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A linguistic information granulation model based on best-worst method in decision making problems[☆]

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ABSTRACT

In the elicitation of decision makers' fuzzy and uncertain assessments, linguistic terms are natural and efficient as the preference modeling tools. Although the linguistic variables are available, they would not be operational without any detailed quantification. Motivated by the flexibility of information granularity, this paper develops information granules to represent linguistic terms in the form of intervals and interval type-2 fuzzy sets (IT2FSs) in best worst method (BWM). The development is aimed at minimizing inconsistency in the decision making (DM) process to ensure the rationality of the assessments provided by decision makers. Furthermore, the input and output based consistencies of BWM are considered. The granulation of entries of pairwise comparison vectors are the foundation of BWM to formulate an optimization problem where particle swarm optimization (PSO) algorithm serves as the optimization framework. Both individual and group decision making (GDM) scenarios are taken into consideration. For the GDM process, a performance index for measuring the group consensus is also proposed. Several examples and validity analysis are covered to illustrate the major ideas of this study. Finally, as a case study, a recommendation of the sequence of visiting tourist attractions in Wuhan and the corresponding comparative analysis are represented.

1. Introduction

In decision making (DM) situations especially for group decision making (GDM), due to their subjectivity, perception of things and domain knowledge, it may seem unreasonable that people make judgements using merely precise numerical assessments [1]. The introduction of fuzzy sets played an important role in solving DM problems [2,3], which serves as the adequate tool to denote the unclear preference information in actual DM scenarios. Generally, people prefer to express their preference information linguistically. The linguistic evaluations have the wide applications in the complex uncertain problems, which can contain more information than single numbers. There are many DM models based on linguistic approaches have been proposed [4–9], such as hesitant fuzzy linguistic information GDM models [4,5], the linguistic computation model based on double hierarchy linguistic preference relations [6], flexible linguistic preference expressions [7], hesitant 2-tuple linguistic terms sets [8,10], discrete fuzzy numbers [9]. Apparently, the qualification of linguistic information is the essential asset in DM models with linguistic evaluation, which can transform the available linguistic information into the formal formalisms of sets [11], fuzzy sets [12] and so on [6,13,14].

Meanwhile, it is worth noting that the transformation of the linguistic information into information granularity becomes an emerging treatment of information processing in the DM situations [10,15–21]. The information granules are extracted in the process of data abstraction and knowledge derivation from the linguistic information. Compared with the distribution and the semantics of the linguistic terms given in advance [4–9], the granulation of linguistic information is processed within a certain level of information granularity. Then, specifies the formal settings of granules and the optimization criteria in the process of dealing with DM problems, which is more flexible and generality [22]. Granular computing makes the linguistic assessments operational, there are abundant of formal framework of information

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granulation. For instance, the granular linguistic information can be qualified in the form of intervals [11,15], fuzzy sets [10,12], and other forms [19,20]. In the current granulation of linguistic terms models [15,17,21], researchers mainly concentrate on the interval form of information granules, while other formalisms are seldom considered. Based on this, two forms of information granules are considered: the classical intervals-based granules and the interval type-2 fuzzy sets (IT2FSs)-based granules to qualify the linguistic terms, and then proceed to the granulation process. The reasons for the selection of the above two formalizations of granules can be summarized as follows: we choose the intervals to construct the interval-based information granules, then this form of information granules is arranged to represent the linguistic terms. It is different from allocating the specific intervals to the corresponding linguistic terms in advance, we input the information granules to turn to the problem into an optimization problem by giving a certain granularity level. Furthermore, we applied the interval-based information granules, which are the most frequently used form in many studies [10,15]. Intervals can intuitively explain the allocation and the optimization process of information granules and allow us to understand the essence of granular computing as well. For the IT2FSs-based granules, compared with the above mentioned interval-based granules, IT2FSs-based granules can cope with uncertainty and fuzziness of the decision making problems [23]. The IT2FSs consist of the imprecise membership functions. In the meanwhile, IT2FSs give rise to the simpler calculation than type-2 fuzzy sets or higher type fuzzy sets. Secondly, for the computation of the IT2FSs-based granules, it is divergent from directly computing the median of centroid of the IT2FSs [24], this proposed model applied the heuristic algorithm PSO algorithm to obtain the optimal value in the centroid interval with considering the flexibility of information granules.

In the multi-criteria decision making (MCDM) scenarios, the consistency of evaluation information of decision makers needs to be measured and optimized. Many researches [10,15–17] focus on the Analytic Hierarchy Process (AHP) method in granular models with constructing the linguistic pairwise comparison matrices to calculate the consistency. While in this paper, we choose the best worst method (BWM) model [25], which reduces the numbers of pairwise comparisons than traditional AHP method to produce the consistent results [25,26]. It is acknowledged that BWM is not a special case of AHP, although they are both the subjective pairwise comparison methods. Intuitively, for the construction of consistency index, in AHP model, the establishment of consistency index is related to the preference matrix by calculating the maximum eigenvalue of matrix. In BWM model, the consistency index is associated with a set of non-linear min–max in-equations. The more differences are discussed in [25]. BWM has been widely applied in various DM problems since its inception for its simplicity and reliability. The improvements for BWM, see in [27,28], and the applications of BWM model in various areas, see in [29–32]. Through the above analysis, it is found that there was no research on BWM model in the perspective of granular computing. To fill this gap, this study constructs an original way to achieve the granulation of linguistic terms set in BWM. In addition, we both consider the input-based and output-based consistencies of BWM [28] in this model.

Recently, in the real-world DM scenarios, considering the complexity and diversity of problems, it is a common phenomenon that a group of experts to make decisions instead of a single individual. Therefore, the discussion on group consistency is necessary and meaningful [33]. There are several common group consistency measurements for different types of preference information. The additive consistency measurement [10,15,33], the multiplicative consistency measurement [34], And other types of consistency measurements can see in [33,35]. In this proposed, we use the average consistency measurement, through the aggregation of individuals' preference information to form the group preference information, thereafter, it is brought into the granular BWM model to calculate the consistency index. The key point is that it is unlike other studies that apply some mathematic operators to gather evaluation information, such the addictive weighted operator [15], we establish a distance-based function to aggregate information based on the granular linguistic terms, which the aggregation process can be transformed into an optimization problem to improve the group consistency by minimizing the distance-based function. PSO algorithm [36] is served as the optimization tool, for its high frequency application in solving complex optimization problems [10,15–17]. In a nutshell overall, the motivation of this paper mainly describes as the construction of granular BWM model in the DM problems both involving a single decision maker and the group scenarios, where for the former situation we can pay more attention to the granulation of linguistic information, and the GDM situation is in accordance with the solution of the DM problems in reality. The granulation formalism of the linguistic terms set discussed in this paper focus on the intervals and the IT2FSs. Different from predefining the semantics in BWM models [37,38], it is worth noting that the input of information granulation provides an operational DM model of the BWM with the linguistic pairwise comparisons. Then, the PSO optimization framework helps transform linguistic qualification into the meaningful information granules in the aim of the achievement of the highest consistency.

The major contributions of this paper are summarized as follows:

- We introduce the granular BWM model through the granulation of the linguistic preference information with both considering the input-based and output-based consistency. Specifically, we establish a suitable mapping of the linguistic terms on information granules in the aim of minimizing the inconsistency in DM process, which is different from giving the distribution and the semantics of the linguistic terms in advance and can be more flexible to reach the higher level of consistency.
- We establish the two forms of information granules to achieve the granulation of linguistic terms. For the interval-based granules, we separate the linguistic terms by allocating a set of cutoff points vectors in a certain range. PSO algorithm is applied to find the optimal cutoff points vector in the aim of maximizing the consistency of this model; for the IT2FSs-based granules, the corresponding membership function (MF) of each granular linguistic term is pre-given, then KM algorithm [39] is used to calculate the centroid of each IT2FS-formed linguistic term. And PSO is also served as the optimization tool to find the suitable value in the above centroids.
- We design the average consistency measurement, the group preference information is obtained by aggregating individual preference information, and then calculate the consistency indexes in the granular BWM model. A standard Euclidean distance-based function is proposed to assist the aggregation process to acquire a high consistent group preference information.

The organization of this paper is as follows, Section 2 overviews the basic knowledge about BWM and the treatment of information granules. Section 3 provides the constructions of granular linguistic information in the form of intervals and IT2FSs in BWM model, the optimization tool PSO algorithm is represented as well. Then, several numerical examples are given to portray the model in detail. Section 4 considers the GDM situations where an Euclidean distance-based objective function is represented to aggregate the individuals' linguistic preference information with maximizing group consistency. Section 5 exhibits the comparative analysis between the model put forward by Pedrycz and Song in [15] and the proposed model to illustrate the reliability of this model. Thereafter, a tourist sites recommendation based on online reviews and the relative comparison results with other fuzzy BWM methods [37,38] are represented in Section 6. In the end, some conclusions are offered in Section 7.

2. Preliminaries

In this section, we briefly present several prerequisites related to BWM and information granules that form the cornerstone of the proposed study.

2.1. BWM model

As a novel pairwise comparison MCDM method, BWM contains two types of pairwise comparison vectors produced by a decision maker, that is, B-O vector (Best to Others) and O-W vector (Others to Worst). The final result can be obtained by some linear or nonlinear models [25,26]. The main steps of the original BWM method are as follows:

Step 1. Construct the criteria set $\{C_1, \dots, C_n\}$

Step 2. Determine the best criterion C_B, and the worst criterion C_W

Step 3. Construct the B-O vector and the O-W vector, which can be expressed as $(a_{B1}, \dots, a_{Bn})$ and $(a_{1W}, \dots, a_{nW})$, where a_{Bj} means the preference value of the best criterion C_B over the criterion C_j $(j = 1, \dots, n)$ and a_{jW} is the preference value of the criterion C_j over the worst criterion C_W.

Step 4. Obtain the weights of criteria $\{w_1^*, w_2^*, \dots, w_n^*\}$ through solving the following nonlinear model.

$$\min \quad \xi$$
$$s.t. \begin{cases} |\frac{w_B}{w_i} - a_{Bi}| \leqslant \xi \\ |\frac{w_i}{w_W} - a_{iW}| \leqslant \xi \\ \sum_{i=1}^{n} w_i = 1 \\ w_i \geqslant 0 \end{cases} \tag{1}$$

Definition 1 (*[28]*)**.** The output-based consistency ratio CR^O is described as:

$$\begin{cases} CR^0 = \frac{\xi^*}{\xi_{\max}} \quad CR^0 \in [0,1] \\ \xi^2 - (1 + 2a_{BW})\xi + (a_{BW}^2 - a_{BW}) = 0 \end{cases} \tag{2}$$

where ξ^* can be obtained by the aforementioned model in Eq. (1) and $\xi_{\max}$ is the maximum possible ξ.

Definition 2 (*[28]*)**.** The input-based consistency ratio CR^I is expressed as follows:

$$CR^I = \max_j CR_j^I$$
$$s.t. \quad CR_j^I = \begin{cases} |\frac{a_{Bj} \times a_{jW} - a_{BW}}{a_{BW} \times a_{BW} - a_{BW}}| & a_{BW} > 1 \\ 0 & a_{BW} = 1 \end{cases} \tag{3}$$

where CR^I stands for the local consistency level in accordance with criterion j and $CR^I \in [0, 1]$.

Remark 1. If the values of CR^O and CR^I are closer to 0, which indicate that the preferences are more consistent. In particular, $CR^O = 0$ and $CR^I = 0$ mean the preferences are cardinal-consistent, which the pairwise comparison system satisfied $a_{Bj} \times a_{jW} = a_{BW}$. The consistency threshold for CR^I and CR^O can be obtained in Tables 1 and 2 in Ref. [28], in which the value of a_{BW} ranges from 3 to 8 with the number of criteria from 3 to 8.

Because of these thresholds, we can judge whether the consistency ratio CR^I and CR^O are acceptable, and it also determines whether decision makers need to modify the pairwise comparison vectors.

2.2. Treatment of information granularity

The major contents of information granules are the formation and the corresponding granularity restrictions [15]. And the information granules in this study are formed and processed in the intervals and IT2FSs framework, with a certain level of information granularity. In what follows, the basic concepts of IT2FSs related to the IT2FSs-based granules are mainly illustrated.

Table 1
Thresholds for the input consistency CR^I.

a_{BW}	Criteria					
	3	4	5	6	7	8
3	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667
4	0.1121	0.1529	0.1898	0.2206	0.2527	0.2577
5	0.1354	0.1994	0.2306	0.2546	0.2716	0.2844
6	0.1330	0.1990	0.2643	0.3044	0.3144	0.3221
7	0.1294	0.2457	0.2819	0.3029	0.3144	0.3251
8	0.1309	0.2521	0.2958	0.3154	0.3408	0.3620

Table 2
Thresholds for the input consistency CR^O.

a_{BW}	Criteria					
	3	4	5	6	7	8
3	0.2087	0.2087	0.2087	0.2087	0.2087	0.2087
4	0.1581	0.2352	0.2738	0.2928	0.3102	0.3154
5	0.2111	0.2848	0.3019	0.3309	0.3479	0.3611
6	0.2164	0.2922	0.3565	0.3924	0.4061	0.4168
7	0.2090	0.3313	0.3734	0.3931	0.4035	0.4108
8	0.2267	0.3409	0.4029	0.4230	0.4379	0.4543

Definition 3 (*[24]*)**.** The general type-2 fuzzy set (T2FS) $\widetilde{A}$, expressed in the universe of discourse X, is defined as:

$$\widetilde{A} = \{((x,u), \mu_{\widetilde{A}}(x,u)) | \forall x \in X, \forall u \in J_x \subseteq [0,1]\} \tag{4}$$

where x is the primary variable, J_x is the primary MF as the restriction on the value of u and $\mu_{\widetilde{A}}(x,u)$ is the secondary MF imposed by x and u.

Definition 4 (*[24]*)**.** Let the secondary MF $\mu_{\widetilde{A}}(x,u) = 1$. IT2FS is the special case of T2FS, which can be written as:

$$\widetilde{A} = \int_{x \in X} \int_{u \in J_x} \frac{1}{(x,u)} \quad J_x \subseteq [0,1] \tag{5}$$

where a trapezoidal IT2FS is depicted in Fig. 1.

Definition 5 (*[40]*)**.** Let $\widetilde{A}$ be an arbitrary non-negative interval type-2 trapezoidal fuzzy set (IT2TrFS) defined in the universe of discourse X and $\widetilde{A}_L$, $\widetilde{A}_U$ be the two type-1 fuzzy sets (T1FSs) satisfied:

$$\begin{aligned} \widetilde{A} &= [\widetilde{A}_L, \widetilde{A}_U] \\ &= [(a_1^L, a_2^L, a_3^L, a_4^L, h_{\widetilde{A}}^L), (a_1^U, a_2^U, a_3^U, a_4^U, h_{\widetilde{A}}^U)] \end{aligned} \tag{6}$$

where $\widetilde{A}_L = (a_1^L, a_2^L, a_3^L, a_4^L, h_{\widetilde{A}}^L)$, $\widetilde{A}_U = (a_1^U, a_2^U, a_3^U, a_4^U, h_{\widetilde{A}}^U)$, and the overall MF (contained $\underline{\mu}_{\widetilde{A}}(x)$ and $\bar{\mu}_{\widetilde{A}}(x)$) of $\widetilde{A}$ can be presented as:

$$\underline{\mu}_{\widetilde{A}}(x) = \begin{cases} \frac{(x-a_1^U)}{(a_2^U-a_1^U)} \cdot h_{\widetilde{A}}^U & a_1^U \leqslant x < a_2^U \\ h_{\widetilde{A}}^U & a_2^U \leqslant x < a_3^U \\ \frac{(a_4^U-x)}{(a_4^U-a_3^U)} \cdot h_{\widetilde{A}}^U & a_3^U \leqslant x < a_4^U \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

and

$$\bar{\mu}_{\widetilde{A}}(x) = \begin{cases} \frac{(x-a_1^L)}{(a_2^L-a_1^L)} \cdot h_{\widetilde{A}}^L & a_1^L \leqslant x < a_2^L \\ h_{\widetilde{A}}^L & a_2^L \leqslant x < a_3^L \\ \frac{(a_4^L-x)}{(a_4^L-a_3^L)} \cdot h_{\widetilde{A}}^L & a_3^L \leqslant x < a_4^L \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

where $a_1^L, a_2^L, a_3^L, a_4^L, a_1^U, a_2^U, a_3^U, a_4^U, h_{\widetilde{A}}^L, h_{\widetilde{A}}^U$ are real numbers and satisfying the conditions $a_1^L \leqslant a_2^L \leqslant a_3^L \leqslant a_4^L$, $a_1^U \leqslant a_2^U \leqslant a_3^U \leqslant a_4^U$, $0 \leqslant h_{\widetilde{A}}^L \leqslant h_{\widetilde{A}}^U \leqslant 1$.

Definition 6 (*[39]*). Let $\widetilde{A}$ be an IT2FS, the left and right end-point of the centroid of $\widetilde{A}$, $c_l(\widetilde{A})$ and $c_r(\widetilde{A})$ can be expressed as follows.

$$c_l(\widetilde{A}) = \min_{\forall \theta(x_i)\in[\bar{\mu}_{\widetilde{A}}(x), \underline{\mu}_{\widetilde{A}}(x)]} \frac{\sum_{i=1}^{N} x_i\theta_i}{\sum_{i=1}^{N}\theta_i} \tag{9}$$

$$c_r(\widetilde{A}) = \max_{\forall \theta(x_i)\in[\bar{\mu}_{\widetilde{A}}(x), \underline{\mu}_{\widetilde{A}}(x)]} \frac{\sum_{i=1}^{N} x_i\theta_i}{\sum_{i=1}^{N}\theta_i} \tag{10}$$

where KM algorithm represented in **Algorithm 1** is introduced to compute the end-points of $c_l(\widetilde{A})$ and $c_r(\widetilde{A})$. And the KM algorithm for $c_r(\widetilde{A})$ is the same as the above procedures.

Algorithm 1 Determination of $c_l(\widetilde{A})$ in KM algorithm

Input: the MF of an IT2F $\bar{\mu}_{\widetilde{A}}(x)$ and $\underline{\mu}_{\widetilde{A}}(x)$
Output: the satisfied left end point $c_l(\widetilde{A})$ of $\widetilde{A}$

```
while not satisfy the following condition do
    find k ∈ [1, N − 1] such that x_k ⩽ c′ ⩽ x_{k+1}
    for i = 1 → N do
        if x_i ⩽ k then
            θ(x_i) ← μ̄_Ã(x)
        else
            θ(x_i) ← μ_Ã(x)
            c″ ← (Σ_{i=1}^{k} x_i μ̄_Ã(x) + Σ_{i=k+1}^{N} x_i μ_Ã(x)) / (Σ_{i=1}^{k} μ̄_Ã(x) + Σ_{i=k+1}^{N} μ_Ã(x))
        end if
        if c″ ≠ c′ then
            c′ ← c″
        else
            c″ ← c_l(Ã)
        end if
    end for
end while
```

3. Granular linguistic BWM method based on PSO

In this section, the entries of pairwise comparison vectors in BWM model are expressed linguistically, the linguistic variables themselves are not operational to further calculate and form the ranking of alternatives. Information granules are provided to qualify these linguistic terms with a certain level of information granularity. Therefore, it can be formulated the granularity optimization problem. The information granules in this proposed model are presented as intervals and IT2FSs. Moreover, PSO is viewed as the optimization tool in the granulation process. Generally, the problem description in this Section can be summarized as supposed that, there are set of criteria: $C = \{c_1, \dots, c_j, \dots, c_n\}$ for a decision maker to obtain the corresponding criteria weight vector $W = (w_1, \dots, w_j, \dots, w_n)^T$ $\sum_j w_j = 1$, where the best criterion c_B and the worst criterion c_W are predetermined. And the preference information is expressed in the linguistic term set $S = \{s_1, \dots, s_l, \dots, s_m\}$, which is qualified by interval-based granules and the IT2FSs-based granules respectively.

3.1. Granular construction process for linguistic terms

In this study, we elaborate on two granulation forms for the qualification linguistic terms: (*A*). the interval-based information granules and (*B*). IT2FSs-based of granules.

(A) Establishment for the linguistic terms as interval-valued information granule

For the linguistic terms set $S = \{s_1, \dots, s_l, \dots, s_m\}$, different from predefining the linguistic terms in the specific intervals, the linguistic terms set is divided by a set of variables (cutoff points) $X = \{x_1, \dots, x_k, \dots, x_{(m-1)}\}$ through a level of information granularity, where $1 < x_1 < x_2 < \cdots < x_m < 9$. Then, the linguistic term s_l $(l = 1, \dots, m)$ can be denoted as $[x_k, x_{(k+1)}]$, more intuitively, we can illustrate the granulation process of the linguistic terms as portrayed in Fig. 2. Therefore, the expression of the linguistic terms (represented in Fig. 2) are as follows: *Low* $= [1, x_1]$, *Medium* $= [x_1, x_2]$, *High* $= [x_2, 9]$. It can be seen that when the number of linguistic terms is m, there are m+1 cutoff points. Afterwards, PSO algorithm is served as the optimal tool to find the suitable cutoff points in a certain range, i.e. 1–9 scale. In addition, intervals are the commonly form of linguistic information granulation in many studies [15,16,21], which can intuitively explain the essence of granular computing. Therefore, the interval-based granules are taken into consideration in the proposed model.

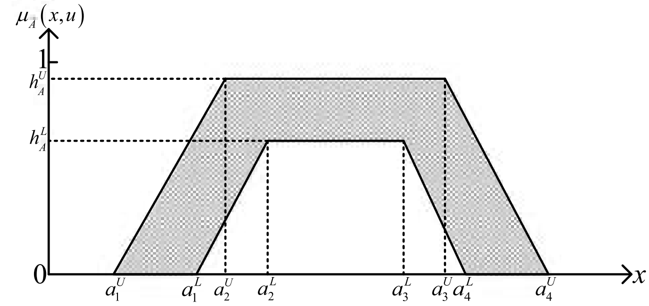


Fig. 1. IT2FSs with the trapezoidal membership function.

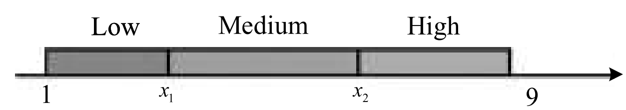


Fig. 2. The interval-based linguistic terms form.

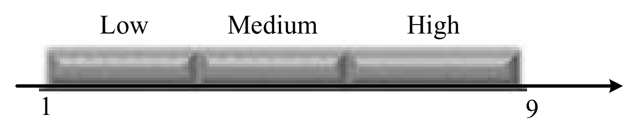


Fig. 3. The IT2FSs-based linguistic terms form.

(B) Qualification of linguistic variables in IT2FSs

In this subsection, the mechanism of the IT2FSs-based information granules to denote the linguistic terms set $S = \{s_1, \dots, s_l, \dots, s_m\}$ are presented as follow: firstly, the MF of IT2FSs-based granules corresponding to the linguistic term s_l $(l = 1, \dots, m)$ is established in advance with a certain level of information granularity. The KM algorithm is applied to calculated the centroid of s_l, denoted as: $[c_l(s_l), c_r(s_l)]$. Then, we use the PSO algorithm to find the optimal value in the specific interval instead of directly taking the median $\left(c_l\left(s_l\right) + c_r\left(s_l\right)\right)/2$. Take an instance, for the linguistic terms set $\{\text{Low}, \text{Median}, \text{High}\}$ in the form of IT2FSs-based granules in Fig. 3, the linguistic term "Low" can be expressed as $[c_l(\text{Low}), c_r(\text{Low})]$, "Median" presents as $[c_l(\text{Median}), c_r(\text{Median})]$ and "High" is $[c_l(\text{High}), c_r(\text{High})]$, where $1 < c_l(\text{Low}) < c_r(\text{Low}) < c_l(\text{Median}) < c_r(\text{Median}) < c_l(\text{High}) < c_r(\text{High}) < 9$.

3.2. The optimization of information granules

After the establishment of information granules, this subsection concentrates on the issue of how to determine the cut-off points of the interval-based granules and the suitable value of the centroid of IT2FSs-based granules, which can both be considered as the optimization problems. The consistency in the decision making process is defined as the optimization criterion, which contains the input-based and output-based consistency measurements. In what follows the optimization process of the interval-based granules and the IT2FSs-based granules are presented.

(A) Optimization for linguistic terms in form of intervals

For the interval form of granular linguistic terms, firstly, the positions of the optimal cutoff points are supposed to be established in a certain range i.e. 1–9 scale, the following objective function Q_1 in this study served as a measurement index is to measure the input-based consistency in DM process, presented as:

$$Q_1 = \max_j CR_j^I = \max_j \frac{|a_{Bj} \times a_{jW} - a_{BW}|}{a_{BW} \times a_{BW} - a_{BW}} \tag{11}$$

where $j = 1, \ldots, n$ and $j \neq B, W$, $a_{BW} > 1$ and CR_j^I ($CR_j^I \in [0,1]$) denotes the input-based consistency index of the *jth* criterion. When the value of CR_j^I is equal to 0, it means that the evaluation process is fully consistent. The aim of the optimization process is to minimize the objective function Q_1, that is, minimize CR^I, where the threshold of CR^I refers to Table 1. Thereafter, the interval-based linguistic terms are the entries of pairwise comparison vectors in the granular BWM model. And a_{Bj}, a_{jW} can be expressed as $[a_{Bj}^L, a_{Bj}^U]$ and $[a_{jW}^L, a_{jW}^U]$. We use Monte Carlo simulation [41] by randomly generating 100 000 particles in the above intervals to search the suitable one. The following linear model is constructed as the output-based consistency measurement expressed as:

$$\min_{k=1,\ldots,N} \{\xi_{(k)}^*\}$$
$$s.t. \begin{cases} w_B^{(k)} - a_{Bj}^{(k)} w_j^{(k)} \leqslant \xi^{(k)} \\ -(w_B^{(k)} - a_{Bj}^{(k)} w_j^{(k)}) \leqslant \xi^{(k)} \\ w_j^{(k)} - a_{jW}^{(k)} w_W^{(k)} \leqslant \xi^{(k)} \\ -(w_j^{(k)} - a_{jW}^{(k)} w_W^{(k)}) \leqslant \xi^{(k)} \\ \sum_j w_j^{(k)} = 1, 0 < w_j^{(k)} < 1 \end{cases} \tag{12}$$

where $a_{Bj}^{(k)} \in [a_{Bj}^L, a_{Bj}^U], a_{jW}^{(k)} \in [a_{jW}^L, a_{jW}^U]$, $a_{Bj}^{(k)}$ and $a_{jW}^{(k)}$ represent the random *kth* point in the interval $[a_{Bj}^L, a_{Bj}^U]$ and $[a_{jW}^L, a_{jW}^U]$ respectively, $w_j^{(k)}$ denotes the corresponding weight of the *kth* criterion. $\xi_{(k)}^*$ is *kth* output-base consistency value. The complete optimization process for linguistic terms set in interval-based granules in BWM model is introduced in **Algorithm 2**.

Algorithm 2 Algorithm for designing interval-based linguistic terms set in BWM

Input: criteria set $C = \{c_1, c_2, ..., c_n\}$, linguistic terms set $S = \{s_1, s_2, ..., s_{m+1}\}$, cutoff points set $X = \{x_1, x_2, ..., x_m\}$
Output: the ranking result of criteria set

while not find the optimal result **do**
  for each linguistic term $s_i (i = 1, ..., m + 1)$ **do**
    $s_i \leftarrow [x_k, x_{k+1}]$, $x_k \in [1, 9]$ $(k = 1, ..., m)$
  end for
  construct the entries of B-O and O-W vectors
  for each a_{Bj} and a_{jW} $(j = 1, ..., n)$ **do**
    minimize fitness function Q_1 in Eq. (11) in PSO framework
  end for
  find the optimal cutoff points
  formalize the interval linguistic term entries
  calculate ξ^* and w_j in Eq. (12) by Monte Carlo simulation
end while

(B) Optimization for linguistic terms in form of IT2FSs

For the IT2FSs form of granular linguistic terms with the corresponding MF given in advance, KM algorithm [39] presented in **Algorithm 1** is applied to calculate these centroids. Thus, the entries of the B-O and O-W vectors are denoted as $[c_l(a_{Bj}), c_r(a_{Bj})]$ and $[c_l(a_j W), c_r(a_j W)]$. And the PSO algorithm is arranged as well to find the optimal values a_{Bj}^*, a_{jW}^* from the above intervals, which the granular BWM model with both considering the input-based and output-based consistencies is follows as:

$$\min \xi$$
$$s.t. \begin{cases} w_B - a_{Bj}^* w_j \leqslant \xi \\ -(w_B - a_{Bj}^* w_j) \leqslant \xi \\ w_j - a_{jW}^* w_W \leqslant \xi \\ -(w_j - a_{jW}^* w_W) \leqslant \xi \\ (a_{Bj}^*, a_{jW}^*) = argmin(\max(\frac{|a_{Bj} \times a_{jW} - a_{BW}|}{a_{BW} \times a_{BW} - a_{BW}})) \\ c_l(a_{Bj}) \leq a_{Bj}^* \leq c_r(a_{Bj}) \\ c_l(a_{jW}) \leq a_{jW}^* \leq c_r(a_{jW}) \\ \sum_j w_j = 1, 0 < w_j < 1 \end{cases} \tag{13}$$

The entire optimization process for IT2FSs-based linguistic terms is represented in **Algorithm 3**. In addition, Remark 2 portrays the detail construction process of B-O and O-W vectors and the connection between these vectors and the decision variables of PSO algorithm, for the convenience of understanding **Algorithms 2** and 3.

Algorithm 3 Algorithm for IT2FSs-based linguistic terms set in BWM

Input: criteria set $C = \{c_1, c_2, ..., c_n\}$, linguistic term set $S = \{s_1, s_2, ..., s_{m+1}\}$, the corresponding MF of IT2FSs
Output: the ranking result of criteria set

while not find the optimal result **do**
  for each linguistic term s_i $(i = 1, ..., m)$ **do**
    construct the IT2FSs of s_i : $\tilde{s}_i$
    compute the centroid of $\tilde{s}_i$ in **KM algorithm**
    let the linguistic term s_i express as $\left[c_l^{\tilde{s}_i}(s_i), c_r^{\tilde{s}_i}(s_i)\right]$
    construct the entries of B-O and O-W vectors
  end for
  for each a_{Bj} and a_{jW} $j = (1, ..., n)$ **do**
    minimize the target model in Eq. (13) in PSO framework
  end for
  find the optimal a_{Bj}, a_{jW}
  compute w_j and CR^O in Eqs.(1) and (2)
end while

Remark 2. In **Algorithm 2**, the preference values of B-O and O-W vectors are denoted by the given linguistic terms set S, The qualification of the linguistic terms in **Algorithm 2** is achieved by the interval-based granules. For instance, $S = L, M, H$ and $L = [1, x_1]$, $M = [x_1, x_2]$, $H = [x_2, 9]$, PSO algorithm is arranged to search the suitable cut-off points x_1, x_2 1–9 scale with the aim of minimizing the inconsistency of BWM model; while, the qualification of the linguistic terms in **Algorithm 3**, is denoted by the IT2FSs-based granules. Simultaneously, for $S = \{L, M, H\}$ with giving rise to the membership function to each linguistic term in advance, then we use the KM algorithm presented in **Algorithm 1** to calculate the centroid intervals, the linguistic terms can be expressed as: $L = [c_l(L), c_r(L)]$, $M = [c_l(M), c_r(M)]$, $H = [c_l(H), c_r(H)]$ satisfied $c_l(L) > 1$ and $c_l(H) < 9$. PSO is applied to find the optimal value from the above centroid intervals.

3.3. Optimization tool

The PSO algorithm [36] as one of population-based algorithms has the wide range of applications in DM problems [10,15–17]. In this paper, we choose the same parameter values of PSO as in many studies [10,15–17,42]. The main formula can be expressed as $\mathbf{V}(t+1) = w\mathbf{V}(t) + c_1 r_1[\mathbf{X}_{pbest}(t) - \mathbf{X}(t)] + c_2 r_2[\mathbf{X}_{gbest}(t) - \mathbf{X}(t)]$ and $\mathbf{X}(t+1) = \mathbf{X}(t) + \mathbf{V}(t+1)$, where t denotes the index of the current generation, $\mathbf{V}(\cdot)$ is the velocity of the particle swarm; w presents the inertia weight; c_1 and c_2 are positive acceleration constants, r_1 and r_2 stand for values coming from an uniform distribution in the range $[0,1]$. $\mathbf{X}(t)$ represents the current position in *tth* generation, $\mathbf{X}_{pbest}$ means the local best position and $\mathbf{X}_{gbest}$ means the global best position. In this study, these parameters mentioned above are designed as $c_1 = c_2 = 2$ and for the inertia weight w, we use a linear form in successive iterations, that is, $w = w_{max} - (w_{max} - w_{min}) \times \boldsymbol{iter-k/iter}$, where $w_{min} = 0.4$ and $w_{max} = 0.9$, ***iter*** is described as the number of iterations, ***iter-k*** is described as the *kth* generation. Moreover, the number of particle swarm is set to 100 distributed in 20 dimensions, the number of iterations is 500. These values have been found that the size of swarms and iterations are appropriate in the search process of PSO framework [15].

3.4. Numerical examples

In this section, several examples are presented to illustrate the proposed model in a single decision maker scenario.

Example 1. There is a car evaluation scenario for a single customer, where the criteria set can be expressed as: $C = \{c_1, c_2, c_3, c_4, c_5, c_6, c_7\}$ with safety (c_1), energy (c_2), space (c_3), appearance(c_4), controllability (c_5), comfort (c_6) and accessories (c_7). And c_1 is the best criterion, c_4 is the worst criterion. The linguistic terms set: $S = \{VL, L, M, H, VH\}$ in 1–9 scale are presented for customer to make preference information. The cutoff points set: $X = \{x_1, x_2, x_3, x_4\}$, which $VL \in [1, x_1]$, $L \in [x_1, x_2]$, $M \in [x_2, x_3]$, $H \in [x_3, x_4]$, $VH \in [x_4, 9]$, then the linguistic evaluation matrix is as follows:

$$\begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_7 \end{array} \\ \begin{pmatrix} - & M & H & VH & M & H & VH \\ & & & H & & & \\ & & & L & & & \\ & & & - & & & \\ & & & H & & & \\ & & & M & & & \\ & & & VL & & & \end{pmatrix} \end{array}$$

where "–" refers to the comparative value of the best criterion and the worst criterion to their own belong to 1. And in the following examples, "–" is also represented as the same meaning. Then, we have sampled 500 times for the above granular pairwise comparisons, and the iterative process of average value of fitness function Q_1 is depicted in Fig. 4, we can witness that the value of Q_1 has been in a downward trend through generation which is in line with our optimization purpose. In the avoidance of the randomness and occasionality in the PSO optimization process, we execute the **Algorithm 2** 500 times and calculate the average values of cutoff points: 1.80, 2.51, 3.87 and 5.26. Hence, the corresponding interval-based linguistic terms are divided as: VL : $[1, 1.80]$, L : $[1.80, 2.51]$, M : $[2.51, 3.87]$, H : $[3.87, 5.26]$, VH : $[5.26, 9]$. The average value of Q_1 is 0.0451 with a standard deviation of 0.0054. And Fig. 5 shows the histogram of distribution of the value of Q_1, which provides the intuitive result that the long tail of PSO-optimized distribution means the high frequency values of the fitness function Q_1.

On the one hand, to examine whether the PSO-optimized result is suitable for reaching high consistency, on the other hand, to testify the effect of constraining the length of the intervals on the final DM result. For comparison, we make the cutoff points follow a uniform distribution in 1–9 scale, the relative cutoff points are: 2.65, 4.13, 5.80, 7.30. And the average value of Q_1 is 0.5519 with a standard deviation of 0.2236, which the distribution of the values of Q_1 are visually shown in Fig. 6. It is worth noting that the cutoff points optimized by PSO

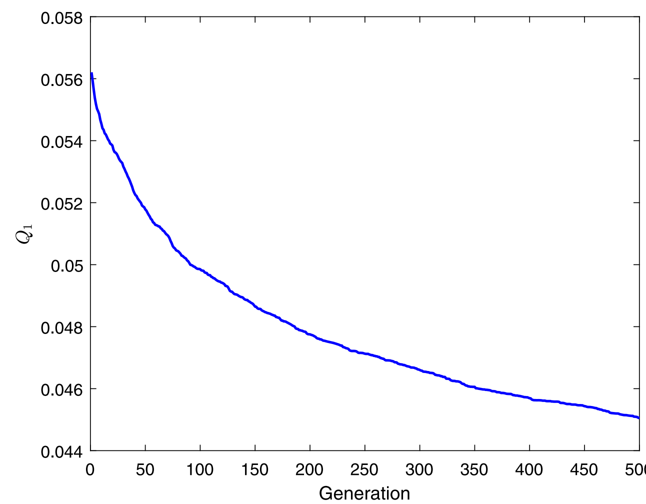


Fig. 4. The trend of Q_1 versus generation by PSO.

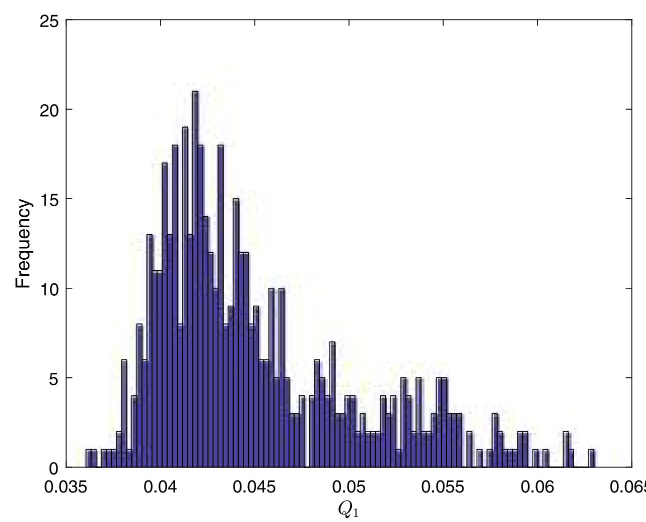


Fig. 5. Histogram of Q_1 after PSO-optimized distribution of the cutoff points.

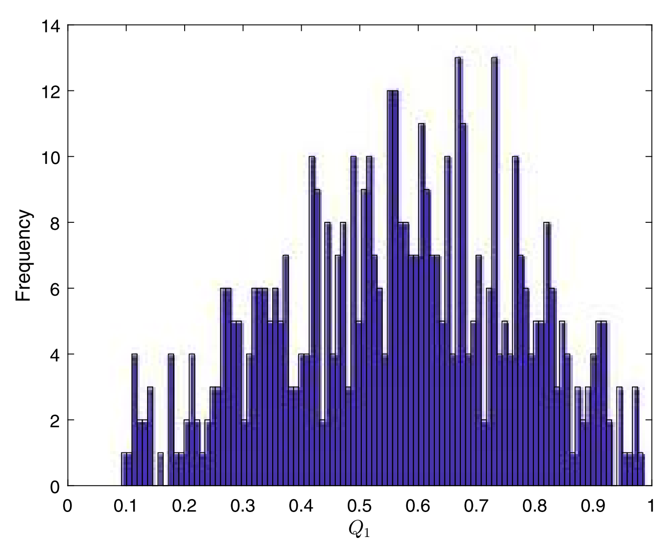


Fig. 6. Histogram of Q_1 after random uniform distribution of the cutoff points.

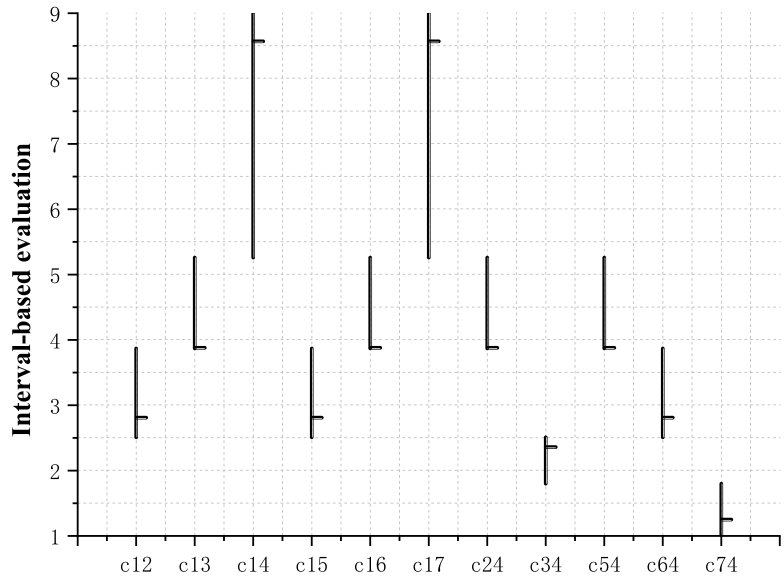


Fig. 7. The optimal numerical evaluation in Example 2.

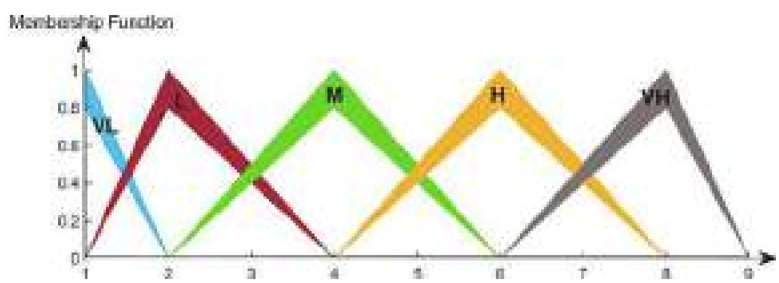


Fig. 8. The distribution of the membership functions of the corresponding terms.

can retain solutions with higher level of consistency. Thereafter, we need to determine the weight of each criterion so that we can make rankings for these criteria. Through Monte Carlo method, we can obtain the following results of pairwise comparison with lowest inconsistency.

$$\begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_7 \end{array} \\ \left(\begin{array}{ccccccc} -2.81 & 3.88 & 8.57 & 2.81 & 3.88 & 8.57 & \\ & & & 3.88 & & & \\ & & & 2.36 & & & \\ & & & - & & & \\ & & & 3.88 & & & \\ & & & 2.81 & & & \\ & & & 1.25 & & & \end{array} \right) \end{array}$$

where the value of ξ^* is 0.0212, according to Eq. (2) the value of CR^O is 0.0047, which is within the threshold in Table 2, the weights of these criteria are as follows: $w_1 = 0.40$, $w_2 = 0.15$, $w_3 = 0.10$, $w_4 = 0.04$, $w_5 = 0.15$, $w_6 = 0.11$, $w_7 = 0.05$. Then the ranking results of these criteria are shown as: $c_1 \succ c_2 \sim c_5 \succ c_6 \succ c_3 \succ c_7 \succ c_4$. And Fig. 7 depicts the optimal numerical assessments obtained from the interval-based linguistic terms, $c_{ij}\,(i = 1 \parallel j = 2, \ldots, 7)$ represents the ratio value of the best criterion to other criteria, $c_{ij}\,(i = 1, \ldots, 7, i \neq 4 \parallel j = 4)$ denotes the ratio value between the worst criterion and other criteria. Moreover, it can be observed that the distribution of the values of cutoff points are not located in the narrow range.

Example 2. We adopt the same pairwise comparison vectors in Example 1 with the IT2FSs-formed linguistic terms. Table 3 shows the linguistic terms and the relative IT2FSs and Fig. 8 portrays the MF of each linguistic term.

Table 3
Linguistic terms set and the corresponding IT2FSs.

Linguistic term	Corresponding IT2FSs
VL (Very Low)	((1,1,1,2;1), (1,1,1,2;0.8))
L (Low)	((1,2,2,4;1), (1,2,2,4;0.8))
M (Medium)	((2,4,4,6;1), (2,4,4,6;0.8))
H (High)	((4,6,6,8;1), (4,6,6,8;0.8))
VH (Very High)	((6,8,8,9;1), (6,8,8,9;0.8))

For the IT2FSs-based linguistic terms set, according to **Algorithm** 3. The first step is to calculate the centroid of each linguistic term by KM algorithm. These centroids come as: $c(VL) = [1.31, 1.36]$, $c(L) = [2.28, 2.39]$, $c(M) = [3.93, 4.07]$, $c(H) = [5.93, 6.07]$, $c(VH) = [7.61, 7.72]$. We use these centroids to represent the terms and construct the related pairwise comparison vectors. Then, PSO algorithm is applied to search the optimal values in these centroids, which are followed as: 1.33, 2.39, 3.93, 5.93 and 7.72. Furthermore, the perform index Q_1 optimized by PSO is 0.2374 ± 0.0017, the histogram of the value of Q_1 through PSO optimization is depicted in Fig. 9. Then, the consistency index ξ^* is 0.1094. The following weights of criteria are: $w_1 = 0.43$, $w_2 = 0.14$, $w_3 = 0.09$, $w_4 = 0.04$, $w_5 = 0.14$, $w_6 = 0.09$, $w_7 = 0.07$, which are close to the values calculated by **Algorithm 2** and the ranking results are the same as the results in Example 1.

From the optimized results in Examples 1 and 2, It can be seen that the IT2FSs-based granules performs better in the optimization of input-based consistency, while the interval-based granules have done well in the optimization of output-based consistency. Furthermore, the criteria weights obtained by the two forms of granules have the similar results.

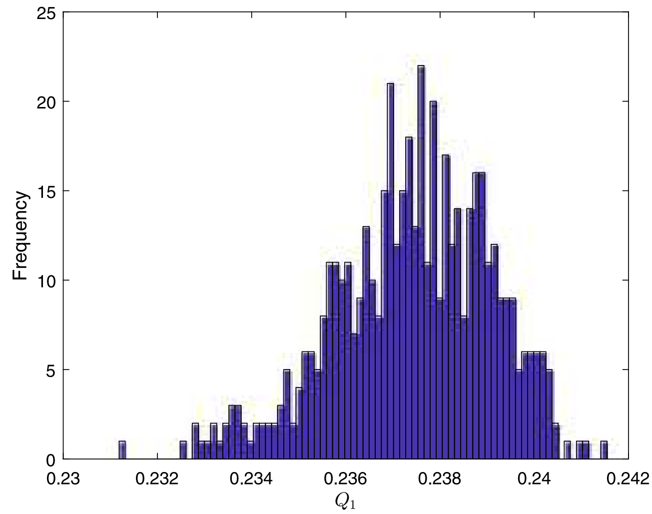


Fig. 9. Histogram of Q_1 after PSO-optimized distribution of IT2FSs-based linguistic terms.

In addition, the time complexity for **Algorithm** 2 is $O(n^4 \times n\log_2 n)$ (When generating cutoff point vectors from 1 to 9 by randomization, we use the fast sorting method and its time complexity is $O(n\log_2 n)$ and the time complexity for **Algorithm** 3 is $O(n^4)$, $O(n^4 \times n\log_2 n) > O(n^4)$, which indicates that the model based on the IT2FSs-based granules has higher efficiency.

4. The group decision scenario

Assumed that, there are k decision makers denoted as : $\{d_1, \dots, d_t, \dots, d_k\}$ with the corresponding decision weight vector $\lambda = (\lambda_1, \dots, \lambda_t, \dots, \lambda_k)^T$ $\sum_t \lambda_t = 1$, gathering together to make evaluations on a set of criteria $C = \{c_1, \dots, c_j, \dots, c_n\}$ to calculate the weights of the criteria $W = (w_1, \dots, w_j, \dots, w_n)^T$ $\sum_j w_j = 1$, each decision maker uses the same linguistic terms set $S = \{s_1, \dots, s_l, \dots, s_m\}$. As for the establishment of the best and worst criteria among the group, in the following numerical examples, it is assumed that the group make an agreement on the establishment of the best and worst criteria by default; while in the real-world case, considering the complexity of practical DM problem, the best and worst criteria in the group are determined by the priorities of the decision makers' weights, that is, they are established by using the additive weighted aggregation (AWA) operators of the decision makers' weights. Here, we distinguish two scenarios: **Scenario 1**. decision makers have different semantics for $S, s_l^{(t)} \neq s_l^{(t+1)}$, where $s_l^{(t)}$ denotes the lth linguistic term in set S from d_t $(t = 1, \dots, k)$ decision maker, and $s_l^{(t+1)}$ stands for the lth linguistic term from d_{t+1} decision maker. **Scenario 2**. decision makers take the same values of linguistic terms in $S, s_l^{(t)} = s_l^{(t+1)}$.

Definition 7. Let A expressed as $A = [A^-, A^+]$ and B expressed as $B = [B^-, B^+]$ where A^-, A^+, B^-, $B^+ \in \mathbf{R}$. The distance function $d(\cdot)$ between A and B can be defined as:

$$d(A, B) = \sqrt{|A^- - B^-|^2 + |A^+ - B^+|^2} \tag{14}$$

Then, we have:

(1) $d(A, B) \geq 0$, that is, $d(\cdot)$ satisfies nonnegativity, when $d(A, B) = 0$ occurs, then it means $A^- = B^-$ and $A^+ = B^+$;
(2) $d(A, B) = d(B, A)$, which indicates that $d(\cdot)$ satisfies symmetry;
(3) $d(A, A) = 0$, it is acknowledged that $d(\cdot)$ meets reflexivity.

Definition 8. Let $a_{Bj}^{(t)}$ and $a_{jW}^{(t)}$ $(j = 1, \dots, n)$ be the entries of B-O and O-W vectors from d_t $(t = 1, \dots, k)$, where $a_{Bj}^{(t)}$ is expressed as: $[\underline{a}_{Bj}^{(t)}, \bar{a}_{Bj}^{(t)}]$ and $a_{jW}^{(t)}$ is expressed as: $[\underline{a}_{jW}^{(t)}, \bar{a}_{jW}^{(t)}]$, let $a_{Bj} = [\underline{a}_{Bj}, \bar{a}_{Bj}]$ and $a_{jW} = [\underline{a}_{jW}, \bar{a}_{jW}]$ be the entries of B-O and O-W vectors from the group, the relationship between $a_{Bj}^{(t)}$, $a_{jW}^{(t)}$ and a_{Bj}, a_{jW} satisfies the condition follows as:

(1) $\min_{j=1,\dots,n}(\underline{a}_{Bj}^{(t)}) \leq \underline{a}_{Bj} \leq \min_{j=1,\dots,n}(\underline{a}_{Bj}^{(t)})$, $\min_{j=1,\dots,n}(\underline{a}_{jW}^{(t)}) \leq \underline{a}_{jW} \leq \min_{j=1,\dots,n}(\underline{a}_{jW}^{(t)})$
(2) $\min_{j=1,\dots,n}(\underline{a}_{Bj}^{(t)}) \leq \underline{a}_{Bj} \leq \min_{j=1,\dots,n}(\bar{a}_{Bj}^{(t)})$, $\min_{j=1,\dots,n}(\bar{a}_{jW}^{(t)}) \leq \bar{a}_{jW} \leq \min_{j=1,\dots,n}(\bar{a}_{jW}^{(t)})$

Generally, the proposed model concerns the information granulation of linguistic terms set within BWM model in GDM situations. In this section, we still concentrate on the two forms of granular linguistic terms set depicted in Section 3. Furthermore, According to Definition 7, we can design a Euclidean distance-based function to aggregate the individual pairwise comparison vectors and find the optimal entries of pairwise comparison vectors of the group through using PSO algorithm. And Definition 8 describes the formation of the preference information from the group. In a nutshell overall, the granular model in GDM situation can be illustrated as follows: firstly, the group members select the quantitative forms (the interval-based or the IT2FSs-based granules) of the linguistic terms set, and construct their preference pairwise comparison vectors respectively. Then, from Definitions 7 and 8, we can construct the model in Eq. (15) to aggregate the individual preference information and obtain group preference information. The problem can be interpreted as finding the optimal value of the group pairwise comparison information a_{Bj} and a_{jW}, PSO algorithm is served as the searching tool to find the suitable results.

$$\min Q_2 = \frac{1}{k}\sum_{t=1}^{k}\lambda_t\Big(\sum_{j=1}^{n} d(a_{Bj}^{(t)}, a_{Bj}) + d(a_{jW}^{(t)}, a_{jW})\Big)$$
$$s.t. \begin{cases} a_{Bj} = [\underline{a}_{Bj}, \bar{a}_{Bj}], a_{jW} = [\underline{a}_{jW}, \bar{a}_{jW}] \\ a_{Bj}^{(t)} = [\underline{a}_{Bj}^{(t)}, \bar{a}_{Bj}^{(t)}], a_{jW}^{(t)} = [\underline{a}_{jW}^{(t)}, \bar{a}_{jW}^{(t)}] \\ \min(\underline{a}_{Bj}^{(t)}) \leq \underline{a}_{Bj} \leq \min(\underline{a}_{Bj}^{(t)}) \\ \min(\underline{a}_{Bj}^{(t)}) \leq \underline{a}_{Bj} \leq \min(\bar{a}_{Bj}^{(t)}) \\ \min(\underline{a}_{jW}^{(t)}) \leq \underline{a}_{jW} \leq \min(\underline{a}_{jW}^{(t)}) \\ \min(\bar{a}_{jW}^{(t)}) \leq \bar{a}_{jW} \leq \min(\bar{a}_{jW}^{(t)}) \end{cases} \tag{15}$$

where $t = 1, \dots, t$, $j = 1, \dots, n$, and for the linguistic terms set in the form of interval-based granules and IT2FSs-based granules, the aggregation model in Eq. (15) can be both adopted, as will be readily seen that the centroid of the IT2FSs-based granules are in the interval form. The following examples are represented to illustrate the model in detail. Finally, the framework of the proposed model in GDM situation is portrayed in Fig. 10.

Example 3. Supposed that there are 3 decision makers, making judgements in the same linguistic terms set to 5 criteria in the form of the following matrices, and each decision maker has different semantics for the five linguistic terms. Furthermore, decision makers make an agreement for the establishment of the best and worst criteria, such that c_1 is the best criterion and c_3 is the worst criterion. The linguistic terms evaluation results are shown as follows:

$$d_1 = \begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{array} \begin{array}{c} \begin{array}{ccccc} c_1 & c_2 & c_3 & c_4 & c_5 \end{array} \\ \begin{pmatrix} - & M & VH & L & VL \\ & & H & & \\ & & - & & \\ & & M & & \\ & & L & & \end{pmatrix} \end{array}$$

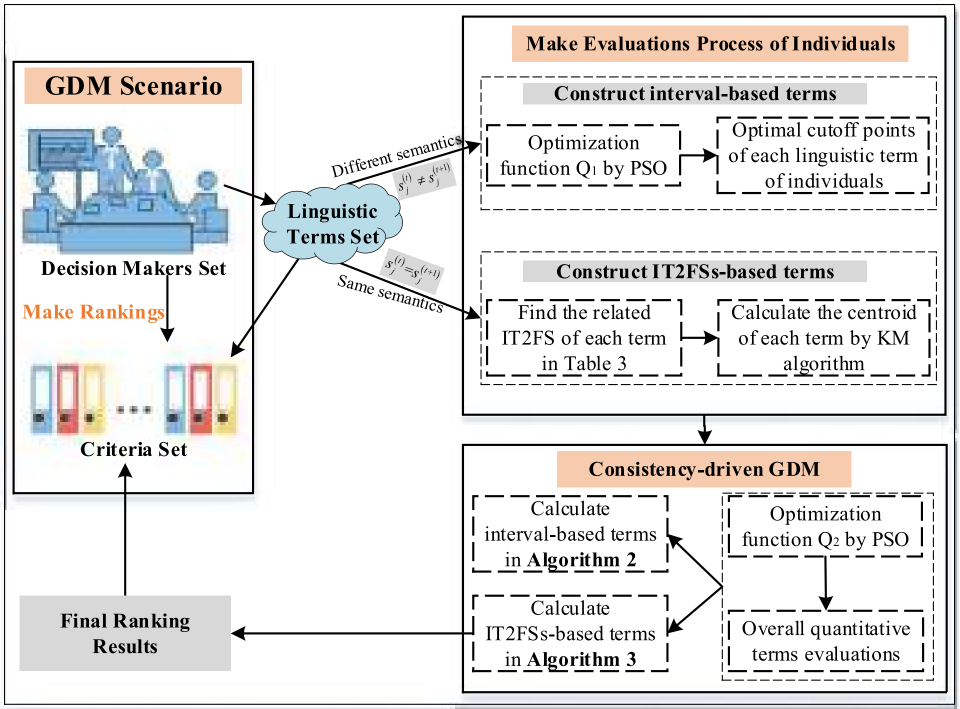


Fig. 10. The framework of granular BWM model in GDM situation.

$$d_2 = \begin{pmatrix} - & M & VH & M & L \\ & & VH & & \\ & & - & & \\ & & H & & \\ & & VL & & \end{pmatrix}$$

$$d_3 = \begin{pmatrix} - & H & VH & L & L \\ & & M & & \\ & & - & & \\ & & M & & \\ & & VL & & \end{pmatrix}$$

The separate PSO-optimized result of each decision maker is as follows: for decision maker d_1, the PSO-optimized cutoff points are: 2.05, 2.79, 3.50, 8.33. The minimized fitness function $Q_1^{(1)} = 0.1052 \pm 0.0039$. For decision maker d_2, the PSO-optimized cutoff points are: 2.46, 3.82, 5.28, 6.85, $Q_1^{(2)} = 0.2285 \pm 0.1074$. For decision maker d_3, the PSO-optimized cutoff points are: 2.02, 2.58, 3.18, 8.41, $Q_1^{(3)} = 0.1047 \pm 0.0035$. The histograms of fitness function $Q_1^{(1)}, Q_1^{(2)}, Q_1^{(3)}$ shown in Fig. 11 provide intuitively to exhibit the results of the realization of linguistic terms division through uniform distribution and PSO-optimized distribution on the 1–9 scales. After the establishment of cutoff points, the individual interval-based evaluation is expressed as:

$$d'_1 = \begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{array} \begin{array}{ccccc} c_1 & c_2 & c_3 & c_4 & c_5 \\ - & [2.79, 3.50] & [8.33, 9.00] & [2.05, 2.79] & [1.00, 2.05] \\ & & [3.50, 8.33] & & \\ & & - & & \\ & & [2.79, 3.50] & & \\ & & [2.05, 2.79] & & \end{array}$$

$$d'_2 = \begin{pmatrix} - & [3.82,5.28] & [6.85,9.00] & [3.82,5.28] & [2.46,3.82] \\ & & [6.85,9.00] & & \\ & & - & & \\ & & [5.28,6.85] & & \\ & & [1.00,2.46] & & \end{pmatrix}$$

$$d'_3 = \begin{pmatrix} - & [3.18,8.41] & [8.41,9.00] & [2.02,2.58] & [2.02,2.58] \\ & & [2.58,3.18] & & \\ & & - & & \\ & & [2.58,3.18] & & \\ & & [1.00,2.02] & & \end{pmatrix}$$

Thereafter, in virtue of Eq. (15), the range of entries d_{ij} of the collective evaluation can be obtained. For instance, $d_{12} = [\underline{d}_{12}, \bar{d}_{12}]$, $2.79 \leq \underline{d}_{12} \leq 3.82$ and $3.50 \leq \bar{d}_{12} \leq 8.41$, then through Eq. (15), the average value of Q_2 optimized by PSO is equal to 1.4073 ± 0.0348, and $\underline{d}_{12} = 3.38$, $\bar{d}_{12} = 5.68$. The collective interval evaluation is presented as follows:

$$d = \begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{array} \begin{array}{ccccc} c_1 & c_2 & c_3 & c_4 & c_5 \\ - & [3.38, 5.68] & [7.88, 9.00] & [2.59, 3.52] & [1.60, 2.85] \\ & & [4.27, 7.01] & & \\ & & - & & \\ & & [3.47, 4.56] & & \\ & & [1.42, 2.38] & & \end{array}$$

Afterwards, **Algorithm 2** is executed according to the framework illustrated in Fig. 10. Hence, we obtain the optimal numeric pairwise assessments in the intervals, which $c_{12} = 5.38$, $c_{13} = 8.85$, $c_{14} = 3.31$, $c_{15} = 2.60$, $c_{23} = 6.01$, $c_{43} = 3.67$ and $c_{53} = 1.43$. Thereafter, the output inconsistency index ξ^* is equal to 0.1270 and the weights of these criteria follow as: $w_1 = 0.48, w_2 = 0.11, w_3 = 0.04, w_4 = 0.18, w_5 = 0.19$. Thus the ranking results are: $c_1 \succ c_5 \succ c_4 \succ c_2 \succ c_3$.

In the next example, we consider the IT2FSs-based linguistic terms, in which decision makers use the same semantics of linguistic terms, and the same evaluation scenario in Example 3 is applied.

Example 4. Here, we use the same IT2FSs-formed linguistic terms set in Example 3, and the pairwise comparison vectors of each decision maker are represented as follows:

$$d'_1 = \begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{array} \begin{array}{ccccc} c_1 & c_2 & c_3 & c_4 & c_5 \\ - & [3.93, 4.07] & [7.61, 7.72] & [2.28, 2.39] & [1.31, 1.36] \\ & & [5.93, 6.07] & & \\ & & - & & \\ & & [3.93, 4.07] & & \\ & & [2.28, 2.39] & & \end{array}$$

$$d'_2 = \begin{pmatrix} - & [3.93,4.07] & [7.61,7.72] & [3.93,4.07] & [2.28,2.39] \\ & & [7.61,7.72] & & \\ & & - & & \\ & & [5.93,6.07] & & \\ & & [1.31,1.36] & & \end{pmatrix}$$

$$d'_3 = \begin{pmatrix} - & [5.93,6.07] & [7.61,7.72] & [2.28,2.39] & [2.28,2.39] \\ & & [3.93,4.07] & & \\ & & - & & \\ & & [3.93,4.07] & & \\ & & [1.31,1.36] & & \end{pmatrix}$$

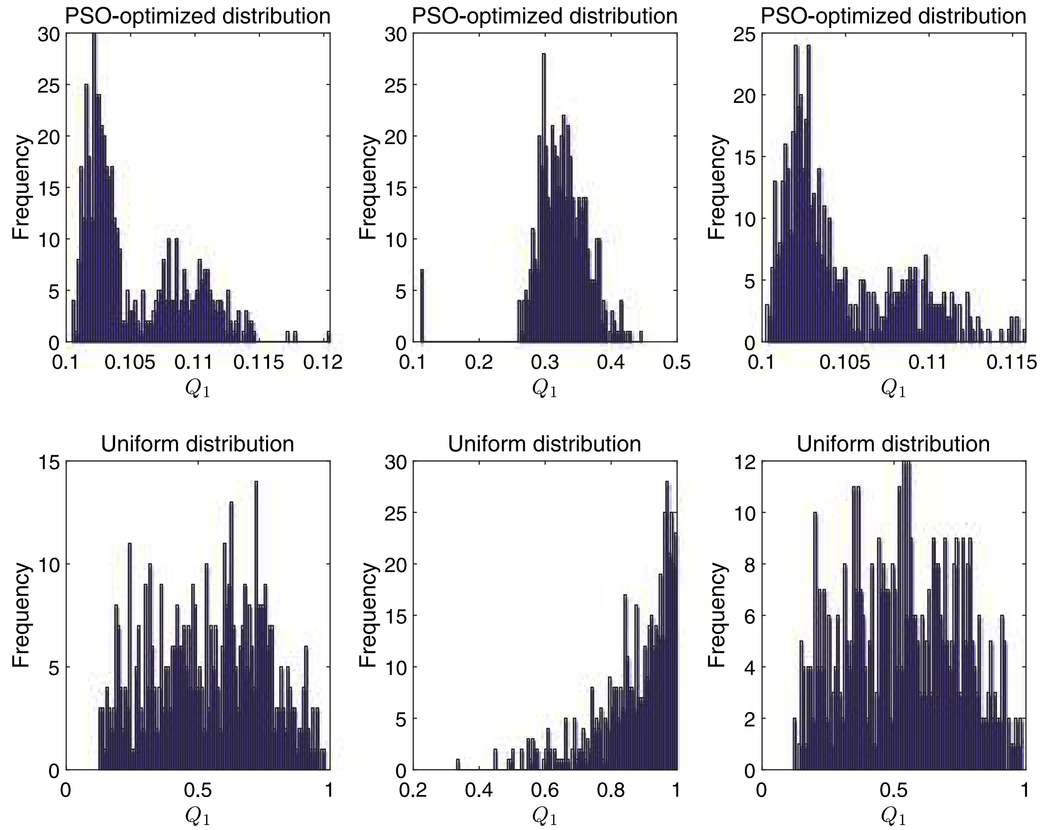


Fig. 11. PSO-optimized and uniform distribution of $Q_1^{(1)}, Q_1^{(2)}, Q_1^{(3)}$.

Then, the minimized value of Q_2 is 0.8541 ± 0.0526. The pairwise comparison evaluation matrix d of the group is given as below:

$$d = \begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{array} \begin{array}{c} \begin{array}{ccccc} c_1 & c_2 & c_3 & c_4 & c_5 \end{array} \\ \begin{pmatrix} - & [4.26, 4.42] & [7.61, 7.72] & [2.59, 2.70] & [2.06, 2.16] \\ & & [5.89, 5.96] & & \\ & & - & & \\ & & [4.29, 4.43] & & \\ & & [1.52, 1.59] & & \end{pmatrix} \end{array}$$

Hence, we can obtain the value of Q_1 is equal to 0.2093 ± 0.0039, the numeric pairwise comparison vectors are: $B-O = (-, 4.26, 7.72, 2.65, 2.08)$; $O-W = (7.72, 5.89, -, 4.35, 1.57)^T$. Finally, we can calculate the inconsistency index ξ^* is equal to 0.1092, the criteria weights are: $w_1 = 0.44, w_2 = 0.13, w_3 = 0.04, w_4 = 0.21, w_5 = 0.18$, which are closed to the results obtained in Example 3, the ranking results of these criteria are: $c_1 \succ c_4 \succ c_5 \succ c_2 \succ c_3$.

5. Comparative analysis

In this section, to examine the effectiveness and feasibility of this proposed method, the comparative analysis of this model and the model proposed by Pedrycz and Song in [15] is presented. And the forms of granule linguistic terms deserve to pay attention. Firstly, the interval-based linguistic information in AHP model and BWM model are taken into consideration, then the T1FSs-based and IT2FSs-based linguistic terms are represented.

5.1. Interval-valued linguistic information

Example 5. Consider the example in [15], that is the 5×5 reciprocal matrix with 5 linguistic terms to make evaluations for five alternatives $\{a_1, a_2, a_3, a_4, a_5\}$. In AHP-based model, the matrix is represented as follows:

$$\begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{array} \begin{array}{c} \begin{array}{ccccc} a_1 & a_2 & a_3 & a_4 & a_5 \end{array} \\ \begin{pmatrix} - & L & M & VL & H \\ 1/L & - & M & 1/VL & VH \\ 1/M & 1/M & - & 1/H & L \\ 1/VL & VL & H & - & H \\ 1/H & 1/VH & 1/L & 1/H & - \end{pmatrix} \end{array}$$

And in BWM-based model, it can be expressed as:

$$\begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{array} \begin{array}{c} \begin{array}{ccccc} a_1 & a_2 & a_3 & a_4 & a_5 \end{array} \\ \begin{pmatrix} - & L & M & VL & H \\ & & & & VH \\ & & & & L \\ & & & & H \\ & & & & - \end{pmatrix} \end{array}$$

According to the **Algorithm** 2, we can obtain the optimal cutoff points in this model followed as: 1.8, 2.3, 8.2, 8.6. And the distribution of these cutoff points and the cutoff points in Pedrycz and Song's method [15] are shown in Fig. 12. It can be intuitively seen that the cutoff points obtained by the two methods are different. The reasons for causing this phenomenon can be expressed as: BWM method and AHP method are essentially different. Particularly, AHP-based method deals with the reciprocal matrix, while BWM method solves the pairwise comparison vectors. Furthermore, the objective functions optimized by PSO algorithm also exist difference, which the values of objective functions are described in Table 4. Thereafter, we can calculate the weights of the five alternatives in our proposed method, which is expressed as: $\mathbf{w} = [1.000, 0.547, 0.435, 1.000, 0.090]^T$. The result in [15] is $\mathbf{w}' = [1.000, 0.932, 0.218, 0.997, 0.172]^T$. Simultaneously, the distribution for theses weights is presented in Fig. 13. Finally, the ranking results in [15] are: $a_1 \succ a_4 \succ a_2 \succ a_3 \succ a_5$. The results in the proposed model

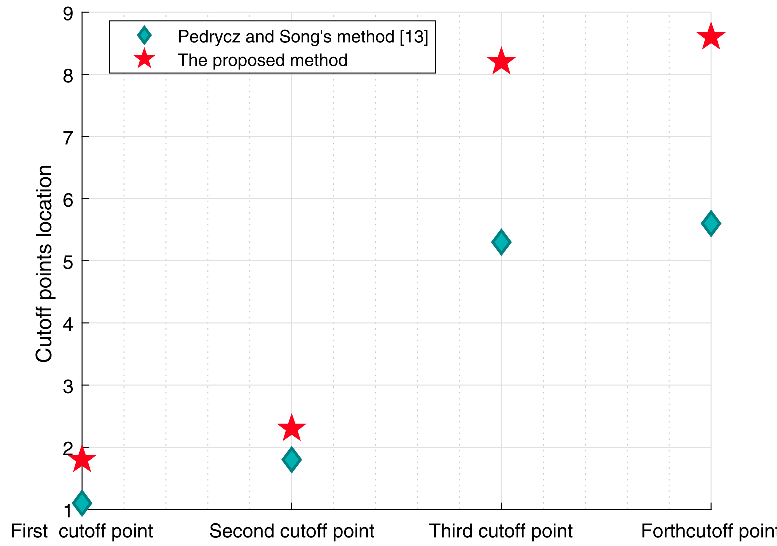


Fig. 12. The positions of cutoff points.

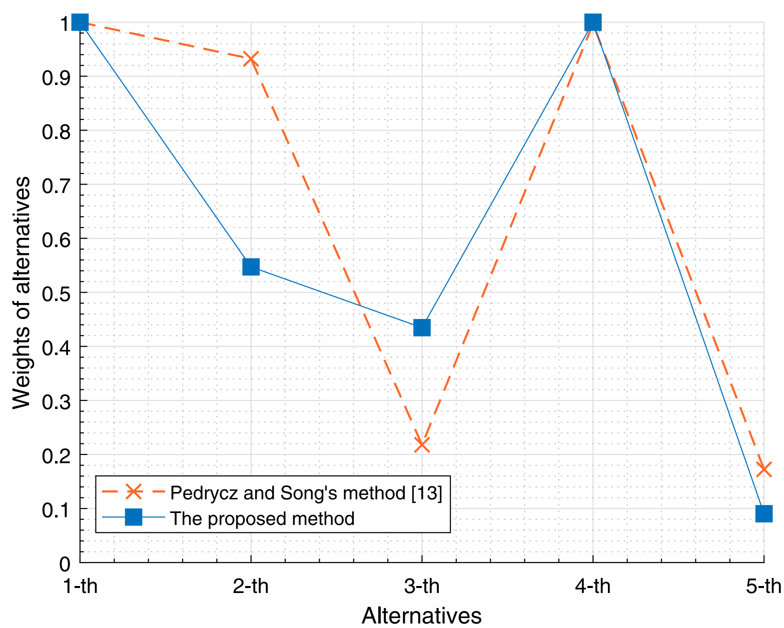


Fig. 13. The distribution of alternatives' weights.

Table 4
The value of index to evaluate the quality of model.

Method	Perform index
Pedrycz and Song's method [15]	$\upsilon = \frac{\lambda_{\max}-n}{n-1} = 0.0205 \pm 0.0102$
The proposed method	$CR^I = 0.0906 \pm 0.0060$ $CR^O = 0.0057$

Table 5
Thresholds for the input consistency CR^O.

Method	Decision making method	Fuzzy linguistic expression
Pedrycz and Song's method [15]	AHP method	T1FSs
The proposed method	BWM method	IT2FSs

are $a_1 \sim a_4 \succ a_2 \succ a_3 \succ a_5$, which are close to the results obtained in [15], which further illustrates the reliability of **Algorithm 2**.

5.2. The T1FSs-based and IT2FSs-based form of linguistic term set

In this subsection, we still adopt the same example in [15]. Firstly, the main constructions of Pedrycz and Song's method and the proposed method for qualification linguistic terms in the fuzzy environment are described in Table 5. Then the establishment of TIFSs MFs given in Table 6 and the IT2FSs in Table 3 are used for comparison. The optimal

Table 6
Linguistic terms set and the corresponding T1FS.

Linguistic term	Corresponding T1FSs
VL (Very Low)	(1,1,1,2;1)
L (Low)	(1,2,2,4;1)
M (Medium)	(2,4,4,6;1)
H (High)	(4,6,6,8;1)
VH (Very High)	(6,8,8,9;1)

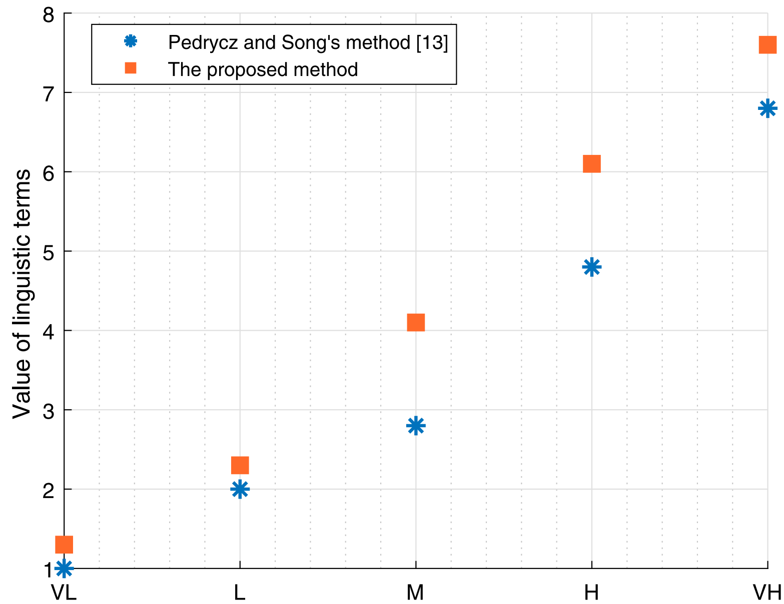


Fig. 14. The distributions of the value of linguistic terms.

values of linguistic terms from [15] come as: 1.0, 2.0, 2.8, 4.8, 6.8, for which the perform index υ is equal to 0.0335, then the results from the proposed model follow as: 1.4, 2.3, 4.1, 6.1, 7.6. And the value of CR^I is 0.1899, and CR^O is equal to 0.0101. The distribution of these values of linguistic terms are presented in Fig. 14. Therefore, we can the compute the weights in the proposed method, expressed as: $\mathbf{w}'_{IT2FSs} = [1.000, 0.571, 0.320, 0.958, 0.115]^T$. The ranking results follow as: $a_1 \succ a_4 \succ a_2 \succ a_3 \succ a_5$. The results in [15] are $\mathbf{w}'_{T1FS} = [1.000, 0.808, 0.284, 0.953, 0.162]^T$, which the ranking of the alternatives is the same with the proposed approach described above.

6. Case study

To demonstrate the practicality of this proposed model, this section exhibits a scenic spots recommendation in Wuhan with online reviews from three websites: tripadvisor.cn, meituan.com and dianping.com.

6.1. Background description

At the beginning of 2020, owing to the sudden eruption of the new corona virus (COVID-19), people had to observe home quarantine and canceled their travel plans, which caused huge losses to the tourism industry in Wuhan. Therefore, after the epidemic ended, Wuhan Municipal Government implemented the public welfare project with free tickets for tourist attractions to promote the economic development of Wuhan.

For this special activity, we have found six popular scenic spots followed by Yellow Crane Tower, Hubei Provincial Museum, Chu River and Han Street, Hankou River Beach, Yangtze River Bridge, and Dong Hu Scenic Resort, which their locations are displayed in Fig. 15. The intention of this study is to assist tourists to arrange the optimal visiting sequence of these scenic spots through online reviews and know the importance of scenic spots. Usually, tourists refer to the comment results from the related travel websites to determine the sequence of these sites. This process can be simplified to the alternative ranking problems in DM. The above six scenic spots can be expressed as a_1 (Yellow Crane Tower), a_2 (Hubei Provincial Museum), a_3 (Chu River and Han Street), a_4 (Hankou River Beach), a_5 (Yangtze River Bridge) and a_6 (Dong Hu Scenic Resort). The travel websites are regarded as decision makers. Fig. 16 shows a tourist's evaluation score of a scenic spot, this paper selects three famous tourism websites TripAdvisor,

Fig. 15. Distribution map of six tourist sites.

Meituan and Dianping which denoted as d_1, d_2 and d_3. Through the proposed model, tourists can obtain the importance degree of these spots denoted as: $W = (w_{a_1}, \ldots, w_{a_6})^T$, $\sum_{j=1}^{6} w_{a_j} = 1$ and find the optimal ranking results to visit these spots.

6.2. Data collection and processing

This paper collected a total of 5449 online reviews, of which 1609 were collected on Meituan, 2700 were collected on Dianping, and 1140 were collected on TripAdvisor. Table 7 represents the detailed evaluation scores of six scenic spots on three websites. Fig. 17 depicts the distribution of points of the six scenic spots on Meituan.com, it is worth noting that the scores of the six scenic spots are mainly concentrated in five points, four points and three points. And for the five level of assessments, there is a set of corresponding linguistic terms $S = \{s_1, s_2, s_3, s_4, s_5\}$, the specific expressions are illustrated in Table 8.

(A) establishment of the best and worst spots

The weights of the three websites d_1, d_2 and d_3 are $\lambda_1 = 0.4$, $\lambda_2 = 0.3$ and $\lambda_3 = 0.3$ respectively. For the selection of the most worthwhile scenic spot a_B and the least worthwhile scenic spot a_W, the ratio r_1 between the number of "five points" and the overall number of

Fig. 16. An example of a review of Dong Hu Scenic Resort on Tripadvisor.cn.

Table 7
Evaluation scores of tourist attractions on the three websites.

		Five points	Four points	Three points	Two points	One point
TripAdvisor	Yellow Crane Tower	106	97	80	12	5
	Hubei Provincial Museum	121	53	4	0	0
	Chu River and Han Street	27	25	10	0	0
	Hankou River Beach	28	35	7	0	0
	Yangtze River Bridge	112	104	13	0	1
	Dong Hu Scenic Resort	147	126	26	0	1
Meituan	Yellow Crane Tower	76	81	94	29	20
	Hubei Provincial Museum	143	73	69	11	4
	Chu River and Han Street	46	37	20	4	2
	Hankou River Beach	98	70	77	34	21
	Yangtze River Bridge	69	83	79	37	32
	Dong Hu Scenic Resort	140	62	52	23	23
Dianping	Yellow Crane Tower	335	100	13	1	1
	Hubei Provincial Museum	306	127	15	1	1
	Chu River and Han Street	312	126	10	0	2
	Hankou River Beach	363	81	6	0	0
	Yangtze River Bridge	373	73	3	1	0
	Dong Hu Scenic Resort	355	83	8	2	2

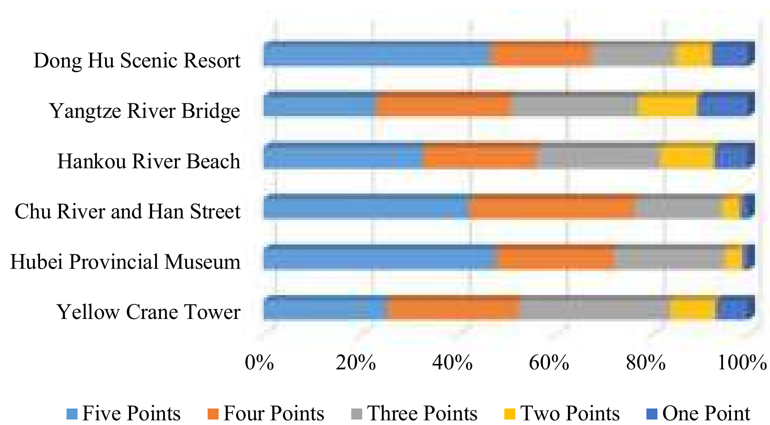


Fig. 17. The distribution of points of the six scenic spots on Meituan.com.

Table 8
The matching of five level assessment and the corresponding linguistic term.

Five level assessment	Expression	The related linguistic term
One point	Terrible	s_1
Two points	Poor	s_2
Three points	Average	s_3
Four points	Very good	s_4
Five points	Excellent	s_5

reviews and ratio r_2 between the number of "one point" and the overall number of reviews are jointly determined. For instance, the r_1 of Hubei Provincial Museum a_1 is $143/300 = 0.4767$ and the corresponding r_2 is $4/300 = 0.0134$ in Meituan, according to the data exhibited in Table 7. The calculation results of r_1 and r_2 of six spots in different sites are depicted in Table 9.

Remark 3. When the r_1 reaches the maximum value and r_2 reaches the minimum value, the corresponding spot is the first attraction worth visiting; on the contrary, while the value of r_1 is the smallest and the value of r_2 is the largest, it is the last attraction visited; or if it does

Table 9
The r_1 and r_2 values of the six attractions on the three websites.

Website	Spot	r_1	r_2	
d_1	a_1	0.3533	0.0167	a_W
	a_2	0.6798	0.0000	a_B
	a_3	0.4355	0.0000	
	a_4	0.4000	0.0000	
	a_5	0.4870	0.0033	
	a_6	0.4900	0.0033	
d_2	a_1	0.2533	0.0667	
	a_2	0.4767	0.0133	a_B
	a_3	0.4220	0.0183	
	a_4	0.3267	0.0700	
	a_5	0.2300	0.1067	a_W
	a_6	0.4667	0.0767	
d_3	a_1	0.7444	0.0022	
	a_2	0.6800	0.0022	a_W
	a_3	0.6933	0.0044	
	a_4	0.8067	0.0000	
	a_5	0.8289	0.0000	a_B
	a_6	0.7889	0.0044	

Table 10
The value of m_B/m_i and m_i/m_W.

Website	Spot	m_B/m_i	m_i/m_W
d_1	a_1	**1.4446**	1.0000
	a_2	1.0000	1.4446
	a_3	1.1655	1.2395
	a_4	1.0861	1.3300
	a_5	**1.0409**	1.3879
	a_6	1.0742	1.3448
d_2	a_1	1.3758	1.0000
	a_2	1.0000	1.3758
	a_3	**0.9455**	**1.4550**
	a_4	1.2857	1.0701
	a_5	1.4211	0.9682
	a_6	1.4109	1.2866
d_3	a_1	0.9954	1.0000
	a_2	1.0000	0.9954
	a_3	0.9886	1.0069
	a_4	0.9752	1.0207
	a_5	**0.9709**	**1.0253**
	a_4	0.9886	1.0069

not belong to the above situations, then calculate the value of formula $0.5 \times r_1 - 0.5 \times r_2$ and select the first and last spots for sightseeing.

From Table 9, a_B and a_W selected by the three websites are different. For d_1, the site a_2 is the a_B and a_1 is the a_W, while for d_2, a_2 is the a_B and a_5 is the a_W, and for d_3, a_5 is the a_B and a_2 is the a_W. Hence, for the agreement of the a_B and a_W, Eq. (16) is constructed, which is shown as follows:

$$r\left(a_i\right) = \lambda_t \sum_t \frac{1}{2}\left(r_1^{(d_t)}\left(a_i\right) - r_2^{(d_t)}\left(a_i\right)\right) \tag{16}$$

where $r\left(a_i\right)$ is the final evaluation ratio of spot a_i, and $\lambda_t (t = 1, 2, 3)$ is the weight of site, $r_1^{(d_t)}\left(a_i\right)$ is the value of r_1 of spot a_i on site d_t and $r_2^{(d_t)}\left(a_i\right)$ is the value of r_2 of the spot a_i on the website d_t. Then based on Eq. (16), $r\left(a_1\right) = 0.2066$, $r\left(a_2\right) = 0.3070$ and $r\left(a_5\right) = 0.2396$, that is, $r\left(a_1\right) < r\left(a_5\right) < r\left(a_2\right)$. Therefore, the final a_B is a_2 and a_W is a_1.

(B) construction of pairwise comparison vectors

After the settlement of best and worst scenic spots, the next procedure is to compare other spots with these two spots by using linguistic terms respectively. Firstly, the "five points" and "four points" are regarded as good reviews, we calculate the ratio $m_i (i = 1, \ldots, 6)$ of the "five points" and "four points" in each attraction, then solve the results of m_B/m_i (m_B the ratio of the best spot) and m_i/m_W (m_W the ratio of the worst spot), the entire results are exhibited in Table 10. Secondly, let

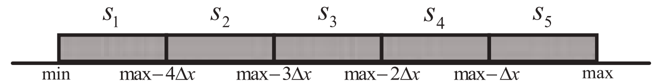


Fig. 18. The division of Linguistic terms used in pairwise comparison.

max and min be the maximum and minimum values of $m_B/m_i (i \neq B)$ and $m_i/m_W (i \neq W)$ respectively, and let $\Delta x = \frac{\max - \min}{length(S)-1}$, where $length(S)$ is the number of linguistic terms set S, Fig. 18 depicts the division of the linguistic terms, when the values of m_B/m_i and m_i/m_W are in the interval $(max - \Delta x, max]$, the corresponding entry of a_B/a_j or a_j/a_W is set to s_5; when it is in $(max-2\Delta x, max-\Delta x]$, the entry is s_4; when it is in $(max-3\Delta x, max-2\Delta x]$, the entry is s_3; when it is in $(max-4\Delta x, max-3\Delta x]$, the entry is s_2; and when it is in $(min, max - 4\Delta x]$, the element value is s_1.

Finally, the pairwise comparison vectors of d_1, d_2 and d_3 can be established as follows:

$$d_1 = \begin{matrix} & a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{matrix} \begin{pmatrix} - & & & & & \\ s_5 & - & s_3 & s_2 & s_1 & s_2 \\ s_3 & & & & & \\ s_4 & & & & & \\ s_5 & & & & & \\ s_5 & & & & & \end{pmatrix}$$

$$d_2 = \begin{pmatrix} - & & & & & \\ s_5 & - & s_2 & s_4 & s_5 & s_5 \\ s_5 & & & & & \\ s_2 & & & & & \\ s_2 & & & & & \\ s_4 & & & & & \end{pmatrix}$$

$$d_3 = \begin{pmatrix} - & & & & & \\ s_5 & - & s_3 & s_2 & s_1 & s_3 \\ s_4 & & & & & \\ s_5 & & & & & \\ s_5 & & & & & \\ s_4 & & & & & \end{pmatrix}$$

Remark 4. For the constructions of the pairwise comparison vectors of d_1, at first, it can be obtained from Table 10 that $max = 1.4446$ related to the ratio of m_2/m_1, and $min = 1.0409$ related to the ratio of m_2/m_5, then the value of Δx is 0.1009. Therefore, when m_2/m_i or m_i/m_1 belongs to the interval $(1.3437, 1.4446]$, the value of the entry of pairwise comparison vector is set to s_5; when it is in the interval $(1.2428, 1.3437]$, the entry is s_4; when it is between $(1.1419, 1.2428]$, the entry is s_3; when in the interval $(1.0410, 1.1419]$, the entry is s_2; and when in the interval $(1.0409, 1.0410]$, the entry is s_1.

6.3. The decision making process

Considering the otherness between the three websites, there are different semantics of the related linguistic terms for each site, which is in accordance with **Scenario 1**, mentioned in Section 4. Furthermore, this subsection transforms the collected numeric-based linguistic terms set into interval-based terms in 1–5 scale, and PSO algorithm is supposed to search the optimal cutoff points (cp_1, cp_2, cp_3, cp_4), the ranges of the linguistic terms are described in Fig. 19. Furthermore, if max is not the value of m_B/m_W, the linguistic term used to evaluate the value of the a_B/a_W is still set to s_5.

Through calculations, for d_1, the optimal cutoff points are followed as: 1.18, 1.61, 2.27 and 2.56, the average value of $Q_1^{(1)}$, that is, CR^I is 0.1016 ± 0.0041, which is within the corresponding threshold of CR^I shown in Table 1, and further reflects the consistency of the

Table 11
The corresponding indices of uniform distribution and PSO optimization process.

	Position of cutoff points	Q_1	Q_2	ξ^*	Weights of spots	Final ranking
PSO-optimized distribution	d_1 : (1.18, 1.61, 2.27, 2.56) d_2 : (1.65, 2.37, 3.02, 3.79) d_3 : (1.03, 1.09, 2.26, 2.37)	$Q_1^{(1)} = 0.1016 \pm 0.0041$ $Q_1^{(2)} = 0.2043 \pm 0.0083$ $Q_1^{(3)} = 0.0122 \pm 0.0065$	1.4465 ± 0.0510	0.0262	$w_{a_1} = 0.06,\ w_{a_2} = 0.29$ $w_{a_3} = 0.20,\ w_{a_4} = 0.18$ $w_{a_5} = 0.14,\ w_{a_6} = 0.15$	$a_2 \succ a_3 \succ a_4 \succ a_6 \succ a_5 \succ a_1$
Uniform distribution	d_1 : (1.05, 1.48, 2.62, 2.78) d_2 : (1.19, 1.35, 1.53, 2.67) d_3 : (1.06, 1.33, 2.20, 2.70)	$Q_1^{(1)} = 0.1216$ $Q_1^{(2)} = 0.4170$ $Q_1^{(3)} = 0.0820$	1.3701 ± 0.0446	0.0145	$w_{a_1} = 0.06,\ w_{a_2} = 0.29$ $w_{a_3} = 0.20,\ w_{a_4} = 0.18$ $w_{a_5} = 0.13,\ w_{a_6} = 0.14$	$a_2 \succ a_3 \succ a_4 \succ a_6 \succ a_5 \succ a_1$

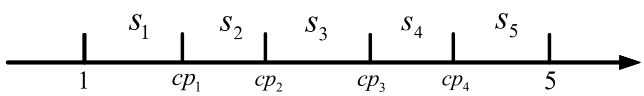


Fig. 19. The range of each linguistic term.

results. The relative cutoff points of d_2 are: 1.65, 2.37, 3.02, 3.79 and $Q_1^{(2)}$ is equal to 0.2043 ± 0.0083. For d_3, the cutoff points are: 1.03, 1.09, 2.26, 2.37 with the value of $Q_1^{(3)}$ being 0.0122 ± 0.0065. Fig. 20 exhibits the PSO-optimized distribution of $Q_1^{(1)}$, $Q_1^{(2)}$ and $Q_1^{(3)}$. After the PSO optimization, the value of Q_2 is 1.4665 ± 0.0510, then following the same procedures as shown in Example 3, the relative pairwise comparison vectors of each website can be constructed, then the group interval-based evaluation information is established as follows:

$$d = \begin{pmatrix} [2.93,5.00] & - & [1.41,2.31] & [1.66,2.30] & [1.78,2.64] & [1.91,3.04] \\ [2.46,3.36] \\ [2.02,3.31] \\ [2.18,4.02] \\ [2.65,3.71] \end{pmatrix}$$

Afterwards, according to the Monte Carlo simulation in **Algorithm** 2, we can obtain the following optimal numerical evaluation information matrix.

$$d = \begin{pmatrix} 4.80 & - & 1.98 & 2.06 & 2.50 & 2.32 \\ 2.90 \\ 2.17 \\ 2.69 \\ 2.78 \end{pmatrix}$$

Finally, the inconsistency index ξ^* is 0.0262 which is within the threshold in Table 2. In virtue of **Algorithm** 2, and the weights of the six scenic spots are: 0.06, 0.32, 0.17, 0.16, 0.14 and 0.15 in order, the final ranking results are: $a_2 \succ a_3 \succ a_4 \succ a_6 \succ a_5 \succ a_1$. Therefore, for tourists with plenty of time, the optimal sequence of sightseeing is Hubei Provincial Museum, Chu River and Han Street, Hankou River Beach, Dong Hu Scenic Resort, Yangtze River Bridge, and Yellow Crane Tower.

6.4. Further discussion

In this subsection, the cutoff points of linguistic terms set S from each website ($d_t(t = 1, 2, 3)$) are uniformly distributed, and then we can analyze the results of the case study thoroughly. Fig. 21 presents the positions of uniform distributed cutoff points and the PSO-optimized distributed cutoff points. Through calculations, the value of Q_1 and Q_2 of each website can be obtained, as well as the weight of scenic spots,etc, which are displayed in Table 11.

From Table 11, the cutoff points of linguistic terms formed by two types of distribution are the same as the final sequence of scenic spots in the proposed model. The distinct difference is that the PSO-optimized distributed cutoff points denote a better consistency at the individual level, that is, the value of Q_1 is smaller, while the uniformly distributed cutoff points perform better at the overall group level for the value of Q_2 and ξ^* being smaller (see Fig. 22).

Table 12
The corresponding TFN of linguistic terms in Ref. [37].

The linguistic term	Corresponding TFN
s_1	(1, 1, 1)
s_2	(2/3, 1, 3/2)
s_3	(3/2, 2, 5/2)
s_4	(5/2, 3, 7/2)
s_5	(7/2, 4, 9/2)

6.5. Comparisons with other methods

In this subsection, to further explain the feasibility and reliability of the proposed method in reality, the comparative analysis is represented for this application in spots recommendation by other types of the personalized individual semantics models based on BWM methods [37,38].

- Gou and Zhao [37] proposed the fuzzy BWM with the pairwise comparison expressed in linguistic terms, which they used triangular fuzzy numbers (TFNs) to qualify the linguistic terms. Furthermore, the graded mean integration representation (GMIR) method was applied to transform the TFNs-formed weights into the numeric-formed weights. The rules of TFNs-formed linguistic terms are displayed in Table 12.
- Dong et al. [38] put forward a novel fuzzy BWM based on TFN, the difference from Gou and Zhao [37] is that they consider the characteristic of decision makers, divided decision makers into three typical types follows as optimistic, pessimistic and neutral, then present the corresponding linear programming model. Furthermore, their model has a unique optimal solution by a proper selection of tolerance parameters. In the following comparative analysis, hybrid approach (the neutral decision maker) is chosen for comparison.

The tourist attractions recommendation displayed in this paper can be solved by these above methods and the ranking results are presented in Table 13 and Fig. 23. The comparison results reflect that the proposed model can be successfully applied to MCDM problems in reality. Meanwhile, owing to the input of information granularity, this model can deal with linguistic terms more flexibly in a fuzzy DM environment, compared with other methods.

7. Conclusions and future studies

In this paper, we have constructed the granular BWM model and the corresponding algorithms of constructing the entries of pairwise comparison vectors in the granular linguistic terms for solving GDM problems. The proposed model consists of two major innovations including the granulation of linguistic terms and the aggregation of individuals' preference. It is acknowledged that linguistic information can express the opinions of experts in a more comprehensive way than numerical numbers. We do not establish the linguistic distribution and semantics in advance, on the contrary, we design the mapping of linguistic evaluations to the corresponding information granules with a pre-given level of information granularity, which the formation and

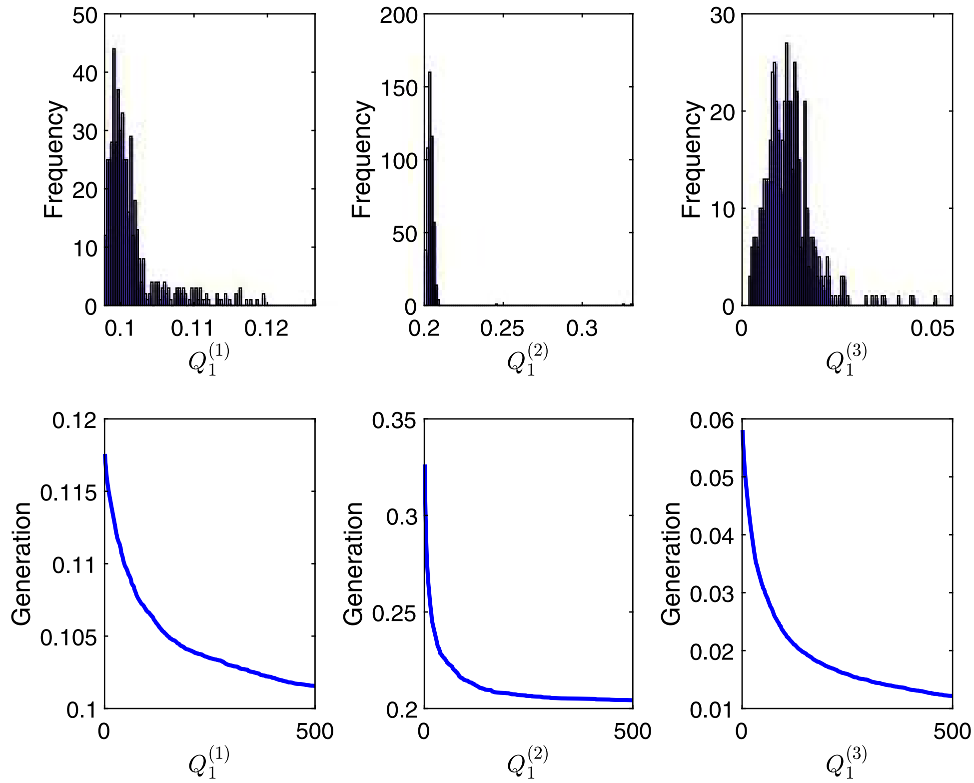


Fig. 20. The distribution and iteration process of $Q_1^{(1)}$, $Q_1^{(2)}$, $Q_1^{(3)}$.

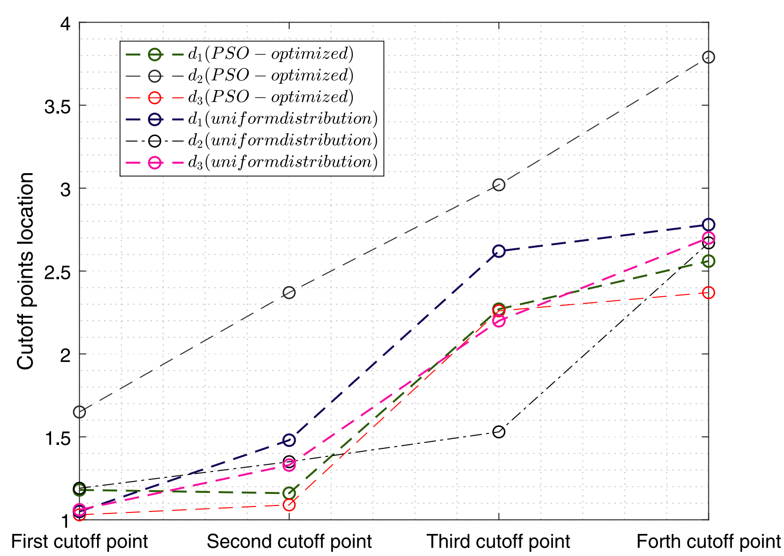
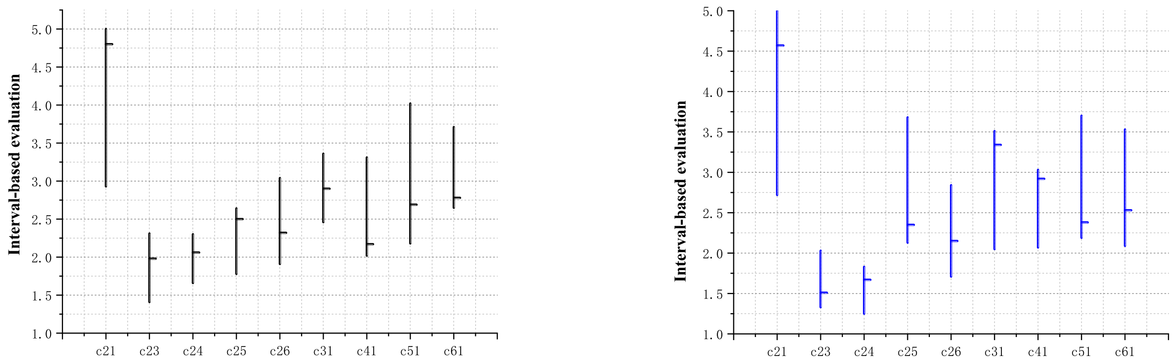


Fig. 21. The positions of cutoff points.

qualification process of granular linguistic information can be view as the optimization process to equip granules with well-articulated semantics in the aim of minimizing the inconsistency of this model and PSO algorithm serves as a suitable optimization tool. This kind of linguistic representation is quite flexible for the expressions of decision makers in the real-world cases. The interval-based granules are the classic and formal setting of information granules [15,43], which we can quickly understand the allocation of information granularity and the essence of granular computing, while this granular type comes with some information loss and cannot be applied in a fuzzy environment. Therefore, the IT2FSs-based granules are proposed to handle the uncertain and fuzzy problems. Meanwhile, through the comparative analysis in Section 5, it is verified that the granular model with two types of granular linguistic terms is feasible to solving DM problems with single decision maker.

This study concerns the GDM situation as well. The Euclidean distance-based function is designed for the aggregation of individual evaluations into the collective evaluations in the aim of maximizing the consistency among the group. For the sake of testifying the usefulness of this model in GDM situations, we apply this model and other existing methods [37,38] to solve the practical problem, which is the sequence recommendation of visiting scenic spots coming from three well-known tourism websites in Wuhan base on massive online reviews, which is proved that this model is effective enough to solve practical GDM problems.



(a) PSO-optimized distribution cutoff points (b) Uniform distribution cutoff points

Fig. 22. The optimal numerical evaluation.

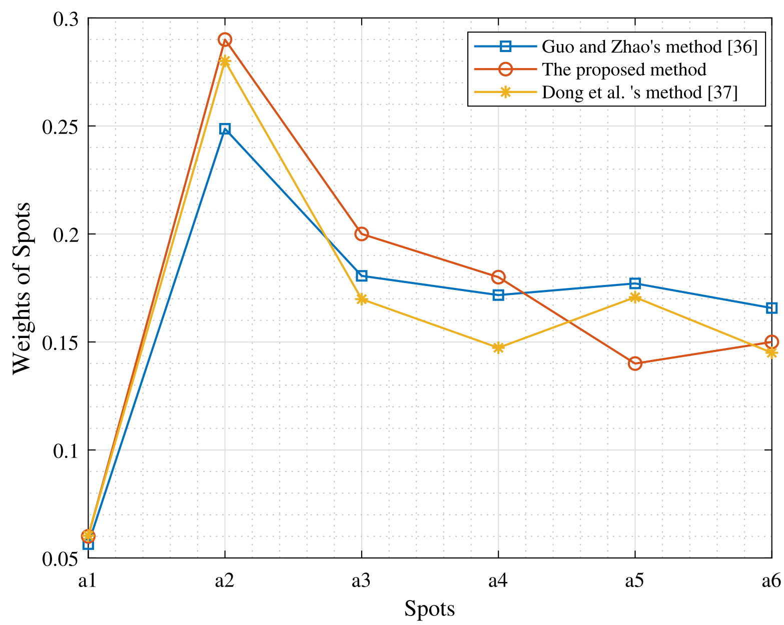


Fig. 23. The distribution of spots' weights.

Table 13
Comparisons with other methods.

Method	Weights of spots	Inconsistency indices	Final ranking
Fuzzy BWM in Guo and Zhao [37]	$w_{a_1}=0.0564,\ w_{a_2}=0.2487$ $w_{a_3}=0.1806,\ w_{a_4}=0.1717$ $w_{a_5}=0.1771,\ w_{a_6}=0.1657$	$\xi^*=0.5469$	$a_2 \succ a_3 \succ a_5 \succ a_4 \succ a_6 \succ a_1$
Fuzzy BWM in Dong et al. [38]	$w_{a_1}=0.0603,\ w_{a_2}=0.2801$ $w_{a_3}=0.1698,\ w_{a_4}=0.1473$ $w_{a_5}=0.1707,\ w_{a_6}=0.1450$	$\xi^*=(0.0185, 0.0274, 0.0331)$ $R(FCR)=0.0160$	$a_2 \succ a_5 \succ a_3 \succ a_4 \succ a_6 \succ a_1$
The proposed method	$w_{a_1}=0.06,\ w_{a_2}=0.29$ $w_{a_3}=0.20,\ w_{a_4}=0.18$ $w_{a_5}=0.14,\ w_{a_6}=0.15$	$\xi^*=0.0262$	$a_2 \succ a_3 \succ a_4 \succ a_6 \succ a_5 \succ a_1$

Future studies may focus on improvements of this research in several directions:

- We would concentrate on the other forms of granular linguistic terms in GDM problems, such as rough sets, hesitate fuzzy sets, shadow set, etc [13,14]. Then, find out the relatively suitable form of granular linguistic information. In the following research of the construction of information granularity, besides the establishment of the level of information granularity, the two evaluation criteria coverage and specificity of granules will be considered to further optimize the information granularity [22].
- In terms of group consistency, this study designs the distance-based function and PSO algorithm is used to improve the agreement level of the group. In the future, for the improvement of group consistency, one could explore the dynamic iteration algorithm to modify the assessments provided by decision makers if the PSO-optimized results are not acceptable.
- In this study, it is can be clearly seen that the PSO algorithm is inclined to being trapped in local optimum with a small search range. Some other alternative algorithms, say, the Differential Evolution (DE) algorithm [43] and Simulated Annealing (SA) algorithm [44] could be explored to optimize granular models.

CRediT authorship contribution statement

Xiaoyu Ma: Conceptualization, Methodology, Data curation, Writing – original draft. **Jindong Qin:** Conceptualization, Methodology, Model design, Writing – original draft, Writing – review & editing, Funding acquisition, Supervision. **Luis Martínez:** Writing – review & editing. **Witold Pedrycz:** Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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