A Model for Linguistic Dynamic Multi-criteria Decision-Making

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Abstract In this contribution, it is introduced a linguistic dynamic multi-criteria decision-making model to solve decision problems in which time dimension is included in the solving process, and the decision-maker provides the assessments by means of linguistic terms. An illustrative example of linguistic dynamic multi-criteria decision-making is exhibited.

Keywords Linguistic dynamic multi-criteria decision-making · Linguistic 2-tuple · Full reinforcement aggregation operator

1 Introduction

Multi-criteria decision-making (MCDM) are very common in real world. It consists of selecting the best alternative from a set of given alternatives or ranking them according to a set of criteria. To solve the MCDM problems, the general process is to aggregate numerical assessments of alternatives with respect to the fixed criteria, then these values are ranked to obtain an order of the alternatives, and the best alternative is selected. In many situations, the MCDM problems need to take into account the time dimension for its resolution. They are called dynamic MCDM (DMCDM) problems. The DMCDM problems have just attracted interests

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of experts, and some models have been introduced from different views [3, 9, 13, 14, 18]. Most of the existing DMCDM models assume that the available alternatives and criteria are fixed during different time periods. Recently, Campanella and Ribeiro [2] proposed a new DMCDM model in which the alternatives and criteria may vary across the time. In their model, the non-dynamic decision-making problem is firstly solved by using the classic MCDM model, which obtains a rating value for each alternative. The evaluations of the alternatives are then calculated by aggregating the current ratings of alternatives with previous evaluations of the alternatives. This stage considers the feedback of previous results to the current evaluations by using an associative, full reinforcement aggregation operator. Several possible situations in which this model may be applied were introduced, such as emergency department operation, medical diagnosis and planetary landing site selection [2, 11, 12].

Sometimes vagueness and uncertainties are contained in MCDM problems, and decision-makers cannot provide their assessments by means of numerical values. They may use the qualitative assessments; in such a case, the fuzzy linguistic approach [19] is used to represent the qualitative aspects of problems by means of linguistic variables to express their assessments. Those problems are called linguistic MCDM problems [1, 7, 15]. Similar to the DMCDM, the alternatives or criteria may vary across the time in the linguistic MCDM problems. To our knowledge, there are no models that are suitable for solving this type of problems. The existing linguistic MCDM methods cannot be applied to this case because their schemes are under the assumption that the alternatives and criteria are fixed across time. Therefore, it is necessary to introduce a new method to solve them. Considering the importance of the linguistic decision-making problems, it would be convenient to extend the quantitative DMCDM model to linguistic environment. In this contribution, it is proposed a novel linguistic DMCDM model to solve the above-mentioned problems. To do so, the rest of the paper is structured as follows: in Sect. 2, the DMCDM model and the linguistic MCDM model are reviewed. In Sect. 3, the new linguistic DMCDM model is introduced. In Sect. 4, an illustrative example is done. Section 5 concludes this contribution.

2 Preliminaries

In this section, we review briefly the DMCDM model, the linguistic decision-making and the 2-tuple linguistic computational model.

2.1 The DMCDM Model

In [2], the DMCDM model which consists of multiple time periods is introduced. The alternatives and the criteria at different periods may be different; hence, the dynamic property is included. In the computational process, the earlier evaluations affect later ones by using an associative, full reinforcement aggregation operator.

Framework of the DMCDM Model. Let $T = \{1, 2, \ldots\}$ be the set of discrete positive time periods, A_t be the set of alternatives at time $t \in T, C_t : A(t) \rightarrow [0,1]^n$ the function mapping each alternative to the corresponding vector of values for the n criteria over which alternatives are evaluated, and Wt be the weighting vector expressing the criteria's relative importance, which satisfies $W_t \in [0,1]^n$ and $\sum_{w \in W_t} w = 1, \forall t \in T$.

At each time $t \in T$, the rating $R_t : A_t \rightarrow [0,1]$ is computed in the enclosed classic MCDM model. It represents the non-dynamic aggregation result of the criteria values for each available alternative.

The remarkable characteristic of this model lies in which the available alternatives and criteria may be different at different time periods, and the previous aggregation results affect the current evaluation results. The historical set of alternatives is defined as

$$H_0 = \emptyset, \text{ and } H_t \subseteq \bigcup_{t' \leq t} A_{t'}, t, t' \in T. \tag{1}$$

A retention policy, which defines the subset of alternatives included in the historical set and carried over to the next iteration, was introduced in [2]. More details can be found in [2].

The final evaluation function $E_t : A_t \cup H_{t-1} \rightarrow [0,1], t \in T$ is obtained as

$$E_t(a) = \begin{cases} R_t(a), a \in A_t \backslash H_{t-1} \\ D_E(E_{t-1}(a), R_t(a)), a \in A_t \cap H_{t-1} \\ E_{t-1}(a), a \in H_{t-1} \backslash A_t \end{cases} \tag{2}$$

where D_E is some aggregation operator.

For each alternative $a \in A_t \cup H_{t-1}$, the above equation means that

1. if the alternative a belongs to the current set of alternatives, but not to the historical set, then its evaluation equals the current rating value;
2. if the alternative a belongs to both the current set of alternatives and the historical set, then its evaluation is obtained by aggregating the current rating value with its evaluation in the previous iteration;
3. if the alternative a does not belong to the current set of alternatives but the historical set, then its evaluation is carried over from the previous iteration.

The dynamic decision process may terminate according to some stopping criterion or may have no end. It depends on the specific problem being solved [2].

Full Reinforcement Aggregation Operators. The above-mentioned operator D_E in Eq. (2) can consider the feedback of the time period $t-1$ to the time period t. It is required that D_E should satisfy associativity and full reinforcement properties [2, 12, 17]. Associativity brings with it an important benefit, that is, when a new element is added to a set of elements, it is not necessary to aggregate them again;

the obtained aggregation result of the past elements can be aggregated with the newly added value directly. Thus, the past values do not need to be stored. Full reinforcement operators include two aspects of reinforcement property: one is upward reinforcement and the other is downward reinforcement. Upward reinforcement means that a collection of high scores can reinforce each other to give an even higher score than any of such scores, while downward reinforcement means the contrary reinforcement. Besides, when aggregating a high value and a low value, the reinforcement operator gives an averaging result. In [2], it is given a comprehensive review of the often-used operators including the upward reinforcement, the downward reinforcement and the averaging aggregation operators. By using this full reinforcement property, the feedback of the evaluation result at the time period $t-1$ can be considered sufficiently in the aggregation process of the next iteration.

In [2, 12, 17] were introduced several kinds of operators that satisfy the full reinforcement property, such as shown below:

1. Additive FIMICA (fixed identity, monotonic, identity, and commutative aggregation) operator [16].
   The additive family of the FIMICA class of aggregation operators is defined as follows [17]:

$$M(A) = f\left(\sum_{i=1}^{n} (a_i - g)\right) \tag{3}$$

   where $g \in [0,1]$, A is a bag, $A = \langle a_1, \ldots, a_n \rangle$, and $f : R \to [0,1]$ is a monotonic mapping which satisfies $f(x) \geq f(y)$ if $x \geq y$.
2. Multiplicative FIMICA operator
   The multiplicative family of the FIMICA class of aggregation operators is defined as follows [17]:

$$M(A) = f\left(\prod_{i=1}^{n} \frac{a_i}{g}\right) \tag{4}$$

   where $g \in [0,1]$, A is a bag, $A = \langle a_1, \ldots, a_n \rangle$, and f is as before.
3. Uninorm
   A uninorm is a mapping $U: [0,1]^2 \to [0,1]$ having the following properties:

(a) $U(a,b) = U(b,a)$
(b) $U(a,b) \geq U(c,d)$ if $a \geq c, b \geq d$
(c) $U(a, U(b,c)) = U(U(a,b),c)$
(d) $U(a,e) = a$ where $e \in [0,1]$ is called the identity element.

The identity element e serves as a boundary which separates the upward reinforcement and downward reinforcement. Values above e have the upward reinforcement, and values below it have the downward reinforcement.

4. Operators generated from fuzzy systems modelling techniques
   An example of this class of operator is called the triple Π aggregation operator [17]:

$$M(\langle a_1, \ldots, a_n \rangle) = \frac{\prod_{j=1}^{n} a_j}{\prod_{j=1}^{n} a_j + \prod_{j=1}^{n} \bar{a}_j} \tag{5}$$

   where $\bar{a}_j = 1 - a_j$, $j = 1, \ldots, n$, and $\langle a_1, \ldots, a_n \rangle$ is a bag. It has been shown in [17] that the triple Π operator is a FIMICA operator [16]. The value $e = 0.5$ serves as the identity element. Those values greater than $e = 0.5$ are upward reinforcing, and those values lower than $e = 0.5$ are downward reinforcing. It is also uninorm. In fact, it is easy to see that it is commutative, monotone, and has the identity element $e = 0.5$. We only need to prove the associativity.
   For $a, b, c \in [0, 1]$, we have

$$M(M(a,b),c) = M\left(\frac{ab}{ab + \bar{a}\bar{b}}, c\right) = \frac{\frac{ab}{ab+\bar{a}\bar{b}} \cdot c}{\frac{ab}{ab+\bar{a}\bar{b}} \cdot c + \frac{\bar{a}\bar{b}}{ab+\bar{a}\bar{b}} \cdot \bar{c}} = \frac{abc}{abc + \bar{a}\bar{b}\bar{c}},$$

$$M(a, M(b,c)) = M\left(a, \frac{bc}{bc + \bar{b}\bar{c}}\right) = \frac{a \cdot \frac{bc}{bc+\bar{b}\bar{c}}}{a \cdot \frac{bc}{bc+\bar{b}\bar{c}} + \bar{a} \cdot \frac{\bar{b}\bar{c}}{bc+\bar{b}\bar{c}}} = \frac{abc}{abc + \bar{a}\bar{b}\bar{c}}.$$

   Thus, $M(M(a,b),c) = M(a, M(b,c))$.
   We see that the uninorm operator and the triple Π operator are associative. Generally speaking, the additive and multiplicative FIMICA operators are not associative.

2.2 Linguistic Decision-Making and 2-Tuple Linguistic Computational Model

As mentioned before, it is convenient to use linguistic approach to represent qualitative aspects of decision-making. When a problem is solved using linguistic information, it implies the need for computing with words (CW) [20]. The CW methodology has been successfully applied to linguistic MCDM problems [8, 10]. In MCDM problems, the assessments of the alternatives with respect to the criteria are often expressed by means of linguistic variables in a given linguistic term set.

According to the fuzzy linguistic approach [19], a linguistic term set is defined by linguistic terms with their syntax and semantics. Here we assume that $S = \{s_0, s_1, \ldots, s_g\}$ is a linguistic term set, where $g + 1$ is an odd number that is called the cardinality of S. It is required that S satisfies the following conditions:

1. There is a negation operator: $Neg(s_i) = s_j$ such that $j = g - i$.
2. $s_i \leq s_j$ iff $i \leq j$.

3. There exists a minimization and a maximization operator

$$\max(s_i, s_j) = s_i, \min(s_i, s_j) = s_j \text{ if } s_i \geq s_j.$$

The linguistic 2-tuple fuzzy representation model [6] was introduced to overcome the limitations like in [4, 5]. We will use this representation in our proposal.

Definition 1 [6] Let $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set and $\beta \in [0, g]$ a numerical value, then the linguistic 2-tuple is obtained by using the following function

$$\begin{aligned} &\Delta: [0, g] \to S \times [-0.5, 0.5) \\ &\Delta(\beta) = (s_i, \alpha), \text{ with } \begin{cases} s_i, \ i = \text{round}(\beta) \\ \alpha = \beta - i, \ \alpha \in [-0.5, 0.5) \end{cases} \end{aligned} \tag{6}$$

where round is the round operation, then α is called a symbolic translation.

We shall denote the set of linguistic 2-tuples as $\bar{S}$.

A linguistic 2-tuple (s_i, α) can be easily transformed to a numerical value $\beta \in [0, g]$ by using the function

$$\begin{aligned} &\Delta^{-1}: S \times [-0.5, 0.5) \to [0, g] \\ &\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta \end{aligned} \tag{7}$$

To aggregate linguistic 2-tuples were introduced different linguistic 2-tuple aggregation operators.

Definition 2 [6] Let $\{(r_1, \alpha_1), \ldots, (r_n, \alpha_n)\}$ be a set of linguistic 2-tuples and $W = (w_1, \ldots, w_n)$ the weights of them, which satisfy $w_i \in [0, 1], \sum_{i=1}^{n} w_i = 1, i = 1, \ldots, n$. The linguistic 2-tuple weighted average operator is defined as

$$\bar{x}^e((r_1, \alpha_1), \ldots, (r_n, \alpha_n)) = \Delta\left(\sum_{i=1}^{n} w_i \Delta^{-1}(r_i, \alpha_i)\right) \tag{8}$$

Definition 3 [6] Let $\{(r_1, \alpha_1), \ldots, (r_n, \alpha_n)\}$ be a set of linguistic 2-tuples and $W = (w_1, \ldots, w_n)$ the weights of them, which satisfy $w_j \in [0, 1], \sum_{j=1}^{n} w_j = 1, j = 1, \ldots, n$. The linguistic 2-tuple ordered weighted average operator is defined as

$$F^e((r_1, \alpha_1), \ldots, (r_n, \alpha_n)) = \Delta\left(\sum_{j=1}^{n} w_j \beta_j\right) \tag{9}$$

where β_j^* denotes the jth largest of the $\beta_i = \Delta^{-1}(r_i, \alpha_i)$ values.

3 The Linguistic DMCDM Model

It has mentioned that the alternatives and criteria may vary across time in DMCDM. Similar things may happen in linguistic MCDM problems. Assuming that at each time $t \in T$, the decision-maker provides the assessment matrix $P^t = \left(p_{ij}^t\right)_{m_t \times n_t}$, where $p_{ij}^t \in \bar{S}$ denotes the achievement of the alternative a_i^t under the criterion c_j^t, $i = 1, \ldots, m_t$, $j = 1, \ldots, n_t$. Note that a_i^t and c_j^t are associated with the period t because they may change in the next period $t + 1$.

In the first step, we compute the rating value $R_t\left(a_i^t\right)$ for each alternative a_i^t. In this stage, the alternatives and criteria are fixed. Thus, we can view this problem as the classic non-dynamic linguistic MCDM problem. If the weights of criteria are given, we use the linguistic 2-tuple weighted average operator to compute the rating values. Otherwise, we will use the linguistic 2-tuple ordered weighted average operator. Assume that $W^t = \left(w_1^t, \ldots, w_{n_t}^t\right)$ are the weights of the criteria, which satisfy $w_i^t \in [0, 1]$, $\sum_{i=1}^{n_t} w_i^t = 1$. We then use the linguistic 2-tuple weighted average operator and obtain the rating as

$$R_t(a_i^t) = \Delta\left(\sum_{j=1}^{n_t} w_j \Delta^{-1}(p_{ij}^t)\right) \tag{10}$$

It is obvious that $R_t(a_i^t) \in \bar{S}$.

Similarly, we define the historical set of alternatives as

$$H_0 = \emptyset, \text{ and } H_t \subseteq \underset{t' \leq t}{\cup} A_{t'}, t, t' \in T. \tag{11}$$

In order to compute the final evaluation of alternatives, we define the evaluation function $E_t : A_t \cup H_{t-1} \rightarrow [0, 1], t \in T$ is obtained as

$$E_t(a_i^t) = \begin{cases} R_t(a_i^t), a_i^t \in A_t \backslash H_{t-1} \\ D'_E\left(E_{t-1}\left(a_i^{t-1}\right), R_t\left(a_i^t\right)\right),\ a_i^t \in A_t \cap H_{t-1} \\ E_{t-1}\left(a_i^{t-1}\right), a_i^t \in H_{t-1} \backslash A_t \end{cases} \tag{12}$$

where D'_E is some aggregation operator.

Here we also require that D'_E should be full reinforcement. Let us explain the reason. For example, when some high scores are given, this means that the results are very “good”; thus, the aggregation result of them should also be very “good”, even “better” than any one of them. In this way, we can guarantee that good scores can bring good result. When some low scores are given, this means that the results are very “bad”; thus, the aggregation result of them should also be very “bad”, even “worse” than any one of them. High scores aggregated with low scores will bring a “medium” result, which is bounded by the maximum and the minimum of the scores. To ensure that the repeated application of the aggregation function D'_E to the past values $E_{t'}, t' \in \{1, \ldots, t\}$ will yield the same result at time t, we require D'_E to be associative. In this case, we have

$$D'_E\left(D'_E(x,y),z\right) = D'_E\left(x, D'_E(y,z)\right) = D'_E(x,y,z) \tag{13}$$

Therefore, it is not necessary to store the past rating values, and the computation can be applied to the evaluation $E_{t-1}\left(a_i^{t-1}\right)$ and the rating $R_t(a_i^t)$ directly.

Unfortunately, none of the existing linguistic aggregation operators has the desired full reinforcement property on the domain $\bar{S}$. All the full reinforcement aggregation operators listed in Sect. 2.1 are defined in [0, 1]; thus, they cannot be applied to our model. To overcome this difficulty, we can transform the values in $\bar{S}$ into one in [0, 1] by using the mapping $h: \bar{S} \to [0,1]$, such that

$$h(s_i, \alpha) = \Delta^{-1}(s_i, \alpha)/g \tag{14}$$

where Δ^{-1} is defined by Eq. (7). Obviously, the function h is linear and satisfies $h(s_0, 0) = 0, h\left(s_g, 0\right) = 1$.

Using the transformation function h, we give the following definition.

Definition 4 The evaluation of the alternative a_i^t at time t is defined as

$$E_t\left(a_i^t\right) = \begin{cases} h\left(R_t\left(a_i^t\right)\right), a_i^t \in A_t \backslash H_{t-1} \\ D_E\left(E_{t-1}\left(a_i^{t-1}\right), R_t\left(a_i^t\right)\right), a_i^t \in A_t \cap H_{t-1} \\ E_{t-1}\left(a_i^{t-1}\right), a_i^t \in H_{t-1} \backslash A_t \end{cases} \tag{15}$$

where D_E is the same as in Eq. (2).

We can see that the obtained evaluation values are in the unit interval [0, 1]. In order to understand the linguistic meanings of the values, they can be trans formed into linguistic 2-tuples by using the function $h^{-1}(\beta) = \Delta(g\beta)$, $\beta \in [0,1]$.

The proposed method will be illustrated by an example in the next section.

4 Illustrative Example

Now, we consider a linguistic DMCDM problem in medical diagnosis. Let $S = \{s_0:$ extremely low, $s_1:$ very low, $s_2:$ low, $s_3:$ medium, $s_4:$ high, $s_5:$ very high, $s_6:$ extremely high$\}$ be a linguistic term set, and $C = \{c_1:$ blood, pressure, $c_2:$ body temperature, $c_3:$ white bloodcell, $c_4:$ blood glucose$\}$ be the set of criteria (here we assume that they are fixed) with the weighting vector $W = (0.1, 0.2, 0.4, 0.3)$. The possible diseases are the alternatives. Suppose that the doctor examines a patient according to the criteria and tries to confirm the disease. The doctor's diagnosis is carried out in three rounds. His/her conclusion

Table 1 Results of the linguistic DMCDM problem

t	c_1	c_2	c_3	c_4	R_t	$h(R_t)$	E_t	E_t^L
1	$a_1^1(s_2,0.3)$	$(s_3,-0.2)$	$(s_4,0.2)$	$(s_5,-0.1)$	$(s_4,-0.06)$	0.66	0.66	$(s_4,-0.06)$
	$a_2^1(s_3,0.1)$	$(s_4,-0.2)$	$(s_3,0.4)$	$(s_5,-0.4)$	$(s_4,-0.19)$	0.64	0.64	$(s_4,-0.19)$
	$a_3^1(s_2,0.4)$	$(s_5,-0.5)$	$(s_1,0.3)$	$(s_3,0.4)$	$(s_3,-0.32)$	0.45	0.45	$(s_3,-0.32)$
2	$a_1^2(s_2,0.4)$	$(s_2,0.2)$	$(s_3,-0.3)$	$(s_4,0.2)$	$(s_3,0.02)$	0.50	0.66	$(s_4,-0.06)$
	$a_2^2(s_3,0.1)$	$(s_4,-0.2)$	$(s_3,0.4)$	$(s_5,-0.4)$	$(s_4,-0.19)$	0.64	0.76	$(s_5,-0.44)$
	$a_3^2(s_2,0.4)$	$(s_5,-0.5)$	$(s_1,0.3)$	$(s_3,0.4)$	$(s_3,-0.32)$	0.45	0.40	$(s_2,-0.40)$
	$a_4^2(s_3,0.1)$	$(s_3,0.2)$	$(s_2,-0.1)$	$(s_4,0)$	$(s_3,-0.09)$	0.49	0.49	$(s_3,-0.06)$
	$a_2^3(s_3,0.2)$	$(s_3,0.3)$	$(s_2,0.1)$	$(s_5,0)$	$(s_2,0.32)$	0.55	0.79	$(s_5,-0.26)$
3	$a_3^3(s_2,0.4)$	$(s_5,-0.5)$	$(s_1,0.3)$	$(s_3,0.4)$	$(s_3,-0.32)$	0.45	0.35	$(s_2,-0.10)$
	$a_4^3(s_3,0.1)$	$(s_3,0.2)$	$(s_2,-0.1)$	$(s_4,0)$	$(s_3,-0.09)$	0.49	0.48	$(s_3,-0.12)$
	$a_5^3(s_3,0)$	$(s_2,0.4)$	$(s_3,-0.4)$	$(s_4,0.1)$	$(s_3,0.05)$	0.51	0.51	$(s_3,0.06)$

may vary across different rounds. The computational results obtained by using the previous method are shown in Table 1.

In the following, some computational steps are further detailed.

1. $t = 1$

   In this stage, the available alternatives are $A_1 = \{a_1^1, a_2^1, a_3^1\}$, and the rating R_t is computed by using the linguistic 2-tuple weighted average operator. For example, the rating $R_1(a_1^1)$ is computed as

$$R_1(a_1^1) = \Delta\begin{pmatrix} 0.1 \times \Delta^{-1}(s_2,0.3) + 0.2 \times \Delta^{-1}(s_2,-0.2) \\ +0.4 \times \Delta^{-1}(s_4,0.2) + 0.3 \times \Delta^{-1}(s_5,-0.1) \end{pmatrix} = (s_4,-0.06)$$

   The rating values R_t are then transformed into $h(R_t) \in [0,1]$ by using Eq. (14). Let $E_1(a_i^1) = h(R_1(a_i^1))$ be the numerical evaluations, and $E_1^L(a_i^1) = R_1(a_i^1)$, $i = 1,2,3$ be the linguistic evaluations.

2. $t = 2$

   A new alternative a_4^2 is added to the available alternative set, and the assessment values of a_1^2 are modified. The rating of a_4^2 is calculated in the enclosed linguistic MCDM problem. The evaluation values of a_1^2, a_2^2, a_3^2 are computed by using an associative, reinforcement operator D_E. In this example, we choose the operator expressed by Eq. (5). Then the linguistic evaluations are computed as $E_2^L(a_i^2) = \Delta(g \cdot E_2(a_i^2))$. As an example, for a_2^2, we have

$$E_2(a_2^2) = D_E(E_1(a_2^1), h(a_2^2)) = \frac{0.64 \times 0.64}{0.64 \times 0.64 + 0.36 \times 0.36} = 0.76$$

and

$$E_2^L\left(a_2^2\right) = \Delta(6 \cdot 0.76) = (s_5, -0.44).$$

3. $t = 3$
In this phase, the alternative a_1^2 is not available any more for some reason; thus, it is removed, and a_5^3 is added as a new alternative. Following the preceding method, it obtains the evaluations as $E_3^L = ((s_5, -0.26), (s_2, -0.10), (s_3, -0.12), (s_3, 0.06))$ and the ranking of alternatives as $a_2^3 \succ a_5^3 \succ a_4^3 \succ a_3^3$. Therefore, the most possible disease is a_2^3.

5 Conclusions

The DMCDM problems are very common in real life. The exploitation of its application to linguistic context is meaningful. In this contribution, we propose a linguistic DMCDM model where the assessments of the alternatives are expressed in terms of linguistic 2-tuples. We introduce the computational method of this model and give an illustrative example to show its feasibility.

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