A Hierarchical Ordinal Model for Managing Unbalanced Linguistic Term Sets based on the Linguistic 2-tuple Model [0]

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Abstract

The use of the Fuzzy Linguistic Approach implies processes of Computing with Words. These processes can be carry out without loss of information using uniformly distributed linguistic term sets represented by a model based on linguistic 2-tuples. However there exist problems when linguistic values can not be adapted to a uniformly distributed ordinal scale. In this contribution we present a method that allows us to manage unbalanced linguistic term sets, such that, the processes of Computing with Words can be carried out without loss of information.

Keywords: linguistic information, unbalanced linguistic term set, computing with words.

1 Introduction

A lot of problems present qualitative or unrigorous aspects (decision making, scheduling, information retrieval, etc.). In these cases the use of the fuzzy linguistic approach [10] has shown itself as a good choice to model the qualitative aspects by means of linguistic variables (see [1], [9],...). These are variables whose values are not numbers but words or sentences in a natural or artificial language.

The use of linguistic variables always implies processes of Computing with Words (CW). Classical processes based on the Extension principle [2] and symbolic one [3] produce a loss of information and hence a lack of precision in the results. In [5] was presented a linguistic computational model based on linguistic 2-tuples that carries out processes of CW in a precise way when the linguistic term sets are symmetrical and uniformly distributed.

However, we can find problems whose linguistic labels are not uniformly distributed, i.e., unbalanced linguistic term sets [7, 8]. As the following example of the school grading system:

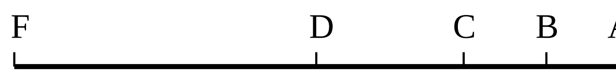

Figure 1: School grading system

With these linguistic values, any linguistic computational model lose information in the processes of CW.

The aim of this contribution is to develop a methodology to choice the semantics of linguistic term sets non symmetrically distributed, operating without loss of information using the linguistic 2-tuple computional model. To do so, we shall use hierarchical linguistic contexts based on the linguistic 2-tuple representation model [6].

In order to do that, the contribution is structured as follows. Section 2 reviews the fuzzy linguistic approach and the 2-tuple linguistic representation model. Section 3 introduces the hierarchical linguistic contexts. Section 4 presents a process to manage unbalanced linguistic term sets. Section 5 solves a education grading system problem. And finally some concluding remarks are pointed out.

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2 Preliminaries

2.1 Fuzzy Linguistic Approach

Usually, we work in a quantitative setting, where the information is expressed by means of numerical values. However, many aspects of different activities in the real world cannot be assessed in a quantitative form, but rather in a qualitative one, i.e., with vague or imprecise knowledge. In that case a better approach may be to use linguistic assessments instead of numerical values. The fuzzy linguistic approach represents qualitative aspects as linguistic values by means of linguistic variables [10].

We have to choose the appropriate linguistic descriptors for the term set and their semantics. In the literature, several possibilities can be found (see [4] for a wide description). In order to accomplish this objective, an important aspect to analyse is the *"granularity of uncertainty"*, i.e., the level of discrimination among different counts of uncertainty. One possibility of generating the linguistic term set consists of directly supplying the term set by considering all terms distributed on a scale on which a total order is defined [9]. For example, a set of seven terms S, could be given as follows:

$$S = \{s_0 : N, s_1 : VL, s_2 : L, s_3 : M, s_4 : H, s_5 : VH, s_6 : P\}$$

Usually, in these cases, it is required that in the linguistic term set there exist:

1) A negation operator: Neg$(s_i) = s_j$ such that $j = g$-i (g+1 is the cardinality).

2) An order: $s_i \leq s_j \Longleftrightarrow i \leq j$. Therefore, there exists a min and a max operator.

The semantics of the linguistic terms is given by fuzzy numbers defined in the [0,1] interval. A computationally efficient way to characterize a fuzzy number is to use a representation based on parameters of its membership function, considering trapezoidal, triangular or gaussian membership functions. Figure 2 presents the above linguistic terms set with labels uniformly distributed as triangular membership functions:

$$\begin{array}{lll} VH = (.67, .83, 1) & P = (.83, 1, 1) & \\ M = (.33, .5, .67) & H = (.5, .67, .83) & \\ N = (0, 0, .17) & VL = (0, .17, .33) & L = (.17, .33, .5) \end{array}$$

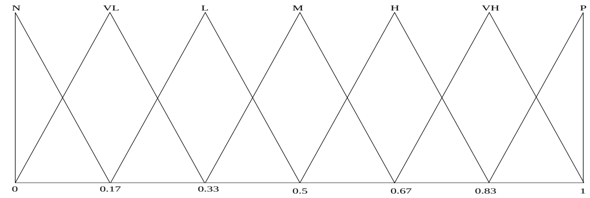

Figure 2: A Set of Seven Terms with its Semantics.

2.2 Linguistic Representation Model Based on 2-tuples

This representation model was presented in [5] and it is based on the concept of symbolic translation and use it for representing the linguistic information by means of 2-tuples, (s_i, α), where s is a linguistic term and α is a numerical value representing the symbolic translation.

Let $S = \{s_0, ..., s_g\}$ be a linguistic term set, and $\beta \in [0, g]$ a numerical value in its interval of granularity (e.g.: let β be a value obtained from a symbolic aggregation operation).

Definition 1. *Let β be the result of an aggregation of the indexes of a set of labels assessed in a linguistic term set S, i.e., the result of a symbolic aggregation operation. $\beta \in [0, g]$, being $g + 1$ the cardinality of S. Let $i = round(\beta)$ and $\alpha = \beta - i$ be two values, such that, $i \in [0, g]$ and $\alpha \in [-.5, .5)$ then α is called a Symbolic Translation.*

From this concept we develop a linguistic representation model which represents the linguistic information by means of 2-tuples (r_i, α_i), $r_i \in S$ and $\alpha_i \in [-.5, .5)$. r_i represents the linguistic label of the information and α_i is the value of the Symbolic Translation.

This representation model defines a set of functions to facilitate computational processes over 2-tuples.

Definition 2. *Let $S = \{s_0, ..., s_g\}$ be a linguistic term set and $\beta \in [0, g]$ a value supporting the result of a symbolic aggregation operation, then the 2-tuple that expresses the equivalent information to β is obtained with the following function:*

$$\Delta : [0, g] \longrightarrow S \times [-0.5, 0.5)$$

$$\Delta(\beta) = (s_i, \alpha), \ with \begin{cases} s_i & i = round(\beta) \\ \alpha = \beta - i & \alpha \in [-.5, .5) \end{cases}$$

where round(·) is the usual round *operation,* s_i *has the closest index label to "*β*" and "*α*" is the value of the symbolic translation.*

Proposition 1. *Let* $S = \{s_0, ..., s_g\}$ *be a linguistic term set and* (s_i, α) *be a 2-tuple. There is always a* Δ^{-1} *function, such that, from a 2-tuple it returns its equivalent numerical value* $\beta \in [0, g] \subset \mathcal{R}$.

Proof.

It is trivial, we consider the following function:

$$\Delta^{-1} : S \times [-.5, .5) \longrightarrow [0, g]$$

$$\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta$$

Remark: From definitions 1 and 2 and from proposition 1, it is obvious that the conversion of a linguistic term into a linguistic 2-tuple consist of adding a value 0 as symbolic translation: $s_i \in S \Longrightarrow (s_i, 0)$.

Different operators over linguistic 2-tuples can be reviewed in [5].

3 Hierarchical Linguistic Contexts

The hierarchical linguistic contexts were introduced in [6] to improve the precision of processes of CW in multigranular linguistic contexts.

Here we use the hierarchical linguistic contexts to build the semantics of unbalanced linguistic term sets as the presented in *Figure 1*.

The following subsections introduce the linguistic hierarchical structure and the way to build it.

3.1 Linguistic Hierarchical Structure

A *Linguistic Hierarchy* is a set of levels, where each level is a linguistic term set with different granularity to the remaining levels. Each level is denoted as:

$$l(t, n(t)),$$

being,

1. t, a number that indicates the level of the hierarchy,
2. *n(t)*, the granularity of the linguistic term set of the level t.

Here, we must point out that in this paper we deal with levels containing linguistic terms whose membership functions are triangular-shaped, symmetrical and uniformly distributed in $[0, 1]$. In addition, the linguistic term sets have an odd value of granularity representing the central label the value of *indifference*.

The levels belonging to a linguistic hierarchy are ordered according to their granularity, i.e., for two consecutive levels t and $t+1$, $n(t+1) > n(t)$. This provides a refinement of the previous level.

From the above concepts, we define a linguistic hierarchy, LH, as the union of all levels t:

$$LH = \bigcup_t l(t, n(t))$$

We are going to show a methodology to build linguistic hierarchies.

3.2 Building Linguistic Hierarchies

To build a linguistic hierarchy, we must take into account that its hierarchical order is given by the increase of the granularity of the linguistic term sets in each level.

We start from a linguistic term set, S, over the universe of the discourse U in the level t:

$$S = \{s_0, ..., s_{n(t)-1}\}$$

being $s_k, (k = 0, ..., n(t) - 1)$ a linguistic term of S. We extend the definition of S to a set of linguistic term sets, $S^{n(t)}$, each term set belongs to a level t of the hierarchy and has a granularity of uncertainty $n(t)$:

$$S^{n(t)} = \{s_0^{n(t)}, ..., s_{n(t)-1}^{n(t)}\}$$

And afterwards, we develop a methodology which satisfies the following rules, that we call, *linguistic hierarchy basic rules*:

1. To preserve all *former modal points* of the membership functions of each linguistic term from one level to the following one.
2. To make *smooth transitions between successive levels.* The aim is to build a new linguistic term set, $S^{n(t+1)}$. A new linguistic term will be added between each pair of terms belonging to the term set of the previous level t. To carry out this insertion, we shall reduce the support of the linguistic labels in order to keep place for the new one located in the middle of them.

Generically, we can say that the linguistic term set of level $t+1$ is obtained from its predecessor as:

$$l(t, n(t)) \rightarrow l(t+1, 2 \cdot n(t) - 1)$$

A graphical example of a linguistic hierarchy is shown in Figure 3:

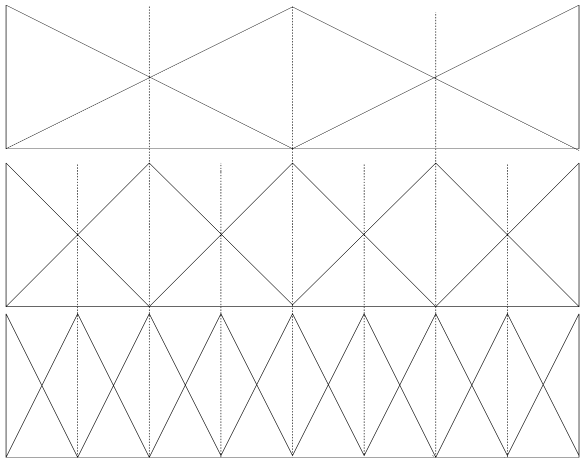

Figure 3: Linguistic Hierarchy of 3, 5 and 9 labels

In [6], transformation functions between labels of different levels were developped. These transformations functions allows make processes of CW in multigranular linguistic contexts without loss of information.

Definition 3 [6]. *Let $LH = \bigcup_t l(t, n(t))$ be a linguistic hierarchy whose linguistic term sets are denoted as $S^{n(t)} = \{s_0^{n(t)}, ..., s_{n(t)-1}^{n(t)}\}$, and let us consider the 2-tuple linguistic representation. The transformation function from a linguistic label in level t to a label in level t+1, satisfying the linguistic hierarchy basic rules, is defined as:*

$$TF_{t+1}^{t} : l(t, n(t)) \longrightarrow l(t+1, n(t+1))$$

$$TF_{t+1}^{t}(s_i^{n(t)}, \alpha^{n(t)}) = \Delta(\frac{\Delta^{-1}(s_i^{n(t)}, \alpha^{n(t)}) \cdot (n(t+1) - 1)}{n(t) - 1})$$

Definition 4 [6]. *Let $LH = \bigcup_t l(t, n(t))$ be a linguistic hierarchy whose linguistic term sets are denoted as $S^{n(t)} = \{s_0^{n(t)}, ..., s_{n(t)-1}^{n(t)}\}$, and let us consider the 2-tuple linguistic representation. The transformation function from a linguistic label in level t to a label in level t-1, satisfying the linguistic hierarchy basic rules, is defined as:*

$$TF_{t-1}^{t} : l(t, n(t)) \longrightarrow l(t-1, n(t-1))$$

$$TF_{t-1}^{t}(s_i^{n(t)}, \alpha^{n(t)}) = \Delta(\frac{\Delta^{-1}(s_i^{n(t)}, \alpha^{n(t)}) \cdot (n(t-1) - 1)}{n(t) - 1})$$

These transformation functions can be generalized to transform linguistic terms between any linguistic level in the linguistic hierarchy.

Definition 5 [6]. *Let $LH = \bigcup_t l(t, n(t))$ be a linguistic hierarchy whose linguistic term sets are denoted as $S^{n(t)} = \{s_0^{n(t)}, ..., s_{n(t)-1}^{n(t)}\}$. The recursive transformation function between a linguistic label that belongs to level t and a label in level t'=t+a, with $a \in Z$, is defined as:*

$$TF_{t'}^{t} : l(t, n(t)) \longrightarrow l(t', n(t'))$$

$If\ |a| > 1\ then$

$$TF_{t'}^{t}(s_i^{n(t)}, \alpha^{n(t)}) = TF_{t'}^{t+\frac{t-t'}{|t-t'|}}(TF_{t+\frac{t-t'}{|t-t'|}}^{t}(s_i^{n(t)}, \alpha^{n(t)}))$$

$If\ |a| = 1\ then$

$$TF_{t'}^{t}(s_i^{n(t)}, \alpha^{n(t)}) = TF_{t+\frac{t-t'}{|t-t'|}}^{t}(s_i^{n(t)}, \alpha^{n(t)})$$

This recursive transformation function can be easily defined in a non recursive way as follows:

$$TF_{t'}^{t} : l(t, n(t)) \longrightarrow l(t', n(t'))$$

$$TF_{t'}^{t}(s_i^{n(t)}, \alpha^{n(t)}) = \Delta(\frac{\Delta^{-1}(s_i^{n(t)}, \alpha^{n(t)}) \cdot (n(t') - 1)}{n(t) - 1})$$

The following result guarantees the transformations between levels of a linguistic hierarchy are carried out without loss of information.

Proposition 2[6]. The transformation function between linguistic terms in different levels of the linguistic hierarchy is bijective:

$$TF_{t}^{t'}(TF_{t'}^{t}(s_i^{n(t)}, \alpha^{n(t)})) = (s_i^{n(t)}, \alpha^{n(t)})$$

4 Semantics of Unbalanced Linguistic Terms Sets based on Linguistic Hierarchies

Here we propose a method to build the semantics of the unbalanced linguistic term set, such that, the linguistic 2-tuple guarantees the precision of the results obtained in the processes of CW.

To do so, we shall use the linguistic hierarchies. The method consists of choosing the semantics of the linguistic terms from different levels of a linguistic hierarchy, accoding to the support of the linguistic terms.

Now, we present an example to a better comprehension of the process. Let us suppose we start from a problem using the linguistic term set shown in Figure 1. According to our process we must choose an adequate linguistic hierarchy as the shown in Figure 3 and select the semantics from the initial terms in different levels.

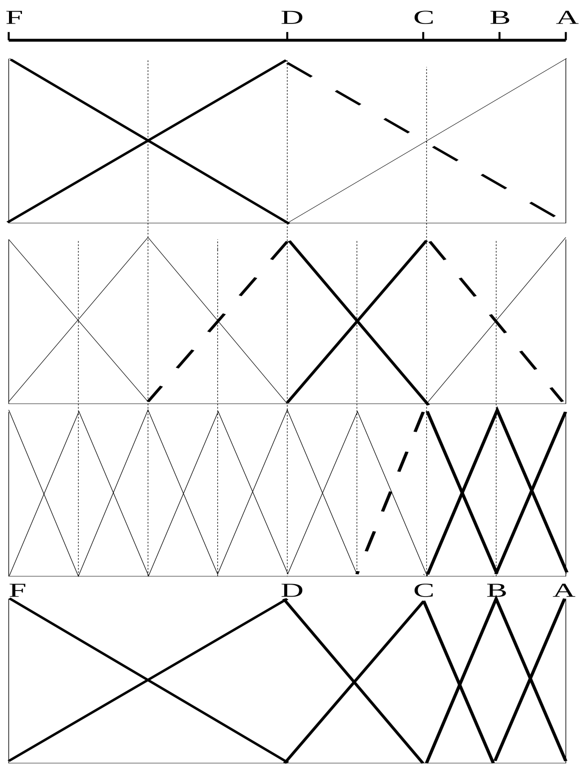

Figure 4: Semantics for a unbalanced term set

Figure 4 shows how we can choose the semantics of the linguistic terms with different support using the different levels of the hierarchy. In this example, we observe that:

- Label **F** is similar to the first label of level one, s_0^3.
- Label **D** is builded from s_1^3 in its upsize and from s_2^5 in its downsize.
- Label **C** is from s_3^5 in its upsize and from s_6^9 in its downsize.
- Labels **B** and **A** are represented by s_7^9, s_8^9 respectively.

Afterwards, we can use the computational technique designed for linguistic 2-tuples and the linguistic hierarchies for designing an aggregation process. To do so, we carry out the following steps:

1. First, linguistic terms of the unbalanced linguistic term set are transformed into a final level (usually with highest granularity, $l(3,9)$ in our example).
2. The 2-tuple computational model is used to make the process of CW.
3. Finally, once it is obtained a result, it is transformed to the correspondent level for expressing the result in the unbalanced linguistic term set.

5 Example: Evaluation from Several Tests

An usual problem in Education is to evaluate different tests to obtain a global evaluation. Let us suppose two pupils have made five tests evaluated by means of the scale presented in Figure 1 and the teacher have to obtain the global evaluation taking into account all the tests are equally important.

John Smith	D	C	B	C	C	C
Martina Johnson	A	D	D	C	B	A

To do so, we shall use the semantics obtained in Figure 4, the transformation functions among levels of the linguistic hierarchy and the 2-tuple computational model.

Firstly, we transform the partial evaluations into the level with nine terms of the hierarchy, using linguistic 2-tuples:

J.S.	$(s_4^9, 0)$	$(s_6^9, 0)$	$(s_7^9, 0)$	$(s_6^9, 0)$	$(s_6^9, 0)$	$(s_6^9, 0)$
M.J.	$(s_8^9, 0)$	$(s_4^9, 0)$	$(s_4^9, 0)$	$(s_6^9, 0)$	$(s_7^9, 0)$	$(s_8^9, 0)$

To obtain the global evaluation for each pupil, we shall use the arithmetic mean operator for 2-tuples, due to the fact all the tests are equally important.

Definition 6 [5]. *Let $x = \{(r_1, \alpha_1), \ldots, (r_n, \alpha_n)\}$ be a set of 2-tuples, the 2-tuple arithmetic mean $\overline{x}^e$ is computed as,*

$$\overline{x}^e = \Delta(\sum_{i=1}^{n} \frac{1}{n} \Delta^{-1}(r_i, \alpha_i)) = \Delta(\frac{1}{n} \sum_{i=1}^{n} \beta_i)$$

We obtain the following global evaluations for each pupil in the third level of the linguistic hierarchy:

John Smith	$(s^9_6, -.16)$
Martina Johnson	$(s^9_6, .16)$

Now these results must be transformed into linguistic values of the educational grading system:

- the value obtained by John is on the upsize of the label C, therefore it must be transformed into the label s^5_3, while
- the value obtained by Martina is on the downsize of the label C, therefore it has not to be transformed because it is represented at the corresponding level.

Hence we obtain the following global evaluation on the unbalanced linguistic term set:

John Smith	$(C, -.08)$
Martina Johnson	$(C, .16)$

6 Concluding Remarks

In this contribution we have developed a method to select the semantics of linguistic labels in problems whose labels are not uniformly distributed. This method build a semantics that allows us to make processes of CW without loss of information. To do so, we have used the linguistic 2-tuple representation model and the linguistic hierarchical contexts together their respective operational techniques.

This method can be useful in a lot of problems that deals with unbalanced linguistic term sets, to improve their final results.

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