Computing with words using the 2-tuple linguistic representation model: Analysis of consistency and description

F. Herrera
Dept. of Computer Science and A.I.
University of Granada, 18071 - Granada
herrera@decsai.ugr.es

L. Martínez
Dept. of Computer Science.
University of Jaén, 23071 - Jaén
martin@ujaen.es

Abstract

The use of linguistic information implies processes of Computing with Words, to accomplish these processes different models have been proposed in the literature. In this contribution, we solve a Multiexpert-Multicriteria Decision-Making problem defined in a multigranularity linguistic context using different computational models. Subsequently, we shall analyze the results obtained to study what linguistic computatinal model is better for Computing with Words processes from the points of view of linguistic description and consistency.

Keywords: Computing with Words, aggregation, linguistic preference variables.

1 Introduction

When a problem is solved using linguistic information, implies the need to use computational techniques that provide linguistic operators for processes of Computing with Words (CW). In the specialized literature there exist different linguistic computational models [5, 6, 9]:

- *The approximative computational model based on the Extension Principle* [4, 6]. This model uses the extension principle to make linguistic computations. And the results can be expressed by means of fuzzy numbers [4] or by means of linguistic terms [6].

- *The ordinal linguistic computational model* [5]. it makes direct computations on labels, using the ordinal structure of the linguistic term sets.

- *The 2-tuple linguistic model* [9]. It uses the 2-tuple linguistic representation model and its characteristics to make linguistic computations.

In [9, 11] we can see that the 2-tuple computational model is an extension of the ordinal one, which using as representation a pair of values for avoid the loss of information, an ordinal value and a numerical translation, therefore always obtains at least the same or better results than the ordinal model as a refinament of it. Hence, in this paper we do not use the ordinal model in the comparative study due to the fact it does not obtain better results than the 2-tuple one as it was shown in [9]

In this paper we shall present a Multiexpert Multicriteria Decision-Making (MEMC-DM) problem, *"Transfer technology strategy selection"*, and solve it using the model based on 2-tuples and the approximative one expressing its results with fuzzy numbers and ranking them [4]. We must remark that there exist another alternative of resolution with the approximative model using linguistic approximation processes [6] that it was studied in [9]. From the solutions obtained by both methods we shall study what model is more clear and consistent.

In order to do that, this contribution is structured as follows: in Section 2 we shall a brief review of the Fuzzy Linguistic Approach, the computational model based on the Extension Principle, and the linguistic 2-tuple representation model. In Section 3 we shall present an MEMC-DM problem and solve it using the two methods reviewed in Section 2. In Section 4 we shall make a comparative study of the results obtained by both methods. Finally, some cloncluding remarks are included.

2 Preliminaries

In this section we shall make a brief review of several concepts and models neccesary to reach the aim of this paper.

2.1 Fuzzy Linguistic Approach

Usually, we work in a quantitative setting, where the information is expressed by means of numerical values. However, many aspects of different activities in the real world cannot be assessed in a quantitative form, but rather in a qualitative one, i.e., with vague or imprecise knowledge. In that case a better approach may be to use linguistic assessments instead of numerical values. The fuzzy linguistic approach represents qualitative aspects as linguistic values by means of linguistic variables [14].

We have to choose the appropriate linguistic descriptors for the term set and their semantics. In the literature, several possibilities can be found (see [8] for a wide description). In order to accomplish this objective, an important aspect to analyze is the *"granularity of uncertainty"*, i.e., the level of discrimination among different counts of uncertainty. One possibility of generating the linguistic term set consists of directly supplying the term set by considering all terms distributed on a scale on which a total order is defined [13]. For example, a set of seven terms S, could be given as follows:

$$S = \{s_0 : N, s_1 : VL, s_2 : L, s_3 : M, s_4 : H, s_5 : VH, s_6 : P\}$$

Usually, in these cases, it is required that in the linguistic term set there exist:

1) A negation operator: $\mathrm{Neg}(s_i) = s_j$ such that $j = g$-i (g+1 is the cardinality).

2) An order: $s_i \leq s_j \Longleftrightarrow i \leq j$. Therefore, there exists a minimization and a maximization operator.

The semantics of the linguistic terms is given by fuzzy numbers defined in the [0,1] interval. A computationally efficient way to characterize a fuzzy number is to use a representation based on parameters of its membership function [1]. The linguistic assessments given by the users are just approximate ones, some authors consider that linear trapezoidal membership functions are good enough to capture the vagueness of those linguistic assessments. The parametric representation is achieved by the 4-tuple (a, b, d, c), where b and d indicate the interval in which the membership value is 1, with a and c indicating the left and right limits of the definition domain of the trapezoidal membership function [1]. A particular case of this type of representation are the linguistic assessments whose membership functions are triangular, i.e., $b = d$, then we represent this type of membership functions by a 3-tuple (a, b, c). An example may be the following :

$$\begin{array}{ll} P = (.83, 1, 1) & VH = (.67, .83, 1) \\ H = (.5, .67, .83) & M = (.33, .5, .67) \\ L = (.17, .33, .5) & VL = (0, .17, .33) \\ N = (0, 0, .17). & \end{array}$$

Other authors use a non-trapezoidal representation, e.g., Gaussian functions [2].

2.2 Linguistic Computational Model based on the Extension Principle

The Extension Principle is a basic concept in the fuzzy sets theory [7] which is used to generalize crisp mathematical concepts to fuzzy sets. The use of extended arithmetic based on the Extension Principle [7] increases the vagueness of the results. Therefore, the results obtained by the fuzzy linguistic operators based on the Extension Principle are fuzzy numbers that usually do not match with any linguistic term in the initial term set.

The main problem of dealing with fuzzy numbers is the lack of a intrinsic order in these numbers, therefore a fuzzy ranking function must be applied to order them. In the specialized literature can be found different fuzzy ranking functions [15].

2.3 The 2-tuple Linguistic Representation Model

This representation model was presented in [9, 11] and it is based on the concept of symbolic translation and use it for representing the linguistic information by means of 2-tuples, (s_i, α), where s is a linguistic term and α is a numerical value representing the symbolic translation.

Let $S = \{s_0, ..., s_g\}$ be a linguistic term set, and $\beta \in [0, g]$ a numerical value in its interval of granularity (e.g.: let β be a value obtained from a symbolic aggregation operation).

Definition 1. *The symbolic translation is a numerical value assessed in $[-.5, .5)$ that supports the "difference of information" between a counting of information β assessed in the interval of granularity ,$[0, g]$, of the term set S and the closest value in $\{0, ..., g\}$ that indicates the index of the closest linguistic term in S.*

From this concept we shall develop a linguistic representation model which represents the linguistic information by means of 2-tuples (r_i, α_i), $r_i \in S$ and $\alpha_i \in [-.5, .5)$. r_i represents the linguistic label center of the information and α_i is the value of the Symbolic Translation.

This representation model defines a set of functions to facilitate computational processes over 2-tuples.

Definition 2. *Let $s_i \in S$ be a linguistic term, then its equivalent 2-tuple representation is obtained by means of the function θ as:*

$$\theta : S \longrightarrow (S \ \times \ [-0.5, 0.5))$$

$$\theta(s_i) = (s_i, 0)$$

Definition 3. *Let $S = \{s_0, ..., s_g\}$ be a linguistic term set and $\beta \in [0, g]$ a value supporting the result of a symbolic aggregation operation, then the 2-tuple that expresses the equivalent information to β is obtained with the following function:*

$$\Delta : [0, g] \longrightarrow S \times [-0.5, 0.5)$$

$$\Delta(\beta) = (s_i, \alpha), \ with \left\{ \begin{array}{cc} s_i & i = round(\beta) \\ \alpha = \beta - i & \alpha \in [-.5, .5) \end{array} \right.$$

where round$(\cdot)$ is the usual round *operation, s_i has the closest index label to "β" and "α" is the value of the symbolic translation.*

Proposition 1.*Let $S = \{s_0, ..., s_g\}$ be a linguistic term set and (s_i, α) be a 2-tuple. There is always a Δ^{-1} function, such that, from a 2-tuple it returns its equivalent numerical value $\beta \in [0, g] \subset \mathcal{R}$.*

Proof.

It is trivial, we consider the following function:

$$\Delta^{-1} : S \times [-.5, .5) \longrightarrow [0, g]$$
$$\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta$$

Here we review the computational technique based on 2-tuples presented in [9, 11]:

1. Comparison of 2-tuples

The comparison of linguistic information represented by 2-tuples is carried out according to an ordinary lexicographic order.

Let (s_k, α_1) and (s_l, α_2) be two 2-tuples, with each one representing a counting of information:

- if $k < l$ then (s_k, α_1) is smaller than (s_l, α_2)
- if $k = l$ then
  1. if $\alpha_1 = \alpha_2$ then (s_k, α_1), (s_l, α_2) represents the same information
  2. if $\alpha_1 < \alpha_2$ then (s_k, α_1) is smaller than (s_l, α_2)
  3. if $\alpha_1 > \alpha_2$ then (s_k, α_1) is bigger than (s_l, α_2)

2. Aggregation of 2-tuples

The aggregation of information consists of obtaining a value that summarizes a set of values, therefore, the result of the aggregation of a set of 2-tuples must be a 2-tuple. In [9] we can find some 2-tuple aggregation operators, that are based on classical aggregation operators.

3. Negation operator of a 2-tuple

The negation operator over 2-tuples is defined as:

$$Neg((s_i, \alpha)) = \Delta(g - (\Delta^{-1}(s_i, \alpha)))$$

where $g+1$ is the cardinality of S, $S = \{s_0, ..., s_g\}$.

3 MultiExpert MultiCritera Decision-Making Problem

Here we shall present the MEMC-DM problem of *"Technology Transfer Strategy selection"*. The transfer of technology from its source to commercial application is a very complex process. It is a multiexpert multicriteria decision-making problem in ill-structured situations. It must be made a careful analysis among criteria, alternatives, weights, and decision makers before making a decision. Using conventional crisp decision methods, we always have to find precise data, but under several conditions, we cannot get precise data because the data are from the experience and the judgment of decision makers. In these cases is more adequated to use the fuzzy linguistic approach instead of precise data to assess the values of the alternatives versus various criteria and the importance weight of criteria.

A general representation of an MEMC-DM problem is introduced. Suppose there is a committee of n experts $(p_1, ..., p_n)$ who assess the appropriateness of m alternatives $(x_1, ..., x_m)$ according to each of k criteria $(C_1, ..., C_k)$ as well as the importance weight of the criteria. Let S_{itj} $(i = 1, ..., m; t = 1, ..., k; j = 1, ..., n)$ be the rating assigned to alternative x_i by the expert p_j according to the criterion C_t. Let W_{tj} be the importance given to C_t by the expert p_j, and W_t the importance of the criterium t according to the whole of experts.

The selection process used for solving this problem is structured as follows:

- **Aggregation Process**. The committee has to aggregate the rating S_{itj} of n experts for each alternative x_i versus each criterion C_t to obtain the rating S_{it}. Each pooled S_{it} can further be weighted by weight W_t according to the relative importance of the k criteria. Then, the final score F_i, fuzzy appropriateness index, of alternative x_i can be obtained by aggregating S_{it} and W_t.
- **Exploitation Process**. Finally, rank the final scores F_i, to obtain the most appropiate alternative.

We consider a committee of four experts, $P = \{p_1, p_2, p_3, p_4\}$, has been formed to determine the most appropiate technology transfer strategy. After screening, four selection criteria are considered:

- *(C_1) Technological availability.*
- *(C_2) Market potencial.*
- *(C_3) Policy support.*
- *(C_4) Management ability.*

For selecting one of four commonly used strategies that were successful in transferring technologies in the case studies are described as follows:

- *(x_1) Purchasing.*
- *(x_2) Working with an industrial partner.*
- *(x_3) Licensing.*
- *(x_4) Cooperative R&D.*

For describing the attitudes of the experts, they will use linguistic variables. Besides each one can give his preferences in his own linguistic term set. Therefore, the definition context of the problem is multigranularity linguistic, different term sets with different granularity or semantics. In our particular case, p_1 uses the term set A (7 labels), p_2 *and* p_3 use the term set B (5 labels) and p_4 uses the term set C (7 labels):

	A		B		C
a_0	$(0,0,0)$	b_0	$(0,0,.25)$	c_0	$(0,0,.16)$
a_1	$(0,0,.25)$	b_1	$(0,.25,.5)$	c_1	$(0,.16,.33)$
a_2	$(0,.25,.5)$	b_2	$(.25,.5,.75)$	c_2	$(.16,.33,.5)$
a_3	$(.25,.5,.75)$	b_3	$(.5,.75,1)$	c_3	$(.33,.5,.67)$
a_4	$(.5,.75,1)$	b_4	$(.75,1,1)$	c_4	$(.5,.67,.84)$
a_5	$(.75,1,1)$			c_5	$(.67,.84,1)$
a_6	$(1,1,1)$			c_6	$(.84,1,1)$.

The preferences provided are the following:

W_{tj}	*Experts*			
Criteria	p_1	p_2	p_3	p_4
C_1	a_3	b_4	b_3	c_3
C_2	a_4	b_3	b_2	c_4
C_3	a_5	b_3	b_4	c_4
C_4	a_4	b_2	b_3	c_3

Table 1. The importance of the criteria

	C_1				C_2			
S_{itj}	p_1	p_2	p_3	p_4	p_1	p_2	p_3	p_4
x_1	a_4	b_4	b_3	c_4	a_3	b_0	b_1	c_1
x_2	a_2	b_2	b_1	c_2	a_3	b_1	b_1	c_4
x_3	a_2	b_1	b_1	c_3	a_4	b_3	b_3	c_4
x_4	a_3	b_2	b_2	c_1	a_4	b_4	b_3	c_5

Table 2. Evaluation under C_1 and C_2

	C_3				C_4			
S_{itj}	p_1	p_2	p_3	p_4	p_1	p_2	p_3	p_4
x_1	a_2	b_0	b_1	c_1	a_4	b_4	b_3	c_4
x_2	a_1	b_1	b_0	c_3	a_3	b_2	b_3	c_5
x_3	a_3	b_2	b_3	c_4	a_3	b_2	b_2	c_3
x_4	a_2	b_2	b_3	c_2	a_3	b_3	b_1	c_3

Table 3. Evaluation under C_3 and C_4

3.1 Solution based on the Approximative model

Aggregation Process

There exist a lot of aggregation operators for combining the experts' preferences. We use the mean operator to aggregate the experts' assessments as in [4]. Let $\oplus$ and $\otimes$ be fuzzy addition and multiplication operators on fuzzy numbers.

$$\mu_{S_{it}} = (\tfrac{1}{n}) \otimes (\mu_{S_{it1}} \oplus \mu_{S_{it2}} \oplus ... \oplus \mu_{S_{itn}})$$
$$\mu_{W_t} = (\tfrac{1}{n}) \otimes (\mu_{W_{t1}} \oplus \mu_{W_{t2}} \oplus ... \oplus \mu_{W_{tn}}),$$

where S_{it} is the average fuzzy appropriateness rating of alternative x_i under criterion C_t and W_t is the average importance weight of criterion C_t.

S_{it}	C_1	C_2
x_1	(.5625,.7925,.9600)	(.0625,.2275,.4575)
x_2	(.1025,.3325,.5625)	(.1875,.4175,.6475)
x_3	(.0825,.3125,.5425)	(.5000,.7300,.9600)
x_4	(.1875,.4150,.6450)	(.6050,.8350,1)
	C_3	C_4
x_1	(0,.1650,.3950)	(.5625,.7925,.9600)
x_2	(.0825,.1875,.4175)	(.4175,.6475,.8750)
x_3	(.3750,.6050,.8350)	(.2700,.5000,.7300)
x_4	(.2275,.4575,.6875)	(.2700.5000,.7300)

Table 4. Fuzzy appropiatness rating of x_i under C_t

	W_t
C_1	(.4575,.6875,.8550)
C_2	(.4375,.6675,.8975)
C_3	(.6250,.8550,.9600)
C_4	(.3950,.6250,.8550)

Table 5. Average importance weight of C_t

Thus, the fuzzy appropiateness index F_i of the ith alternative is obtained using:

$$F_i = (\frac{1}{k}) \otimes [(S_{i1} \otimes W_1) \oplus ... \oplus (S_{ik} \otimes W_k)].$$

Alternatives	*Indices* F_i
x_1	$F_1 = (.1267, .3332, .6078)$
x_2	$F_2 = (.0863, .2680, .5527)$
x_3	$F_3 = (.1493, .3829, .6877)$
x_4	$F_4 = (.1498, .3865, .6832)$

Table 6. Appropiateness indices

Exploitation Process

There exist many methods for ranking fuzzy numbers [15], in this case we shall use the Kim and Park [12] and Chang [3] methods to compute the ranking values of fuzzy appropriateness indices under a group of experts. Obtaining the following indices:

Alternatives	*Kim&Park*	Chang
x_1	.4540	.0856
x_2	.3828	.0705
x_3	.5132	.1094
x_4	.5151	.1084

Table 7. Ranking indices

Therefore, according to the *Kim&Park* order the best selection of technology transfer strategy is x_4, i.e., **"Cooperative R&D"**, while according to *Chang* order the best selection is x_3 , i.e., **"Licensing"**.

3.2 Solution based on 2-tuples

Aggregation Process

To carry out this process using linguistic 2-tuples, we use the process presented in [10]:

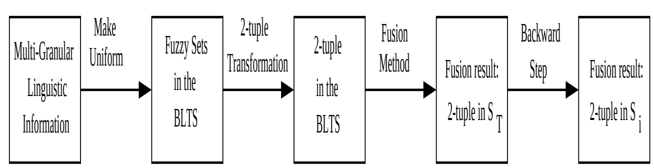

Figure 1: Fusion of multi-granularity linguistic information

The information is unified over the following term set, S_T:

s_0	$(0, 0, .07)$	s_1	$(0, .07, .14)$	s_2	$(.07, .14, .21)$
s_3	$(.14, .21, .28)$	s_4	$(.21, .28, .35)$	s_5	$(.28, .35, .42)$
s_6	$(.35, .42, .5)$	s_7	$(.42, .5, .58)$	s_8	$(.5, .58, .65)$
s_9	$(.58, .65, .72)$	s_{10}	$(.65, .72, .79)$	s_{11}	$(.72, .79, .86)$
s_{12}	$(.79, .86, .93)$	s_{13}	$(.86, .93, 1)$	s_{14}	$(.93, 1, 1)$

Being the preferences assessed as:

	p_1	p_2	p_3	p_4
C_1	$(s_7, 0)$	$(s_{13}, -.22)$	$(s_{10}, .5)$	$(s_7, 0)$
C_2	$(s_{10}, .5)$	$(s_{10}, .5)$	$(s_7, 0)$	$(s_9, .36)$
C_3	$(s_{13}, -.22)$	$(s_{10}, .5)$	$(s_{13}, -.22)$	$(s_9, .36)$
C_4	$(s_{10}, .5)$	$(s_7, 0)$	$(s_{10}, .5)$	$(s_7, 0)$

Table 8. Importance of the criteria, W_{tj}

	p_1	p_2	p_3	p_4
x_1	$(s_{10}, .5)$	$(s_{13}, -.22)$	$(s_{10}, .5)$	$(s_9, .36)$
x_2	$(s_3, .23)$	$(s_7, 0)$	$(s_3, .23)$	$(s_5, -.37)$
x_3	$(s_3, .23)$	$(s_3, .23)$	$(s_3, .23)$	$(s_7, 0)$
x_4	$(s_7, 0)$	$(s_7, 0)$	$(s_7, 0)$	$(s_2, .36)$

Table 9. Evaluation under C_1, S_{i1j}

	p_1	p_2	p_3	p_4
x_1	$(s_7, 0)$	$(s_1, .25)$	$(s_3, .23)$	$(s_2, .36)$
x_2	$(s_7, 0)$	$(s_3, .23)$	$(s_3, .23)$	$(s_9, .36)$
x_3	$(s_{10}, .5)$	$(s_{10}, .5)$	$(s_{10}, .5)$	$(s_9, .36)$
x_4	$(s_{10}, .5)$	$(s_{13}, -.22)$	$(s_{10}, .5)$	$(s_{12}, -.37)$

Table 10. Evaluation under C_2, S_{i2j}

	p_1	p_2	p_3	p_4
x_1	$(s_3, .23)$	$(s_1, .25)$	$(s_3, .23)$	$(s_2, .36)$
x_2	$(s_1, .25)$	$(s_3, .23)$	$(s_1, .21)$	$(s_7, 0)$
x_3	$(s_7, 0)$	$(s_7, 0)$	$(s_{10}, .5)$	$(s_9, .36)$
x_4	$(s_3, .23)$	$(s_7, 0)$	$(s_{10}, .5)$	$(s_5, -.37)$

Table 11. Evaluation under C_3, S_{i3j}

	p_1	p_2	p_3	p_4
x_1	$(s_{10}, .5)$	$(s_{13}, -.22)$	$(s_{10}, .5)$	$(s_9, .36)$
x_2	$(s_7, 0)$	$(s_7, 0)$	$(s_{10}, .5)$	$(s_{12}, -.37)$
x_3	$(s_7, 0)$	$(s_7, 0)$	$(s_7, 0)$	$(s_7, 0)$
x_4	$(s_7, 0)$	$(s_{10}, .5)$	$(s_3, .23)$	$(s_7, 0)$

Table 12. Evaluation under C_4, S_{i4j}

Now we are combining the 2-tuples using the arithmetic mean for 2-tuples:

$$S_{it} = \Delta(\frac{1}{n}\sum_{j=1}^{n} \beta_{itj}) = (s_{it}, \alpha_{it}),\ s_{it} \in S_T$$

$$W_t = \Delta(\frac{1}{n}\sum_{j=1}^{n} \beta_{tj}) = (w_t, \alpha_t),\ w_t \in S_T,$$

with $\beta_{itj} = \Delta^{-1}(S_{itj})$ and $\beta_{tj} = \Delta^{-1}(W_{tj})$.

Obtaining the following collective values:

S_{it}	C_1	C_2	C_3	C_4
x_1	$(s_{11}, -.21)$	$(s_3, .46)$	$(s_3, -.49)$	$(s_{11}, -.21)$
x_2	$(s_5, . - .5)$	$(s_6, -.3)$	$(s_3, .17)$	$(s_9, .03)$
x_3	$(s_4, .17)$	$(s_{10}, .21)$	$(s_8, .46)$	$(s_7, 0)$
x_4	$(s_6, -.16)$	$(s_{11}, .35)$	$(s_6, .34)$	$(s_7, -.07)$

Table 13. Fuzzy appropiatness rating of x_i under C_t

C_1	C_2	C_3	C_4
$(s_9, .32)$	$(s_9, .34)$	$(s_{11}, .35)$	$(s_9, -.25)$

Table 14. Average importance weight of C_t, W_t

The appropiateness index F_i for each alternative x_i is obtained with the following expression:

$$F_i = \Delta(\frac{\sum_{t=1}^{k} \beta_{it} \cdot \beta_{W_t}}{\sum_{t=1}^{k} \beta_{W_t}}),$$

with $\beta_i = \Delta^{-1}(s_i, \alpha_i)$ and $\beta_{w_i} = \Delta^{-1}(w_i, \alpha_i)$.

The indices obtained are:

	$F_i \in S_T$	$F_i \in A$	$F_i \in B$	$F_i \in C$
x_1	$(s_7, -.41)$	$(a_3, -.15)$	$(b_2, -.15)$	$(c_3, -.2)$
x_2	$(s_5, .42)$	$(a_3, -.43)$	$(b_2, -.43)$	$(c_2, .31)$
x_3	$(s_8, -.48)$	$(a_3, .17)$	$(b_2, .17)$	$(c_3, .24)$
x_4	$(s_8, -.45)$	$(a_3, .19)$	$(b_2, .19)$	$(c_3, .27)$

Table 15. F_i in the BLTS and initial domains.

Exploitation Process

This representation of the information, 2-tuples, has defined a total order over itself. Therefore, we can order the results without need of any computation.

According to the results of the *Table 15*, the best selection of technology transfer strategy is x_4, i.e., **"Cooperative R&D"**.

4 Comparative study

The problem of "Technology transfer strategy selection" has been solved using: (i) The approximative model based on the extension principle, and (ii) the model based on linguistic 2-tuples.

Obtaining the following results in the aggregation process:

	Ext. Prin.	2-tuples
x_1	$F_1 = (.1267, .3332, .6078)$	$(s_7, -.41)$
x_2	$F_2 = (.0863, .2680, .5527)$	$(s_5, .42)$
x_3	$F_3 = (.1493, .3829, .6877)$	$(s_8, -.48)$
x_4	$F_4 = (.1498, .3865, .6832)$	$(s_8, -.45)$

Table 16. Results with fuzzy numbers and 2-tuples

While in the explotation process we have obtained the following orders:

	Approx. model		2-tuples		
	K&P	Chang	A	B	C
x_1	.4540	.0856	$(a_3, -.15)$	$(b_2, -.15)$	$(c_3, -.2)$
x_2	.3828	.0705	$(a_3, -.43)$	$(b_2, -.43)$	$(c_2, .31)$
x_3	.5132	**.1094**	$(a_3, .17)$	$(b_2, 17)$	$(c_3, .24)$
x_4	**.5151**	.1084	$(\mathbf{a_3}, \mathbf{.19})$	$(\mathbf{b_2}, \mathbf{.19})$	$(\mathbf{c_3}, \mathbf{.27})$

Table 17. Solution set of alternatives

Our objective is to analyze the approximative and 2-tuple linguistic computational models from the following points of view:

1. Linguistic description.

2. Consistency of the results.

4.1 Linguistic description

From a review of the results of the table 17, we can observe that approximative model obtains real numbers for expressing the appropriatness of each alternative, these values are far from the initial expression domains used by the experts, while the 2-tuple model expresses its results with a near domain from the initial ones. Therefore the results obtained by the 2-tuple model are easier to understand by the experts.

4.2 Consistency Analysis

When we talk about consistency, it means to obtain the same solution from the same inputs. Following, we review the consistency of the different linguistic computational models:

1. *Approximative model.* We can see in table 17 that this model from the same inputs obtains different solution sets depending on the fuzzy ranking selected to order the collective preference values.

2. *2-tuple model.* This model always obtains the same solution set of alternatives indepently of the expression domain as it was proved in [10] and the ranking function is inherent to the structure of the 2-tuple.

5 Concluding Remarks

In this contribution in order to clarify the improvements of the linguistic 2-tuple computational model over the classical linguistic computational models, we have shown that the 2-tuple model is more descriptive and consistent than the model based on the Extension Principle included when it does not use linguistic approximation processes.

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