nine and sixty-six percent of the articles, respectively. Finally, the utility value and fuzzy estimation formats were used in sixty-two percent of the articles. The quantity and frequency with which each of the preference formats is identified in the articles are displayed in Table 2

At this point, it is appropriate to present the basic properties of the preference formats listed in Table 1. Furthermore, further details are provided on the four preference formats most used in the articles studied.

D. BASIC PROPERTIES OF PREFERENCE FORMATS

This section presents the basic properties of the preference formats used in the thoroughly studied articles. The names of the preferred formats used in this section were defined to establish a standard that facilitates understanding and distinction.

Multiplicative preference relations: the decision maker evaluates the alternatives or criteria relationally. In other words, alternatives or criteria are compared with each other, which results in the construction of an $n \times n$ matrix in which the intensity (strength) of the relationship between the alternatives or criteria can be discriminated by the decision maker or not. In the first case, the intensity of the relationship between alternatives or criteria can be defined using Saaty's scale [54] or linguistic terms [3]. In the latter case, the decision maker only defines whether an alternative or criterion is more or less important than the others without indicating the intensity of the preference relationship [34], [36]. It is important to add that it is possible to extract the weights from the multiplicative preference relations matrix using incomplete information, that is, using a matrix in which not all alternatives or criteria were compared to each other [45]. Some studies consider this preference format a multiplicative pairwise comparison matrix [34], [55].

Ordered preferences: the decision maker ranks the alternatives or criteria from most important to least important [22]. This ordering is strict if the alternatives or criteria occupy a single position and not strict when alternatives or criteria can share the same position in the ranking [13]. This format is mentioned in some works as ordering alternatives [2], ordering vectors [52], and ordinal weights [13].

Valued preferences: the decision maker expresses his preferences through a vector of values, which may represent the weights precisely or not. In the latter case, transformation functions must be used to obtain the weights. Techniques such as budget allocation [56] and the Likert scale [13] can help the decision-maker define the vector of values. This preference format is also known as importance degree vector [34], numerically valued [57], precise weights [13], utility function [22], utility value vector [55], utility values [58], or utility vector [59].

Fuzzy preference relations: the decision maker linguistically evaluates how much better or worse an alternative or criterion is [1]. In other words, alternatives or criteria are evaluated relationally with each other, and the estimation of this relationship can be processed by different fuzzy preference relations, such as nonreciprocal and additive reciprocal [36], [52].

Fuzzy estimates preferences: the decision maker evaluates the alternatives linguistically through an importance vector, for example, {Low, Medium, High}, with these linguistic terms converted into fuzzy estimates through membership functions [34]. Some researchers also refer to this preferred format as linguistic terms [58].

Selected preferences subset: the decision-makers select only the subset of alternatives and criteria that are important to them. The alternatives and criteria in the selected subset can be considered equally important [33]. However, the decision-maker can employ other preference formats to evaluate the importance of the alternatives or criteria of the selected subset [38].

Interval preferences: the decision maker performs two evaluations of the alternatives or criteria, establishing an interval of importance corresponding to the uncertainty associated with the importance of each alternative or criterion. The estimation of interval values can be carried out individually (ordering or valuation) or relationally (comparison), expressed numerically or linguistically [13], [29]. Note that the estimation of intervals through vectors of linguistic values is mathematically equivalent to intuitionistic fuzzy sets [9]. The interval preferences nomenclature introduced here is universal, covering the other nomenclatures found in the literature, such as numerical-valued, interval-valued, and linguistic-valued preference relations [57].

Intuitionistic fuzzy preference relations: decision-makers use adherence, non-adherence, and hesitation levels in the relational assessment of preference intensities between alternatives or criteria [9].

Hesitant fuzzy preference relations: decision-makers express their hesitation in the relational evaluation of alternatives or criteria using different linguistic terms, which allows cognitive uncertainty to be considered in the decision process [9], [51].

Ratio bounds: the decision maker defines an acceptance threshold for the difference between the best and worst evaluated alternatives or criteria. When this threshold is not satisfied, deviation variables are inserted [13].

E. MOST FREQUENTLY USED FORMATS AND THEIR CONVERSIONS TO IMPORTANCE COEFFICIENTS OR WEIGHTS

This subsection outlines the four preference formats most commonly used in group decision-making problems involving heterogeneous information. Firstly, the preferred formats are categorized according to their mathematical structures (vector or matrix), means of expression (linguistic or numerical), and estimation process (individual or relational). Second, the advantages and disadvantages of each preferred format are discussed. Thirdly, transformation functions are presented that allow the evaluation of these preference formats to be converted into importance coefficients or weights. The considerations below refer to the set of alternatives $X = \{x_1, x_2, , \ldots, x_n\}$ for a criterion Y.

In the ordered preferences format, the decision maker evaluates alternatives X individually by numbers that represent the position of each alternative or criterion x. This evaluation results in a vector of importance $O = \{o(x_1), o(x_2), \ldots, o(x_k), \ldots, o(x_n)\}$ where $o(x_k)$ is a rearrangement function that defines the position of the alternative or criterion x_k among the integer values $\{1, 2, \ldots, k, \ldots, n\}$. The following transformation function performs the conversion of alternatives ordered by importance into importance coefficients or weights [46], [60]:

$$w_{x_k} = \frac{n - o(x_k) + 1}{\sum_{k=1}^{n} (n - o(x_k) + 1)} = \frac{2(n+1-o(x_k))}{n(n+1)}, \quad k = 1, 2, .., n. \tag{1}$$

where $o(x_k)$ is the order of importance of the k-th sub-indicator.

Note that the evaluation becomes easier with each definition of the most important alternative or criterion, as this definition reduces the set of alternatives to be evaluated. This characteristic makes this format valuable for problems with a high number of alternatives. This format's weaknesses include the lack of distinction in the importance or weights between neighboring alternatives, which means that the interval between alternatives is uniform. Furthermore, all alternatives are considered important or have weight. The decision maker cannot assign zero importance to alternatives.

In the valued preference format, the decision maker evaluates alternatives X individually by numbers, resulting in a vector $U = \{u(x_1), u(x_2), \ldots, u(x_k), \ldots, u(x_n)$, where $u(x_k) \in [0, 1]$, and corresponds to the importance or weight of the alternative or criterion x_k. Note that the sum of the values in vector U can be equal to one or not. In the latter case, obtaining the weights of the vector U must be normalized through the following transformation function:

$$w_{x_k} = \frac{u(x_k)}{\sum_{k=1}^{n} u(xk)}, \quad k, 1, 2, \ldots, n \tag{2}$$

This format allows the decision maker to assign importance or zero weights to one or more alternatives. Furthermore, the decision maker has the flexibility to define the difference between the importance or weights between alternatives. However, this format requires greater cognitive effort from the decision-maker than when dealing with the ordered preferences format. Note that the decision-maker evaluates all alternatives with non-zero importance or weight simultaneously.

In the multiplicative preference relations format, the decision maker evaluates alternatives X relationally through a linguistic scale [54]. Through this scale, the decision maker informs how many times x_k is preferable to x_l. This evaluation results in an $M_{n\times n}$ matrix of positive and reciprocal preference relations $m(x_k, x_l)$. The importance or weights of the alternatives can be extracted based on the normalized eigenvector associated with the maximum eigenvalue of each preference relation in the matrix. This processing is assumed to be a transformation function between formats for didactic and research purposes.

There is an agreement in the literature about the positive aspects of relational evaluation, by which two alternatives are compared. This pairwise comparison allows the decision maker to focus on just two alternatives at a time, favoring the accuracy of evaluations. However, this preferred format also has important disadvantages. Paired assessment is susceptible to intransitivity such as A>B; A<C; C<B, as well as A=2×B; B=2×C; A=3×C, when it should be A=4×C. This inconsistency (intransitivity) of judgments grows with the number of evaluations [5]. Although it is the most frequently used format in the studies investigated, these characteristics limit the applicability of the multiplicative preference relations format to problems with few alternatives, requiring the application of additional methods to reduce judgment inconsistency.

In the fuzzy preference relations format, the decision maker evaluates the alternatives. Although some works show the possibility of generating fuzzy preference relations automatically using linguistic or fuzzy estimates of alternatives [61], [62], the way to evaluate alternatives in this preference format is almost always done through pairwise comparisons. This similarity with the format of multiplicative preference relations implies the same advantages (accuracy and reduction of evaluation errors) and disadvantages (cognitive stress and inconsistency of evaluations) of this type of evaluation.

F. METHODS FOR AGGREGATING HETEROGENEOUS INFORMATION

Converting evaluations in different formats to importance coefficients or weights still does not solve the decision-making problem with heterogeneous information. After homogenizing the assessments, it is necessary to aggregate the information to obtain a collective assessment.

Two strategies are obtained to realize collective assessments from heterogeneous information. The first strategy involves homogenizing assessments in different formats and aggregating them into a single, collective assessment [9], [43]. This strategy is employed in 68% of the articles studied. It includes the average of homogenized individual evaluations [45,47), ordered weighted averaging [6], [41] and ordered weighted geometric geometry [33], [38], and, within the others category, fuzzy nonstrict preference relation intersection fuzzy sets [52], hesitant fuzzy weighted aggregation operators [43], and aggregation by group according to preference format and then global aggregation [9].

The second strategy does not require homogenizing assessments in different formats. Heterogeneous information is incorporated into an optimization model that minimizes the discrepancies between individual and collective assessments [7], [11], [48], [49]. Figure 5 presents the frequencies of each approach used to aggregate individual assessments.

Forty-five percent of the articles adopt an ordered aggregation operator to aggregate the homogenized information in the collective evaluation. In this sense, it is relevant to present the main properties of these operators, including the linguistic operator "at least half," used in seventy percent of articles that employ ordered weighted averaging or geometric.

The operationalization of ordered weighted averaging [27] and ordered weighted geometric [23] is carried out in five stages. First, normalize information in the range [0,1]. Second, order the information from largest to smallest. Third, set the weights for the aggregation operators. Fourth, consider information ordered by importance. Fifth, aggregate the values weighted by the average.

In short, the fundamental difference between the two aggregation operators (hereinafter OWs) lies in defining the type of mean (arithmetic or geometric) used in step five. Thus, the operationalization of operators OWs of dimension n, is a mapping $OWs{:}R^n \rightarrow R$ with a weighting vector $w = (w_1, w_2, \ldots, w_n)^T$ where $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_i = 1$. Thus, the ordered weighted averaging is obtained by:

$$OWA_w(a_1, a_2, \ldots, a_n) = \sum\nolimits_{j=1}^{n} w_j b_j \tag{3}$$

and, the ordered weighted geometric obtained by:

$$OWG_w(a_1, a_2, \ldots, a_n) = \left(\prod_{j=1}^{n} b_j^{w_j}\right)^{1/\sum_{j=1}^{n} w_j} \tag{4}$$

where b_j is the j-th highest performing element of $a_1, a_2, \ldots, a_n$.

The weights in (4) and (5) of the OWs are typically defined using linguistic quantifiers. In particular, a linguistic quantifier Q reflects a concept of fuzzy majority in aggregating operator elements. For example, a nondecreasing proportional fuzzy linguistic quantifier Q, is given by:

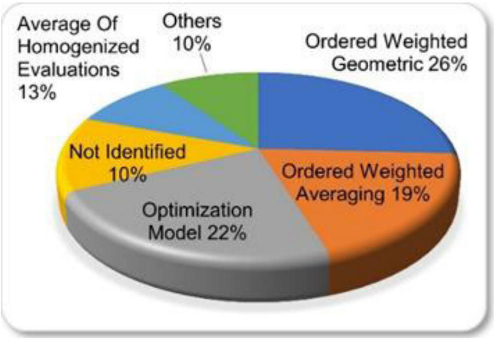
$$w_i = Q\left(\frac{i}{n}\right) - Q\left(\frac{i-1}{n}\right), i = 1, 2, \ldots, n \tag{5}$$

where Q is defined as:

$$Q(r) = \begin{cases} 0, & \text{if } r < a \\ \dfrac{r-a}{b-a}, & \text{if } a \le r \le b \\ 1, & \text{if } r > b \end{cases} \tag{6}$$

where $a, b, r \in [0, 1]$.

The membership function (6) can be operationalized by the linguistic quantifiers "most," "as many as possible," and "at least half" [63], the latter being given by [0,0.5] and which occupies a prominent position among the articles studied.



Fuzzy linguistic operator used in Ordered Weighted Averaging (OWA) and in Ordered Weighted Geometric (OWG) operators

Linguistic operator	Frequency
At least half	11
Max. / Min.	1
Most	1
As many as possible	1

FIGURE 5. **The aggregation approach used in the articles analyzed and the linguistic quantifier used in aggregation approaches by ordered weighted averaging or geometric.**

G. METHODS FOR MEASURING CONSENSUS LEVEL

Consensus levels reflect the degree of agreement, measured as the average deviation or distance between individual preferences and collective preferences [1], [64], [65]. In the specialized literature, there are two main approaches to computing and measuring the consensus level between the individual and the collective assessments [66]. The first measures consensus by computing the distances between individuals' preferences and the collective assessment, and the second measures consensus by calculating the distances between individuals. Both approaches are briefly revised below:

Consensus measures based on distances to the collective assessment: A collective assessment represents the global opinion of the group, and it is usually computed by fusing all individual homogenized preferences of experts by an aggregation operator. Consensus measurements are then obtained by computing the distances between individual and collective preferences. In Spillman [67], an early consensus measure based on fuzzy set theory was introduced, measuring the consensus level for each expert as the distance between their fuzzy preferences and an "ideal" agreement with a maximum consensus degree. In such a way, the consensus notion was more flexible and realistic in practice than the idea of consensus as unanimous agreement, as considered in other earlier works [68]. In Herrera and Herrera-Viedma [69] and inspired by Kacprzyk's soft consensus approach [70], two different consensus measures were assessed linguistically employing linguistic terms.

Consensus measures based on distances between experts: For each pair of experts in the group, the degrees of similarity between their opinions are computed based on distance metrics. Such similarity values between each pair of experts are then aggregated to obtain a consensus measurement. Kacprzyk and Fedrizzi [70] conducted extensive research into human-consistent consensus measures to better understand the concept of consensus as it is perceived by human beings in practice, rather than relying solely on unanimity as a definition of consensus. As a result, they proposed the notion of soft consensus based on the concept of fuzzy majority [70]. Based on this notion, one of the first consensus measures for fuzzy preference relations was formalized by Kacprzyk and Fedrizzi [70]. The consensus degree was hierarchically computed at multiple levels. The concept of fuzzy majority was defined at the consensus level by applying a fuzzy logic-based calculus of linguistically quantified propositions [70].

Only eight articles measure the consensus levels between collective and individual assessments. This is an unexpected finding, considering that the articles are inserted within the group decision-making literature. Furthermore, the optimization modeling developed in 22% of the articles is based on collective evaluation [29], [50]. The lack of measurement of consensus levels in these works is critical, specifically in these works, as it does not allow the consistency of the collective assessment obtained to be assessed. Only three articles use optimization modeling to measure consensus levels. The methods used in these articles include cluster analysis [7], deviation analysis between variables [11], and a cosine similarity measure [48]. However, most articles that measure consensus between individual assessments use the score similarity method [10] or rankings [5], [43], [46], [47].

III. FRAMEWORK FOR DEALING WITH HETEROGENEOUS INFORMATION IN MULTI-CRITERIA GROUP DECISION-MAKING PROBLEMS

Firstly, it is appropriate to analyze the preference format categories that exist in the literature. The categorization of preference formats proposed by Wang and Chin [13] is guided by the definition of weights. In this sense, the authors categorize preference formats into precise weights, intervals, ordinals, ratio bounds, pairwise comparisons, and fuzzy preference relations. Wu and Liao [9] are based on how decision-makers conduct evaluations and categorize the preferred formats into ordering, numerical, interval, and linguistic. Note that these categorizations do not cover all preference formats and are contradictory. Valued preferences are not necessarily precise weights, nor do they fall into the interval, ordinal, ratio bounds, pairwise comparison, and fuzzy preference relations categories. Evaluations conducted by precise weights and ordering formats are carried out using numerical values. By this logic, these formats should not be categorized differently. Furthermore, classifying new preference formats into the categories indicated in the studies by Wang and Chin [13] and Wu and Liao [9] is not challenging as the logic behind these categorizations is unclear.

In this scenario, a universal, comprehensive, and easy-to-follow categorization requires understanding the preferred formats in terms of their mathematical structure (vector or matrix), means of expression (linguistic or numerical), and estimation process (individual or relational). The categorization framework, based on these elements, allows for the creation of categories to cover both mapped and unmapped preference formats.

In particular, the mathematical structure, means of expression, and estimation process of alternatives indicate the need to establish the following categories: individual numeric vector, relational numeric vector, individual linguistic vector, relational linguistic vector, individual numeric matrix, relational numeric matrix, individual linguistic matrix, relational linguistic matrix. For example, following this logic to categorize the most used preference formats in the literature is possible, as shown in Figure 6.

Based on the proposed framework, it is possible to categorize the four most used preference formats in the literature as follows:

- Multiplicative preference relation: relational numeric matrix (RNM);
- Ordered preferences: individual numeric vector (INV);
- Valued preferences: individual numeric vector (INV);
- Fuzzy preference relation: relational linguistic matrix (RLM).

A key advantage of the categorization framework is its ability to develop and categorize new preference formats. For example, it is possible to develop a preference format that combines the ease and agility of the ordered preferences format with the prediction of the multiplicative relations preference format in the so-called related ordered preferences format.

A. RELATIONAL ORDERED PREFERENCE FORMAT

The evaluation of alternatives in this new preference format is carried out in four stages. In the first stage, the decision maker orders the alternatives, generating an individual numerical vector:

$$O = \{o(x_3), o(x_2), o(x_1), o(x_4)\} \tag{7}$$

In the second stage, the decision maker uses the Saaty [54] scale to determine how much the first alternative in the vector is more important than the second alternative in the vector, repeating the evaluation, comparing the second with the third, and so on. This comparison is simplified as the assessment is based on alternatives ordered by importance, eliminating all negative values on the scale (e.g., "less important than"). The result of this operation is a second numeric vector:

$$O_k/O_l = \left\{\frac{o(x_3)}{o(x_2)}, \frac{o(x_2)}{o(x_1)}, \frac{o(x_1)}{o(x_4)}\right\} \tag{8}$$

Note that the first and last vector alternatives are compared only once, so the vector obtained in the first step has one more value than the vector obtained in the second step. The missing value of the second vector is obtained in the third stage.

In the third stage, the decision maker evaluates the intensity of importance of the least important alternative from the vector obtained in the first stage. In short, x_4/c represents the importance of the least important alternative in the problem, so c is the decision-making problem. This intensity of importance is quantified by the modified Saaty's [54] scale, in which the value one is replaced by zero. In this sense, the decision maker can assign a zero weight to one or more alternatives. In this case, the second alternative of the vector obtained in the first stage will have zero weight if its relationship with the last variable is equal to one (equal importance).

The fourth stage consists of constructing the vector of multiplicative relations, joining the results of stages two and three:

$$Ro = \left\{\frac{o(x_3)}{o(x_2)}, \frac{o(x_2)}{o(x_1)}, \frac{o(x_1)}{o(x_4)}, \frac{o(x_4)}{c}\right\} \tag{9}$$

Finally, the weights of the alternatives can be obtained by the appropriate transformation function. In this case, equation (1), which is used to transform the ordered preferences format, can also be applied to the relational ordered preference format. The results of applying this transformation function to the vector of ordered alternatives $\{x_3, x_2, x_1, x_4\}$ and related by the Saaty [54] scale with values $\{5, 7, 3, 3\}$ are presented in Table 3.

TABLE 3. Weights converted from evaluations in ordered preferences and relational ordered preference formats.

Alternative	Ordered preferences	Relational ordered preference
x_1	0.200	0.150
x_2,	0.300	0.325
x_3	0.400	0.450
x_4	0.100	0.075

This example demonstrates the ability of the relational ordered preference format to increase precision in defining the weights of alternatives and reflect the preferences of decision-makers. In short, the related ordered preferences format offers four advantages over the ordered preferences and multiplicative preference relation formats. First, agility and simplicity in evaluation do not require paired comparison between all alternatives. Second, it is not susceptible to inconsistency in assessments; the intensity relationship is always positive, and it is carried out only once. Third, forecast in differentiating weights between alternatives, the distance between each pair of alternatives varies according to the interest of the decision maker. Fourth, flexibility to assign identical or zero weights so that one or more alternatives can have identical weights, including zero weights.

At this point, it is necessary to consider the possibility of a group of experts evaluating the alternatives using the relational ordered preference format or even multiple

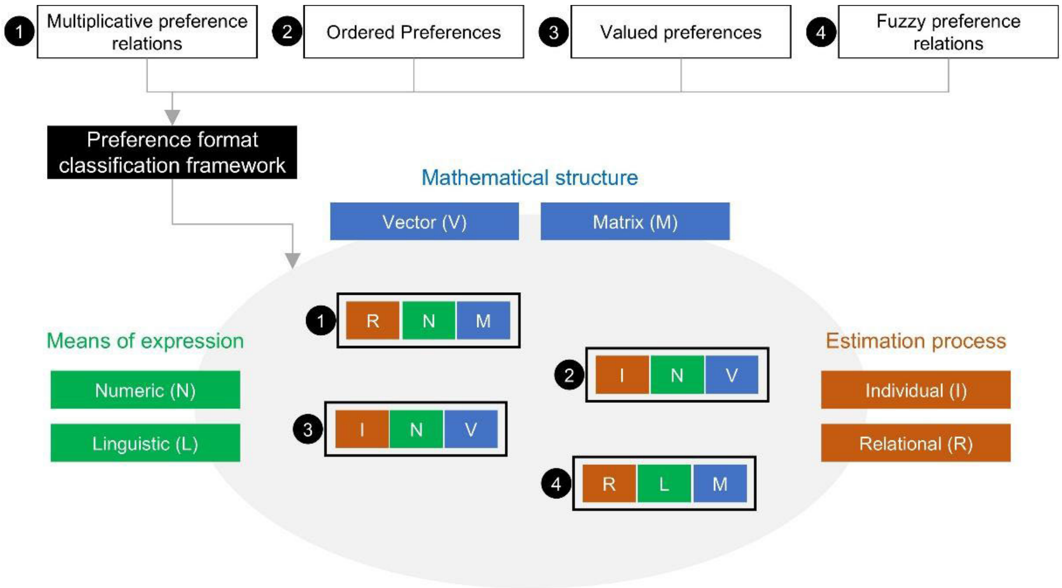


FIGURE 6. Preference format categorization framework.

preference formats. In this situation, ensuring that the assessments aggregated in the collective assessment present consensus levels that signal convergence around the final assessment is necessary. For this purpose, a consensus-based ordered weighted averaging operator is proposed.

B. CONSENSUS-BASED ORDERED WEIGHTED AVERAGING OPERATOR

Section II demonstrates that individual evaluations can be aggregated into collective evaluations in several ways, such as ordered weighted averaging/geometric averaging (e.g., [6], [38]) and optimization models (e.g., [7]), which are the most frequently employed methods. On the one hand, decision-making models minimize the difference between individual and collective evaluation, but the importance coefficients or individual weights are unknown. Furthermore, some models distort the influence of experts on collective evaluation by aggregating individual evaluations in equal formats before optimization (e.g., [49]). On the other hand, ordered weighted averaging/geometric is operationalized after obtaining individual assessments that have been converted to the same format. This pre-processing enables the comparison of importance coefficients or weights of alternatives or criteria among experts. However, obtaining the collective assessment by ordered weighted averaging or geometric averaging completely ignores its consensus level with the individual assessments.

Consensus-based ordered weighted averaging is designed to overcome these limitations and capitalize on the unique aspects of the two previous approaches. In short, the method utilizes transposed individual assessments, ordered by their dissimilarity with the collective assessment, and weights are defined to minimize absolute deviations between individual and collective assessments.

The consensus-based ordered weighted averaging can be carried out by similarity or agreement measures. Among the best-known measures are the Pearson [71] and Spearman [72] correlation coefficients and the kappa [73] and pi [74] concordance coefficients. Other very popular measures in the decision-making literature are much simpler to implement. They are summarized as the mean absolute deviation of individual and collective assessments [10] or the mean absolute deviation of the rankings of individual and collective assessments [5]. For illustrative purposes, the consensus-based ordered weighted averaging applied in this research adopts the absolute mean deviation between individual assessments and collective assessments, being operationalized in three steps:

Step 1: Convert evaluations in different formats into weights by the appropriate transformation function, as shown in Section III-B. Build the matrix of evaluations of alternatives (row) carried out by each specialist (column), as shown in Table 4. Transpose the matrix of homogenized assessment as shown in Table 5.

TABLE 4. Matrix of homogenized assessments.

	S_1	S_2	S_3	S_4
x_1	0.15	0.15	0.24	0.12
x_2,	0.33	0.28	0.26	0.24
x_3	0.45	0.44	0.38	0.29
x_4	0.08	0.13	0.12	0.35

Step 2: Calculate the absolute deviation between individual and collective assessments (Table 6). Order the assessments based on the absolute deviations classified from highest to lowest, as shown in Table 7.

TABLE 5. Transposed matrix of homogenized assessments.

	x_1	x_2,	x_3	x_4
S_1	0.15	0.33	0.45	0.08
S_2	0.15	0.28	0.44	0.13
S_3	0.24	0.26	0.38	0.12
S_4	0.12	0.24	0.29	0.35

TABLE 6. Absolute deviations between individual and collective assessments.

	x_1	x_2,	x_3	x_4
S_1	0.014	0.050	0.058	0.093
S_2	0.013	0.003	0.051	0.042
S_3	0.074	0.013	0.011	0.049
S_4	0.047	0.040	0.098	0.185
Sum				0.84

TABLE 7. Ordered difference between individual and collective assessments.

	x_1	x_2,	x_3	x_4
a_1	0.24	0.33	0.29	0.35
a_2	0.12	0.24	0.45	0.08
a_3	0.15	0.26	0.44	0.12
a_4	0.15	0.28	0.38	0.13

Note: from largest to smallest deviation.

Note that from this step onwards, it is impossible to associate the assessment with the specialist. This property offers transparency and impartiality in defining assessments, which will be weighted according to the lowest and highest weights of the consensus-based ordered weighted averaging.

Step 3: Define the consensus-based ordered weighted averaging weights that minimize the absolute deviations between the collective and individual assessments.

To this end, the Microsoft Excel evolutionary optimization algorithm (Figure 7) can be used. The genetic algorithm finds the optimal solutions considering random mutation and natural selection. The algorithm preserves the best solutions from each generation, allowing the solution to improve over time, and the best solutions represent optimal or near-optimal points [75].

In short, the optimization results are presented in Tables 8 and 9.

TABLE 8. Consensus-based ordered weighted averaging weights.

	w_j	x_1	x_2,	x_3	x_4
a_1	0.05	0.01	0.02	0.01	0.02
a_2	0.32	0.04	0.07	0.14	0.02
a_3	0.32	0.05	0.08	0.14	0.04
a_4	0.32	0.05	0.09	0.12	0.04
Sum	1.00	0.14	0.26	0.42	0.12

TABLE 9. Absolute deviations of individual and collective assessments.

	x_1	x_2	x_3	x_4
a_1	0.003	0.048	0.007	0.051
a_2	0.001	0.001	0.000	0.000
a_3	0.085	0.016	0.062	0.007
a_4	0.036	0.042	0.149	0.227
Sum				0.735

Note that the sum of the deviations between the individual and collective assessments in Table 9 is 14.4% lower than the corresponding value in Table 6. This result demonstrates the effectiveness of consensus-based ordered weighted averaging in maximizing the consensus level while providing the decision maker with individual assessments in a comparable format. In addition to these advantages, consensus-based ordered weighted averaging eliminates subjectivity in defining the weights of the ordered weighted averaging/geometric operator. Its consensus maximization prevents experts from being convinced to re-evaluate alternatives. The method disregards the most divergent assessments, regardless of the expert, unlike other methods that exclude a particular expert. Finally, consensus-based ordered weighted averaging does not reveal which opinion was excluded or maintained, thereby offering impartiality in the exclusion and inclusion of opinions.

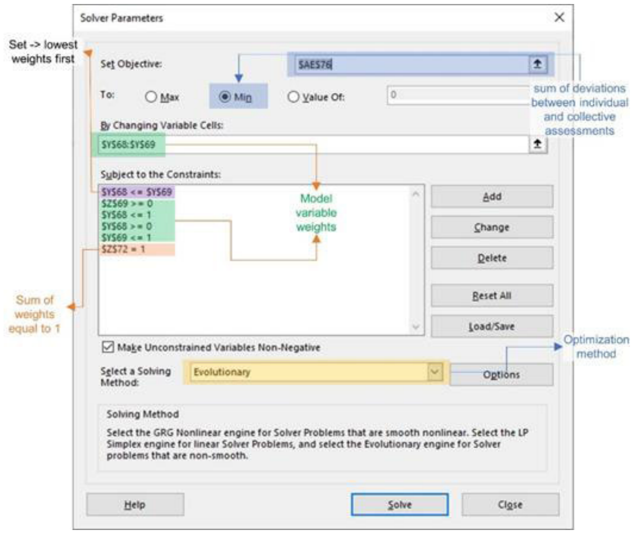


FIGURE 7. Microsoft Excel optimization algorithm for performing the consensus-based ordered weighted averaging operator.

C. ILLUSTRATIVE APPLICATION EXAMPLE

Governance is a multidimensional construct composed of several indicators that signal the quality of the institutions and regulatory environment of countries [76]. The World Bank's Worldwide Governance Indicators provide relevant information on this multidimensional phenomenon, assisting researchers and public managers in conducting analyses and defining public policies [77], [78].

The Worldwide Governance Indicators structure is organized into six criteria that represent key elements of governance: control of corruption (x_1); government effectiveness (x_2); rule of law (x_3); quality of the business environment (x_4); regulatory quality (x_5); and voice and accountability (x_6).

Not discussing its usefulness and importance, understanding governance based on the simultaneous interpretation of its various criteria requires great cognitive effort [5], which can be avoided by constructing a composite indicator [56].

An example of a governance composite indicator is presented in [5]. Although it proves that the composite indicator simplifies the interpretation of complex realities, the study highlights the problem of weighting its criteria [76]. In this case, evaluating the weights of the criteria is challenging because experts are not familiar with the realities of all countries, which impacts the quality of the assessments and consensus levels [46]. This problem makes the governance composite indicator an appropriate example to verify the precision gain in assessments carried out using the relational ordered preference format and the increase in consensus degree achieved by applying the consensus-based ordered weighted averaging operator.

To this end, 17 experts on governance with publications in journals were interviewed. First, they ranked the six governance criteria in order of importance. These evaluations were transformed into weights to facilitate comparisons and are presented in Table 10.

TABLE 10. Criteria weights by the ordered preferences format.

Experts	x_1	x_2,	x_3	x_4	x_5	x_6
1	0.19	0.10	0.29	0.05	0.14	0.24
2	0.14	0.19	0.24	0.05	0.10	0.29
3	0.19	0.10	0.29	0.05	0.14	0.24
4	0.14	0.24	0.29	0.19	0.10	0.05
5	0.14	0.05	0.29	0.10	0.19	0.24
6	0.24	0.10	0.29	0.14	0.05	0.19
7	0.05	0.19	0.29	0.14	0.24	0.10
8	0.29	0.14	0.24	0.19	0.05	0.10
9	0.05	0.19	0.29	0.14	0.24	0.10
10	0.19	0.10	0.29	0.05	0.14	0.24
11	0.14	0.29	0.10	0.05	0.24	0.19
12	0.24	0.19	0.05	0.14	0.10	0.29
13	0.19	0.24	0.29	0.10	0.14	0.05
14	0.19	0.14	0.24	0.10	0.29	0.05
15	0.10	0.05	0.29	0.24	0.14	0.19
16	0.14	0.10	0.05	0.29	0.19	0.24
17	0.05	0.10	0.19	0.14	0.24	0.29

Second, the experts compared and assessed the intensity of the difference between pairs of ranked criteria, always in pairs. For example, comparing the criteria ordered in first and second place, the criteria ordered in second and third place, and so on. The results of this relational evaluation are presented in Table 11.

TABLE 11. Criteria weights by the relational ordered preferences formats.

Experts	x_1	x_2,	x_3	x_4	x_5	x_6
1	0.15	0.15	0.31	0.08	0.15	0.15
2	0.14	0.18	0.27	0.05	0.09	0.27
3	0.15	0.15	0.31	0.08	0.15	0.15
4	0.17	0.17	0.17	0.17	0.17	0.17
5	0.20	0.07	0.20	0.13	0.20	0.20
6	0.24	0.10	0.29	0.14	0.05	0.19
7	0.04	0.21	0.25	0.17	0.25	0.08
8	0.36	0.14	0.21	0.14	0.07	0.07
9	0.04	0.21	0.25	0.17	0.25	0.08
10	0.13	0.25	0.13	0.13	0.25	0.13
11	0.12	0.32	0.08	0.01	0.23	0.23
12	0.22	0.20	0.07	0.16	0.13	0.22
13	0.17	0.23	0.29	0.11	0.17	0.05
14	0.17	0.17	0.23	0.02	0.38	0.02
15	0.06	0.02	0.34	0.26	0.14	0.18
16	0.13	0.08	0.01	0.34	0.20	0.25
17	0.15	0.15	0.31	0.08	0.15	0.15

The differences between the weights obtained in the two evaluation stages are presented in Table 12.

TABLE 12. Difference between weights assigned by the ordered and the relational ordered preferences formats.

Experts	x_1	x_2,	x_3	x_4	x_5	x_6
1	0.00	-0.02	0.01	-0.00	-0.02	0.03
2	0.04	-0.06	-0.02	-0.03	-0.01	0.08
3	0.01	0.01	-0.03	0.00	0.00	0.01
4	0.04	-0.06	-0.02	-0.03	-0.01	0.08
5	-0.02	0.07	0.12	0.02	-0.07	-0.12
6	-0.06	-0.02	0.09	-0.04	-0.01	0.04
7	0.00	0.00	0.00	0.00	0.00	0.00
8	0.01	-0.02	0.04	-0.02	-0.01	0.01
9	-0.07	0.00	0.02	0.05	-0.02	0.02
10	0.01	-0.02	0.04	-0.02	-0.01	0.01
11	0.07	-0.15	0.16	-0.08	-0.11	0.11
12	0.03	-0.04	0.02	0.03	0.00	-0.04
13	0.02	-0.01	-0.03	-0.02	-0.03	0.07
14	0.02	0.01	0.00	-0.01	-0.02	0.00
15	0.02	-0.03	0.00	0.07	-0.10	0.03
16	0.03	0.03	-0.05	-0.02	0.00	0.01
17	0.01	0.02	0.04	-0.05	-0.01	-0.01
Mean	0.00	-0.02	0.01	0.00	-0.02	0.03

Note that only nine weights are the same in both formats. In other words, 91% of the ratings were adjusted, accurately capturing the difference between the criterion weights. Particularly the weights of criteria x_2, and x_5, were adjusted 0.02 downward on average, while criterion x_6, was adjusted 0.03 upward on average.

Third, the consensus degree among experts was calculated by the consensus-based ordered weighted averaging operator for the weights in Tables 10 and 11. Five simulations demonstrated the algorithm's evolution based on the weights (number) change of the model's opinions. Then, the results were compared to the consensus degree obtained by extreme value reduction [79], [80], [81] to verify the effectiveness of the proposed method.

Extreme value reduction is based on the ordered weighted averaging operator [80], like the consensus-based method. While the extreme value reduction focuses on extremely positive or negative opinions [79], [81], the consensus-based method focuses on more divergent opinions (i.e., distant) from the collective opinion. This difference results in higher consensus levels when applying the consensus-based method than the extreme value reduction method.

The results in Table 13 show that the consensus degree of consensus-based surpasses that of extreme value reduction in the first iteration. In this case, consensus-based considers the 16 most convergent opinions and disregards the most divergent opinion. In turn, extreme value reduction disregards the most extreme positive or negative opinion without considering its distance from the collective opinion.

The results also show that the consensus degree is lower for the weights in the ordered preferences format assessed. This finding highlights the limitation of this format in providing an accurate portrayal of the criteria weights, especially regarding the impossibility of assigning equal weights and unidentical differences between the criteria weights.

TABLE 13. Consensus degree by the different approaches.

Opinions	Weights	Consensus-based		Extreme value reductions	
		ROP	OP	ROP	OP
17	0.059	0.70	0.68	0.70	0.68
16	0.063	0.73	0.71	0.71	0.69
15	0.067	0.75	0.73	0.75	0.72
14	0.071	0.78	0.76	0.76	0.72
13	0.077	0.81	0.77	0.79	0.75

Note: ROP stands for relational ordered preference, while OP stands for ordered preference.

Fourth, the governance composite indicator was constructed based on the weights obtained in the relational ordered format from the thirteen most convergent evaluations. This number of assessments was chosen to get a consensus degree of 0.80, as shown in Table 13.

Finally, the sensitivity of the scores of the governance composite indicator G-CI were estimated based on their absolute difference to the alternative composite indicators: A-1 (consensus-based AND ordered preferences), A-2 (extreme value reduction AND relational ordered preferences), and A-3 (extreme value reduction AND ordered preferences).

Table 14 shows the governance composite indicator scores, alternative composite indicators, and the sensitivity of the scores for the twenty largest economies in the world.

TABLE 14. Scores and sensitivity of the worldwide governance index obtained by different approaches.

Country	G-CI	A-1	A-3	A-4	Sensitivity
Australia	0.91	0.92	0.92	0.92	0.00
Brazil	0.34	0.35	0.35	0.36	0.05
Canada	0.92	0.92	0.92	0.92	0.01
China	0.36	0.34	0.34	0.33	0.06
France	0.78	0.78	0.78	0.78	0.01
Germany	0.89	0.90	0.90	0.90	0.01
India	0.38	0.39	0.39	0.39	0.02
Indonesia	0.40	0.40	0.40	0.40	0.02
Iran	0.01	0.01	0.01	0.01	0.01
Italy	0.62	0.64	0.64	0.65	0.05
Japan	0.88	0.88	0.88	0.88	0.00
Korea	0.76	0.76	0.76	0.76	0.00
Mexico	0.25	0.26	0.26	0.27	0.04
Netherlands	0.96	0.96	0.96	0.96	0.00
Russian Federation	0.20	0.20	0.20	0.21	0.00
Spain	0.71	0.71	0.71	0.72	0.02
Switzerland	0.99	0.99	0.99	0.99	0.00
Turkey	0.25	0.25	0.25	0.24	0.00
United Kingdom	0.84	0.84	0.84	0.84	0.01
United States	0.74	0.74	0.74	0.73	0.01

The results show that changes in the criteria assessment format and consensus-building significantly affect the governance composite indicator scores of some countries. The scores of China and Mexico vary by 0.06 and 0.05, equivalent to a 15% variation in the composite indicator scores of these countries. As another example, Brazil's governance composite indicator score varies by an average of 0.05. This variation represents 13% of Brazil's governance composite indicator score, which is 0.34.

IV. CONCLUSION

This research demonstrates that multiplicative preference relations, ordered preferences, valued preferences, and fuzzy preference relations are the preference formats most commonly employed in studies of group decision-making that utilize heterogeneous information. These four preference formats are used in 93% of studies that employ multiple preference formats for group decision-making. In turn, the most frequently used aggregation methods are the weighted average and geometric mean operators, which are used in forty-five percent of the studies analyzed.

Another interesting finding of this research is the low frequency of measuring consensus levels of collective assessments in articles on group decision-making with heterogeneous information. Only twenty-one percent of the articles investigated measure the level of consensus of experts with the group's opinion.

Among other discoveries and novelties of the study, it is possible to highlight five of them. First, a transparent, comprehensive, and intuitive framework for dealing with heterogeneous information in multi-criteria group decision-making problems. Second, the formulation of a standardized nomenclature of preference formats. Third, a pioneering model for categorizing mapped and unmapped preference formats. Fourth, a new preference format that combines simplicity, agility, and precision in evaluating criteria and alternatives. Fifth, a novel method that maximizes the consensus levels between individual and collective assessments. Sixth, a data-driven approach for defining the ordered weighted averaging operator weights.

Furthermore, applying the new preference format of the consensus-based ordered weighted average operator in the governance composite indicator case demonstrates the effectiveness and usefulness of the proposed framework. In particular, the consensus-based ordered weighted average operator emphasizes the most convergent assessments. The method has proven more effective than the extreme value reduction ordered weighted average, which emphasizes assessments with intermediate values. However, the consensus-based method overlooks the fact that not all divergent assessments have a low degree of consensus, which affects the overall consensus degree.

In addition to exploring how to overcome this limitation, future lines of research include consideration of expert hesitation [31], [51], [82], adaptations of the consensus-based ordered weighted averaging operator to other consensus metrics [83], applications to composite indicators [84], [85], as well as to other group decision-making problems with multiple criteria and alternatives [86], such as resource allocation [87], supplier evaluation [88], and prioritization of renewable energy systems [89].

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