Group Decision Making Based on the Linguistic 2-tuple Model in Heterogeneous Contexts

Herrera F.
Dept. of Computer Science and A.I. University of Granada
18071 Granada - Spain
Email: herrera@decsai.ugr.es

Martínez L.[0]
Dept. of Computer Science. University of Jaén
23071 - Jaén, Spain
Email: martin@ujaen.es

Keywords: decision making, interval valued, intuitionistic fuzzy sets, linguistic variables, aggregation

Abstract: Lot of activities carried out in the enterprise implies Group Decision Making processes. In Group Decision Making is difficult that all experts have an exact knowledge about the problem. At the begining, Group Decision Making problems manage uncertainty with real values within a predefined range, soon interval valued approaches were proposed and more recently fuzzy-interval valued and linguistic approaches have obtained successfull results. In this paper, we shall deal with Group Decision Making problems in which the experts can express their knowledge over the alternatives using different types of information: numerical, interval valued, fuzzy-interval valued or the linguistic one, that is called *Heterogeneous Information*. The main problem to deal with heterogeneous information is: *how to aggregate it?*. The aim of the contribution is to develop an aggregation method able to combine all different types of information in the decision process. To do so, we shall use the the linguistic 2-tuple representation model.

1 Introduction

In the enterprise processes, there exist a wide range of activities that can involve imprecision and vague information. In this contribution, we focus in Group Decision Making (GDM) problems that consist of a decision situation in which two or more individuals express their preferences over some set of alternatives to obtain a solution (an alternative or set of alternatives). It is supposed there is a finite set of alternatives $X = \{x_1, ..., x_n\}$ $n \geq 2$, as well as a finite set of experts $E = \{e_1, ..., e_m\}$ $m \geq 2$. In the fuzzy context, the resolution process starts from, a set of fuzzy preference relations where each expert e_k provides his/her preferences on X, i.e. $P_{e_k}(x_i, x_j) = p_{ij}^k$ the degree of preference of alternative x_i over x_j, to obtain a solution either from the individual preference relations, without constructing a social preference relation, or by computing first a social fuzzy preference relation and then using it to find a solution (Kacprzyk and Fredizzi, 1986). Any of the above approaches, called direct and indirect approaches respectively, the resolution process for reaching a solution of the GDM problems is composed by two phases (Roubens, 1997):

1. Aggregation phase: that combines the expert preferences, and
2. Exploitation one: that obtains a solution set of alternatives from a preference relation.

The nature of the preference values , p_{ij}^k, provided by the experts depends on the knowledge of the experts over the alternatives. This knowledge is not precise and usually present uncertainty. Early this uncertainty were expressed in the preference values by means of real values assessed in a predefined range (Kacprzyk and Fredizzi, 1986; Yager, 1988), soon another approaches based on interval valued (Kundu, 1997; Kuchta, 2000; Téno and Mareschal, 1998), fuzzy-interval valued (Atanassov, 1999; Szmidt and Kacprzyk, 1996) and linguistic one (Buckley, 1984; Delgado et al., 1993) were proposed.

The most of the proposals for solving GDM problems are focused in cases where all the experts provide their preferences in an unique way, either real values, or interval values, or fuzzy-interval values, or linguistic labels. But not always, all the experts involve in the GDM problem can express their preferences in the same way, it could be each one express his/her preferences with different types of information: real values, interval-valued, fuzzy-interval val-

[0]This work has been supported by Research Project PB98-1305

ues, linguistic labels, that we shall call as *Heterogeneous Information*. Hence, we say that the GDM problem is defined in a heterogeneous context.

The main difficulty for dealing with GDM problems defined in a heterogenous context is how to aggregate the preferences?. Because of, there not exist operators or processes for combining that type of information.

The linguistic 2-tuple model (Herrera and Martínez, 1999b) has shown itself as a good choice to manage non-homogeneous information in aggregation processes (Herrera and Martínez, 1999a; Herrera and Martínez, 2000). According to the ideas expressed in those papers, we propose an improved aggregation process that is able to manage preferences expressed by means of *numercial values, linguistic labels, interval-valued* and *Intuitionistic Fuzzy Sets*. This aggregation process will be developed according to the following scheme:

1. The heterogeneous information is unified by means of fuzzy sets on a specific domain, called Basic Linguistic Term Set (BLTS).

2. The fuzzy sets will be aggregated by means of an aggregation operator to obtain collective preference fuzzy sets.

3. These collective fuzzy sets will be transformed into linguistic 2-tuples.

Once the heterogeneous information has been aggregated and expressed by means of linguistic 2-tuples, the exploitation step of the GDM process (rank alternatives) is easy to carry out for obtaining a solution set of alternative/s.

In order to do that, this paper is structured as follows: in Section 2 we shall review different approaches to express the preferences in the decision making problems and the linguistic 2-tuple representation model; in Section 3 we shall propose an aggregation process for combining heterogeneous information; in Section 4 we shall present an example of a GDM problem defined in a heterogeneous context and finally, some concluding remarks are pointed out.

2 Preliminaries

In decision making problems the experts express their preferences depending on their knowledge over the alternatives by means of preference relations. In this section, we shall review different approaches that we can find in the literature to express those preferences. And afterwards, we shall review the 2-tuple linguistic representation model, that plays a key role in the aggregation process proposed in this contribution.

2.1 Approaches for Modelling Preferences

2.1.1 Fuzzy Binary Relations

A (fuzzy) binary relation R on X is defined as a fuzzy subset of the direct product $X \times X$ with values in $[0,1]$, i.e, $R : X \times X \rightarrow [0,1]$, where, $R(x_i, x_j) = p_{ij}$, denotes the degree to which $x_i R x_j$. Particularly, in preference analysis, p_{ij}, denotes the *degree to which an alternative x_i is preferred to alternative x_j* (Kacprzyk and Fredizzi, 1986; Yager, 1988).

2.1.2 Interval-valued Relations

The approaches based on fuzzy binary relations had serious problems, in particular it has been argued that the most of experts are unable to make a fair estimation of the inaccuracy of their judgements, making far larger estimation errors that the boundaries accepted by them as feasible (de Mántaras and Godó, 1997).

An approach to overcome this problem is to add some flexibility to the uncertainty representation problem by means of interval-valued relations:

$$R : X \times X \rightarrow \wp([0,1]).$$

Where $R(x_i, x_j) = p_{ij}$ denotes the interval-valued preference degree of the alternative x_i over x_j . In these approaches (Kundu, 1997; Kuchta, 2000; Téno and Mareschal, 1998), the preferences provided by the experts consist of interval values assessed in $\wp([0,1])$, where the preference is expressed as $[\underline{a}, \overline{a}]_{ij},\ with\ \underline{a} \leq \overline{a}$.

2.1.3 Intuitionistic Fuzzy Sets

The Intuitionistic Fuzzy Sets (IFS) (Atanassov, 1999; Szmidt and Kacprzyk, 1996) are a tool based on fuzzy sets used to represent uncertainty.

Definition 1. (Atanassov, 1999) *An IFS A in E is defined as an object of the following form:*

$$A = \{< x, \mu_a(x), \nu_A(x) > / x \in E\}$$

where the function:

$$\mu_A(x) : E \rightarrow [0,1]$$

and,

$$\nu_A(x) : E \rightarrow [0,1]$$

define the degree of membership and the degree of non-membership of the element $x \in E$, respectively. And for every $x \in E$:

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1$$

Definition 2. (Atanassov, 1999)*The value of,*

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$$

is called the degree of non-determinacy (or uncertainty) of the element $x \in E$ to the IFS A.

In (Kreinovich et al., 1999) is showed that an IFS is equivalent to an interval composed by two real numbers.Therefore, in this contribution when we shall deal indistinctly with IFS or interval-valued preferences.

2.1.4 Linguistic Approach

Usually, we work in a quantitative setting, where the information is expressed by means of numerical values. However, many aspects of different activities in the real world cannot be assessed in a quantitative form, but rather in a qualitative one, i.e., with vague or imprecise knowledge. In that case a better approach may be to use linguistic assessments instead of numerical values. The fuzzy linguistic approach represents qualitative aspects as linguistic values by means of linguistic variables (Zadeh, 1975). This approach is adequate in some situations where the information may be unquantifiable due to its nature, and thus, it may be stated only in linguistic terms.

We have to choose the appropriate linguistic descriptors for the term set and their semantics. In order to accomplish this objective, an important aspect to analyse is the *"granularity of uncertainty"*, i.e., the level of discrimination among different counts of uncertainty. Typical values of cardinality used in the linguistic models are odd ones, such as 7 or 9, where the mid term represents an assessment of "approximately 0.5", and with the rest of the terms being placed symmetrically around it (Bonissone and Decker, 1986).

One possibility of generating the linguistic term set consists of directly supplying the term set by considering all terms distributed on a scale on which a total order is defined. For example, a set of seven terms S, could be given as follows:

$$\{s_0 : N, s_1 : VL, s_2 : L, s_3 : M, s_4 : H, s_5 : VH, s_6 : P\}.$$

Usually, in these cases, it is required that in the linguistic term set there exist:

1. A negation operator: Neg$(s_i) = s_j$ such that $j = g$-i (g+1 is the cardinality).
2. $s_i \leq s_j \iff i \leq j$. Therefore, there exists a *min* and a *max* operator.

The semantics of the linguistic terms is given by fuzzy numbers defined in the [0,1] interval. A way to characterize a fuzzy number is to use a representation based on parameters of its membership function (Bonissone and Decker, 1986). The linguistic assessments given by the users are just approximate ones, some authors consider that linear trapezoidal membership functions are good enough to capture the vagueness of those linguistic assessments. The parametric representation is achieved by the 4-tuple (a, b, d, c), where b and d indicate the interval in which the membership value is 1, with a and c indicating the left and right limits of the definition domain of the trapezoidal membership function (Bonissone and Decker, 1986). A particular case of this type of representation are the linguistic assessments whose membership functions are triangular, i.e., $b = d$, then we represent this type of membership functions by a 3-tuple (a, b, c). An example may be the following (Figure 1) :

$N = (0, 0, .17)$ $VL = (0, .17, .33)$
$L = (.17, .33, .5)$ $M = (.33, .5, .67)$
$H = (.5, .67, .83)$ $VH = (.67, .83, 1)$
$P = (.83, 1, 1)$.

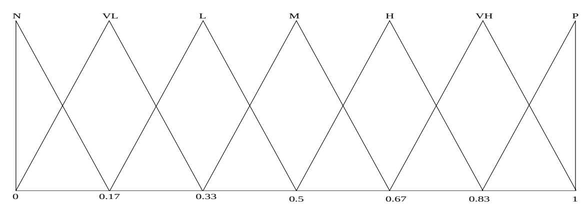

Figure 1: A set of seven linguistic terms with its semantics

Other authors use a non-trapezoidal representation, e.g., Gaussian functions (Bordogna and Passi, 1993).

2.2 The 2-tuple Linguistic Representation Model

In this subsection we review the 2-tuple linguistic representation model, presented in (Herrera and Martínez, 1999b), that we shall use to manage the heterogeneous information, therefore it plays a central role in the aim of this contribution.

This linguistic model takes as a basis the symbolic model and in addition defines the concept of Symbolic Translation and uses it to represent the linguistic information by means of a pair of values called linguistic 2-tuple, (s, α), where s is a linguistic term and α is a numeric value representing the symbolic translation.

Definition 3. *Let β be the result of an aggregation of the indexes of a set of labels assessed in a linguistic term set S, i.e., the result of a symbolic aggregation operation. $\beta \in [0, g]$, being $g + 1$ the cardinality of S. Let $i = round(\beta)$ and $\alpha = \beta - i$ be two values, such that, $i \in [0, g]$ and $\alpha \in [-.5, .5)$ then α is called a Symbolic Translation.*

This model defines a set of transformation functions between numeric values and linguistic 2-tuples.

Definition 4. *Let $S = \{s_0, ..., s_g\}$ be a linguistic term set and $\beta \in [0, g]$ a value representing the result of a symbolic aggregation operation, then the 2-tuple that expresses the equivalent information to β is obtained with the following function:*

$$\Delta : [0, g] \longrightarrow S \times [-0.5, 0.5)$$
$$\Delta(\beta) = (s_i, \alpha), \ with \left\{ \begin{array}{cc} s_i & i = round(\beta) \\ \alpha = \beta - i & \alpha \in [-.5, .5) \end{array} \right.$$

where $round(\cdot)$ is the usual round *operation, s_i has the closest index label to "β" and "α" is the value of the symbolic translation.*

Proposition 1.*Let $S = \{s_0, ..., s_g\}$ be a linguistic term set and (s_i, α) be a 2-tuple. There is always a Δ^{-1} function, such that, from a 2-tuple it returns its equivalent numerical value $\beta \in [0, g] \subset \mathcal{R}$.*

Proof. It is trivial, we consider the following function:

$$\Delta^{-1} : S \times [-.5, .5) \longrightarrow [0, g]$$
$$\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta$$

Remark: From definitions 2 and 3 and from proposition 1, it is obvious that the conversion of a linguistic term into a linguistic 2-tuple consist of adding a value 0 as symbolic translation: $s_i \in S \Longrightarrow (s_i, 0)$

Toghether with the 2-tuple representation model was developed a computational technique to operate with the 2-tuples without loss of information (Herrera and Martínez, 1999b).

3 Aggregation Process for a Heterogeneous GDM problem

In this section we present our purpose to carry out the *aggregation phase* of a decision making process in a GDM problem defined in a heterogeneous context.

A GDM problem defined in a heterogeneous context has a finite set of alternatives, $X = \{x_1, x_2, \ldots, x_n\}$ $(n \geq 2)$, as well as a finite set of experts $E = \{e_1, e_2, \ldots, e_m\}$ $(m \geq 2)$. Each expert, $e_k \in E$, provides his/her preferences on X using preference relations. We focus in GDM problems in which the preference relations provided by the experts can be:

1. Fuzzy preference relations (Kacprzyk and Fredizzi, 1986), $P^n_{e_k} : X \times X \rightarrow [0, 1]$, where $P^n_{e_k}(x_i, x_j) = p^k_{ij}$ denotes the preference degree of the alternative x_i over x_j provided by the expert e_k.
2. Interval-valued preference relation (Téno and Mareschal, 1998), $P^I_{e_k} : X \times X \rightarrow \wp([0, 1])$, where $P^I_{e_k}(x_i, x_j) = p^k_{ij}$ denotes the interval-valued preference degree of the alternative x_i over x_j provided by the expert e_k. These relations can be obtained from preferences provided by experts using IFS.
3. Linguistic preference relation assessed in a pre-established label set (Herrera et al., 1995), $S = \{s_0, \ldots, s_g\}$, $P^S_{e_k} : X \times X \rightarrow S$, where $P^S_{e_k}(x_i, x_j) = p^k_{ij}$ denotes the preference degree of the alternative x_i over x_j linguistically expressed provided by the expert e_k.

Following, we present our proposal for combining this heterogeneous information. This aggregation process is composed by the following phases:

1. *Making the information uniform.* The heterogeneous information will be unified into a specific linguistic domain, which is a *Basic Linguistic Term Set* (BLTS). Each numerical, interval-valued and linguistic performance value is expressed by means of a fuzzy set on the BLTS, $F(S_T)$. The process is carried out in the following order:
   (a) Transforming numerical values in $[0, 1]$ into $F(S_T)$.
   (b) Transforming linguistic terms into $F(S_T)$.
   (c) Transforming interval-valued into $F(S_T)$.
2. *Aggregating individual performance values.* For each alternative, a collective performance value is obtained aggregating the above fuzzy sets on the BLTS, that represents the individual performance values assigned by the experts according to his/her preference. Therefore, each collective performance value is a new fuzzy set on the specific linguistic domain, the BLTS.

   It is clear that the information must be unified to be manageable. The fuzzy sets are useful to unify and aggregate the information at the beggining, but in processes of decision making (exploitation phase) that the preference values must be ranked are not a good solution. In (Herrera and Martínez, 1999a; Herrera and Martínez, 2000) were shown that the conversion of the fuzzy sets into linguistic 2-tuple provides good results.
3. *Transforming fuzzy sets into linguistic 2-tuples.* The collective performance values (fuzzy sets) are transformed into linguistic 2-tuples in the BLTS. Obtaining a collective preference relation expressed by means of linguistic 2-tuples.

Following we shall show in deep each step of the different phases of the aggregation process.

3.1 Making the information uniform

Firstly, the heterogeneous information is unified in an unique expression domain, the BLTS. Before to unify the heterogeneous information, we have to decide how to choose the BLTS, S_T. We study the linguistic term set S that belongs to the definition context of the GDM problem. If:

1. *S is a fuzzy partition* (Ruspini, 1969),
2. *and the membership functions of its terms are triangular,* i.e., $s_i = (a_i, b_i, c_i)$

then we select S as BLTS, due to the fact that, these conditions are necessary and sufficient for the transformation between values in $[0,1]$ and 2-tuples, being them carried out without loss of information (Herrera and Martínez, 2000).

If the linguistic term set S, used in the definition context of the problem, does not satisfy the above conditions then we shall choose as BLTS a term set with a larger number of terms than the number of terms that a person is able to discriminate (normally 11 or 13, see (Bonissone and Decker, 1986)) and satisfies the above conditions. We choose the BLTS with 15 terms symmetrically distributed, with the following semantics (graphically, Figure 2).

s_0	(0,0,0.07)	s_1	(0,0.07,0.14)
s_2	(0.07,0.14,0.21)	s_3	(0.14,0.21,0.28)
s_4	(0.21,0.28,0.35)	s_5	(0.28,0.35,0.42)
s_6	(0.35,0.42,0.5)	s_7	(0.42,0.5,0.58)
s_8	(0.5,0.58,0.65)	s_9	(0.58,0.65,0.72)
s_{10}	(0.65,0.72,0.79)	s_{11}	(0.72,0.79,0.86)
s_{12}	(0.79,0.86,0.93)	s_{13}	(0.86,0.93,1)
s_{14}	(0.93,1,1)		

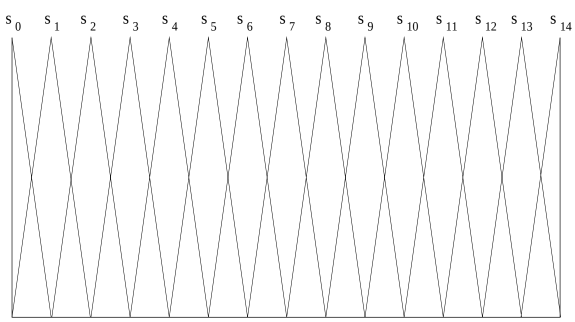

Figure 2: A BLTS with 15 terms symmetrically distributed

Once we have chosen the BLTS we shall define the transformation functions that unify the heterogeneous information by means of fuzzy sets over the BLTS, denoted as $F(S_T)$.

3.1.1 Transforming numerical values in $[0,1]$ into $F(S_T)$.

Let $F(S_T)$ be the set of fuzzy sets in $S_T = \{s_0, \ldots, s_g\}$, we shall transform a numerical value $\vartheta \in [0,1]$ into a fuzzy set in $F(S_T)$ computing the membership value of ϑ in the membership functions associated with the linguistic terms of S_T.

Definition 5. (Herrera and Martínez, 2000) *The function τ transforms a numerical value into a fuzzy set in S_T:*

$$\tau : [0,1] \rightarrow F(S_T)$$
$$\tau(\vartheta) = \{(s_0, \gamma_0), ..., (s_g, \gamma_g)\}, s_i \in S_T \, and \, \gamma_i \in [0,1]$$
$$\gamma_i = \mu_{s_i}(\vartheta) = \begin{cases} 0, & if\, \vartheta \notin Support(\mu_{s_i}(x)) \\ \frac{\vartheta - a_i}{b_i - a_i}, & if\, a_i \leq \vartheta \leq b_i \\ 1, & if\, b_i \leq \vartheta \leq d_i \\ \frac{c_i - \vartheta}{c_i - d_i}, & if\, d_i \leq \vartheta \leq c_i \end{cases}$$

Remark: We consider membership functions, $\mu_{s_i}(\cdot)$, for linguistic labels, $s_i \in S_T$, that achieved by a parametric function (a_i, b_i, d_i, c_i). A particular case are the linguistic assessments whose membership functions a triangular, i.e., $b_i = d_i$.

3.1.2 Transforming linguistic terms in S into $F(S_T)$.

Definition 6.(Herrera and Martínez, 1999a) *Let $S = \{l_0, \ldots, l_p\}$ and $S_T = \{s_0, \ldots, s_g\}$ be two linguistic term sets, such that, $g \geq p$. Then, a multi-granularity transformation function, τ_{SS_T}, is defined as:*

$$\tau_{SS_T} : A \rightarrow F(S_T)$$
$$\tau_{SS_T}(l_i) = \{(c_k, \gamma_k^i) \,/\, k \in \{0, ..., g\}\}, \forall l_i \in S$$
$$\gamma_k^i = \max_y \min\{\mu_{l_i}(y), \mu_{c_k}(y)\}$$

where $F(S_T)$ is the set of fuzzy sets defined in S_T, and $\mu_{l_i}(\cdot)$ and $\mu_{c_k}(\cdot)$ are the membership functions of the fuzzy sets associated with the terms l_i and c_k, respectively.

Therefore, the result of τ_{SS_T} for any linguistic value of S is a fuzzy set defined in the BLTS, S_T.

3.1.3 Transforming interval-valued into $F(S_T)$.

Let $I = [\underline{i}, \overline{i}]$ be an interval-valued in $[0,1]$, to carry out this transformation we assume that the interval-valued has a representation, inspired in the membership function of fuzzy sets (Kuchta, 2000), as follows:

$$\mu_I(\vartheta) = \begin{cases} 0, & if\, \vartheta < \underline{i} \\ 1, & if\, \underline{i} \leq \vartheta \leq \overline{i} \\ 0, & if\, \overline{i} < \vartheta \end{cases}$$

where ϑ is a value in $[0,1]$. In Figure 3 can be observed the graphical representation of an interval.

Definition 7. *Let $S_T = \{s_0, \ldots, s_g\}$ be a BLTS. Then, the function τ_{IS_T} transforms a interval-valued I in $[0,1]$ into a fuzzy set in S_T.*

$$P_{e_k} = \begin{pmatrix} p^k_{11} = \{(s_0, \gamma^{11}_{k_0}), \ldots, (s_g, \gamma^{11}_{k_g})\} & \cdots & p^k_{1n} = \{(s_0, \gamma^{1n}_{k_0}), \ldots, (s_g, \gamma^{1n}_{k_g})\} \\ \vdots & \cdots & \vdots \\ p^k_{n1} = \{(s_0, \gamma^{n1}_{k_0}), \ldots, (s_g, \gamma^{n1}_{k_g})\} & \cdots & p^k_{nn} = \{(s_0, \gamma^{nn}_{k_0}), \ldots, (s_g, \gamma^{nn}_{k_g})\} \end{pmatrix}$$

Table 1: Preference Relation of Fuzzy Sets

$$\tau_{IS_T} : I \rightarrow F(S_T)$$
$$\tau_{IS_T}(I) = \{(c_k, \gamma^i_k) \,/\, k \in \{0, ..., g\}\},$$
$$\gamma^i_k = \max_y \min\{\mu_I(y), \mu_{c_k}(y)\}$$

where $F(S_T)$ is the set of fuzzy sets defined in S_T, and $\mu_I(\cdot)$ and $\mu_{c_k}(\cdot)$ are the membership functions associated with the interval-valued I and terms c_k, respectively.

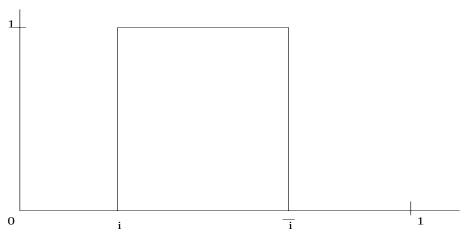

Figure 3: Membership function of $I = [\underline{i}, \overline{i}]$

3.2 Aggregating individual performance values

Using the above transformation functions we express the input information by means of fuzzy sets on the BLTS, $S_T = \{s_0, \ldots, s_g\}$, i.e, we have the input information in an unique expression domain. Now we use an aggregation function for combining the fuzzy sets on the BLTS to obtain a collective performance for each alternative that will be a fuzzy set on the BLTS.

For the heterogeneous GDM problem, the preference relations are expressed by means of fuzzy sets on the BLTS as can be seen in Table 1, where p^k_{ij} is the preference degree of the alternative x_i over x_j provides by the expert e_k.

We shall represent each fuzzy set, p^k_{ij}, as $r^k_{ij} = (\gamma^{ij}_{k_0}, \ldots, \gamma^{ij}_{k_g})$ being the values of r^k_{ij} their respective membership degrees. Then, the collective performance value of the preference relation according to all preference relations provided by experts $\{r^k_{ij}, \forall e_k\}$ is obtained aggregating these fuzzy sets. This collective performance value, denoted r_{ij}, is a new fuzzy set defined in S_T, i.e.,

$$r_{ij} = (\gamma^{ij}_0, \ldots, \gamma^{ij}_g)$$

characterized by the following membership function:

$$\gamma^{ij}_v = f(\gamma^{ij}_{1_v}, \ldots, \gamma^{ij}_{k_v}),$$

where f is an "aggregation operator" and k is the number of experts.

3.3 Transforming Fuzzy Sets into 2-tuple

In this phase we transform the fuzzy sets on the BLTS into linguistic 2-tuples over the BLTS. In (Herrera and Martínez, 1999a) was presented a function χ that transforms a fuzzy set in a linguistic term set into a numerical value in the interval of granularity of S_T, $[0, g]$:

$$\chi : F(S_T) \rightarrow [0, g]$$
$$\chi(\tau(\vartheta)) = \chi(\{(s_j, \gamma_j),\, j : 0, ..., g\}) = \frac{\sum_{j=0}^{g} j\gamma_j}{\sum_{j=0}^{g} \gamma_j} = \beta$$

Therefore, applying the Δ function to β we shall obtain a collective preference relation whose values are linguistic 2-tuples.

4 Example

Let us suppose that an enterprise want to renove its cars. There exist four models of car available, {CAR1, CAR2, CAR3 and CAR4} and three experts provide his/her preference relations over the four cars. The first expert expresses his/her preference relation using numerical values in $[0, 1]$, P^n_1. The second one expresses the preferences by means of linguistic values in a linguistic term set S (see Figure 1), P^S_2. And the third expert can express them using interval-valued in $[0, 1]$, P^I_3, or equivalently using IFS, P^{IFS}_3, that will be represented by means of pairs of values (μ, ν). The three experts attempt to reach a collective decision.

$$P^n_1 = \begin{pmatrix} - & .5 & .8 & .4 \\ .5 & - & .9 & .5 \\ .8 & .9 & - & .4 \\ .4 & .5 & .4 & - \end{pmatrix}$$

$$P^S_2 = \begin{pmatrix} - & H & VH & M \\ H & - & H & VH \\ VH & H & - & VH \\ M & VH & VH & - \end{pmatrix}$$

$$P_3^I = \begin{pmatrix} - & [.7,.8] & [.65,.7] & [.8,.9] \\ [.7,.8] & - & [.6,.7] & [.8,.85] \\ [.65,.7] & [.6,.7] & - & [.7,.9] \\ [.8,.9] & [.8,.85] & [.7,.9] & - \end{pmatrix} \equiv$$

$$\equiv \quad P_3^{IFS} = \begin{pmatrix} - & (.7,.2) & (.65,.3) & (.8,.1) \\ (.7,.2) & - & (.6,.3) & (.8,.15) \\ (.65,.3) & (.6,.3) & - & (.7,.1) \\ (.8,.1) & (.8,.15) & (.7,.9) & - \end{pmatrix}$$

Table 2: Preferences Relations

4.1 Decision Process

We shall use the following decision process to solve this problem:

A) Aggregation Phase

We use the aggregation process presented in this paper.

1. **Making the information uniform**
   (a) *Choose the BLTS.* It will be S, due to the fact, it satisfies the conditions showed in Section 3.1.
   (b) *Transforming the input information into* $F(S_T)$. Applying the transformation functions defined in Section 3 we obtain the results showed in Table 3.
   (c) *Aggregating individual performance values.* When all information is expressed by means of fuzzy sets defined in a BLTS we use a aggregation operator for combining it. In this example we shall use as aggregation operator, f, the arithmetic mean obtaining the collective preference relation showed in Table 4.
2. **Transforming into 2-tuple**. In this step we transform the fuzzy sets in a BLTS into 2-tuples with the function χ (Herrera and Martínez, 2000) and Δ(Herrera and Martínez, 1999b). The collective preference relation P is:

$$\begin{pmatrix} - & (H,.31) & (VH,-.43) & (H,-.18) \\ (H,.31) & - & (H,.33) & (H,.38) \\ (VH,-.43) & (H,.33) & - & (H,.29) \\ (H,-.18) & (H,.38) & (H,.29) & - \end{pmatrix}$$

B) Exploitation Phase

To solve the GDM problem, finally we calculate the dominance degree for the alternative x_i over the rest of alternatives. To do so, we shall use the following function:

$$\Lambda(x_i) = \frac{1}{n-1} \sum_{j=0 \mid j \neq i}^{n} \beta_{ij}$$

where n is the number of alternatives and $\beta_{ij} = \Delta^{-1}(p_{ij})$ being p_{ij} a lingusitic 2-tuple. Then, we shall choose as solution set of alternatives those with a bigger value of dominance degree.

In this phase we shall calculate the dominance degree for this preference relation showed in Table 5.

$CAR1$	$CAR2$	**CAR3**	$CAR4$
$(H,.23)$	$(H,.34)$	**(H,.4)**	$(H,.16)$

Table 5: Dominance degree of the alternatives

Then, the dominance degree rank the alternatives and we choose the best alternatives how solution set of GDM problem, in this example the solution set is {**CAR3**}.

5 Concluding Remarks

We have developed an aggregation process for aggregating heterogeneous information composed by *interval valued, Intuitionistic Fuzzy Sets, numerical and linguistic values* and it is based on the transformation of the heterogeneous information into fuzzy sets and finally into linguistic 2-tuples. This aggregation process has been applied it to a GDM problem defined in a heterogeneous context.

In the future we want to apply this aggregation process to other types of information used in the literature to express preference as can be Interval-Valued Fuzzy Sets.

REFERENCES

Atanassov, K. (1999). *Intuitionistic Fuzzy Sets. Theory and Applications*. Phisyca-Verlag.

Bonissone, P. and Decker, K. (1986). *Selecting Uncertainty Calculi and Granularity: An Experiment in Trading-Off Precision and Complexity*. Uncertainty in Artificial Intelligence. North-Holland.

Bordogna, G. and Passi, G. (1993). A fuzzy linguistic approach generalizing boolean information retrieval: A model and its evaluation. *J. of the American Society for Information Science*, 44:70–82.

Buckley, J. (1984). The multiple judge, multiple criteria ranking problem: A fuzzy set approach. *Fuzzy Sets and Systems*, 13:23–37.

de Mántaras, R. L. and Godó, L. (1997). From intervals to fuzzy truth-values: Adding flexibility to reasoning under uncertainty. *Int. Jou. of Uncertainty, Fuzziness and Knowledge-Based Systems*, 5(3):251–260.

$$P_1^n = \begin{pmatrix} - & (0,0,0,1,0,0,0) & (0,0,0,0,.19,.81,0) & (0,0,.59,.41,0,0,0) \\ (0,0,0,1,0,0,0) & - & (0,0,0,0,0,.59,.41) & (0,0,0,1,0,0,0) \\ (0,0,0,0,.19,.81,0) & (0,0,0,0,0,.59,.41) & - & (0,0,.59,.41,0,0,0) \\ (0,0,.59,.41,0,0,0) & (0,0,0,1,0,0,0) & (0,0,.59,.41,0,0,0) & - \end{pmatrix}$$

$$P_2^S = \begin{pmatrix} - & (0,0,0,0,1,0,0) & (0,0,0,0,0,1,0) & (0,0,0,1,0,0,0) \\ (0,0,0,0,1,0,0) & - & (0,0,0,0,1,0,0) & (0,0,0,0,0,1,0) \\ (0,0,0,0,0,1,0) & (0,0,0,0,1,0,0) & - & (0,0,0,0,0,1,0) \\ (0,0,0,1,0,0,0) & (0,0,0,0,0,1,0) & (0,0,0,0,0,1,0) & - \end{pmatrix}$$

$$P_3^I = \begin{pmatrix} - & (0,0,0,0,.81,.81,0) & (0,0,0,.12,1,.19,0) & (0,0,0,0,.81,1,.41) \\ (0,0,0,.12,1,.19,0) & - & (0,0,0,.41,1,.19,0) & (0,0,0,0,.19,1,.12) \\ (0,0,0,0,.19,.81,0) & (0,0,0,.41,1,.19,0) & - & (0,0,0,0,.81,1,.41) \\ (0,0,0,0,.81,1,.41) & (0,0,0,0,.19,1,.12) & (0,0,0,0,.81,1,.41) & - \end{pmatrix}$$

Table 3: Fuzzy sets in a BLTS

$$P = \begin{pmatrix} - & (0,0,0,0,.6,.27,0) & (0,0,0,.04,.4,.67,0) & (0,0,.2,.47,.27,.33,.14) \\ (0,0,0,0,.6,.27,0) & - & (0,0,0,.14,.67,.26,.14) & (0,0,0,.33,.06,.67,.04) \\ (0,0,0,.04,.4,.67,0) & (0,0,0,.14,.67,.26,.14) & - & (0,0,.2,.14,.27,.67,.14) \\ (0,0,.2,.47,.27,.33,.14) & (0,0,0,.33,.06,.67,.04) & (0,0,.2,.14,.27,.67,.14) & - \end{pmatrix}$$

Table 4: The collective Preference relation.

Delgado, M., Verdegay, J., and Vila, M. (1993). Linguistic decision making models. *Int. Journal of Intelligent Systems*, 7:479–492.

Herrera, F., Herrera-Viedma, E., and Verdegay, J. (1995). A sequential selection process in group decision making with linguistic assessment. *Information Sciences*, 85:223–239.

Herrera, F. and Martínez, L. (1999a). A fusion method for multi-granularity linguistic information based on the 2-tuple fuzzy linguistic representation model. Technical Report Technical Report#DECSAI-990107, Dept. Computer Sciences and A.I., Universidad de Granada.

Herrera, F. and Martínez, L. (1999b). A selection method based on the 2-tuple linguistic representation model for decision-making with multigranularity linguistic information. In *Proceedings of the EUSFLAT-ESTYLF Joint Conference 99*, pages 453–456, Palma de Mallorca (Spain).

Herrera, F. and Martínez, L. (2000). An approach for combining linguistic and numerical information based on 2-tuple fuzzy representation model in decision-making. *Int. J. of Uncertainty, Fuzziness and Knowledge-Based Systems*, 8(5):539–562.

Kacprzyk, J. and Fredizzi, M. (1986). Group decision making with a fuzzy linguistic majority. *Fuzzy Sets and Systems*, 18:105–118.

Kreinovich, V., Mukaidono, M., and Atanassov, K. (1999). From fuzzy values to intuitionistic fuzzy values to intuitionistic fuzzy intervals etc.: Can we get an arbitrary ordering? *Notes on Intuitionistic Fuzzy Sets*, 5(3):11–18.

Kuchta, D. (2000). Fuzzy capital budgeting. *Fuzzy Sets and Systems*, 111:367–385.

Kundu, S. (1997). Min-transitivity of fuzzy leftness relationship and its application to decision making. *Fuzzy Sets and Systems*, 86:357–367.

Roubens, M. (1997). Fuzzy sets and decision analysis. *Fuzzy Sets and Systems*, 90:199–206.

Ruspini, E. (1969). A new approach to clustering. *Inform. Control*, 15:22–32.

Szmidt, E. and Kacprzyk, J. (1996). Intuitionistic fuzzy sets in group decision making. *Notes on Intuitionistic Fuzzy Sets*, 2(1):15–32.

Téno, J. L. and Mareschal, B. (1998). An interval version of PROMETHEE for the comparison of building products' design with ill-defined data on environmental quality. *European Journal of Operational Research*, 109:522–529.

Yager, R. (1988). On ordered weighted averaging aggregation operators in multicriteria decision making. *IEEE Transactions on Systems, Man, and Cybernetics*, 18:183–190.

Zadeh, L. (1975). The concept of a linguistic variable and its applications to approximate reasoning. *Information Sciences, Part I, II, III*, 8,8,9:199–249,301–357,43–80.