An Extended Version of the Fuzzy Multicriteria Group Decision-Making Method in Evaluation Processes

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Abstract. Evaluation processes are a key element used in quality inspection, marketing and other fields in industrial companies. In these processes, it is very common that a group of evaluators assess a set of evaluated elements, according to a set of criteria, which may have different nature and usually present uncertainty. In this context, the fuzzy multicriteria group decision-making (FMCGDM) method has been successfully applied to different evaluation problems. This method provides a closeness coefficient of each evaluated element in order to generate a final raking. However, its applications to complex evaluation processes that requires the understandability of the closeness coefficient drive us to propose the use of the linguistic 2-tuple representation model to extend the FMCGDM method, in order to provide linguistic closeness coefficients, which are easy to understand. Moreover, we apply the extended version of the FMCGDM method in an evaluation process of fabric hand.

1 Introduction

Evaluation is a complex cognitive process that involves different mechanisms in which it is necessary to define the elements to evaluate, fix the evaluation framework, gather the information and obtain an evaluation assessment. The aim of any evaluation process is to obtain information about the worth of an element (product, service, material, etc.) and a complete description of different aspects, indicators, criteria in order to improve or compare, and in this way identify which are the best ones [9,13].

The fuzzy multicriteria group decision-making (FMCGDM) method [15] has been successfully applied in different complex evaluation processes such as new product development [6,7], sustainable energy [12] and power distribution system planning [16]. As result, FMCGDM method has been developed in a decision support system called *Decider* [8].

The FMCGDM method is an interesting option in evaluation processes because it has many desirable features in complex evaluation processes, since this method can

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structure the criteria within a hierarchy and these criteria can be model considering their nature, objective or subjective.

In order to deal with both objective and subjective assessments, the FMCGDM method transforms the assessments into linguistic terms represented by triangular fuzzy numbers. These fuzzy numbers are computed to obtain a closeness coefficient for each evaluated element, considering the distance measure between the fuzzy collective assessment of each evaluated element and both a general ideal solution and a general negative ideal solution. These closeness coefficients are expressed in the unit interval and are used to generate the ranking for the set of evaluated elements. These values cannot be interpreted beyond the order set of evaluated elements. However, there are complex evaluation processes that require the understandability of the closeness coefficient further than order.

In this context, the Computing with Words (CWW) methodology considers that inputs and output results should be expressed in a linguistic domain in order to be close to human natural language, providing interpretable and understandable results [10,11,14]. Fig. 1 provides a schematic view of the CWW methodology.

Fig. 1. Computing with words methodology

The aim of this contribution is to extend the FMCGDM method in order to cope the understanding of closeness coefficients that will allow an interpretable ranking, beyond the order. To do so, we propose to include a final stage in the FMCGDM method where the linguistic 2-tuple representation model [3] is used in order to provide linguistic closeness coefficients. Finally, we present an application in a hand-based textile material evaluation to show the usefulness and effectiveness of the proposed extended version of the FMCGDM method.

This contribution is structured as follows: Section 2 reviews the FMCGDM method in detail. Section 3 presents the proposed extension that includes a final stage to provide linguistic closeness coefficients. Section 4 shows an application in an evaluation process where the proposed extended version of the FMCGDM method is applied. Finally, Section 5 points out concluding remarks.

2 FMCGDM Method

This FMCGDM method is composed of three stages:

Stage one: Determination of weights of evaluators and criteria

- Step 1: Identify evaluators, evaluated elements, and criteria. Suppose a group of n evaluators $E = \{e_1, e_2, \ldots, e_n\}$. Let $S = \{S_1, S_2, \ldots, S_m\}$, be a set of elements to evaluate. Finally, suppose a two-level criteria hierarchy with $C = \{C_1, C_2, \ldots, C_t\}$ criteria and $C_i = \{C_{i1}, C_{i2}, \ldots, C_{ij_i}\}$ subcriteria.

- Step 2: Determine weights of criteria. Let $WC = (WC_1, WC_2, \ldots, WC_t)$ be the weights for all criteria, where $WC_i \in \{$*Absolutely unimportant, Unimportant, Less important, Important, More important, Strongly important, Absolutely important*$\}$, being described these weights by fuzzy numbers. In the same way, let $WC_i = (WC_{i1}, WC_{i2}, \ldots, WC_{ij_i})$ $i = 1, 2, \ldots, t$ be the weights for the set of subcriteria, using the same linguistic term set as WC_i.
- Step 3: Identify weights for evaluators. Each evaluator, e_y is assigned a weight denoted by a linguistic term $we_y \in \{$*Normal, Important, More important, Most important*$\}$, $y = 1, 2, \ldots n$. These linguistic terms are determined through discussions in the group or assigned by a higher management level at the beginning of the evaluation process.

Stage two: Individual preference generation

- Step 4: Set up the relevance degree (score or assessment) of each criterion. For a subjective criterion, let $SC_{ij}^{yk} = \{SC_{i1}^{yk}, SC_{i2}^{yk}, \ldots, SC_{ij_i}^{yk}\}$ be the relevance degree (scores) of the evaluated element S_k on criterion C_{ij}, $i = 1, 2, \ldots, t$, $j = 1, 2, \ldots, j_i$, $k = 1, 2, \ldots, m$, where $SC_{ij_z}^{yk} \in B = \{$*Very low, Low, Medium low, Medium, Medium high, High, Very high*$\}$. The set of objective criteria are evaluated in a domain numerical or interval-value. These objective assessments are conducted into a triangular fuzzy number in B, using an adequate transformation function [7].
- Step 5: Calculate the relevance degrees of all criteria. The relevance degree CS_i^{yk} of the C_i on the evaluated element S_k is calculated by $CS_i^{yk} = \sum_{j=1}^{j_i} WC_{ij} \times SC_{ij}^{yk}$ where $SC_{ij}^{yk} = \sum_{j_z=1}^{j_i} WC_{ij_z} \times SC_{ij_z}^{yk}$; $i = 1, 2, \ldots, t; k = 1, 2, \ldots, m; y = 1, 2, \ldots, n$.
- Step 6: Calculate the relevance degrees of the set of evaluated elements. The relevance degree S_k^y of the evaluated element $S_k (k = 1, 2, \ldots, m)$ by evaluator e_y is calculated by $S_k^y = \sum_{i=1}^{t} CS_i^{yk} \times WC_i$.
- Step 7: Normalize the relevance degrees of the set of evaluated elements. The normalized relevance degrees of the k-th evaluated elements by evaluator e_k are normalized by

$$\overline{S}_k^y = \frac{S_k^y}{\sum_{k=1}^{m} (S_k^y)}$$

where S_k^y is the relevance degree obtained in Step 6.

Stage three: Aggregation of group evaluation

- Step 8: Normalize weights of each evaluator. Each evaluator e_y, is assigned a weight denoted by a linguistic term $we_y (y = 1, 2, \ldots, n)$. The normalized weight of an evaluator e_y is denoted as

$$we_y^* = \frac{we_y}{\sum_{y=1}^{n} (we_y)}$$

– Step 9: Calculate normalized weighted fuzzy decision vector. Considering the normalized weights of all evaluators, a weighted normalized fuzzy decision vector is obtained by

$$(\tilde{r}_1, \tilde{r}_2, \ldots, \tilde{r}_m) = (we_1^*, we_2^*, \ldots, we_n^*) \times \begin{pmatrix} \overline{S}_1^1 & \overline{S}_2^1 & \ldots & \overline{S}_m^1 \\ \overline{S}_1^2 & \overline{S}_2^2 & \ldots & \overline{S}_m^2 \\ \vdots & \vdots & \ddots & \vdots \\ \overline{S}_1^n & \overline{S}_2^n & \ldots & \overline{S}_m^n \end{pmatrix}$$

where $\tilde{r}_k = \sum_{y=1}^{n} we_y^* \times \overline{S}_k^y$

– Step 10: Calculate the distance between the evaluation result and positive- and negative-ideal solution. In the weighted normalized fuzzy decision vector the elements $(\tilde{r}_k, k = 1, 2, \ldots, m)$ are normalized as positive fuzzy numbers and their ranges belong to the closed unit interval. We define a fuzzy positive-ideal solution $(FPIS, r^* = 1)$ and a fuzzy negative-ideal solution $(FNIS, r^- = 0)$. Then the positive and negative solution distances between each $\tilde{r}_k$ and r^*, and r^- can be calculated as:

$$d_k^* = d(\tilde{r}_k, r^*), \quad and \quad d_k^- = d(\tilde{r}_k, r^-), k = 1, 2, \ldots, m$$

where d is a distance between two fuzzy numbers [1,5].

– Step 11: Calculate closeness coefficients of the set of evaluated elements. A closeness coefficient is defined to determine the ranking order of the set of evaluated elements once both d_k^* and d_k^- of each $S_k (k = 1, 2, \ldots, m)$ are obtained. The closeness coefficient of each evaluated element is calculated based on:

$$CC_k = \frac{1}{2}\left(d_k^- + (1 - d_k^*)\right) \in [0, 1]$$

where the best evaluated element corresponds to $\max(CC_k, k = 1, \ldots, m)$.

3 An Extended Version of FMCGDM Method

Our proposal consists of extending the FMCGDM method, including a final stage in order to obtain understandable closeness coefficients further than the sort order. To do so, we use the linguistic 2-tuple representation model [3].

The extended version of FMCGDM keeps the three initial stages of the FMCGDM method and it includes a new stage called *verbalization of closeness coefficients* with three steps. Fig. 2 illustrates the extended version of the FMCGDM method. Now, we present the new stage that extends the FMCGDM method.

Stage four: Verbalization of closeness coefficients

– Step 12: Determine the linguistic domain of expression of closeness coefficients. Here must be fixed the linguistic domain, $S_T = \{s_0, ..., s_g\}$, i.e., the linguistic term set where the closeness coefficients will be expressed. Thereby, it is establishes a linguistic domain that should be understandable and interpretable by the group involved in order to make right decisions.

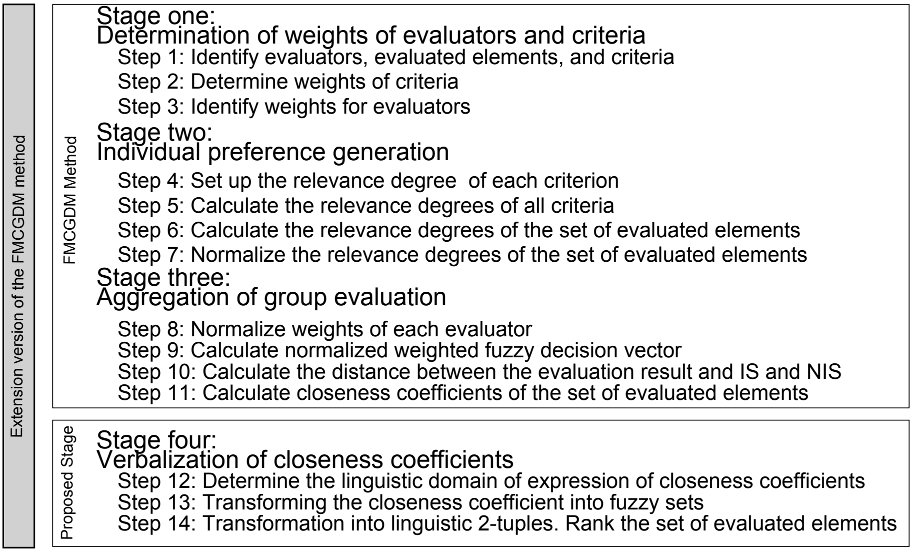

Fig. 2. Extended version of the FMCGDM method

– Step 13: Transforming the closeness coefficient into fuzzy sets. The closeness coefficients obtained in the step 10 are expressed through fuzzy sets in S_T. To do so, the closeness coefficient of each evaluated element is transformed, using a transformation function for numerical values defined as follows:

Definition 1 [4]. Let $v \in [0, 1]$ be a numerical value and $S_T = \{s_0, s_1, \ldots, s_g\}$ a S_T. The *numerical-linguistic transformation function* $T_{NS_T} : [0, 1] \longrightarrow \mathcal{F}(S_T)$ is defined by:

$$T_{NS_T}(v) = \sum_{i=0}^{g} (s_i / \gamma_i) \tag{1}$$

$$\gamma_i = \mu_{s_i}(v) = \begin{cases} 0, \text{ if } v < a \text{ o } v > c, \\ \frac{v-a}{b-a}, \text{ if } a < v < b, \\ 1, \text{ if } b = v, \\ \frac{c-v}{c-b}, \text{ if } b < v < c \end{cases}$$

where $\gamma_i \in [0, 1]$ and $\mathcal{F}(S_T)$ is the fuzzy sets on S_T, and μ_{s_i} is the membership function of the linguistic label $s_i \in S_T$. Usually, the fuzzy sets on S_T are expressed in short as the vector format: $(\gamma_0, \gamma_1, ..., \gamma_g)$.
Therefore, each closeness coefficient for each evaluated element, CC_k, is conducted in the following way.

$$T_{NS_T}(CC_k) = (\gamma_0, \gamma_1, \ldots, \gamma_g)_k$$

– Step 14: Transformation into linguistic 2-tuples and rank the set of evaluated elements. Our extended version of the FMCGDM method is based on the use of the

linguistic 2-tuple representation model. Thereby, the previous fuzzy sets are conducted into linguistic 2-tuples, which facilitate its interpretation [3].
The linguistic 2-tuple representation model is based on the concept of symbolic translation and represents the linguistic information through a 2-tuple (s, α), where s is a linguistic term and α is a numerical value that represents the symbolic translation [3]. Therefore, being $\beta \in [0, g]$ the value generated by a symbolic aggregation operation, we can assign a 2-tuple (s, α) that expresses the equivalent information of this β.

Definition 2 [3]. Let $S = \{s_0, \ldots, s_g\}$ be a set of linguistic terms. The 2-tuple set associated with S is defined as $\langle S \rangle = S \times [-0.5, 0.5)$. We define the function $\Delta_S : [0, g] \longrightarrow \langle S \rangle$ given by,

$$\Delta_S(\beta) = (s_i, \alpha), \quad \text{with} \quad \begin{cases} i = \text{ round } (\beta), \\ \alpha = \beta - i, \end{cases} \tag{2}$$

where *round* assigns to β the integer number $i \in \{0, 1, \ldots, g\}$ closest to β.

We note that Δ_S is bijective [3] and $\Delta_S^{-1} : \langle S \rangle \longrightarrow [0, g]$ is defined by $\Delta_S^{-1}(s_i, \alpha) = i + \alpha$. In this way, the 2-tuples of $\langle S \rangle$ will be identified with the numerical values in the interval $[0, g]$.
As aforementioned, in this step, fuzzy sets are transformed into linguistic 2-tuples in the S_T by using the function χ defined as:

Definition 3 [4]. Given the linguistic term set $S = \{s_0, s_1, \ldots, s_g\}$, the function $\chi : \mathcal{F}(S) \longrightarrow \langle S \rangle$ is defined by

$$\chi\left((\gamma_0, \gamma_1, \ldots, \gamma_g)\right) = \Delta_S \left(\frac{\sum_{j=0}^{g} j\, \gamma_j}{\sum_{j=0}^{g} \gamma_j} \right) = (s_i, \alpha) \in S. \tag{3}$$

Thereby, the fuzzy sets in S_T of the set of evaluated elements are transformed into linguistic 2-tuples in S_T, using the functions Δ_S and χ presented in Definitions 2 and 3.

$$\chi : \mathcal{F}(S_T) \longrightarrow \langle S_T \rangle$$
$$\chi((\gamma_0, \gamma_1, \ldots, \gamma_g)_k) = (s_i, \alpha)_k$$

Finally, the ranking of evaluated elements is obtained by the comparison of the corresponding linguistic 2-tuple in the S_T. It provides a complete ranking of the set of evaluated elements from the best to the worst according to linguistic closeness coefficients. The better evaluated element corresponds to the maximum linguistic closeness coefficient $max\{((s_i, \alpha)_k), k = 1, 2, ..., m\}$.

It is noteworthy that these linguistic 2-tuples can be transformed to another linguistic term set in a quick and easy way, following the extended approach based on linguistic 2-tuples to deal with multiple linguistic scales presented in [2].

4 An Application to a Fabric Material Evaluation Process

In this section, we show an application in a hand-based textile material evaluation process that was presented in [7] with the aim to show the usefulness and effectiveness of the extended version of the FMCGDM method.

To do so, we introduce the context of the evaluation process and then, we apply the extended version of the FMCGDM method in order to obtain linguistic closeness coefficients that provide an interpretable ranking, further than the order.

4.1 Fabric Hand-Based Textile Material Evaluation

A garment company wants to develop a new series of sports jackets. There are six possible textile fabrics to be evaluated, using fabric hand, in order to choose the most suitable one. This evaluation process is involved a group of five evaluators, $E = \{e_1, \ldots, e_5\}$, each of them will evaluate the six textile fabrics $S = \{S_1, S_2, S_3, S_4, S_5, S_6\}$.

Table 1. Hierarchy of criteria and weights

Criteria	Weights	Criteria	Weights
C_1: Dimensional properties	*Important*	C_2: Mechanical properties	*Strongly important*
c_{11}: Thickness	*More important*	c_{21}: Extensibility	*Strongly important*
c_{12}: Density	*Important*	c_{22}: Compressibility	*Important*
C_3: Surface properties	*More important*	c_{23}: Flexibility	*Strongly important*
c_{31}: Surface friction	*Less important*	c_{24}: Resilience	*More important*
c_{32}: Surface contour	*Important*	C_4: Thermal-wet sensation	*Absolutely important*

Each material of the textile fabric is characterized by a hierarchy of criteria. The set of evaluators determine the weights of all the criteria that are described by a linguistic term set $\{$*Absolutely unimportant, Unimportant, Less important, Important, More important, Strongly important, Absolutely important*$\}$. Furthermore, a set of machines is also used to measure these textile fabrics under the criteria. The hierarchy of criteria and weights determined by the group of evaluators are shown in the Table 1.

The set of evaluators gives their assessment for each textile fabric on each criterion, using the following linguistic term set: $\{$*Very low, Low, Medium low, Medium, Medium high, High, Very high*$\}$. The assessments provided for the set of evaluators and the data collected for the set of machines can be consulted in [7].

4.2 Extended version of FMCGDM Method

In this subsection, we show the application of the extended version of the FMCGDM method in the evaluation process mentioned in order to obtain linguistic closeness coefficients that provide an interpretable ranking, further than the order.

Due to space limitations, we show closeness coefficients obtained carrying out the three initial stages of the extended version of FMCGDM method [7] in the Table 2.

Now, we focus on the application of the new proposed stage *Verbalization of closeness coefficients*, which is composed of three steps. This stage is carried out as follows:

Table 2. Closeness coefficients for each material

Material	Closeness coefficients
S_1	.424
S_2	.568
S_3	.538
S_4	**.603**
S_5	.560
S_6	.543

Stage four: Verbalization of closeness coefficients

– Step 12: Determine the linguistic domain of expression of closeness coefficients. The garment company fixes the linguistic domain, i.e., S_T, where closeness coefficients will be expressed. In this case, the company establishes the linguistic term set shown in Fig. 3.

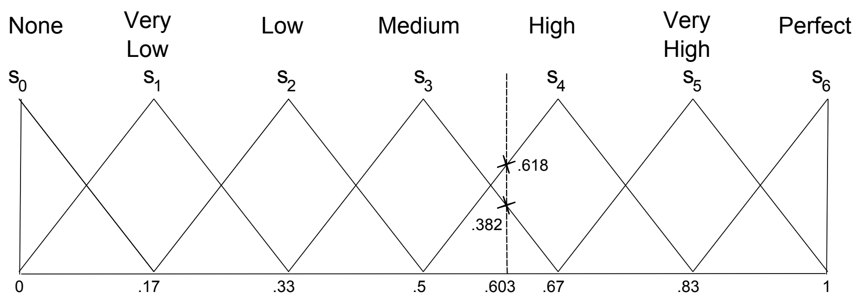

Fig. 3. Transforming the closeness coefficient, .603, into $\mathcal{F}(S_T)$

– Step 13: Transforming the closeness coefficient into fuzzy sets. In this step, the closeness coefficients are transformed in fuzzy sets in S_T, using the function T_{NS_T}. In Table 3 (third column), we show closeness coefficients in fuzzy set in S_T of the set of material. Furthermore, Fig. 3 illustrates the transformation of the closeness coefficients of the material S_4.

– Step 14: Transformation into linguistic 2-tuples and rank the set of evaluated elements. The fuzzy sets in S_T are transformed into linguistic 2-tuples using the functions Δ_S and χ. In Table 3 (fourth column), we illustrate closeness coefficients in linguistic 2-tuples in S_T . Following, as example, we show the transformation to the fuzzy set obtained for S_4 over the S_T showed in Fig. 3.

$$\chi\left((0, 0, 0, .382, .618, 0, 0)\right) = \left(\frac{3\times.382+4\times.618}{.382+.618}\right)$$
$$= \Delta_S\left(3.618\right) = (s_4, -.382) = (High, -.382)$$

Now, closeness coefficients have been expressed in a linguistic domain, facilitating the interpretation of the final ranking, further than order. Moreover, the proposed extended

Table 3. Closeness coefficients for each material

Material	Closeness coefficients	$\mathcal{F}(S_T)$	$\langle S_T \rangle$	Ranking
S_1	.424	$(0, 0, .456, .544, 0, 0, 0)$	(Low,-.456)	6
S_2	.568	$(0, 0, 0, .592, .408, 0, 0)$	(Medium,.408)	2
S_3	.538	$(0, 0, 0, .772, .228, 0, 0)$	(Medium,.228)	5
S_4	.603	$(0, 0, 0, .382, .618, 0, 0)$	(High,-.382)	1
S_5	.560	$(0, 0, 0, .640, .360, 0, 0)$	(Medium,.36)	3
S_6	.543	$(0, 0, 0, .742, .258, 0, 0)$	(Medium,.258)	4

version of the FMCGDM method follows the CWW methodology, since inputs and output are expressed in a linguistic domain in order to be close to human natural language.

5 Concluding Remarks

Several evaluation processes implicate complex situations in which criteria are structured in a hierarchy and both objective and subjective criteria are involved. The FMCGDM method is an adequate option to model these processes, providing a final ranking based on closeness coefficients expressed in the unit interval for the set evaluated elements. However, some complex evaluation processes are beyond a ranking, in order to understand the interpretation of the values that generate the ranking. In this contribution, we have proposed an extension version of the FMCGDM method that uses the linguistic 2-tuple representation model to cope the understanding of closeness coefficients. This new version includes a final stage that provides linguistic closeness coefficients that facilitate the interpretation of the ranking, further than order. Moreover, we have applied the extension of the FMCGDM method in order to show the usefulness and effectiveness of the proposed extended version.

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