

A Multi-granular Linguistic Promethee Model

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Abstract— In Multi-criteria Decision Making (MCDM) problems dealing with qualitative criteria and uncertain information the use of linguistic values is suitable for the experts in order to express their judgments. It is common that the group of experts involved in such problems have different degrees of knowledge about the criteria, so we propose a multi-granular linguistic framework such that each expert can provide his/her evaluations in different linguistic term sets according to his/her knowledge. MCDM problems have been solved in the literature by using different methods, in this contribution we focus on PROMETHEE method and our proposal consists of developing tools and operators for the PROMETHEE method to deal with multi-granular linguistic information.

Keywords— Multi-criteria decision making, linguistic hierarchies, Promethee.

1 Introduction

Decision-making is a common human activity and its multidimensional nature of real world decision problems is well addressed by multi-criteria decision aid (MCDA). The focal point of interest within the methodological framework of MCDA is the analysis and the modelling of the multiple decision makers' preferences. This special characteristic of MCDA implies that a comprehensive model of a decision situation cannot be developed, but instead the model should be developed to meet the expert's requirements. However, sometimes tradeoffs between some criteria may also be too difficult to define for the experts, and they can be then reluctant to express any measurable opinions.

Different methods belonging to MCDA are the following [1]: the *Electre* family developed by Roy and his co-workers, *Promethee* (Brans, Mareshal, Vincke), *Oreste* (Pastijn and Leyson), *Melchior* (Leclercq), *Qualifex* (Paelink), *Regime* (Hinloopen, Nijkamp, Rietvald), *Macbetch* (Bana e Costa, Vansnick), *Ahp* (Saaty), *Topsis* (Hwang and Yoon). Often these methods require a group of experts to express their preferences over the criteria involved in the decision process.

In the real-world, many decision problems are characterized by two overarching concerns: to consider conflict between the criteria of the problem, and to take into account the uncertainty inherent in decision making that depends on the outcome of unknown future events.

To deal with these concerns, the Promethee method is used. Promethee [2] is a popular decision method that has been successfully applied in the selection of the final solution of a problem. It generates a ranking of available

alternatives, according to the expert's preferences, and the best ranked one is considered the favourite final solution.

In this paper, we focus on decision under uncertainty because is one of the most frequent situations in practical decision making, namely in planning activities in many fields. Traditional studies of such issues are conducted by using probabilistic tools and techniques. However, it is not difficult to see in many problems that aspects related to imprecision or vagueness clearly have a non probabilistic character since they are related to imprecision of meanings. Usually, when we deal with certain knowledge in a quantitative setting the information provided by the experts is expressed by means of numerical precise values. However, when we work in a qualitative setting, that is, with vague or imprecise knowledge, it could not be estimated with an exact numerical value. Then, a more realistic approach may be to use linguistic assessments instead of numerical values [11].

The use of linguistic variables makes experts' evaluations more flexible and reliable, but implies processes of computing with words (CW). The main problem that presents the traditional linguistic approaches to carry out the CW processes is the loss of information and hence a lack of precision in the final results. Different linguistic computational models have been developed Semantic model [4], Symbolic [5] or the 2-tuple one [3] that provides a model to deal with CW processes in a precise way.

Our aim in this contribution is focused on MCDM problems where different experts can have different degree of knowledge about the criteria so they can use different linguistic term sets to provide their information defining a multi-granular linguistic context. Again, the main problem is to carry out the CW processes in such a context, in order to overcome this drawback, Herrera and Martinez [6] have developed a model based on linguistic hierarchies. Thus, the CW processes in such contexts can be carried out without loss of information.

Accordingly, a flexible and realistic multi-granular hierarchical linguistic approach based on Promethee method is presented in this paper. The main advantage of this approach is to tackle the uncertainty of both performance of criteria and experts' knowledge without loss of information.

The structure of the paper is the following one. Basic concepts about Promethee method are introduced in section 2. A brief linguistic background is presented in section 3. An aggregation process for multi-granular linguistic information in PROMETHEE is proposed in section 4. In section 5, we

apply this approach to an example. The conclusions are pointed out in section 6.

2 The Promethee Method

The PROMETHEE method (Preference Ranking Organization METHod for Enrichment Evaluation) is a multicriteria decision-making method, belonging to the family of outranking methods [2]. It is a ranking method quite simple in conception and application compared to other methods for multicriteria analysis. It is well adapted to problems where a finite number of alternatives are ranked considering several conflicting criteria. The evaluation table is the starting point of this method. In this table, the alternatives are evaluated according to different criteria. The implementation of Promethee requires two additional types on information, namely: information about the relative importance, w_j , (i.e. the weights) of the criteria considered, and information on the expert's preference modeling, which it uses when comparing the contribution of the alternatives in terms of each separate criterion.

The Promethee method encompasses two phases: (i) the aggregation of information about the alternatives and the criteria, (ii) the exploitation of the outranking relation for decision aid.

The aggregation phase requires that each point of view would be associated with a generalized criterion to assess the preference for an alternative a_i with regards to a_k as a function of $P_j(a_i, a_k) = H_j(f_j(a_i) - f_j(a_k))$. A generalized criterion is thus a function $H_j(f_j(a_i) - f_j(a_k))$ which is null when $(f_j(a_i) - f_j(a_k))$ is negative, non-decreasing with $(f_j(a_i) - f_j(a_k))$ varying between 0 and 1. Six different types of generalized criteria (for a further description see [2]) are proposed to experts, in each case at most two parameters from these thresholds q, p and s have to be fixed. Indifference threshold, q , is the largest deviation to consider as negligible on that criterion. It is a small value with regards to the scale of measurement. Preference threshold, p , is the smallest deviation to consider decisive in the preference of one alternative over another. It is a large value with respect to the scale of measurement. Gaussian threshold, s , is only used with the Gaussian preference function. It is usually fixed as intermediate value between an indifference and a preference threshold.

The outranking relation can be then represented by an oriented graph. The value of each arc is the multi-criteria preference index $\pi(.,.)$, which is defined for all ordered pairs of alternatives. These indices that may take any value in the interval $[0,1]$ define a fuzzy outranking relation. For each $(a_i, a_k) \in A \times A$, Promethee permits the computation of the following quantities for alternatives a_i and a_k :

$$\pi(a_i, a_k) = \frac{\sum_{j=1}^n w_j P_j(a_i, a_k)}{\sum_{j=1}^n w_j} \quad (1)$$

$$\begin{aligned} \phi^+(a_i) &= \sum_{a_k \in A} \pi(a_i, a_k), \\ \phi^-(a_i) &= \sum_{a_k \in A} \pi(a_k, a_i), \\ \phi(a_i) &= \phi^+(a_i) - \phi^-(a_i) \end{aligned} \quad (2)$$

For each alternative a_i , belonging to the set A of alternatives, $\pi(a_i, a_k)$ is an overall preference index of a_i over a_k . The leaving flow $\phi^+(a_i)$ defines the strength of the alternative a_i , how much a_i dominates all the other alternatives of A . Symmetrically, the entering flow $\phi^-(a_i)$ defines the weakness of the alternative, how much a_i is dominated by all the other alternatives of A . $\phi(a_i)$ represents a value function, whereby a higher value reflects a higher attractiveness of alternative a_i . We call $\phi(a_i)$ the net flow of alternative a_i .

According to Promethee I, alternative a_i is better than a_k if the leaving flow of a_i ($\phi^+(a_i)$) is greater than the leaving flow of a_k ($\phi^+(a_k)$) and the entering flow of a_i ($\phi^-(a_i)$) is smaller than the entering flow of a_k ($\phi^-(a_k)$).

Equality in $\phi^+(a_i)$ and $\phi^-(a_i)$ indicates indifference between the two alternatives. In the case where the leaving flows indicate that a_i is better than a_k , while the entering flows indicate the reverse, a_i and a_k are considered incomparable. Therefore, the Promethee I provide a partial ranking of the alternatives.

In Promethee II, the net flow $\phi(a_i)$ is used in order to obtain a complete ranking of all alternatives. The alternative with the higher net flow is better.

3 Linguistic Background

Due to the fact that, our proposal consists in dealing with MCDM problems defined in multi-granular linguistic contexts that implies processes of CW, here we review briefly the 2-tuple linguistic representation model and the linguistic hierarchies structure that are necessary concepts to achieve our aim.

3.1 The 2-tuple Fuzzy Representation Model

This model was presented in [3] for overcoming the drawback of the loss of information presented by the classical linguistic computational models [12], i.e., (i) the semantic model [4], and (ii) the symbolic one [5].

The 2-tuple fuzzy linguistic representation model is based

on the symbolic method and takes as the base of its representation the concept of Symbolic Translation.

Definition 1. *The Symbolic Translation of a linguistic term $s_i \in S = \{s_0, \dots, s_g\}$ is a numerical value assessed in $[-0.5, 0.5]$ that supports the “difference of information” between an amount of information $\beta \in [0, g]$ and the closest value in $\{0, \dots, g\}$ that indicates the index of the closest linguistic term in S (s_i), being $[0, g]$ the interval of granularity of S .*

From this concept a new linguistic representation model is developed, which represents the linguistic information by means of 2-tuples (s_i, α_i) , $s_i \in S$ and $\alpha_i \in [-0.5, 0.5]$.

Definition 2. *Let $S = \{s_0, \dots, s_g\}$ be a linguistic term set and $\beta \in [0, g]$ a value supporting the result of a symbolic aggregation operation. Then the 2-tuple that expresses the equivalent information to β is obtained with the following function:*

$$\Delta : [0, g] \rightarrow S \times [-0.5, 0.5] \quad (3)$$

$$\Delta(\beta) = (s_i, \alpha), \begin{cases} s_i & i = \text{round}(\beta) \\ \alpha = \beta - i & \alpha \in [-0.5, 0.5] \end{cases}$$

where $\text{round}(\cdot)$ is the usual round operation, s_i has the closest index label to “ β ” and “ α ” is the value of the symbolic translation.

Proposition 1. Let $S = \{s_0, \dots, s_g\}$ be a linguistic term set and (s_i, α_i) be a linguistic 2-tuple. There is always a Δ^{-1} function, such that, from a 2-tuple it returns its equivalent numerical value $\beta \in [0, g]$ in the interval of granularity of S .

Proof. It is trivial, we consider the following function:

$$\Delta^{-1} : S \times [-0.5, 0.5] \rightarrow [0, g] \quad (4)$$

$$\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta$$

Remark 1. From Definitions 1 and 2 and Proposition 1, it is obvious that the conversion of a linguistic term into a linguistic 2-tuple consists of adding a value 0 as symbolic translation: $s_i \in S \Rightarrow (s_i, 0)$.

This model has a computational technique based on the 2-tuples [3]:

- Aggregation of 2-tuples
The aggregation of linguistic 2-tuples consist of obtaining a value that summarizes a set of values, therefore, the result of the aggregation of a set of 2-tuples must be a linguistic 2-tuple. We can find several 2-tuple aggregation operators in [5] based on classical aggregation operators as the arithmetic mean and weighted mean operators.
- Comparison of 2-tuples

The comparison of information represented by 2-tuples is carried out according to an ordinary lexicographic order.

Let (s_k, α_1) and (s_l, α_2) be two 2-tuples represented two assessments:

- If $k < l$ then (s_k, α_1) is smaller than (s_l, α_2) ;
- If $k = l$ then
 - 1) If $\alpha_1 = \alpha_2$ then (s_k, α_1) and (s_l, α_2) represent the same value;
 - 2) If $\alpha_1 < \alpha_2$ then (s_k, α_1) is smaller than (s_l, α_2) ;
 - 3) If $\alpha_1 > \alpha_2$ then (s_k, α_1) is bigger than (s_l, α_2) .

- Negation Operator of a 2-tuple

The negation operator over 2-tuples is defined as:

$$\text{Neg}(s_i, \alpha) = \Delta(g - \Delta^{-1}(s_i, \alpha)) \quad (5)$$

where $g+1$ is the cardinality of S .

3.2 Linguistic Hierarchies

The Linguistic Hierarchies were introduced in [6] in order to accomplish processes of CW with multi-granular linguistic information in a precise way. A Linguistic Hierarchy is a set of levels, where each level represents a linguistic term set with different granularity to the remaining levels. Each level is denoted as $l(t, n(t))$:

- t a number that indicates the level of the hierarchy
- $n(t)$ the granularity of the term set of the level t

We assume that levels containing linguistic terms are triangular shaped, symmetrical and uniformly distributed. In addition, the linguistic term sets have an odd number of linguistic terms being the middle one the value of indifference.

The levels belonging to a linguistic hierarchy are ordered according to their granularity, i.e., for two consecutive levels t and $t+1$, $n(t+1) > n(t)$. Therefore, the level $t+1$ is a refinement of the previous level t .

From the above concepts, we define a linguistic hierarchy, LH, as the union of all levels t :

$$LH = \bigcup_t l(t, n(t)) \quad (6)$$

Given an LH, we denote as $S^{n(t)}$ the linguistic term set of LH corresponding to the level t of LH characterized by a granularity of uncertainty $n(t)$:

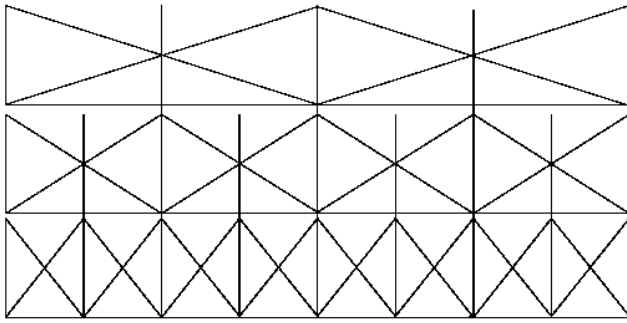
$$S^{n(t)} = \{s_0^{n(t)}, \dots, s_{n(t)-1}^{n(t)}\} \quad (7)$$

Generically, we can say that the linguistic term set of level $t + 1$ is obtained from its predecessor as:

$$l(t, n(t)) \rightarrow l(t + 1, 2 \bullet n(t) - 1) \quad (8)$$

A graphical example of a linguistic hierarchy can be seen in Figure 1.

Fig. 1. Linguistic Hierarchy with term sets of 3,5 and 9 terms.



In [6] different transformation functions between labels of different levels were developed without loss of information. To understand how these functions are working, there were defined transformation functions between two consecutive levels and afterwards between any levels of the hierarchy, those transformation functions use the linguistic 2-tuple computational model. Here, we present the transformation function between any levels.

Definition 3. Let $LH = \bigcup_t I(t, n(t))$ be a linguistic hierarchy whose linguistic term sets are denoted as $S^{n(t)} = \{s_0^{n(t)}, \dots, s_{n(t)-1}^{n(t)}\}$, and let us consider the 2-tuple linguistic representation. The transformation function from a linguistic label in level t to a label in level t' is defined as:

$$TF_{t'}^t : I(t, n(t)) \rightarrow I(t', n(t')) \quad (9)$$

$$TF_{t'}^t(s_i^{n(t)}, \alpha^{n(t)}) = \Delta_{n(t)} \left(\frac{\Delta_{n(t)}^{-1}(s_i^{n(t)}, \alpha^{n(t)}) \bullet (n(t') - 1)}{n(t) - 1} \right)$$

Proposition 2. The transformation function between linguistic terms in different levels of the linguistic hierarchy is bijective:

$$TF_{t'}^{t'}(TF_{t'}^t(s_i^{n(t)}, \alpha^{n(t)})) = (s_i^{n(t)}, \alpha^{n(t)}). \quad (10)$$

4 A Linguistic Multi-Granular Promethee model

The model developed in this paper is the result of the integration between the aggregation operators of the PROMETHEE method and the multi-granular linguistic model to combine multiple experts' assessments defining a multi-granular linguistic framework. So, each expert can express his/her evaluations in the suitable scale according to his/her knowledge in the table 1.

Table 1. Multiple experts' assessments scheme.

| | Experts | | | | | | | | |
|----------------|------------------------------|------------------------------|------------------------------|------------------------------|------------------------------|------------------------------|------------------------------|------------------------------|------------------------------|
| | e ₁ | | | e ₂ | | | e _E | | |
| Alt. | c ₁ | c _i | c _n | c ₁ | c _i | c _n | c ₁ | c _i | c _n |
| a ₁ | C ₁₁ ¹ | C _{1j} ¹ | C _{1n} ¹ | C ₁₁ ² | C _{1j} ² | C _{1n} ² | C ₁₁ ^E | C _{1j} ^E | C _{1n} ^E |
| a _j | C _{j1} ¹ | C _{ij} ¹ | C _{jn} ¹ | C _{j1} ² | C _{ij} ² | C _{jn} ² | C _{j1} ³ | C _{ij} ³ | C _{jn} ³ |
| a _m | C _{m1} ¹ | C _{mj} ¹ | C _{mn} ¹ | C _{m1} ² | C _{mj} ² | C _{mn} ² | C _{m1} ³ | C _{mj} ³ | C _{mn} ³ |

Being $A = \{a_1, \dots, a_j, \dots, a_m\}$ a set of alternatives, $C = \{c_1, \dots, c_i, \dots, c_n\}$ a set of criteria, $E = \{e_1, \dots, e_l, \dots, e_E\}$ a set of experts. The assessments C_{ij}^e provided by the experts, e_e , can be assessed in a linguistic term sets of the linguistic hierarchy that can have different granularity of uncertainty. Therefore $C_{ij}^e \in S^e$ and $S^e \in LH$.

The proposed model to deal with MCDM problems defined in multi-granular linguistic contexts consists of three steps. Due to the use of multi-granular linguistic information, the aggregation step is divided in two steps:

- *Normalization step.* The multi-granular linguistic information is expressed in a unique linguistic expression domain.
- *Aggregation step.* The unified information expressed in a unique linguistic term set is aggregated.
- *Exploitation step.* The collective preference values are ordered in a decreasing way and the solution set is composed of the best alternative/s.

4.1 Normalization step

At the beginning, a linguistic term set to unify the multi-granular linguistic information must be selected called S_T . Any linguistic term set to do it can be chosen because the transformations between levels in a LH are bijective (see Proposition 2). In order to reduce the number of computations, the linguistic term set that the most of experts express their preferences in it shall be chosen.

Let us suppose that $S^e = S^{n(t')}$ and $S_T = S^{n(t')}$, so a transformation function between linguistic terms in different levels of the linguistic hierarchy is obtained as follows:

$$TF_{t'}^{t'}(C_{ij}^e) = (s_{ij}^e, \alpha). \quad (11)$$

with, $s_{ij}^e \in S_T$.

4.2 Aggregation step

In this step, two types of preference indexes, individual ($\pi^e(a_i, a_k)$) and collective ($\pi(a_i, a_k)$), will be computed. The individual preference index which translates the intensity of the preference of the alternative a_i compared to the alternative a_k according to the point of view of each expert is expressed as:

$$\pi^e : A \times A \rightarrow S^{n(t)} \times [-0.5, 0.5] \quad (12)$$

$$\pi^e(a_i, a_k) = \Delta \left[\frac{\sum_{j=1}^n \Delta^{-1}(w_j^e \cdot P_j^e(a_i, a_k))}{\Delta^{-1}(W^e)} \cdot n(t) \right] = (s_i^{e, n(t)}, \alpha_i^e)$$

w_j^e is the weight assigned by each expert to each criterion,

$W^e = \sum_{j=1}^n w_j^e$, $P_j^e(a_i, a_k)$ is a preference function among the

six types of functions proposed in [2], a_i and a_k are two alternatives belonging to A , $n(t)$ is the cardinality of the chosen linguistic term set, $(s_i^{e, n(t)}, \alpha_i^e)$ is a linguistic

2-tuple, $i, k \in \{1, \dots, m\}$ alternatives, $j \in \{1, \dots, n\}$ criteria, $e \in \{1, \dots, E\}$ experts.

Thus, the collective preference index corresponds to the aggregation of the linguistic values computed. The aggregation operator could be different in each problem. Different aggregation operators were defined in [3] to deal with linguistic 2-tuples, for the sake of simplicity in this case we have chosen the arithmetic mean. The expression of the collective preference index is:

$$\pi : A \times A \rightarrow S^{n(t)} \times [-0.5, 0.5] \quad (13)$$

$$\pi(a_i, a_k) = \Delta \left[\sum_{e=1}^E \frac{1}{E} \Delta^{-1}(\pi^e(a_i, a_k)) \right] = (s_i^{n(t)}, \alpha_i)$$

With, $E = \{e_1, \dots, e_E\}$ is the set of experts and $(s_i^{n(t)}, \alpha_i)$ is a 2-tuple.

4.3 Exploitation step

The exploitation step generates a solution set of alternatives (the best ones) for the decision problem. To do so, this step uses a total ranking of the alternatives in a decreasing way according to a choice function. Different choices functions have been proposed in the literature [8-10]. In this paper, a choice function that computes the dominance degree for each alternative, a_i , over the other alternatives is used as follows:

$$\Lambda(a_i) = \frac{1}{m-1} \sum_{k, i=1, k \neq i}^m \pi(a_i, a_k) \quad (14)$$

Then, the best alternative(s) are in the head of ranking should be chosen as solution set of alternatives.

5 Numerical Example

In this section, we present an investment example to show the integration between the aggregation operators of PROMETHEE method and the linguistic hierarchies.

5.1 Input Data

An investment company wants to invest a sum of money in the best option. There is a panel with four possible alternatives $A = \{a_1, \dots, a_4\}$ of investment possibilities. a_1 is a car industry, a_2 is a food company, a_3 is a computer company, and a_4 is an arms industry. The investment company chooses four experts $E = \{e_1, \dots, e_4\}$ from four consultancy departments: risk analysis, growth analysis, social-political analysis, and environmental impact analysis departments respectively, to construct a decision group throughout a set of three criteria $C = \{c_1, c_2, c_3\}$ being, c_1 profit, c_2 pollution, and c_3 employment.

These experts use different linguistic term sets from the LH (showed in Fig. 1) to provide their preferences over the

alternative set as following: e_1 provides his preferences in $l(3,9)$, e_2 provides his preferences in $l(2,5)$, e_3 provides his preferences in $l(1,3)$, and e_4 provides his preferences in $l(3,9)$.

After a deep study, each expert provides the following preference values:

Table 2. Input data of each expert.

| | e1 | | | e2 | | | e3 | | | e4 | | |
|------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| | c1 | c2 | c3 | c1 | c2 | c3 | c1 | c2 | c3 | c1 | c2 | c3 |
| a1 | S_6^9 | S_3^9 | S_2^9 | S_4^5 | S_2^5 | S_1^5 | S_2^3 | S_1^3 | S_1^3 | S_8^9 | S_1^9 | S_3^9 |
| a2 | S_7^9 | S_2^9 | S_4^9 | S_3^5 | S_3^5 | S_2^5 | S_1^3 | S_1^3 | S_1^3 | S_5^9 | S_2^9 | S_1^9 |
| a3 | S_8^9 | S_5^9 | S_5^9 | S_3^5 | S_1^5 | S_2^5 | S_2^3 | S_1^3 | S_2^3 | S_7^9 | S_3^9 | S_5^9 |
| a4 | S_8^9 | S_6^9 | S_1^9 | S_4^5 | S_3^5 | S_2^5 | S_2^3 | S_2^3 | S_2^3 | S_8^9 | S_5^9 | S_2^9 |
| wj | S_8^9 | S_6^9 | S_4^9 | S_3^5 | S_4^5 | S_1^5 | S_1^3 | S_2^3 | S_1^3 | S_7^9 | S_1^9 | S_5^9 |
| Type | II | III | IV | II | III | IV | II | III | IV | II | III | IV |
| pj | S_4^9 | S_2^9 | S_5^9 | S_3^5 | S_3^5 | S_2^5 | S_2^3 | S_1^3 | S_2^3 | S_5^9 | S_2^9 | S_4^9 |
| qj | S_2^9 | S_1^9 | S_3^9 | S_1^5 | S_2^5 | S_1^5 | S_1^3 | S_0^3 | S_1^3 | S_3^9 | S_1^9 | S_3^9 |

5.2 Normalization Step

In this example, the linguistic term set $l(3,9)$ shall be chosen to unify the multi-granular linguistic information, since the most of experts have expressed their preferences in it.

5.3 Aggregation Step

This step is carried out by the computation of both individual and collective preference indexes. The obtained results are shown in the tables below:

Table 3. Individual preference index of each expert.

| π^1 | a1 | a2 | a3 | a4 |
|---------|--------------|--------------|--------------|--------------|
| a1 | - | $(S_3^9, 0)$ | $(S_0^9, 0)$ | $(S_0^9, 0)$ |
| a2 | $(S_1^9, 0)$ | - | $(S_0^9, 0)$ | $(S_0^9, 0)$ |
| a3 | $(S_8^9, 0)$ | $(S_3^9, 0)$ | - | $(S_1^9, 0)$ |
| a4 | $(S_7^9, 0)$ | $(S_3^9, 0)$ | $(S_3^9, 0)$ | - |

Table 4. Collective preference index.

| π | a_1 | a_2 | a_3 | a_4 |
|-------|-----------------|------------------|-----------------|------------------|
| a_1 | - | $(S_3^9, -0.24)$ | $(S_1^9, 0.4)$ | $(S_0^9, 0)$ |
| a_2 | $(S_3^9, 0.27)$ | - | $(S_3^9, 0.26)$ | $(S_0^9, 0.25)$ |
| a_3 | $(S_2^9, 0.14)$ | $(S_0^9, 0.75)$ | - | $(S_3^9, -0.27)$ |
| a_4 | $(S_4^9, 0.12)$ | $(S_4^9, 0.5)$ | $(S_2^9, 0.33)$ | - |

5.4 Exploitation Step

This step provides a total ranking of the alternatives in a decreasing way (Table 6) according to a choice function (Table 5).

Table 5. Dominance degree.

| | |
|--------------------------------|--------------------------------|
| $\Lambda(a_1) = (S_1^9, 0.04)$ | $\Lambda(a_2) = (S_2^9, -0.3)$ |
| $\Lambda(a_3) = (S_1^9, 0.4)$ | $\Lambda(a_4) = (S_3^9, -0.3)$ |

Where the degrees are computed as:

$$\begin{aligned} \Lambda(a_1) &= \frac{1}{3} \sum_{k,i=1, k \neq i}^3 \pi(a_1, a_k) = \frac{1}{3} (\pi(a_1, a_2) + \pi(a_1, a_3) + \pi(a_1, a_4)) \\ &= \frac{1}{3} ((S_3^9, -0.24) + (S_1^9, 0.4) + (S_0^9, 0)) \\ &= (S_1^9, 0.4) \end{aligned}$$

Table 6. Alternatives ranking.

| | | | |
|-------|-------|-------|-------|
| a_4 | a_2 | a_3 | a_1 |
|-------|-------|-------|-------|

According to the ranking of the alternatives, the company should choose the alternative, a_4 , for its investment.

6 Conclusions

In order to manage multi-granular linguistic information in MCDM problems, we extended aggregation operators of PROMETHEE method for combining the linguistic values by the direct computation on labels. In this paper, a multi-criteria, multi-expert method has been presented to obtain the overall linguistic value without loss of information, taking into account the particular nature of the criteria and the specific differences among the experts through the aggregation process. The proposed model is computationally simple and quick.

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