Managin Heterogeneus Information in Group Decision Making

F. Herrera
Dept. of Computer Sci. & A.I.
University of Granada, 18071
Granada, Spain
herrera@decsai.ugr.es

L. Martínez[0], **P.J. Sánchez**
Dept. of Computer Science
University of Jaén, 23071
Jaén, Spain
martin,pedroj@ujaen.es

E. Herrera-Viedma
Dept. of Computer Sci. & A.I.
University of Granada, 18071
Granada, Spain
viedma@decsai.ugr.es

Abstract

In Group Decision Making is difficult that all experts have the same knowledge over the problem. In this contribution we shall focus in Group Decision Making problems in which the experts can express their knowledge over the alternatives using different types of information that will be called *Heterogeneous Information*. The aim of this contribution is to develop an aggregation process able to manage all different types of information. As mean to agregate the heterogeneous information we shall use the linguistic 2-tuple model.

Keywords: decision making, aggregation, linguistic 2-tuples, heterogenous information.

1 Introduction

Group Decision Making (GDM) problems have a finite set of alternatives $X = \{x_1, ..., x_n\}$ $n \geq 2$, as well as a finite set of experts $E = \{e_1, ..., e_m\}$ $m \geq 2$. Each expert e_k provides his/her preferences on X by means of a preference relation, i.e. $P_{e_k}(x_i, x_j) = p_{ij}^k$ the degree of preference of alternative x_i over x_j.

A solution is derived either from the individual preference relations, without constructing a social preference relation (direct approach), or by computing first a social fuzzy preference relation and then using it to find a solution (indirect approach) [8], in this contribution we sahll use the last one. In any of the above approaches the decision process is composed by two steps [12]: (i) Aggregation phase: that combines the expert preferences, and (ii) Exploitation one: that obtains a solution set of alternatives from a preference relation.

It seems dificult that the nature of the preference values , p_{ij}^k, provided by the experts would be the same because it depends on the knowledge of them over the alternatives (usualy it is not precise). Therefore, the preference has been expressed in different domains. Early in DM problems, the uncertainty were expressed in the preference values by means of real values assessed in a predefined range [8], soon other approaches based on interval valued [9, 13] and linguistic one [3, 14] were proposed. The most of the proposals for solving GDM problems are focused in cases where all the experts provide their preferences in an unique domain, usually the experts are from different knowledge field and could express his/her preferences with different types of information depending on their kholedge. We shall call this type of information as *Heterogeneous Information*. Hence, we say that the GDM problem is defined in a heterogeneous context.

The main difficulty for dealing with GDM problems defined in a heterogenous context is how to aggregate the preferences. Because of, there not exists operators or processes for combining that information. The linguistic

[0]This work has been supported by Research Project PB98-1305

2-tuple model [4] has shown itself as a good choice to manage non-homogeneous information in aggregation processes [5, 6]. In this contribution we shall present an aggregation process for heterogeneous information based on the 2-tuple model.

In order to do that, this paper is structured as follows: in Section 2 we shall review different basic concepts; in Section 3 we shall propose an aggregation process for combining heterogeneous information; in Section 4 we shall solve an example of a GDM problem defined in a heterogeneous context and finally, some concluding remarks are pointed out.

2 Preliminaries

We have just seen that in decision making problems the experts express their preferences depending on their knowledge over the alternatives by means of preference relations. Here we review different approaches to express those preferences. And afterwards, we shall review the 2-tuple linguistic representation model.

2.1 Approaches for Modelling Preferences

2.1.1 Fuzzy Binary Relations

A valued (fuzzy) binary relation R on X is defined as a fuzzy subset of the direct product $X \times X$ with values in $[0,1]$, i.e, $R : X \times X \rightarrow [0,1]$. The value, $R(x_i, x_j) = p_{ij}$, of a valued relation R denotes the degree to which $x_i R x_j$. These were the first type of relations used in decicion making [8].

2.1.2 Interval-valued Relations

The above approach has serious problems, in particular it has been argued that the most experts are unable to make a fair estimation of the inaccuracy of their judgements, making far larger estimation errors that the boundaries accepted by them as feasible [10].

A first approach to overcome this problem is to add some flexibility to the uncertainty representation problem by means of interval-valued relations:

$$R : X \times X \rightarrow \wp([0,1]).$$

Where $R(x_i, x_j) = p_{ij}$ denotes the interval-valued preference degree of the alternative x_i over x_j . In these approaches [9, 13], the preferences provided by the experts consist of interval values assessed in $\wp([0,1])$, where the preference is expressed as $[\underline{a}, \overline{a}]_{ij},\ with\ \underline{a} \leq \overline{a}$.

2.1.3 Linguistic Approach

Many aspects cannot be assessed in a quantitative way, but rather in a qualitative one, i.e., with vague or imprecise knowledge. In that case a good approach may be to use linguistic assessments instead of numerical values. The fuzzy linguistic approach represents qualitative aspects as linguistic values by means of linguistic variables [15].

We have to choose the appropriate linguistic descriptors for the term set and their semantics. In the literature, several posibilities can be found (see [7] for a wide description). An important aspect to analyce is the *"granularity of uncertainty"*, i.e., the level of discrimination among different counts of uncertainty. The *"granularity of uncertainty"* for the linguistic term set $S = \{s_0, ..., s_g\}$ is $g+1$, while its *"interval of granularity"* is $[0, g]$.

One possibility of generating the linguistic term set consists of directly supplying the term set by considering all terms distributed on a scale on which a total order is defined [14]. For example, a set of seven terms S, could be given as follows:

$$S = \{s_0 : N, s_1 : VL, s_2 : L, s_3 : M, s_4 : H, s_5 : VH, s_6 : P\}$$

Usually, in these cases, it is required that in the linguistic term set satisfy the following additional characteristics:

1. There is a negation operator: $\mathrm{Neg}(s_i) = s_j$ such that $j = g - i$ (g+1 is the cardinality).
2. $s_i \leq s_j \iff i \leq j$. Therefore, there exists a *min* and a *max* operator.

The semantics of the linguistic terms are given by fuzzy numbers defined in the [0,1] interval. A way to characterize a fuzzy number is to use a representation based on parameters of its membership function [1]. The linguistic assessments given by the users are just approximate ones, some authors consider that linear trapezoidal membership functions are good enough to capture the vagueness of those linguistic assessments. The parametric representation is achieved by the 4-tuple (a, b, d, c), where b and d indicate the interval in which the membership value is 1, with a and c indicating the left and right limits of the definition domain of the trapezoidal membership function [1]. A particular case of this type of representation are the linguistic assessments whose membership functions are triangular, i.e., $b = d$, then we represent this type of membership functions by a 3-tuple (a, b, c). A possible semantics for the above term set, S, may be the following (Figure 1):

$$\begin{array}{ll} P = (.83, 1, 1) & VH = (.67, .83, 1) \\ H = (.5, .67, .83) & M = (.33, .5, .67) \\ L = (.17, .33, .5) & VL = (0, .17, .33) \\ N = (0, 0, .17) & \end{array}$$

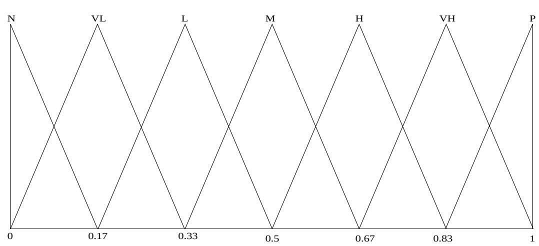

Figure 1: A set of seven linguistic terms with its semantics

2.2 The 2-tuple Linguistic Representation Model

This model was presented in [4], for overcoming the drawback of the loss of information presented by the classical linguistic computational models: (i) The model based on the Extension Principle [1], (ii) and the symbolic one [2]. The 2-tuple fuzzy linguistic representation model is based on symbolic methods and takes as the base of its representation the concept of Symbolic Translation.

Definition 1. *The Symbolic Translation of a linguistic term $s_i \in S = \{s_0, ..., s_g\}$ is a numerical value assessed in $[-.5, .5)$ that suport the "difference of information" between a counting of information $\beta \in [0, g]$ and the closest value in $\{0, ..., g\}$ that indicates the index of the closest linguistic term in $S(s_i)$, being [0,g] the interval of granularity of S.*

From this concept a new linguistic representation model is developed, which represents the linguistic information by means of 2-tuples (r_i, α_i), $r_i \in S$ and $\alpha_i \in [-.5, .5)$. r_i represents the linguistic label center of the information and α_i is the Symbolic Tranlation.

This model defines a set of functions between linguistic 2-tuples and numerical values.

Definition 2. *Let $S = \{s_0, ..., s_g\}$ be a linguistic term set and $\beta \in [0, g]$ a value rsupporting the result of a symbolic aggregation operation, then the 2-tuple that expresses the equivalent information to β is obtained with the following function:*

$$\Delta : [0, g] \longrightarrow S \times [-0.5, 0.5)$$
$$\Delta(\beta) = (s_i, \alpha), \; with \begin{cases} s_i & i = round(\beta) \\ \alpha = \beta - i & \alpha \in [-.5, .5) \end{cases}$$

where $round(\cdot)$ is the usual round *operation, s_i has the closest index label to "β" and "α" is the value of the symbolic translation.*

Proposition 1. *Let $S = \{s_0, ..., s_g\}$ be a linguistic term set and (s_i, α) be a linguistic 2-tuple. There is always a Δ^{-1} function, such that, from a 2-tuple it returns its equivalent numerical value $\beta \in [0, g]$ in the interval of granularity of S.*

Proof. It is trivial, we consider the following function:

$$\Delta^{-1} : S \times [-.5, .5) \longrightarrow [0, g]$$
$$\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta$$

Remark: From Definitions 1 and 2 and Proposition 1, it is obvious that the conversion of a linguistic term into a linguistic 2-tuple consist of adding a value 0 as symbolic translation: $s_i \in S \Longrightarrow (s_i, 0)$

3 Aggregation Process for a Heterogeneous Information in a GDM problem

In this section we present our purpose to carry out the *aggregation step* of a decision making process in a GDM problem defined in a heterogeneous context.

We focus in GDM problems in which the preference relations provided by the experts can be:

1. Fuzzy preference relations [8]
2. Interval-valued preference relation [13]
3. Linguistic preference relation assessed in a pre-established label set [3]

Following, we present our proposal for combining this heterogeneous information. This aggregation process is composed by the following phases:

1. *Making the information uniform.* The heterogeneous information will be unified into a specific linguistic domain, which is a *Basic Linguistic Term Set* (BLTS). The process is carried out in the following order:
   (a) Transforming numerical values in $[0,1]$ into $F(S_T)$.
   (b) Transforming linguistic terms into $F(S_T)$.
   (c) Transforming interval-valued into $F(S_T)$.
2. *Aggregating individual performance values.* For each alternative, a collective performance value is obtained by means of the aggregation of the above fuzzy sets on the BLTS that represents the individual performance values assigned by the experts according to his/her preference.
3. *Transforming into 2-tuple.* The collective performance values (fuzzy sets) are transformed into linguistic 2-tuples in the BLTS and obtained a collective 2-tuple preference relation.

Following we shall show in deep each step of the above phases of the aggregation process.

3.1 Making the information uniform

Firstly, the heterogeneous information is unified in an unique expression domain. In this case, we shall use fuzzy sets over a BLTS, denoted as $F(S_T)$. We study the linguistic term set S that belongs to the definition context of the GDM problem. If:

1. *S is a fuzzy partition,*
2. *and the membership functions of its terms are triangular,* i.e., $s_i = (a_i, b_i, c_i)$

then we select S as BLTS, due to the fact that, these conditions are necessary and sufficient for the transformation between values in $[0,1]$ and 2-tuples, being them carried out without loss of information [6].

If the linguistic term set S, used in the definition context of the problem, does not satisfy the above conditions then we shall choose as BLTS a term set with a larger number of terms than the number of terms that a person is able to discriminate (normally 11 or 13, see [1]) and satisfies the above conditions. We choose the BLTS with 15 terms symmetrically distributed, with the following semantics (graphically, Figure 2).

s_0	(0,0,0.07)	s_1	(0,0.07,0.14)
s_2	(0.07,0.14,0.21)	s_3	(0.14,0.21,0.28)
s_4	(0.21,0.28,0.35)	s_5	(0.28,0.35,0.42)
s_6	(0.35,0.42,0.5)	s_7	(0.42,0.5,0.58)
s_8	(0.5,0.58,0.65)	s_9	(0.58,0.65,0.72)
s_{10}	(0.65,0.72,0.79)	s_{11}	(0.72,0.79,0.86)
s_{12}	(0.79,0.86,0.93)	s_{13}	(0.86,0.93,1)
s_{14}	(0.93,1,1)		

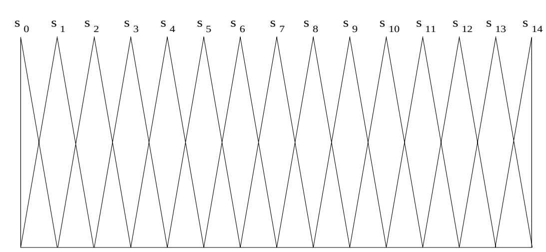

Figure 2: A BLTS with 15 terms symmetrically distributed

The process of unifying the information involves the comparison between fuzzy sets.

Comparisons are usually carried out by means of a measure of comparison. We focus in measures of comparison which evaluate the resemblance of likeness of two objects (fuzzy sets in our case). These type of measures are called *measures of similitude* [11]. For simplicity, in this contribution we sall choose a measure of similitude based on a possibility funcion $S(A,B) = max_x min(\mu_A(x), \mu_B(x))$, where μ_A and μ_B are the membership function of the fuzzy set A and B respectively.

3.1.1 Transforming numerical values in $[0,1]$ into $F(S_T)$.

Let $F(S_T)$ be the set of fuzzy sets in $S_T = \{s_0, \ldots, s_g\}$, we shall transform a numerical value $\vartheta \in [0,1]$ into a fuzzy set in $F(S_T)$ computing the membership value of ϑ in the membership functions associated with the linguistic terms of S_T.

Definition 3. [6] *The function τ transforms a numerical value into a fuzzy set in S_T:*

$$\tau : [0,1] \rightarrow F(S_T)$$
$$\tau(\vartheta) = \{(s_0, \gamma_0), ..., (s_g, \gamma_g)\}, s_i \in S_T \, and \, \gamma_i \in [0,1]$$
$$\gamma_i = \mu_{s_i}(\vartheta) = \begin{cases} 0, & if\, \vartheta \notin Support(\mu_{s_i}(x)) \\ \frac{\vartheta - a_i}{b_i - a_i}, & if\, a_i \leq \vartheta \leq b_i \\ 1, & if\, b_i \leq \vartheta \leq d_i \\ \frac{c_i - \vartheta}{c_i - d_i}, & if\, d_i \leq \vartheta \leq c_i \end{cases}$$

Remark: We consider membership functions, $\mu_{s_i}(\cdot)$, for linguistic labels, $s_i \in S_T$, that achieved by a parametric function (a_i, b_i, d_i, c_i). A particular case are the linguistic assessments whose membership functions a triangular, i.e., $b_i = d_i$.

Example 1

Let $\vartheta = 0.78$ be a numerical value to be transformed into a fuzzy set in $S = \{s_0, ..., s_4\}$. The semantic of these term set is:

$s_0 = (0, 0, 0.25)$ $s_1 = (0, , 0.25, 0.5)$
$s_2 = (0.25, 0.5, 0.75)$ $s_3 = (0.5, 0.75, 1)$
$s_4 = (0.75, 1, 1)$

$$\tau(0.78) = \{(s_0, 0), (s_1, 0), (s_2, 0), (s_3, 0.88), (s_4, 0.12)\}$$

3.1.2 Transforming linguistic terms in S into $F(S_T)$.

Definition 5.[5] *Let $S = \{l_0, \ldots, l_p\}$ and $S_T = \{s_0, \ldots, s_g\}$ be two linguistic term sets, such that, $g \geq p$. Then, a multi-granularity transformation function, τ_{SS_T}, is defined as:*

$$\tau_{SS_T} : A \rightarrow F(S_T)$$
$$\tau_{SS_T}(l_i) = \{(c_k, \gamma_k^i) \,/\, k \in \{0, ..., g\}\}, \, \forall l_i \in S$$
$$\gamma_k^i = \max_y \min\{\mu_{l_i}(y), \mu_{c_k}(y)\}$$

where $F(S_T)$ is the set of fuzzy sets defined in S_T, and $\mu_{l_i}(\cdot)$ and $\mu_{c_k}(\cdot)$ are the membership functions of the fuzzy sets associated with the terms l_i and c_k, respectively.

Therefore, the result of τ_{SS_T} for any linguistic value of S is a fuzzy set defined in the BLTS, S_T.

Example 2

Let $S = \{l_0, l_1, \ldots, l_4\}$ and $S_T = \{s_0, s_1, \ldots, s_6\}$ be two term set, with 5 and 7 labels, respectively, and with the following semantics associated:

$l_0 = (0, 0, 0.25)$ $s_0 = (0, 0, 0.16)$
$l_1 = (0, , 0.25, 0.5)$ $s_1 = (0, 0.16, 0.34)$
$l_2 = (0.25, 0.5, 0.75)$ $s_2 = (0.16, 0.34, 0.5)$
$l_3 = (0.5, 0.75, 1)$ $s_3 = (0.34, 0.5, 0.66)$
$l_4 = (0.75, 1, 1)$ $s_4 = (0.5, 0.66, 0.84)$
$s_5 = (0.66, 0.84, 1)$
$s_6 = (0.84, 1, 1)$

The fuzzy set obtained after applying τ_{SS_T} for l_1 is:

$$\tau_{AS_T}(l_1) = \{(s_0, 0.39), (s_1, 0.85), (s_2, 0.85), (s_3, 0.39), (s_4, 0), (s_5, 0), (s_6, 0)\}$$

3.1.3 Transforming interval-valued into $F(S_T)$.

Let $I = [\underline{i}, \overline{i}]$ be an interval-valued in $[0,1]$, to carry out this transformation we assume that the interval-valued has a representation, inspired in the membership function of fuzzy sets [9], as follows:

$$\mu_I(\vartheta) = \begin{cases} 0, & if\, \vartheta < \underline{i} \\ 1, & if\, \underline{i} \leq \vartheta \leq \overline{i} \\ 0, & if\, \overline{i} < \vartheta \end{cases}$$

Table 1: The preference relation

$$P_{e_k} = \begin{pmatrix} p^k_{11} = \{(s_0, \gamma^{11}_{k_0}), \ldots, (s_g, \gamma^{11}_{k_g})\} & \cdots & p^k_{1n} = \{(s_0, \gamma^{1n}_{k_0}), \ldots, (s_g, \gamma^{1n}_{k_g})\} \\ \vdots & \ldots & \vdots \\ p^k_{n1} = \{(s_0, \gamma^{n1}_{k_0}), \ldots, (s_g, \gamma^{n1}_{k_g})\} & \cdots & p^k_{nn} = \{(s_0, \gamma^{nn}_{k_0}), \ldots, (s_g, \gamma^{nn}_{k_g})\} \end{pmatrix}$$

Table 2: Example preference relations

$$P^n_1 = \begin{pmatrix} - & .5 & .8 & .4 \\ .5 & - & .9 & .5 \\ .8 & .9 & - & .4 \\ .4 & .5 & .4 & - \end{pmatrix} \quad P^S_2 = \begin{pmatrix} - & H & VH & M \\ H & - & H & VH \\ VH & H & - & VH \\ M & VH & VH & - \end{pmatrix} \quad P^I_3 = \begin{pmatrix} - & [.7, .8] & [.65, .7] & [.8, .9] \\ [.7, .8] & - & [.6, .7] & [.8, .85] \\ [.8, .9] & [.6, .7] & - & [.7, .9] \\ [.8, .9] & [.8, .85] & [.7, .9] & - \end{pmatrix}$$

where ϑ is a value in $[0, 1]$. In Figure 3 can be observed the graphical representation of an interval.

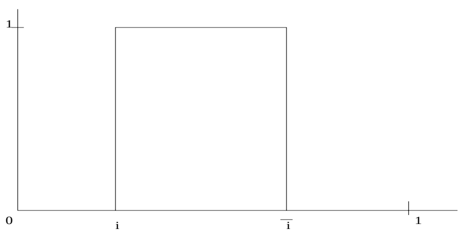

Figure 3: Membership function of $I = [\underline{i}, \overline{i}]$

Definition 5. *Let $S_T = \{s_0, \ldots, s_g\}$ be a BLTS. Then, the function τ_{IS_T} transforms a interval-valued I in $[0, 1]$ into a fuzzy set in S_T.*

$$\tau_{IS_T} : I \to F(S_T)$$
$$\tau_{IS_T}(I) = \{(c_k, \gamma^i_k) \,/\, k \in \{0, ..., g\}\},$$
$$\gamma^i_k = \max_y \min\{\mu_I(y), \mu_{c_k}(y)\}$$

where $F(S_T)$ is the set of fuzzy sets defined in S_T, and $\mu_I(\cdot)$ and $\mu_{c_k}(\cdot)$ are the membership functions associated with the interval-valued I and terms c_k, respectively.

Example 3

Let $I = [0.6, 0.78]$ be an interval-valued to be transformed into a fuzzy set in S_T. The semantic of these term set is the same of Example 3.1.1. The fuzzy set obtained after applying τ_{IS_T} is:

$$\tau_{IS_T} = \{(s_0, 0), (s_1, 0), (s_2, 0.6), (s_3, 1), (s_4, 0.2)\}$$

3.2 Aggregating individual performance values

Using the above transformation functions we express the input information by means of fuzzy sets on the BLTS, $S_T = \{s_0, \ldots, s_g\}$,. Now we use an aggregation function for combining the fuzzy sets on the BLTS to obtain a collective performance for each alternative that will be a fuzzy set on the BLTS.

For the heterogeneous GDM the preference relations are expressed by means of fuzzy sets on the BLTS as Table 1, where p^k_{ij} is the preference degree of the alternative x_i over x_j provides by the expert e_k.

We shall represent each fuzzy set, p^k_{ij}, as $r^k_{ij} = (\gamma^{ij}_{k_0}, \ldots, \gamma^{ij}_{k_g})$ being the values of r^k_{ij} their respective membership degrees. Then, the collective performance value of the preference relation according to all preference relations provided by experts $\{r^k_{ij}, \forall e_k\}$ is obtained aggregating these fuzzy sets. This collective performance value, denoted r_{ij}, is a new preference relation of fuzzy set defined in S_T, i.e.,

$$r_{ij} = (\gamma^{ij}_0, \ldots, \gamma^{ij}_g)$$

characterized by the following membership function:

$$\gamma^{ij}_v = f(\gamma^{ij}_{1_v}, \ldots, \gamma^{ij}_{k_v}),$$

where f is an "aggregation operator" and k is the number of experts.

3.3 Transforming into 2-tuple

In this phase we transform the fuzzy sets on the BLTS into linguistic 2-tuples over the BLTS. In [5] was presented a function χ that transforms a fuzzy set in a linguistic term set into a numerical value in the interval of granularity of S_T, $[0, g]$:

$$\chi : F(S_T) \to [0, g]$$
$$\chi(\tau(\vartheta)) = \chi(\{(s_j, \gamma_j),\, j = 0, ..., g\}) = \frac{\sum^g_{j=0} j\gamma_j}{\sum^g_{j=0} \gamma_j} = \beta$$

Table 3: Fuzzy sets in a BLTS

$$P_1^n = \begin{pmatrix} - & (0,0,0,1,0,0,0) & (0,0,0,0,.19,.81,0) & (0,0,.59,.41,0,0,0) \\ (0,0,0,1,0,0,0) & - & (0,0,0,0,0,.59,.41) & (0,0,0,1,0,0,0) \\ (0,0,0,0,.19,.81,0) & (0,0,0,0,0,.59,.41) & - & (0,0,.59,.41,0,0,0) \\ (0,0,.59,.41,0,0,0) & (0,0,0,1,0,0,0) & (0,0,.59,.41,0,0,0) & - \end{pmatrix}$$

Table 4: The collective Preference relation.

$$P = \begin{pmatrix} - & (0,0,0,0,.6,.27,0) & (0,0,0,.04,.4,.67,0) & (0,0,.2,.47,.27,.33,.14) \\ (0,0,0,0,.6,.27,0) & - & (0,0,0,.14,.67,.26,.14) & (0,0,0,.33,.06,.67,.04) \\ (0,0,0,.04,.4,.67,0) & (0,0,0,.14,.67,.26,.14) & - & (0,0,.2,.14,.27,.67,.14) \\ (0,0,.2,.47,.27,.33,.14) & (0,0,0,.33,.06,.67,.04) & (0,0,.2,.14,.27,.67,.14) & - \end{pmatrix}$$

Therefore, applying the Δ function to β we shall obtain a collective preference relation whose values are linguistic 2-tuples.

4 Example

Let us suppose that an enterprise want to renove its computers. There exist four models of computers available, {HP, IBM, COMPAQ and DELL} and three experts provide his/her preference relations over the four cars. The first expert expresses his/her preference relation using numerical values in $[0,1]$, P_1^n. The second one expresses the preferences by means of linguistic values in a linguistic term set S (see Figure 1), P_2^S. And the third expert can express them using interval-valued in $[0,1]$, P_3^I. The three experts attempt to reach a collective decision.

4.1 Decision Process

We shall use the following decision process to solve this problem:

A) Aggregation Phase

We use the aggregation process presented in this paper.

1. **Making the information uniform**
   - (a) *Choose the BLTS.* It will be S, due to the fact, it satisfies the conditions showed in Section 3.1.
   - (b) *Transforming the input information into $F(S_T)$.* (see Table 3)
   - (c) *Aggregating individual performance values.* In this example we use as aggregation operator, f, the arithmetic mean obtaining the collective preference relation shows in Table 4

2. **Transforming into 2-tuple**. The result of this transformation is:

$$P = \begin{pmatrix} - & (H,.31) & (VH,-.43) & (H,-.18) \\ (H,.31) & - & (H,.33) & (H,.38) \\ (VH,-.43) & (H,.33) & - & (H,.29) \\ (H,-.18) & (H,.38) & (H,.29) & - \end{pmatrix}$$

B) Exploitation Phase

To solve the GDM problem, finally we calculate the dominance degree for the alternative x_i over the rest of alternatives. To do so, we shall use the following function:

$$\Lambda(x_i) = \frac{1}{n-1} \sum_{j=0 \,|\, j \neq i}^{n} \beta_{ij}$$

where n is the number of alternatives and $\beta_{ij} = \Delta^{-1}(p_{ij})$ being p_{ij} a lingusitic 2-tuple.

In this phase we shall calculate the dominance degree for this preference relation showed in Table 5.

Table 5: Dominance degree of the alternatives

HP	*IBM*	**COMPAQ**	*DELL*
$(H,.23)$	$(H,.34)$	**(H,.4)**	$(H,.16)$

Then, the dominance degree rank the alternatives and we choose the best alternatives how solution set of GDM problem, in this example the solution set is **{COMPAQ}.**

5 Concluding Remarks

We have developed an aggregation process for aggregating heterogeneous information composed by numerical, interval valued and linguistic values. This aggregation process is based on the transformation of the information into fuzzy sets and afterwards into linguistic 2-tuples. This aggregation process has been applied it to a GDM problem defined in a heterogeneous context.

In the future we want to apply this aggregation process to other types of information used in the literature to express preference as can be Interval-Valued Fuzzy Sets, Intuitionistic Fuzzy Sets.

References

[1] P.P. Bonissone, K.S. Decker, Selecting Uncertainty Calculi and Granularity: An Experiment in Trading-off Precision and Complexity, in: L.H. Kanal and J.F. Lemmer, Eds., *Uncertainty in Artificial Intelligence* (North-Holland, 1986) 217-247.

[2] M. Delgado, J.L. Verdegay and M.A. Vila, Linguistic Decision Making Models, *Int. Journal of Intelligent Systems* **7** (1993) 479-492.

[3] F. Herrera, E. Herrera-Viedma, J.L. Verdegay, A Linguistic Decision Process in Group Decision Making, *Group Decision and Negotiation* **5** (1996) 165-176.

[4] F. Herrera and L. Martínez, A 2-tuple Fuzzy Linguistic Representation Model for Computing with Words, *IEEE Trans. on Fuzzy Systems* **8:6** (2000) 746-752.

[5] F. Herrera and L. Martínez, A Fusion Method for Multi-Granularity Linguistic Information based on the 2-tuple Fuzzy Linguistic Representation Model, Dept. Computer Science and A.I., University of Granada, Technical Report#DECSAI-990107, 1999. (http://decsai.ugr.es/~herrera/public.html)

[6] F. Herrera and L. Martínez, An Approach for Combining Linguistic and Numerical Information Based on 2-tuple Fuzzy Representation Model in Decision-Making, Int. J. of Uncertainty, *Fuzziness and Knowledge-Based Systems* **8:5** (2000) 539-562.

[7] F. Herrera, E. Herrera-Viedma, Linguistic Decision Analysis: Step for Solving decision problems under linguistic information, *Fuzzy Sets and Systems* **115** (2000) 67-82.

[8] J. Kacprzyk and M. Fedrizzi, *Multiperson Decision Making Models Using Fuzzy Sets and Possibility Theory* (Kluwer Academic Publishers, Dordrecht, 1990).

[9] D. Kuchta, Fuzzy Capital Budgeting, *Fuzzy Sets and Systems* **111** (2000) 367-385.

[10] R. López de Mántaras, L. Godó, From Intervals to Fuzzy Truth-Values: Adding Flexibility to Reasoning Under Uncertainty, *Int. Jou. of Uncertainty, Fuzziness and Knowledge-Based Systems* **5:3** (1997) 251-260.

[11] M. Rifiqi, V. Berger, B. Bouchon-Meunier, Discrimination Power of Measures of Comparison, *Fuzzy Sets and Systems* **90** (2000) 189-196.

[12] Roubens M., Fuzzy Sets and Decision Analysis, *Fuzzy Sets and Systems* **90** (1997) 199-206.

[13] J.F. Le Téno, B. Mareschal, An Interval Version of PROMETHEE for the Comparison of Building Products' Design with Ill-defined Data on Environmental Quality, *European Journal of Operational Research* **109** (1998) 522-529.

[14] R.R. Yager, An Approach to Ordinal Decision Making, *Int. Journal of Approximate Reasoning* **12** (1995) 237-261.

[15] L.A. Zadeh, The Concept of a Linguistic Variable and Its Applications to Approximate Reasoning. Part I, *Information Sciences* **8** (1975) 199-249, Part II, *Information Sciences* **8** (1975) 301-357, Part III, *Information Sciences* **9** (1975) 43-80.