Fusion of Multigranular Linguistic Information based on the 2-tuple Fuzzy Linguistic Representation Model

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Abstract

The Fuzzy Linguistic Approach has been applied successfully to many problems, its use implies processes of *Computing with Words* (CW). One important limitation of the fuzzy linguistic approach appears when these processes are applied to problems defined in multigranular linguistic contexts. This limitation consists of the difficulty in dealing with this type of information in processes of CW, due to the fact, that there is no standard normalization process for this type of information as in the numerical domain. In this contribution, taking as base the 2-tuple fuzzy linguistic representation model and its computational technique, we shall present a method for easily dealing with multigranular linguistic information in fusion processes.

Keywords: Linguistic variables, fusion processes, granularity of uncertainty.

1 Introduction

On many occasions we find problems that present several sources of information to qualify their phenomena. When these phenomena present quantitative aspects, they can be assessed by means of precise numerical values, however when the aspects presented by the phenomena are qualitative may be difficult to qualify using precise values. So, the use of the fuzzy linguistic approach [13] has shown itself as a good choice to model these phenomena.

The use of the fuzzy linguistic approach implies processes of *Computing with Words*. In the specialized literature can be found three different linguistic computational models that provide linguistic operators for CW:

- Model based on the Extension Principle [3]
- The symbolic one [2]
- Model based on the 2-tuple representation [7]

An important aspect when the fuzzy linguistic approach is used, is to determine the "granularity of uncertainty" of the linguistic term set used to assess the linguistic variables. When a problem presents multigranular linguistic information, the classical computational techniques presented in [2, 3] have an important limitation because in these computational methods, neither a standard normalization process nor fusion operators are defined for this type of information. Therefore, the results obtained are not fitted (loss of information) and are expressed by values in domains far removed from the inital expression domains.

The aim of this paper is to develop an aggregation process, for multigranular linguistic information, that overcomes the above limitations. To do so, we shall use the 2-tuple fuzzy linguistic representation model and its computational technique [7], together with the multigranular linguistic information

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fusion ideas presented in [5]. Finally, we shall solve a Multi-Expert Decision-Making (MEDM) problem defined in a multigranular linguistic context.

To do so, the paper is structured as follows: in Section 2, we shall make a brief review of some preliminaries. In Section 3, we develop a fusion method for multigranular linguistic information. In Section 4, a decision process over an MEDM problem with multigranular linguistic information is presented. Finally, some concluding remarks are pointed out in Section 5.

2 Preliminaries

In this section we briefly review the fuzzy linguistic approach together with the three linguistic computational techniques and present a general scheme for MEDM problems.

2.1 Fuzzy Linguistic Approach

Many aspects of different activities in the real world cannot be assessed in a quantitative form, but rather in a qualitative one, i.e., with vague or imprecise knowledge. In that case a better approach may be to use linguistic assessments instead of numerical values. The fuzzy linguistic approach represents qualitative aspects as linguistic values by means of linguistic variables [13].

We have to choose the appropriate linguistic descriptors for the term set and their semantics [5]. One possibility of generating the linguistic term set consists of directly supplying the term set by considering all terms distributed on a scale on which a total order is defined [12]. For example, a set of seven terms S, could be:

$$\{s_0 : N, s_1 : VL, s_2 : L, s_3 : M, s_4 : H, s_5 : VH, s_6 : P\}$$

Usually, in these cases, it is required that in the linguistic term set there exist:

1. A negation operator: Neg(s_i) $= s_j$ such that $j = g$-i (g+1 is the cardinality).
2. An order: $s_i \leq s_j \iff i \leq j$. Therefore, there exists a min and a max operator.

The semantics of the terms are given by fuzzy numbers defined in the [0,1] interval, which are usually described by membership functions. For example, we may assign the following semantics to the set of seven terms:

$$\begin{array}{ll} P = (.83, 1, 1) & VH = (.67, .83, 1) \\ H = (.5, .67, .83) & M = (.33, .5, .67) \\ L = (.17, .33, .5) & VL = (0, .17, .33) \\ N = (0, 0, .17). & \end{array}$$

which is graphically shown in Figure 1.

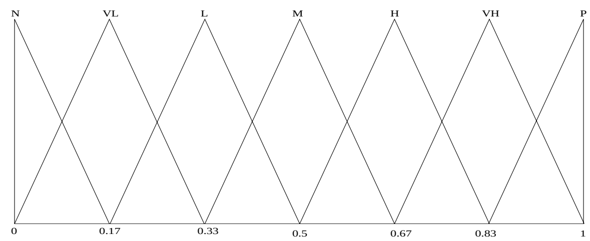

Figure 1: A Set of 7 Terms with its Semantic

2.2 Linguistic Computational Models

The linguistic variables are used in processes of CW that imply their fusion, aggregation, comparison, etc. To perform these computations have been developed three techniques in the literature. (i) The model based on the Extension Principle, (ii) the symbolic one, and (iii) the model based on the linguistic 2-tuple. Here we briefly review the two first, and the third one will be review in deep.

1. *The linguistic computational methods based on the Extension Principle* [3]. These methods use the extended arithmetic, based on the Extension Principle [4], on the membership functions associated to the linguistic terms to make linguistic computations. The use of extended arithmetic based on the Extension Principle increases the vagueness of the results. Therefore, the results obtained are counts of information that usually do not match any linguistic term in the initial term set, so a linguistic approximation process [3] is needed to express the results in the original expression domain.

2. *The linguistic computational symbolic models* [2]. These methods do not use the membership functions of the labels to perform the computations, but use the order index and properties of such linguistic assessments to

make direct computations on labels. The results are numerical values which must be approximated to obtain a value that indicates the index of the associated linguistic term.

For a more detailed description of these linguistic computational models see [2, 3].

3. *The 2-tuple Fuzzy Linguistic Representation Model.* This model, has been presented in [7], is based on the symbolic one and in a concept called *Symbolic Translation.*

Definition 1. *Let β be the result of an aggregation of the indexes of a set of labels assessed in a linguistic term set $S = \{s_0, ..., s_g\}$, i.e., the result of a symbolic aggregation operation. $\beta \in [0, g]$, being $g+1$ the cardinality of S. Let $i = round(\beta)$ and $\alpha = \beta - i$ be two values, such that, $i \in [0, g]$ and $\alpha \in [-.5, .5)$ then α is called a Symbolic Translation.*

From this concept in [7] it was developed a linguistic representation model which represents the linguistic information by means of a pair of values (s_i, α_i), $s_i \in S$ and $\alpha_i \in [-.5, .5)$.

This model defines a set of functions to deal with 2-tuples.

Definition 2. *Let $S = \{s_0, ..., s_g\}$ be a linguistic term set and $\beta \in [0, g]$ a value supporting the result of a symbolic aggregation operation, then the 2-tuple that expresses the equivalent information to β is obtained with the following function:*

$$\Delta : [0, g] \longrightarrow S \times [-0.5, 0.5)$$

$$\Delta(\beta) = \begin{cases} s_i & i = round(\beta) \\ \alpha = \beta - i & \alpha \in [-.5, .5) \end{cases}$$

where s_i has the closest index label to "β" and "α" is the value of the symbolic translation.

Proposition 1. *Let $S = \{s_0, ..., s_g\}$ be a linguistic term set and (s_i, α) be a 2-tuple. There is always a Δ^{-1} function, such that, from a 2-tuple it returns its equivalent numerical value $\beta \in [0, g] \subset \mathcal{R}$.*

Proof.
It is trivial, we consider the following function:

$$\Delta^{-1} : S \times [-.5, .5) \longrightarrow [0, g]$$

$$\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta$$

2.3 A General Scheme of an Muti-Expert Decision-Making problem

A Muti-Expert Decision-Making (MEDM) problem can be defined as follows. Let $A = \{a_1, ..., a_n\}$ be a set of alternatives, each one assessed by a set of experts $\{e_1, ..., e_m\}$. This scheme is shown in Table 1:

Table 1: A general MEDM problem

Alternatives	*Experts*			
(a_i)	e_1	e_2	...	e_m
a_1	y_{11}	y_{12}	...	y_{1m}
...	...	...	...	...
a_n	y_{n1}	y_{n2}	...	y_{nm}

We focus in MEDM problems defined over multigranular linguistic term sets, i.e., problems where their preference values y_{ij} can be assessed in linguistic term sets S_j that can have different granularity of uncertainty and/or semantics.

Decision-making problems that manage preferences from different experts follow a common resolution scheme [10] composed by two phases:

1. *Aggregation phase:* It combines the individual preferences to obtain a collective preference value for each alternative.
2. *Exploitation phase:* It orders the collective preference values according to a given criterion to obtain the best alternative/s.

In problems defined in multigranular linguistic contexts the aggregation phase is carried out in two steps [5]:

- *Normalization step.* The multigranular linguistic information is expressed in an unique linguistic term set.
- *Combination step.* The unified linguistic information expressed in an unique linguistic term set is aggregated.

3 Fusion Method for Multigranular Linguistic Information based on the 2-tuple Representation

This fusion method for multigranular linguistic information is developed according to the following processes.

1. **Making the information uniform**. The multigranular linguistic information is unified into "fuzzy sets" in a Basic Linguistic Term Set (BLTS).
2. **Transforming fuzzy sets into 2-tuples**. The fuzzy sets in the BLTS are transformed into 2-tuples in the BLTS.
3. **Fusion of 2-tuples**. We apply a 2-tuple aggregation operator in order to obtain aggregated values expressed by means of 2-tuples assessed in the BLTS.
4. **Backward step**. The 2-tuples obtained by the aggregation method, are assessed in the BLTS, and can be distant from the linguistic term sets used by the sources of information. Therefore, it is offered the option to express the results in the initial term sets for a better comprehensiveness of them. This step is not neccesary, it is simply convenient.

Subsequently, we shall develop the above method over an MEDM problem.

3.1 Making the Information Uniform

With a view to manage the information we must make it uniform. The multigranular linguistic information provided by all the sources must be transformed into a unified linguistic term set, called BLTS and denoted as S_T.

Before defining a transformation function into this BLTS, S_T, we have to decide how to choose S_T. We take into consideration two possibilities:

- If there is only one term set with the maximum granularity, then, it is chosen as S_T.
- If we have two or more linguistic term sets with maximum granularity then, S_T is chosen depending on the semantics of these linguistic term sets, finding two possible situations to establish S_T:

  1. All the linguistic term sets have the same semantics, then S_T is any one of them.

  2. There are some linguistic term sets with different semantics. Then, S_T is a basic linguistic term set with a larger number of terms than the number of terms that a person is able to discriminate (normally 11 or 13, see [9]). We define a BLTS with 15 terms and the following semantics (see Figure 2):

s_0	$(0, 0, .07)$	s_1	$(0, .07, .15)$	s_2	$(.07, .15, .22)$
s_3	$(.15, .22, .29)$	s_4	$(.22, .29, .36)$	s_5	$(.29, .36, .43)$
s_6	$(.36, .43, .5)$	s_7	$(.43, .5, .57)$	s_8	$(.5, .57, .64)$
s_9	$(.57, .64, .71)$	s_{10}	$(.64, .71, .78)$	s_{11}	$(.71, .78, .85)$
s_{12}	$(.78, .85, .93)$	s_{13}	$(.85, .93, 1)$	s_{14}	$(.93, 1, 1)$

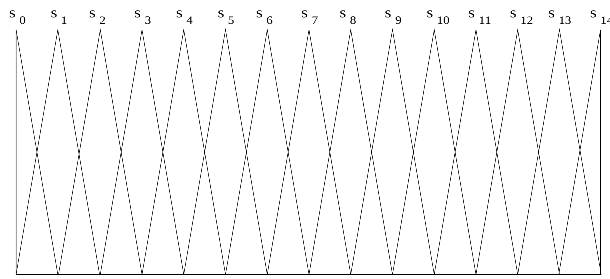

Figure 2: Term set with 15 terms

Once the BLTS has been chosen, the multigranular linguistic information is unified. This process involves the comparison between fuzzy sets representing the semantics of the initial terms assessed in S_j and the fuzzy sets of the linguistic terms of the BLTS. Comparisons are usually carried out by means of a measure of comparison. Depending on the framework, the measure of comparison can have different forms [8, 11]. We focus on measures of comparison which evaluate the resemblance or likeness of two objects (fuzzy sets in our case). These type of measures are called *"measures of similitude"* [1].

Measures of similitude are very general and different classes can be identified [1]. For simplicity, in this paper we shall choose a measure of similitude based on a possibility function $S(A, B) = \max_x \min(\mu_A(x), \mu_B(x))$, where μ_A and μ_B are the membership functions of the fuzzy sets A and B respectively. Therefore, to make the information uniform, we shall use the following function:

Definition 3 [5]. *Let $A = \{l_0, \ldots, l_p\}$ and $S_T = \{c_0, \ldots, c_g\}$ be two linguistic term sets, such that, $g \geq p$. Then, a multigranular transformation function, τ_{AS_T} is defined as:*

$$\tau_{AS_T} : A \longrightarrow F(S_T)$$

$$\tau_{AS_T}(l_o) = \{(c_k, \alpha_k^o) \,/ k \in \{0, \ldots, g\}\}, \; \forall l_j \in A$$

$$\alpha_k^o = \max_x min\{\mu_{l_o}(x), \mu_{c_k}(x)\}$$

where $F(S_T)$ is the set of fuzzy sets defined in S_T, and $\mu_{l_o}(x)$ and $\mu_{c_k}(x)$ are the membership functions of the fuzzy sets associated to the terms l_o and c_k, respectively.

The result of τ_{AS_T} for any linguistic value of A is a fuzzy set defined in the BLTS, S_T.We shall denote each $\tau_{S_j S_T}(y^{ij})$ with $y^{ij} \in S_j$, as r^{ij}, and represents each fuzzy set of performance, r^{ij}, by means of its respective membership degrees, i.e.,

$$r^{ij} = (\alpha_0^{ij}, \ldots, \alpha_g^{ij}).$$

3.2 Transforming Fuzzy Sets into 2-tuples

So far, we have unified the multigranular linguistic information transforming each linguistic term "y^{ij}" provided by the sources in a fuzzy set by means of $\tau_{S_j S_T}(y^{ij})$ over the BLTS S_T, such that,$\tau_{S_j S_T}(y^{ij}) = \{(c_0, \alpha_0^{ij}), ..., (c_g, \alpha_g^{ij})\}$. To deal with this type of information, we shall transform each fuzzy set into a linguistic 2-tuple using a central value computed by means of a weighted average, where the weights are the membership degrees of the fuzzy set. We shall define the function χ that computes a value $\beta \in [0, g]$ that represents a central value of the information in the fuzzy set $\tau_{S_j S_T}(y^{ij})$.

Definition 4. *Let $\tau_{S_j S_T}(l_o) = \{(c_0, \alpha_0^o), ..., (c_g, \alpha_g^o)\}$ be a fuzzy set that represents a linguistic term $l_o \in S_j$ over S_T. We shall obtain a numerical value, that supports the information of the fuzzy set, assessed in the interval $[0, g]$ by means of the following function:*

$$\chi : F(S_T) \longrightarrow [0, g]$$

$$\chi(\tau_{S_j S_T}(l_o)) = \frac{\sum_{k=0}^{g} k\alpha_k^o}{\sum_{k=0}^{g} \alpha_k^o} = \beta$$

This value β is easy to transform into a linguistic 2-tuple using the function Δ:

$$\Delta(\chi(\tau_{S_j S_T}(y^{ij}))) = \Delta(\chi(r^{ij})) = (s_k, \alpha)^{ij}$$

3.3 Fusion of 2-tuples

Here we shall obtain the result we are looking for, an aggregated value from the multigranular linguistic information.

Our objective is to aggregate the information associated to the alternative i. In [7] a wide range of 2-tuple linguistic aggregation operators were presented. To aggregate the 2-tuples, $(s_k, \alpha)^{ij}, \; j = 1, ..., m$, we shall choose one of these linguistic 2-tuple aggregation operators and we shall apply it to combine the 2-tuples, obtaining as a result an aggregated linguistic 2-tuple assessed in S_T.

Formally, it can be expressed as:

$$FO((s_k, \alpha)^{i1}, ..., (s_k, \alpha)^{im})) = (s_k, \alpha_k)^i$$

where FO is any 2-tuple fusion operator.An example of a 2-tuple aggregation operator can be:

Definition 5.[7] *Let $x = \{(r_1, \alpha_1), ..., (r_m, \alpha_m)\}$ be a set of 2-tuples, the 2-tuple arithmetic mean $\overline{x}^e$ is computed as,*

$$\overline{x}^e\{(r_1, \alpha_1), \ldots, (r_m, \alpha_m)\} =$$
$$= \Delta(\sum_{i=1}^{m} \frac{1}{m} \Delta^{-1}(r_i, \alpha_i)) = \Delta(\frac{1}{m} \sum_{i=1}^{m} \beta_i)$$

Therefore, an example can be:

$$\overline{x}^e\{(M, 0), (L, 0), (VL, 0), (H, 0)\} = (M, -.5)$$

3.4 The Backward Step

This is an optional step in the fusion process that offers the possibility of making a transformation to the initial term sets, for improving the comprehensiveness of the results. To accomplish the backward step we shall present a transformation function, that obtains a 2-tuple in any initial linguistic term set $S_j = \{s_0, ..., s_{g_j}\}$ from a 2-tuple expressed in the BLTS, $S_T = \{s_0, ..., s_g\}$. This function will carry out the following processes:

1. In first place, it transforms each 2-tuple $(s_k, \alpha_k) \in S_T$ into a fuzzy set in S_T with an only two values of membership degree different from 0:

$$\delta : S_T \times [-.5, .5) \longrightarrow \{S_T \times [0,1]\} \times \{S_T \times [0,1]\}$$

$$\delta(s_k, \alpha_k) = \{(s_h, 1 - \gamma_h), (s_{h+1}, \gamma_h)\}$$

where
$$h = trunc(\Delta^{-1}(s_k, \alpha_k))$$
$$\gamma_h = \Delta^{-1}(s_k, \alpha_k) - h$$

2. Following, it is applied the measure of similitude $\tau_{S_T S_j}$ to the above fuzzy set, obtaining two fuzzy sets in S_j:

$$\tau_{S_T S_j}(s_h) = \{(s_0, \alpha_0^h), ..., (s_{g_j}, \alpha_{g_j}^h)\}$$
$$\tau_{S_T S_j}(s_{h+1}) = \{(s_0, \alpha_0^{h+1}), ..., (s_{g_j}, \alpha_{g_j}^{h+1})\}$$

3. The fuzzy sets in the initial linguistic term set, S_j, are converted into numerical values assessed in $[0, g_j]$ by means of the χ function, obtaining β_h and $\beta_{h+1} \in [0, g_j]$, such that,

$$\chi(\tau_{S_T S_j}(s_h)) = \beta_h$$
$$\chi(\tau_{S_T S_j}(s_{h+1})) = \beta_{h+1}$$

4. To achieve our objective, we need to obtain a value $\beta_k^j \in [0, g_j]$ that represents the amount of information of (s_k, α_k). We have β_h *and* $\beta_{h+1} \in [0, g_j]$, that represent the information supportted by s_h *and* s_{h+1}, now we make a linear combination (LC) using the degrees of membership of the fuzzy set to obtain the value that we are looking for:

$$LC[(\beta_h, 1 - \gamma_h), (\beta_{h+1}, \gamma_h)] =$$
$$= (\beta_h * (1 - \gamma_h)) + (\beta_{h+1} * \gamma_h) = \beta_k^j \in [0, g_j]$$

Then, applying Δ to β_k^j we shall obtain the linguistic 2-tuple assessed in S_j that we were looking for:

$$\Delta(\beta_k^j) = (s_k^j, \alpha_k^j)$$

Now we define the function $\mathbf{\Gamma}$ that accomplish the whole process of the backward step:

Definition 6: *Let (s_k, α_k) be a 2-tuple assessed in the BLTS, therefore its equivalent 2-tuple in S_j is computed as:*

$$\Gamma : S_T \times [-.5, .5) \longrightarrow S_j \times [-.5, .5)$$

$$\Gamma(s_k, \alpha_k) = \Delta(LC(\chi(\tau_{S_T S_j}(\delta(s_k, \alpha_k))))) = (s_k^j, \alpha_k^j)$$

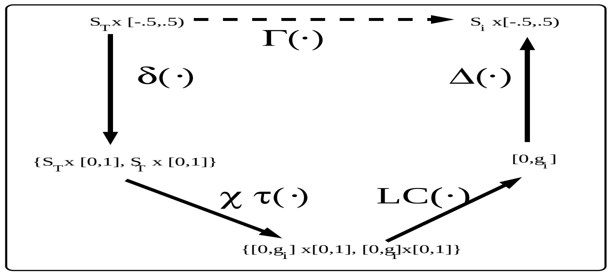

Figure 3: Backward step. Γ function

This process will be carried out for all source term sets S_j, therefore each source can easily understand the results. In *Figure 3* we can see graphically the whole process.

Obviously, the backward step has sense only if the order of the alternatives is not altered during the process. In [6] is proved that Γ does not alter the order.

4 Example

Here we shall apply the 2-tuple multigranular fusion method to a decision process over the following MEDM problem.

A distribution company needs to renew its computing system, so it contracts a consulting company to carry out a survey of the different possibilities existing on the market, to decide which is the best option for its needs. The alternatives are the following:

x_1	x_2	x_3	x_4
UNIX	WINDOWS-2000	AS/400	VMS

The consulting company has a group of four consultancy departments (experts).

p_1	p_2	p_3	p_4
Cost analysis	Systems analysis	Risk analysis	Technology analysis

Each department (expert) provides a performance vector expressing its preferences for each alternative assessed in linguistic term sets with a different granularity and/or semantics:

- p_1: preferences in the set of 9 labels, S_1.
- p_2: preferences in the set of 7 labels, S_2.
- p_3: preferences in the set of 5 labels, S_3.
- p_4: preferences in the set of 9 labels, S_4.

S_1		**S_4**	
a_0	$(0,\ 0,\ .12)$	d_0	$(0,0,0,0)$
a_1	$(0,\ .12,\ .25)$	d_1	$(0,.01,.02,.07)$
a_2	$(.12,\ .25,\ .37)$	d_2	$(.04,.1,.18,.23)$
a_3	$(.25,\ .37,\ .5)$	d_3	$(.17,.22,.36,.42)$
a_4	$(.37,\ .5,\ .62)$	d_4	$(.32,.41,.58,.65)$
a_5	$(.5,\ .62,\ .75)$	d_5	$(.58,.63,.80,.86)$
a_6	$(.62,\ .75,\ .87)$	d_6	$(.72,.78,.92,.97)$
a_7	$(.75,\ .87,\ .1)$	d_7	$(.93,.98,.99,1)$
a_8	$(.87,\ 1,\ 1)$	d_8	$(1,1,1,1)$

S_2		**S_3**	
b_0	$(0,\ 0,\ .16)$	c_0	$(0,0,.25)$
b_1	$(0,\ .16,\ .33)$	c_1	$(0,.25,.5)$
b_2	$(.16,\ .33,\ .5)$	c_2	$(.25,.5,.75)$
b_3	$(.33,\ .5,\ .66)$	c_3	$(.5,.75,1)$
b_4	$(.5,\ .66,\ .83)$	c_4	$(.75,1,1)$
b_5	$(.66,\ .83,\ .1)$		
b_6	$(.83,\ 1,\ 1)$		

The performance vectors provided by the experts are the following:

		experts			
		p_1	p_2	p_3	p_4
alternatives	x_1	a_4	b_3	c_2	d_4
	x_2	a_6	b_4	c_3	d_5
	x_3	a_3	b_3	c_2	d_3
	x_4	a_5	b_5	c_1	d_5

where $y_{ij} \in S_j$ is the performance value given by the expert p_j over the alternative x_i.

We shall apply the decision process presented in section 2.3 to solve this MEDM problem with multigranular linguistic information.

A. Collective Performance Vector.

1. **Making the Information Uniform**

   We have to choose the BLTS. In this case S_T is the term set of 15 labels given in Figure 2. All the assessments are converted to S_T by means of $\tau_{S_i S_T}$. We obtain fuzzy sets as results:

$$r^{11}(0,0,0,0,.05,.45,.8,.82,.48,.23,0,0,0,0,0)$$

2. **Transforming r^{ij} into 2-tuples**

   Using the functions χ and Δ. As example we obtain:

$$\Delta(\chi(r^{11})) = (s_7,-.32)^{11}\Delta(\chi(r^{21})) = (s_7,-.05)^{21}$$

   After this transformation, we manage 2-tuples assessed in the BLTS, S_T.

3. **Computing the collective performance values**

   For each alternative x_i we compute its collective performance value using a 2-tuple linguistic aggregation operator, in this case we choose the 2-tuple mean operator:

$$\begin{aligned} x_1 &\longrightarrow \overline{x}^e((s_j,\alpha)^{1i})) = (s_8,-.46)^1 \\ x_2 &\longrightarrow \overline{x}^e((s_j,\alpha)^{2i})) = (s_9,-.32)^2 \\ x_3 &\longrightarrow \overline{x}^e((s_j,\alpha)^{3i})) = (s_7,-.16)^3 \\ x_4 &\longrightarrow \overline{x}^e((s_j,\alpha)^{4i})) = (s_8,-.25)^4 \end{aligned}$$

   Then the collective vector is:

$$\{(s_8,-.46)^1,(s_9,-.32)^2,(s_7,-.16)^3,(s_8,-.25)^4\}$$

4. **The Backward step**

   Now we can make the backward step to express the collective performance vector in the linguistic term sets used by the experts, i.e., S_1, S_2, S_3, S_4. To do so, we shall use the Γ function:

   (a) First, the collective values are transformed into fuzzy sets in S_T.

$$\begin{aligned} \delta(s_8,-.46)^1 &= \{(s_7,.46),(s_8,.54)\}^1 \\ \delta(s_9,-.32)^2 &= \{(s_8,.32),(s_9,.68)\}^2 \\ \delta(s_7,-.16)^3 &= \{(s_6,.16),(s_7,.84)\}^3 \\ \delta(s_8,-.25)^4 &= \{(s_7,.25),(s_8,.75)\}^4 \end{aligned}$$

   (b) Following, we shall apply the functions $\tau_{S_T S_1}, \tau_{S_T S_2}, \tau_{S_T S_3}, \tau_{S_T S_4}$ to the above fuzzy sets. For example:

$$\tau_{S_T S_3}(s_6) = \{(c_0,0)(c_1,.4)(c_2,.79)(c_3,0)(c4,0)\}$$

   (c) Transforming the fuzzy sets into numerical values by means of the χ function. An example can be:

$$\chi(\tau_{S_T S_3}(s_6)) = \quad 1.66$$

   (d) Expressing the collective vector in all initial linguistic term sets:

S_1 :$\{(a_4, .34)^1, (a_5, -.02)^2, (a_4, -.09)^3, (a_5, -.14)^4\}$
S_2 : $\{(b_3, .3)^1, (b_4, -.23)^2, (b_3, -.08)^3, (b_4, -.33)^4\}$
S_3 : $\{(c_2, .15)^1, (c_2, .44)^2, (c_2, -.06)^3, (c_2, .21)^4\}$
S_4 : $\{(d_4, .17)^1, (d_5, -.46)^2, (d_4, .02)^3, (d_4, .24)^4\}$

B. Selection Process

Finally, we shall apply a choice degree to the collective performance vector to obtain the solution set of alternatives. In this problem the solution set obtained is $\{\mathbf{x_2}\}$. Then, the best option for the distribution company according to the experts' opinion is the Windows-2000 based system.

5 Concluding Remarks

In this paper we have presented a fusion method based on the 2-tuple fuzzy linguistic representation that allows us to easily deal with multigranular linguistic information in fusion processes. The development of this method takes as base the 2-tuple linguistic representation model and its computational technique.

This new fusion method is useful for problems with multiple sources of information that express their knowledge with linguistic information assessed in several linguistic term sets with different cardinality and/or semantics. We have applied this fusion method to a decision process as an application example.

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