UNIVERSITY OF JAÉN

School of Engineering and Computing

Computer Science Department

Emergency Decision Making Under Uncertainty

Thesis Memory Presented by

Liang Wang

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to obtain the PhD Degree in Computer science

Supervisors

Dr. Luis Martínez López
Dra. Rosa Mª Rodríguez Domínguez

Jaén, October de 2018

The Thesis entitled *Emergency decision making under uncertainty*, presented by D. Liang Wang to obtain the PhD degree in Computer Science, has been carried out in the Computer Science Department of the University of Jaén with the supervisors Dr. Luis Martínez López and Dra. Rosa María Rodríguez Domínguez. To be evaluated, this research memory is presented as a set of published articles, according to Article 23, point 3, Regulation of Doctoral Studies of the University of Jaén, approved in February 2012.

Jaén, 1st October 2018

PhD student:

Fdo. D. Liang Wang

Supervisors:

Fdo. Dr. Luis Martínez López   Fdo. Dra. Rosa María Rodríguez Domínguez

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Contents

Chapter 1

Introduction

1.1 Motivation

Emergency events (EEs) are defined as events that suddenly take place, causing or having the possibility of provoking intense death and injury, property losses, ecological damages and social hazards [105], which are often characterized as destructiveness, abruptness, complexity, changeability, diffuseness and so on. By its nature, EEs can be divided into four categories [63]: natural disasters, accident disasters, public health incidents and social hazards.

In recent years, various EEs, such as earthquakes, floods, hurricanes, and terrorist attacks, etc., have caused huge losses and severe negative impacts on human life and socio-economic development. When an EE occurs, Emergency decision making (EDM) is a very important activity, in which some measures should be taken to mitigate and reduce the damages or losses (property, lives, environment etc.) caused by EEs [136]. In real world situation, a decision maker (DM) is usually in charge of the EDM process, who takes the responsibility for the decision outcomes and plays a very crucial role in dealing successfully with EEs.

Since DM and EDM play crucial roles in mitigating the damages or losses caused by EEs, it has become a very active and important research direction in current emergency management study [33, 54, 69, 109, 126, 129].

As a result of the study of EDM, different works have been proposed in the literature to discuss related topics, such as:

- ◇ Number of involved individuals in the EDM process, i.e., classical EDM problems in which just one DM involved in the decision process [27, 54, 57, 68], and group emergency decision making (GEDM) problems [39, 60, 96, 117, 129, 137] in which multiple experts involved who play a role of think tank to support the DM to make a decision. The general schemes of them are shown in Figure

1.1 and Figure 1.2, respectively.

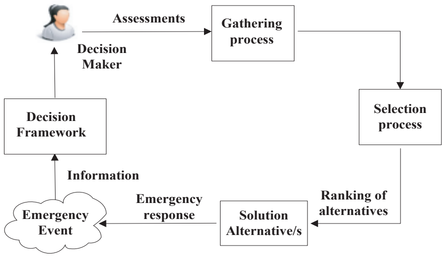

Figure 1.1: General scheme of classic EDM

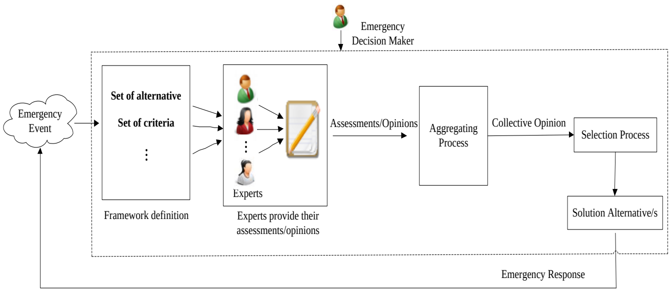

Figure 1.2: General scheme of GEDM

- ◇ DM's psychological behavior in the EDM process [33, 69, 107, 109], and aggregation of experts' assessments/opinions in the GEDM process [123, 124, 125, 126, 127]

- ◇ Elements of EEs in the EDM process [61], specifically uncertain, incomplete information [54, 64, 91, 109, 122], and dynamic evolution of EEs [40, 53, 107],

With the rapid development of technology, economy, and society in last decade, EEs are increasingly diversified and complicated, hence it is a big challenge for an individual DM to deal with real world complicated EEs, this is particularly true when the decision environment becomes highly complex and uncertain [70]. However, using group wisdom in the EDM process might be a powerful and effective way to cope with complex and damaging EEs, in which multiple experts with diverse professional background (e.g., hydrological, meteorological, sociological, and demographic) act as a think tank supporting the DM in the decision process, these methods lead to group emergency decision making (GEDM) problems, and the general scheme of GEDM has been shown in Figure 1.2.

Due to the fact that in other decision making problems, behavior experiments [14, 55, 103] have shown that human beings are usually bounded rational under risk and uncertainty, particularly, when they are under pressure, time restriction and risk decision environments, their psychological behavior will affect their decision behavior (risk-seeking, risk-aversion, neutral) directly [55]. Therefore, human beings psychological behavior also should play an important role in the decision making precess, and must be considered in all types of EDM problems.

There is a proverb in Chinese culture, i.e., "*Know yourself and know your enemy, you will win every war*", this proverb means that if someone wants to win every war, he/she must know not only himself/herself, but also know his/her enemy's features or characteristics. Similarly, for EDM problems, such proverb does also work. In order to make the emergency response more pertinently, effectively, and successfully, a large number of EDM approaches have considered the features of EEs [61] from various aspects, such as uncertain, incomplete information [54, 64, 91, 109, 122], dynamic evolution [40, 53, 107, 128], historical records [91, 137], domino effects [138] and so forth.

Despite the large amount of models and approaches have been proposed by a variety of authors to deal with EDM relevant problems, and have made significant contributions to emergency management. Up to date, the research results obtained in this field of study are not sufficient when dealing with real world complicated EDM problems: new difficulties and challenges arise, which require further and deep study for the improvements of existing studies. Some of these difficulties and challenges described below are the main motivation of this research memory:

- ◇ *Inclusion of experts' psychological behavior in GEDM process*: As aforementioned previously, due to human beings are usually bounded rational under risk and uncertainty, their psychological behavior is a very crucial factor in decision making processes, however, such an important issue has not been yet considered in existing GEDM approaches [39, 60, 96, 117, 129, 137]. For this practical and important problem, it seems necessary to propose a model, which is capable of dealing with GEDM problems more effectively.

- ◇ *Experts' hesitation in the GEDM process*: Hesitation is a quite common and inevitable behavior in our daily life, especially, in real world situations, due to the complexity and time restriction of decision problems, and the possibility of decision outcomes resulting in serious consequences, when experts are not good at or not familiar with one specifical aspect of the given decision problem, they might hesitate to provide either their assessments or opinions. Nevertheless, such practical and interesting topic has not been discussed in existing GEDM approaches [39, 60, 96, 117, 129, 137] so far. Therefore, it is a big challenge and meaningful to consider such a practical and inevitable topic.

◇ *Experts' opinions fusion*: Due to the importance of experts' opinions to deal with the decision problems successfully or not, it is necessary to handle experts' opinions quickly, properly and keep as much knowledge as possible. Existing GEDM approaches [123, 124, 126, 127] show that different consensus models and methods are employed to aggregate experts' opinions from various perspectives, however, there are obvious shortcomings that existing models and methods are not suitable for dealing with GEDM problems, i.e., loss of information in early stages of the decision problem [126, 127], time cost consensus models [123, 124] and information domains not suitable for handling fuzzy information [60, 39, 125, 129]. While, information is extremely valuable, because it means lives and chances. Thus, it might be difficult to obtain accurate decision results without a proper fusion model for GEDM problems.

◇ *Dynamic evolution considered in GEDM problems*: As a result obtained through analysis on existing dynamic studies [53, 107], it was found that only the time changes is considered in existing dynamic studies, however, with the evolution of EEs, not only changes the time, but also the information related to the EEs (alternative, criteria, etc.). It needs to point out that dynamic evolution of EEs is related to various aspects rather than just with time changes, which is a practical issue in real world, and should be also considered.

◇ *Information types tackled in GEDM problems*: Information plays a crucial part in all different types of decision problems no exception for GEDM problems. Existing GEDM approaches deal with the information employing only one expression domain: numerical values [123], interval values [109] or linguistic information [54]. Nevertheless, the information about the EEs in real world includes various types (numerical, interval, linguistic, hesitant information) at the same time rather than one specifical type, but none existing GEDM proposals considers multiple types of information at the same time.

◇ *Determination of criteria weights in GEDM problems*: There are three categories of methods that have been proposed to determine criteria weights [38, 112]: subjective, objective and hybrid methods. Subjective methods use the preferences of a DM to determine criteria weights [36, 110]; objective methods use a decision matrix to determine attribute weights [21, 22]; hybrid methods combine the preferences of a DM with a decision matrix to determine criteria weights [72, 114]. Subjective methods are widely used in existing EDM studies [33, 53, 68, 69, 107, 109], in which DM provides the criteria weights. When facing complex EEs in real world, it is difficult for DM to provide reasonable criteria weights, particularly, when DM is under pressure and hesitate in EDM problems. Therefore, it is a challenge to find out a more effective and suitable way to determine the criteria weights for GEDM problems.

Previous challenges found in existing GEDM problems make that current EDM approaches can not satisfy the situations and needs demanded by real world GEDM problems, such as *experts' psychological behavior and hesitation*, *experts' opinions fusion*, *dynamic evolution*, *heterogeneous information*, *determination of criteria weights* as mentioned above. For those reasons and challenges, this research memory conducts further and deep researches to fill those gaps.

1.2 Objectives

According to the challenges pointed out in existing GEDM approaches stated at the previous section, the purpose of this research is focused on the improvements of current GEDM approaches.

Based on such a purpose, the following four research objectives are considered:

1. To develop a novel GEDM approach that considers experts' psychological behavior [108] neglected in current studies. To illustrate the advantages, validity and feasibility of the new proposed method, a case study and related comparisons with existing EDM approaches are carried out.

2. To define a new perspective of dynamic evolution that considers not only the time changes, information related to EEs updated, but also problem structure changes (alternative, criteria, etc.) and considers different types of uncertainty in the EEs. Afterwards, a new dynamic GEDM approach [106] will be proposed for overcoming the limitations in current GEDM approaches by including this new dynamic view and heterogeneous information that will be applied to an explosion emergency decision problem illustrating its novelty, advantages, and validity.

3. To define a GEDM framework in which multiple types of uncertainty will be modelled by intervals, fuzzy and hesitant information. Additionally, a consensus model with low-time cost will be explored to aggregate experts' opinions that is also suitable for dealing with fuzzy information. A new way for determination criteria weights will be defined. Afterwards, a novel GEDM method will be proposed [105] that deals with heterogeneous information and looks

for agreed solution in a low time cost way together with taking into account experts' psychological behavior. Such a proposal will be applied to a real world case study of GEDM to show its validity and performance.

4. Improving information fusion in GEDM and considering hesitant uncertainty together experts' behavior [136] in order to build a GEDM method able to obtain better results because the information fusion process keeps more information than classical aggregation processes. Such a GEDM method will be applied to a real-world emergency problem.

1.3 Structure

To achieve these objectives presented in Section 1.2, and taking into account the article 23, point 3, of the current regulations for Doctoral Studies at the University of Jaén, in accordance with the program established in the RD 99/2011, this research memory will be presented as a compendium of published articles by the Ph.D student during his Ph.D period.

Three articles have been published in international journals indexed by JCR database, produced by ISI. And one article has been published in the international journal, *Complex Intelligent Systems* indexed in the Emerging Sources Citation Index. In summary, the report is composed of a total of four articles which have been published in high quality international journals.

The structure of this research memory is briefly described below:

- ◇ Chapter 2: Some basic concepts and methods such as, related concepts of decision making, emergency decision making, prospect theory, fuzzy TODIM method, hesitant fuzzy sets (HFS), hesitant fuzzy linguistic term sets (HFLTS) and so on; that are used in our proposals to achieve the different objectives pointed out in Section 1.2 are revised.
- ◇ Chapter 3: The published proposals that compose the research memory will be introduced briefly, in addition, discussions of each result obtained are presented in short to clarify the achievements reached in our research.
- ◇ Chapter 4: This chapter acts as the core of the doctoral thesis, which includes the publications obtained as the research results. For each publication, the detailed information of the journals in which the proposals have been published is indicated.
- ◇ Chapter 5: Final conclusions regarding this research and possible promising future works are pointed out.

Chapter 2

Basics Concepts and Background

This chapter establishes the framework of concepts and tools related to our research across this memory. Due to the fact that, the different papers that composed this research memory introduce and revise the necessary background for understanding our proposals, in this chapter we have just provided a brief and structured revision of the main necessary concepts related to our proposals including some related concepts about decision making and emergency decision making, the modelling and managing of uncertainty in decision making and several multicriteria decision-making methods under uncertainty, such as Fuzzy TOPSIS and Fuzzy TODIM. All these concepts, tools and methods are further detailed in each specific paper of the compendium provided in this research memory (Chapter 4), when they are required.

2.1 Decision Making: Introduction and Classification

In this section, a brief introduction and a classification of decision making are revised as the basic knowledge of this thesis, which pave the way for our coming researches.

2.1.1 Introduction

Decision making involves the selection of a course of action from among two or more possible alternatives in order to arrive at a solution for a given problem [100]. According to the definition of decision making, it not only exists in human beings daily life, but also includes modern management both organizational and managerial activities [100]. Based on the foregone definition, a decision can be understood as a course of action purposely chosen from a set of alternatives to achieve organizational or managerial objectives or goals.

A decision making process is a continuous and indispensable component of managing any organization or business activities in which organizational or business goals are achieved [50]. To achieve specific goals, either organisations or companies may face lots of obstacles in administrative, marketing wings and operational domains. Such problems are sorted out through a comprehensive decision making process. No decision comes as end in itself, since it may evolve new problems to be solved. When one problem is solved another arises and so on, such that this is why decision making, as aforementioned, is a continuous and dynamic process.

Generally speaking, a decision cannot be taken abruptly in a management setting, it should follow a series of steps as the following ones that can be shown in Figure 2.1 [50]:

1. Defining the problem: The structure of given problem, features, terminology etc. are defined in this phase;

2. Gathering information and collecting data: Information and data about given problem are gathered;

3. Developing and weighting the options: Alternatives and criteria are developed, and criteria weights are determined;

4. Choosing the best possible option: Specifical method is employed to select the best alternative concerning different criteria;

5. Planning and executing: To plan and execute the selected best alternative for solving the given problem;

6. Taking follow up action: According to the executing performance of the alternative, follow up action is taken to improve next possible decision problems.

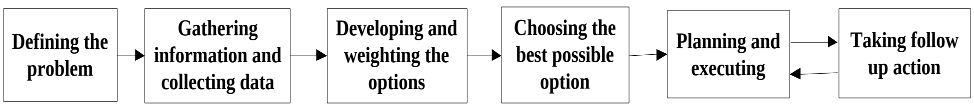

Figure 2.1: General decision making process

Even though, there are different types of decision problems, the general processes are almost the same like those ones shown in Figure 2.1.

2.1.2 Classification

Decision making is a quite common mankind activity in daily life. Human beings usually face different situations in which there exist several options or alternatives, in some situations, they must choose one among them as the best option or alternative.

Such activities widely exist in various fields, such as economy, engineering, health etc.

Despite there are various decision making problems, they share the following common features [49]:

- multiple criteria: each problem has multiple criteria, which can be objectives or attributes;
- conflicting criteria: multiple criteria may conflict with each other;
- incommensurable units: criteria may have different units of measurement;
- design/selection: solutions to multicriteria decision-making (MCDM) problems are either to design the best alternative(s) or to select the best one among previously specified finite alternatives.

According to the different situations or contexts in which the decision problem is conducted, decision problems can be classified into different types, such as based on *types of criteria* [70], *number of involved individuals* [76], *decision environment* [119] and so on.

(1) Types of criteria

Considering the common features shared in various decision problems mentioned above, two *types of criteria* can be distinguished: *objectives* and *attributes*. Therefore, MCDM problems can be classified into two wide classes [70]:

- multiobjective decision making;
- multiattribute decision making.

The main difference between these two classes is that the first concentrates on continuous solution decision spaces and the second focuses on problems with discrete solution decision spaces.

Multiobjective decision-making is known as the continuous type of multicriteria decision making and its main characteristics are that DMs need to achieve multiple objectives while these objectives are noncommensurable and conflict with each other. A multiobjective decision-making model includes a vector of decision variables, objective functions that describe the objectives, and constraints. DMs attempt to maximize or minimize the objective functions.

Multiattribute decision-making is related to making a preference decision (that is, comparison, choice, prioritization, and/or ordering) over the available alternatives that are characterized by multiple, usually conflicting, attributes. The main peculiarity of multiattribute decision making problems is that there are usually a limited number of predetermined alternatives (solutions), which are associated with attribute values and involve the selection of the best alternative from a pool of preselected alternatives described in terms of their attributes.

In almost all multiobjective decision making models, the alternatives can be generated automatically by the models. However, in the case of multiattribute decision making models, it is necessary to generate alternatives manually.

(2) Number of involved individuals

According to the involved number of individuals, decision making can be classified into two categories [76]:

- Individual decision making: such decisions are usually taken by a single individual, which can be quickly taken and less costly. Although decisions are based on individual thinking and limited information gathered by decision makers (managers), they are high-quality if the individual has expertise and experience in making such decisions.

- Group decision making: it has been defined in various ways, including as "*a decision situation in which more than one individual is involved, each with their own attitudes and viewpoints, recognizing the existence of a common problem and attempting to make a common decision together*" [70]. Such decisions are usually smarter than individual decision making, and they are frequently utilized in many complex real-life decision situations that are difficult for individual decision making. The solution to a group decision making (GDM) problem can be obtained by applying either a direct approach or an indirect approach in the selection process [44]. A direct approach obtains the solution directly from experts' information, whereas in an indirect approach, collective information is computed before the solution is determined. Regardless of the approach considered, the selection process to solve GDM problems consists of two phases (see Figure 2.2) [79]: (1) an aggregation phase, in which individual information is aggregated, and (2) an exploitation phase, in which an alternative or subset of alternatives is obtained as the solution to the problem.

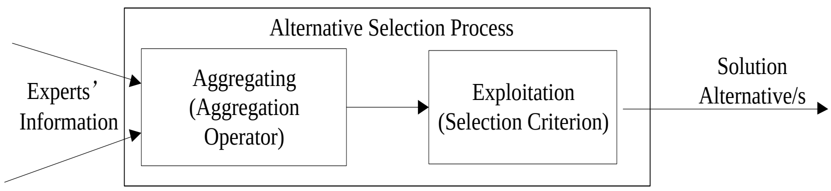

Figure 2.2: General scheme of group decision making

(3) Decision environment

According to different decision environments in which the decision problem is carried out, it can be classified into three types [119]:

- Decision making under certain environment: in decision making problems defined in such environment, the DM will have a clear state of alternatives, for example, what the alternatives are, what conditions are associated with each alternative, and the outcome of each alternative. In such decision environment, accurate, measurable, and reliable information on which to base decisions is available. In addition, the cause and effect relationships are known and the future is highly predictable.

- Decision making under risk environment: in decision making problems defined under risk environment, the DM has incomplete information about available alternatives but has a good idea of the probability of outcomes for each alternative. When making a decision under risk environment, DM must determine the probability associated with each alternative on the basis of the available information and his/her experiences.

- Decision making under uncertainty environment: in these decision making problems, the DM has no complete information about problems, the future environment is unpredictable and everything is in a state of flux. DM is not aware of all available alternatives, the risks associated with each alternative, and the outcome of each alternative or their probabilities.

Due to the complex situations and incomplete information, most decisions made in real world are usually under uncertainty in which DMs make decisions depending on their judgments, knowledge and experiences. Because EDM problems are typically characterized by at least uncertainty, time pressure, and lack of information, resulting in potentially serious consequences [60, 105], they are typical decision problems defined in the uncertainty context. Therefore, *decision making under uncertainty* is the main topic discussed in this research memory, which will be emphatically introduced in next section.

2.2 Decision making under uncertainty

As aforementioned, most decision problems in real world are defined in uncertainty context, this is particularly true for EDM problems. Therefore, this subsection introduces the basic knowledge of *decision making under uncertainty* in short in order that readers understand the principles comprehensively.

It needs to clarify that decision making under uncertainty can be divided into: *pure uncertainty* and *uncertainty* [28].

Under the *pure uncertainty* environment, DM has absolutely no knowledge, even not about the likelihood of occurrence for any state of nature. In such situation, decisions made are usually based on DM's expertise, experiences, and attitude toward the unknown [28, 31]. According to DM's attitudes toward the unknown, the decision problems can be carried out as follows [46]:

1. Pessimism, or Conservation (MaxMin Criterion): This model selects the alternative or option with the result that is the maximum of the minimum rewards. In this situation, DM assumes that the minimum reward occurs for each alternative or option, and then selects the alternative or option with the maximum of these minimum rewards;

2. Optimism, or Aggressive (MaxMax Criterion): This model selects the alternative or option with the result that is the maximum of the maximum rewards. In this situation, DM assumes that the most favorable state of nature for each alternative or option will occur;

3. Equal Likelihood Criterion (Laplace decision criterion): Assuming that all states of nature are equally likely to occur, the decision is made based on the highest *average reward* of alternatives or options. *Average reward* is calculated as: the sum of all rewards divided by the number of states of nature;

4. Coefficient of Optimism, or weighted average (Hurwicz Criterion): This decision model is a compromise between an optimistic and pessimistic decision. A coefficient, α, is selected by the DM to indicate the degree of optimism or pessimism about the future, $0 \leq \alpha \leq 1$. When α is equal to 1, the DM is purely optimistic; when α is equal to 0, the DM is purely pessimistic. The weighted reward is determined as: α(maximun rewards)+$(1-\alpha)$(minimum rewards), then the alternative or option with the highest weighted reward is selected;

5. Minimize Regret (Regret/Opportunity Loss): This decision model focuses on the difference between the optimal reward and the actual reward received. It determines the maximum regret for each alternative, and selects the alternative with the minimum value.

Previous decision models under uncertainty from 1 to 4 can be represented by an OWA operator [113].

Definition 1 *[113] An OWA operator of dimension n is a mapping $F : \Re^n \twoheadrightarrow \Re$ with an associated weight vector $W = (w_1, \ldots, w_n)^T$ such that $\sum_{i=1}^n w_i = 1$, $0 \leq w_i \leq 1$, and $F(a_1, \ldots, a_n) = \sum_{i=1}^n w_i b_i$, where b_i is the* i-th *largest of $a_1, \ldots, a_n$.*

Different OWA operators are distinguished by their weight vectors. The following OWA operators lead to the well-known decision criteria for decision making under uncertainty:

1. F_*: *In this case* $W = W_* = (0, \ldots, 0, 1)^T$ *and* $F_*(a_1, \ldots, a_n) = \min_i(a_i)$, *which is the purely pessimistic decision (maxmin criterion);*

2. F^*: *In this case* $W = W^* = (1, 0, \ldots, 0)^T$ *and* $F^*(a_1, \ldots, a_n) = \max_i(a_i)$, *which is the purely optimistic decision (maxmax criterion);*

3. F_A: *In this case* $W = W_A = (1/n, \ldots, 1/n)^T$ *and* $F_A(a_1, \ldots, a_n) = \frac{1}{n}\sum_{i=1}^{n}(a_i)$, *which is the equally likely decison (Laplace decision criterion);*

4. F_H: *In this case* $W = W_H = (\alpha, 0, \ldots, 0, 1-\alpha)^T$ *and* $F_H(a_1, \ldots, a_n) = \alpha \max_i(a_i) + (1-\alpha)\min_i(a_i)$, *which is the Hurwicz criterion.*

Under the *uncertainty* environment, the problems defined are different from the ones in *pure uncertainty* environment, in which the information about the problem is vague and imprecise [74], this situation is also known as decision making problems in a fuzzy context or fuzzy decision making [9]. Fuzzy sets theory [58, 131], intervals [75], intuitionistic fuzzy sets [3], hesitant fuzzy sets (HFS) [98], fuzzy linguistic approach [130], hesitant fuzzy linguistic terms sets (HFLTS) [88], etc. have proven to be effective ways to cope with uncertain information in decision problems.

In this memory, the EDM problems are defined in uncertainty environment, i.e., fuzzy context, in which different extensions of fuzzy sets [98, 130, 88] will be employed in our proposals to overcome the difficulties and challenges pointed out in Section 1.1.

2.3 Emergency decision making: State of art and limitations

In this section, the state of art of EDM is briefly revised because EDM will be the driving force of our research, and the limitations in current EDM approaches are then pointed out to highlight the importance and necessity of our proposals.

2.3.1 Emergency decision making

When an EE occurs, how to take effective and appropriate measures to make an emergency response immediately, to mitigate or reduce the losses caused by EE has drawn great attention of researchers all over the world. Through a plenty collection of literature, reading and a comprehensive review, the following main topics related to EDM have been discussed in current EDM studies [2, 23, 51, 52, 84]:

(1) Related studies on emergency plan

Emergency plan [84] is a formal written plan based on identified potential accidents together with their consequences. This plan describes how such accidents and their consequences should be handled both on-site and off-site. The main aim of the plan is to limit the negative effects of an accident by being prepared with a plan and facilities ready to react without delay. Because emergency plans play an extremely important role in the process of dealing with the accidents, it has drawn great attention and actively explored all over the world.

Using Web of Science Core Collection and Science Citation Index Expanded (SCIE) & Social Sciences Citation Index (SSCI) index database, searching 'emergency plan' or 'emergency planning' as the title keywords from January 2000 to June 2018, all the publication results of each year are shown in Figure 2.3 and Table 2.1.

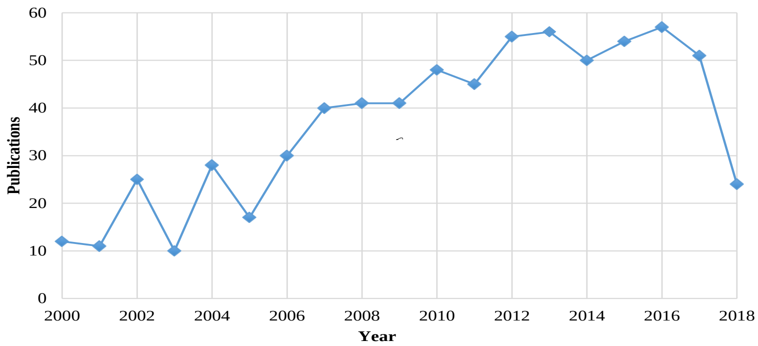

Figure 2.3: Publications of each year on emergency plan

Table 2.1: Publications of each year on emergency plan

Year	Publications	Year	Publications	Year	Publications	Year	Publications
2000	12	2001	11	2002	25	2003	10
2004	28	2005	17	2006	30	2007	40
2008	41	2009	41	2010	48	2011	45
2012	55	2013	56	2014	50	2015	54
2016	57	2017	51	2018	10		

It can be seen clearly that the related studies on emergency plan represents increasing tendency in recent years, and has become one of the active research topics in emergency management.

By analytical induction of current studies on emergency plan, it is found that the main researches focus on the following topics:

(a) Research on the formulation of emergency plans for different subjects

Some studies focus on the researches on the formulation of emergency plans for toxic gas (materials) release, such as ammonia release accident [84], toxic gas release accident occurred in big cities [135], chlorine gas release in processing plant [101], high temperature gas release in nuclear power plant [30]. Some studies are about the researches on the formulation of emergency plan for specific industry or area, such as chemical industry accidents [47], carbon capture and storage in North American [93], hospitality and tourism industry in Malaysian [2], metro operation accidents [71], bombing terrorist attack on Manchester's city centre [118]. There are others researches on the standard (framework) about the formulation of emergency plan for different objectives, such as the framework of national gas emergency plan in European Union [134], top-level design of the emergency plan framework for China [67], framework of compilation of emergency plan for work accidents in enterprise [19], framework of emergency plan for different objectives (individual, regional, and national) in Brazil [13].

Previous works clearly show that current studies on the formulation of emergency plan have obtained fruitful results, and related research results have been widely applied in practical production or real life. The EDM based on emergency plan has been widely used in practical coping processes of EEs, which establishes the solid foundation for emergency management.

(b) Research on the affected elements and supporting tools used in the design of emergency plans

Some studies focus on the different methods or ways to design the emergency plan considering related features of EEs such as, design the emergency plan based on hierarchical task network (HTN) planning considering features of incomplete information, concurrent execution and uncertain execution durations [64], considering resources and temporal constraints [82], and considering emergency command operation requirements [97], design the emergency plans for different scenarios considering limited human resources and materials [51, 52], design the emergency plans for civil protection considering land use and related elements [81], design the emergency plans based on test and practice experiences of on-site and off-site [85]. Others researches on the design of emergency plan by using computers or related assistant systems, such as design the emergency plan through the computer simulations of different scenarios [45], design the emergency plan for urban fire risk by using geographical information systems [35], design and improve the capacity and validity of emergency plan by using self-protection management support system [15].

According to above review, we can see that current studies on emergency plan have achieved abundant and relevant results from different perspectives by using various methods and ways.

(c) Research on the evaluation about emergency plan performance

When emergency plans are designed, it is necessary to evaluate their feasibility and validity in order to find out immediately the problems and modify/improve the plans to ensure their effectiveness [40]. Therefore, different evaluation approaches are proposed, such as combination of analytic hierarchy process (AHP) and fuzzy theories [18], fuzzy comprehensive evaluation method [23], combination of fuzzy cognitive maps and AHP method [57], fuzzy AHP and 2-tuple fuzzy linguistic approach [54], timed colored hybrid Petri-net based method [138].

Current evaluation studies have evaluated emergency plans by using various methods from different perspectives, the problems in existing emergency plans are immediately modified and improved through the evaluation process to improve the coping capacity of emergency plans. Such kind of research contents replenish and perfect the current related studies of emergency plans.

(2) Studies on related features of emergency events

Similarly, using Web of Science Core Collection and Science Citation Index Expanded (SCIE) & Social Sciences Citation Index (SSCI) index database, searching 'emergency decision making' as the title keywords from January 2000 to June 2018 in "Operations research management science or Management", all the publication results of each year are shown in Figure 2.4 and Table 2.2

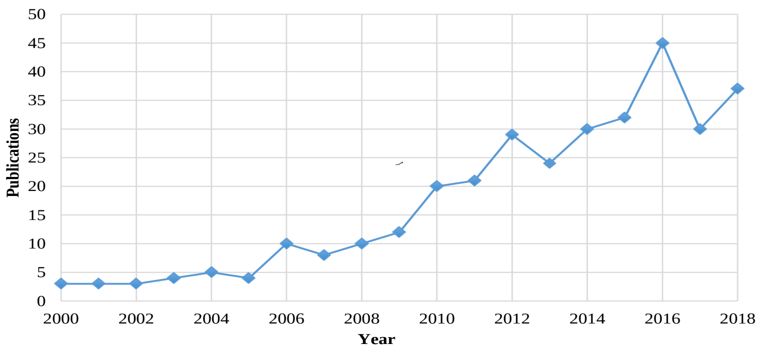

Figure 2.4: Publications of each year on EDM

EEs are usually featured by uncertain, incomplete and inadequate information, dynamic evaluation and so on. Existing EDM approaches have considered these features from various aspects in order to make the emergency response pertinently, effectively, and successfully.

Some studies have proposed various methods to deal with the related problems

Table 2.2: Publications of each year on EDM

Year	Publications	Year	Publications	Year	Publications	Year	Publications
2000	3	2001	3	2002	3	2003	4
2004	5	2005	4	2006	10	2007	8
2008	10	2009	12	2010	20	2011	21
2012	29	2013	24	2014	30	2015	32
2016	45	2017	30	2018	37		

of information features, such as incomplete and inadequate information problems [32, 54, 91, 109, 137]. Others have developed different approaches on the related problems of dynamic evaluation of EEs, such as dynamic decision models based on game theory [128], fault tree analysis method [68], the combination of entropy principle and dissipative structure theory [61], machine learning [139] and so forth.

Through the previous review, it can be clearly seen that different studies have discussed the related features of EEs from different points of view and achieved successful achievements, which make significant and important contributions to the development of emergency management. However, based on this review, it is found that there are some problems that have not been solved yet and also there are still some limitations in current studies. For sake of clarity, the coming subsection describes in further detail such limitations.

2.3.2 Limitations in current emergency decision making

As it has been previously pointed out, there are several limitations in current EDM studies, which are listed as follows:

1. Regarding the related information features of EEs, current studies only consider single type of information (numerical values [123], interval values [109], linguistic variables [54]) to deal with the uncertain information, it is seldom to consider different types of information at the same time. However, in real world EE, it is common that heterogeneous information contexts appear in the EE management.

2. Regarding the dynamic evaluation of EEs, current studies just discuss the dynamic feature only from the perspective of time changes [53, 107], do not consider the updated information along with the evolution of EEs, that it is common in EEs because not only the time changes but also information related the EEs (alternatives, criteria, experts etc.).

3. It is seldom to consider the DM's or experts' personal different behaviors (bounded rational, hesitation) in current EDM approaches [39, 60, 117, 123],

however, such behaviors are key and inevitable problems in real world and must be considered in EDM problems.

With respect to the previous limitations, this research memory will conduct a series of deep researches on these limitations and related topics in order to fill these gaps and enrich the theoretical basis and methods of current EDM.

2.4 Group emergency decision making and consensus reaching process

We have pointed out in Section 2.1.2 the usefulness and necessity of GDM process in those decision situations in which the group is smarter than the individual and several points of view can contribute to achieve a better solution. Therefore, when in real world, it is facing complex EEs, the difficulty to make comprehensive and reasonable judgements in short time based on individual DM's wisdoms increase, while using group wisdoms to solve such kind of complex problems is a reasonable and effective way because of including a wide knowledge and expertise in addition to multiple perspectives to solve the EDM. Regarding the diversity and complexity of EEs, related problems about group emergency decision making (GEDM) have drawn increasing attention from all over the world [105, 106, 123, 126, 127].

In this section, some basic concepts and knowledge about GEDM are reviewed in order to show the tendency and developments of current GEDM.

2.4.1 Group emergency decision making

Group emergency decision making consists of the evolution of the emergency decision making to a scheme that considers multiple experts with diverse professional backgrounds (e.g., hydrological, geological, meteorological, sociological, and demographic) who act as a think tank supporting the DM in the decision process.

Unlike other group decision-making (GDM) problems, such as supplier selection [24, 83], supply-chain risk management [6, 11], and large construction projects [56], GEDM problems are always defined under strong time constraints [27], which implies high risk and uncertainty [70], and their outcomes might result in extremely serious social consequences in the form of loss of lives and property [60]. Therefore, a GEDM process whose decisions are made among all experts should avoid conflicts and useless solutions [70] in order to obtain higher-quality decisions with timely response [12, 94]. Usually, in GEDM problems, experts play the role of think tank in supporting the DM who is in charge of the EE by means of a two-step decision solving process that consists of [79]:

- An aggregation process, in which individual information provided by experts is aggregated;
- A selection process, in which alternatives are obtained as solutions to the problems.

Such a general scheme of GEDM process is shown in Figure 1.2.

Through a comprehensive analysis on current GEDM studies, there are two main topics that have been discussed so far in specialized literature. One is related to studies on how to deal with group opinions [39, 96, 125, 127, 129], the another one is related to studies on large-scale GEDM problems [122, 123, 124, 126].

The main limitations in current GEDM studies have been pointed out in Section 1.1, such as *experts' psychological behavior*, *experts' hesitation*, *experts' opinions fusion* etc., this research memory will carry out a series of related studies to overcome them by using prospect theory, HFS, HFLTS and so forth.

2.4.2 Consensus reaching process

Experts' opinions play an important and crucial role in GEDM process, which determines the quality and impacts of decision making, therefore, their opinions should be properly considered. There are two different kinds of ways for dealing with a GDM problem (see Figure 2.5): a direct approach and an indirect approach [79]. A direct approach obtains the solution directly from experts' information, whereas in an indirect approach, collective information is computed before the solution is determined.

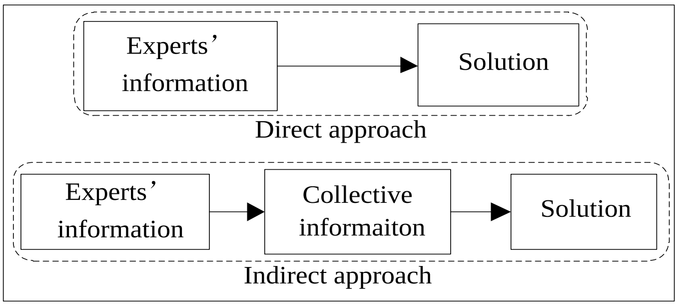

Figure 2.5: Two different ways for dealing with a GDM problem

In both cases to obtain a useful solution in the selection process, it is convenient to reach an acceptable agreement among all involved experts, otherwise, some experts might refuse the solution or feel frustrated on the decision results because they can think that their opinions have not been considered sufficiently. Therefore, how to achieve an acceptable agreement among all involved experts is an interesting and worthy topic to study. Consensus reaching process (CRP) is the process in which experts' opinions bring closer to each other and finally obtain an acceptable agreement among all involved experts, which is defined [12] as a dynamic and iterative process consisting of several rounds of discussion in which experts adjust their initial opinions in order to make themselves closer to each other and then reach a collective opinion that is used to make the final decision [12, 94]. A general scheme of CRP is shown in Figure 2.6, which consists of four main phases:

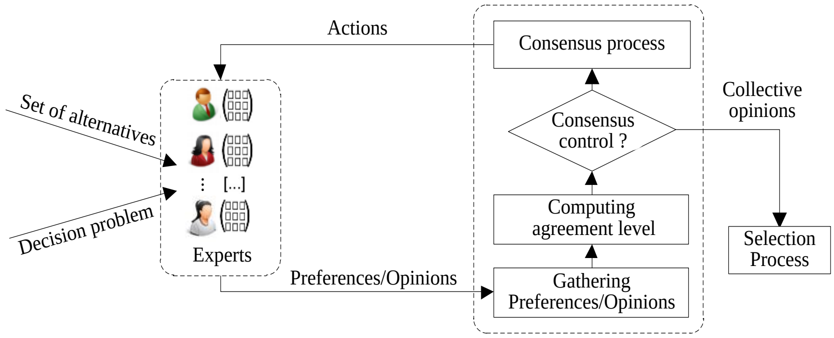

Figure 2.6: General scheme of CRP

1. Gathering preferences/opinions

   The preferences/opinions regarding the given decision problem provided by experts are gathered in this phase.

2. Computing agreement level

   There are two different measures that can be used to compute agreement level [78]: one is based on distances from individual expert to collective preference; another is based on distances between preferences of different pairs of experts.

3. Consensus control

   Consensus threshold is set in this phase, which denotes the minimum value of acceptable agreement. If the agreement level obtained in previous phase is greater than the consensus threshold that means the collective opinions have been reached, and then the process moves into selection process; otherwise, it moves into consensus process.

4. Consensus process

   If the agreement level is not enough, a procedure should be conducted to increase the level of agreement in another discussion round. The procedure can be classified into two categories [78]: 1) with feedback procedure [73, 94], in which a moderator identifies the farthest assessments from consensus in the current round, and then some suggestions for experts are generated to modify their assessments to get closer to the rest of the group in a new round; 2) without feedback procedure [66, 120], in which experts' assessments are updated automatically to increase the agreement level, in such procedure, experts only provide their initial preferences/opinons, experts are no necessary to be involved in a new discussion round. In this phase, the number of maximum rounds should be set in order to avoid an endless process.

CRP is one of the most studied topics [78] in current GDM and GEDM problems, important and relevant results have been recently achieved [39, 78, 96, 125, 127, 129].

CRPs in GEDM is not our main aim in this research but in Section 4.3 it will be introduced a novel CRP model for GEDM under uncertain information that overcome several limitations of current CRPs in GEDM for dealing timely with different types of uncertainty.

2.5 Methods and models

Here, different methods and models used across this research memory are briefly revised in following subsections, including prospect theory, fuzzy TODIM, fuzzy TOPSIS based on alpha-level sets, and hesitant fuzzy linguistic term sets and so on. All of them are relevant for the different proposals that will be developed in this research to achieve our goals.

2.5.1 Prospect theory

Prospect theory (PT) was first introduced in 1979 [55] and extended in 1992 [103] by Daniel Kahneman and Amos Tversky as a behavior economic theory, which describes the way in which people choose between probabilistic alternatives that involve risk when the probabilities of outcomes are known. According to this theory, people make decisions based on the potential value of losses and gains rather than the final outcome.

PT is regarded as the most influential theory among different behavioral decision making theories such as regret theory [7], disappointment theory [8], third-generation PT [95]. Since PT was proposed, it has been studied [95, 102, 103] and widely applied to solve various decision-making problems, such as asset allocation [10], health domains [4, 5], portfolio insurance [29], traffic management [62, 140], multi-attribute decision making [34], and emergency decision making [69, 109], considering humans' psychological behavior under uncertainty.

The first key concept in PT is the "*reference point*" (RP), which is defined as a neutral position asset or expectation value of people who want to gain an amount or not to lose it. RP determines the feeling of gains or losses based on the difference between expectation and outcomes; the value of the RP is affected by the expectations of people [55].

In multi-attribute decision-making problems, the attributes can be classified into two types: benefits and costs [72]. The higher a benefit attribute is, the better the situation is, while the higher a cost attribute is, the worse the situation is.

According to different types of attributes, a RP changes with people's expectations with respect to the predefined amounts to gains or losses. For a better understanding of the RPs in PT, see Figure 2.7 (benefit attributes versus cost attributes). According to Figure 2.7, RP is analogously defined for cost attributes.

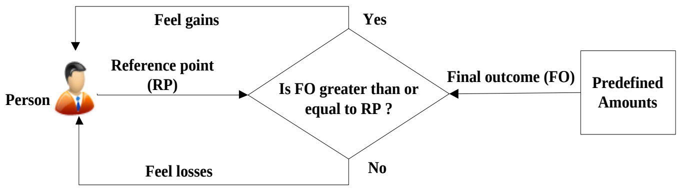

Figure 2.7: Gains and losses based on reference point and predefined amounts

In Kahneman and Tversky [55] and Tversky and Kahneman [103], it has shown that people's psychological behavior exhibits a risk-aversion tendency for gains and a risk-seeking tendency for losses. Therefore, PT describes the decision process in the following three stages.

1. In the editing phase, people decide which outcomes they consider are equivalent, set an RP, consider less outcomes as losses and greater outcomes as gains for the benefit attribute, and consider less outcomes as gains and greater outcomes as losses for the cost attribute.

2. In the evaluation phase, people behave as if they would compute a prospect value by using a value function based on the potential outcomes and then calculate the overall prospect values.

3. In the selection phase, the alternative of having a higher overall prospect value is finally selected.

The PT involves the following three important principles [55]:

- Reference dependence. Experts perceive gains and losses according to an RP. Thus, the prospect value function can be divided into a gain domain and a loss domain regarding the RP.

- Diminishing sensitivity. Experts exhibit a risk-aversion tendency for gains and a risk-seeking tendency for losses. According to the principle of diminishing sensitivity, the prospect value function is concave in the loss domain and convex in the gain domain, that is, the marginal value of both gains and losses is decreasing with size.

- Loss aversion. The experts are more sensitive to losses than to equal gains [1]. In accordance with the principle of loss aversion, the prospect value function is steeper in the loss domain than in the gain domain.

According to these three principles, an S-shaped value function is proposed in PT (see Figure 2.8), which shows a prospect value function with a convex S-shape for losses and a concave S-shape for gains. Prospect values are calculated for measuring the magnitude of gains and losses by using a value function in PT, which is defined on deviations from the RP, and expressed in the form of a power law according to the following expression [103].

$$v(x) \in \begin{cases} x^{\alpha}, x \geq 0 \\ -\lambda(-x)^{\beta}, x < 0 \end{cases} \tag{2.1}$$

where α and β are power parameters related to gains and losses, respectively, $0 \leq \alpha, \beta \leq 1$, where x denotes the gains or losses with $x \geq 0$ or $x < 0$, respectively, and λ is the risk-aversion parameter, which has the characteristic of being steeper for losses than for gains, $\lambda > 1$.

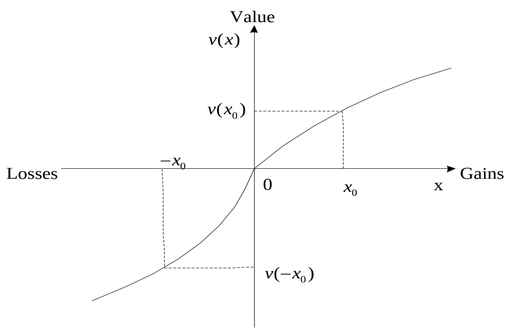

Figure 2.8: S-shaped value function of prospect theory

2.5.2 TODIM and Fuzzy TODIM method

TODIM (an acronym in Portuguese "TOmada de Decisão Iterativa Multicritério") method, proposed by Gomes and Lima [42, 43], is a popular MCDM method based on prospect theory [55] to capture human being's psychological behavior, which is defined for dealing with the MCDM problems in which the criteria representatives are in the format of crisp values. It has been widely applied to solve different decision problems [41, 80]. The TODIM [42, 43] method is briefly reviewed as follows.

2.5.2.1 TODIM method

Let $A = \{a_1, a_2, \ldots, a_m\}$ be a set of alternatives, a_i denotes the *i-th* alternative, $i = 1, 2, \ldots, m$. $C = \{c_1, c_2, \ldots, c_n\}$ be a set of criteria/attributes, $w_c = (w_{c_1}, w_{c_2}, \ldots, w_{c_n})$ be the weighting vector of criteria/attributes, w_{c_j} denotes the weight of *j-th* criterion/attribute, c_j, $j = 1, 2, \ldots, n$. Let $X = (x_{ij})_{m\times n}$ be the decision matrix, x_{ij} denotes the assessment provided by decision maker regarding alternative a_i concerning criterion c_j.

Step 1: Normalize decision matrix, $X = (x_{ij})_{m\times n}$ into $\bar{X} = (\bar{x}_{ij})_{m\times n}$, using the normalization method regarding different types of criterion (cost or benefit).

Step 2: The reference criterion c_r is determined and the relative weight w_{jr} of c_j can be obtained, i.e.,

$$w_{jr} = \frac{w_{c_j}}{w_r} \tag{2.2}$$

where $w_r = max\{w_{c_j} | j = 1, 2, \ldots, n\}$.

Step 3: The dominance degree, $\Phi_j(a_i, a_k)$, of alternative $a_i (i = 1, 2, \ldots, m)$ over the rest of alternatives $a_k (k = 1, 2, \ldots, m)$ regarding criterion $c_j (j = 1, 2, \ldots, n)$ is calculated, i.e.,

$$\Phi_j(a_i, a_k) = \begin{cases} \sqrt{(\bar{x}_{ij} - \bar{x}_{kj}) w_{jr} / (\sum_{j=1}^{n} w_{jr})}, (\bar{x}_{ij} - \bar{x}_{kj}) \geqslant 0 \\ \\ -\frac{1}{\theta} \sqrt{(\bar{x}_{ij} - \bar{x}_{kj}) (\sum_{j=1}^{n} w_{jr}) / w_{jr}}, (\bar{x}_{ij} - \bar{x}_{kj}) < 0 \end{cases} \tag{2.3}$$

where $(\bar{x}_{ij} - \bar{x}_{kj}) \geq 0$ and $(\bar{x}_{ij} - \bar{x}_{kj}) < 0$ represents the gain and loss of alternative a_i over a_k regarding criterion c_j, respectively. θ denotes the attenuation factor of the losses, $\theta > 0$.

Step 4: The dominance degree, $\delta(a_i, a_k)$, of alternative a_i over the rest of alternatives a_k is calculated, i.e.,

$$\delta(a_i, a_k) = \sum\nolimits_{j=1}^{n} \Phi_j(a_i, a_k) \tag{2.4}$$

Step 5: The overall dominance degree, $\eta(a_i)$, of alternative a_i is calculated, i.e.,

$$\eta(a_i) = \frac{\sum_{k=1}^{m} \delta(a_i, a_k) - \min_i\{\sum_{k=1}^{m} \delta(a_i, a_k)\}}{\max_i\{\sum_{k=1}^{m} \delta(a_i, a_k)\} - \min_i\{\sum_{k=1}^{m} \delta(a_i, a_k)\}} \tag{2.5}$$

Step 6: According to the overall dominance degree of each alternative, the corresponding ranking can be determined. The greater $\eta(a_i)$, the better alternative a_i.

2.5.2.2 Fuzzy TODIM method

To cope with complex problems and uncertain information in the real world, the TODIM method has been extended to deal with fuzzy MCDM problems [99, 115]. The fuzzy TODIM method [99, 59] is briefly reviewed below.

Let $P = (p_{ij})_{m\times n}$ be a fuzzy decision matrix, $p_{ij} = (p^1_{ij}, p^2_{ij}, p^3_{ij}, p^4_{ij})$ denotes the rating of alternative a_i concerning c_j. Due to the main differences between TODIM and fuzzy TODIM are Step 1 and Step 3, to save space, only those two steps are introduced as follows:

Step 1: The fuzzy decision matrix, $P = (p_{ij})_{m\times n}$, is normalized into $\bar{P} = (\bar{p}_{ij})_{m\times n}$, according to the cost and benefit criteria.

Step 3: The dominance degree, $\Phi_j(a_i, a_k)$, of a_i over the rest of alternatives $a_k (k = 1, 2, \ldots, m)$ regarding c_j is calculated, i.e.,

$$\Phi_j(a_i, a_k) = \begin{cases} \sqrt{w_{jr}/(\sum_{j=1}^n w_{jr}) d(\bar{p}_{ij}, \bar{p}_{kj})}, \\ F(\bar{p}_{ij}) - F(\bar{p}_{kj}) \geqslant 0 \\ \\ -\frac{1}{\theta}\sqrt{(\sum_{j=1}^n w_{jr})/w_{jr} d(\bar{p}_{ij}, \bar{p}_{kj})}, \\ F(\bar{p}_{ij}) - F(\bar{p}_{kj}) < 0 \end{cases} \tag{2.6}$$

where $d(\bar{p}_{ij}, \bar{p}_{kj})$ represents the distance between two fuzzy numbers $\bar{p}_{ij}$ and $\bar{p}_{kj}$. θ denotes the attenuation factor of the losses, $\theta > 0$. $F(*)$ is a defuzzification function [59].

In this memory, we will use the fuzzy TODIM method [99] based on prospect theory [55] to consider experts' psychological behavior in fuzzy environment because of its advantage and capability of capturing such behavior under fuzzy environment.

2.5.3 TOPSIS and Fuzzy TOPSIS method based on alpha-level sets

For easy understanding of fuzzy TOPSIS method based on alpha-level sets, the TOPSIS method is first reviewed.

2.5.3.1 TOPSIS method

TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) method was first proposed by Huwang and Yoon [48], which is a very popular MADM method and has been widely applied to solve different decision problems [16, 17, 48, 111].

To avoid repeated expressions, the notations and related meanings about alternatives, criteria, and criteria weights are as same as that defined in Section 2.5.2.1. Let $X = (x_{ij})_{m\times n}$ be the decision matrix, x_{ij} denotes the values/ratings assigned to alternative a_i with respect to criterion c_j. Then, the TOPSIS method is briefly reviewed below.

Step 1: Normalize the decision matrix $X = (x_{ij})_{m\times n}$ using the following equation:

$$\bar{x}_{ij} = \frac{x_{ij}}{\sqrt{\sum\limits_{i=1}^{n} x_{ij}^2}}, \quad i = 1, \ldots, m; j = 1, \ldots, n \tag{2.7}$$

Normalized decision matrix $\bar{X} = (\bar{x}_{ij})_{m\times n}$, where $\bar{x}_{ij}$ is the normalized criteria rating.

Step 2: Calculate the weighted normalized decision matrix $V = (v_{ij})_{m\times n}$

$$v_{ij} = w_{c_j}\bar{x}_{ij}, i = 1, \ldots, m; j = 1, \ldots, n \tag{2.8}$$

where w_{c_j} is the relative weight of the *j-th* criterion.

Step 3: Determine the ideal and negative-ideal solutions:

$$\begin{aligned} A^* &= \{v_1^*, \ldots, v_m^*\} \\ &= \{(\max_j v_{ij}, j \in \Omega_b), (\min_j v_{ij}, j \in \Omega_c)\} \end{aligned} \tag{2.9}$$

$$\begin{aligned} A^- &= \{v_1^-, \ldots, v_m^-\} \\ &= \{(\min_j v_{ij}, j \in \Omega_b), (\max_j v_{ij}, j \in \Omega_c)\} \end{aligned} \tag{2.10}$$

where Ω_b and Ω_c are the sets of benefit criteria/attributes and cost criteria/attributes, respectively.

Step 4: Calculate the Euclidean distances of each alternative from the ideal solution and the negative-ideal solution, respectively:

$$D_i^* = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^*)^2}, \quad i = 1, \ldots, m \tag{2.11}$$

$$D_i^- = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^-)^2}, \quad i = 1, \ldots, m \tag{2.12}$$

Step 5: Calculate the relative closeness RC_i for each alternative with respect to the ideal solution. The relative closeness of the alternative a_i with respect to A^* is defined as:

$$RC_i = \frac{D_i^-}{D_i^* + D_i^-}, \quad i = 1, \ldots, m \tag{2.13}$$

Step 6: Rank the alternatives according to their relative closeness to the ideal solution. The greater RC_i is, the better alternative a_i. The best alternative is the one with the greatest relative closeness to the ideal solution.

2.5.3.2 Fuzzy TOPSIS method based on alpha-level sets

The fuzzy TOPSIS method based on alpha-level sets [111] is a distinctive and powerful approach among other fuzzy TOPSIS versions [16, 17, 25, 26, 111] due to its prominent advantages of keeping the uncertain information in a better way. This is the significant difference between the fuzzy TOPSIS method based on alpha-level sets and other versions. Due to such advantages, the fuzzy TOPSIS method based on alpha-level sets will be used in this research, therefore, it will be reviewed in short [111].

Let $\tilde{X} = (\tilde{x}_{ij})_{m\times n}$ be a fuzzy decision matrix characterized by membership functions $\mu_{\tilde{x}_{ij}}(x)\,(i = 1,\ldots,m,\ j = 1,\ldots,n)$ and $\tilde{W} = (\tilde{w}_1,\ldots,\tilde{w}_n)$ be the fuzzy weights characterized by $\mu_{\tilde{w}_j}(x)\,(j = 1,\ldots,n)$. If all the criteria/attributes, $\{c_1,\ldots,c_n\}$, are assessed by using linguistic term sets with the same syntax and semantics, then the fuzzy decision matrix $\tilde{X}$ has the same dimension and therefore it is not necessary any normalization. Otherwise, $\tilde{X}$ has to be normalized.

If $\tilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij}, d_{ij})\,(i = 1,\ldots,m,\ j = 1,\ldots,n)$ are trapezoidal fuzzy numbers, then the normalization process can be carried out by (the same normalization process for triangular fuzzy numbers),

$$\tilde{r}_{ij} = (\frac{a_{ij}}{d_j^*}, \frac{b_{ij}}{d_j^*}, \frac{c_{ij}}{d_j^*}, \frac{d_{ij}}{d_j^*}), i = 1,\ldots,m; j \in \Omega_b \tag{2.14}$$

$$\tilde{r}_{ij} = (\frac{a_j^-}{d_{ij}}, \frac{a_j^-}{c_{ij}}, \frac{a_j^-}{b_{ij}}, \frac{a_j^-}{a_{ij}}), i = 1,\ldots,m; j \in \Omega_c \tag{2.15}$$

where

$$d_j^* = \max_i d_{ij}, j \in \Omega_b, \tag{2.16}$$

$$a_j^- = \min_i a_{ij}, j \in \Omega_c \tag{2.17}$$

where Ω_b and Ω_c denotes the sets of benefit and cost criteria/attributes, respectively.

It can be seen that $\tilde{r}_{ij}$ belong to [0,1], thus, positive and negative ideal solutions can be defined as $A^* = \{1, \ldots, 1\}$ and $A^- = \{0, \ldots, 0\}$, respectively. For a fuzzy decision matrix $\tilde{X} = (\tilde{x}_{ij})_{m\times n}$ without normalization, the positive and negative ideal solutions can be obtained as follows:

$$\begin{aligned} P^* &= \{x_1^*, \ldots, x_m^*\} \\ &= \{(\max_j d_{ij}, j \in \Omega_b), (\min_j a_{ij}, j \in \Omega_c)\} \end{aligned} \tag{2.18}$$

$$\begin{aligned} P^- &= \{x_1^-, \ldots, x_m^-\} \\ &= \{(\min_j a_{ij}, j \in \Omega_b), (\max_j d_{ij}, j \in \Omega_c)\} \end{aligned} \tag{2.19}$$

Let $(r_{ij})_\alpha = [(r_{ij})_\alpha^L, (r_{ij})_\alpha^U]$ and $(w_j)_\alpha = [(w_j)_\alpha^L, (w_j)_\alpha^U]$ be alpha-level sets of $\tilde{r}_{ij}$ and $\tilde{w}_j$, respectively. Then, Eq. (2.13), the relative closeness (RC), RC_i of the alternative a_i with respect to A^* can be written as:

$$RC_i = \frac{\sqrt{\sum_{j=1}^{n} (w_j r_{ij})^2}}{\sqrt{\sum_{j=1}^{n} (w_j r_{ij})^2} + \sqrt{\sum_{j=1}^{n} (w_j (r_{ij} - 1))^2}} \tag{2.20}$$

where

$$(w_j)_\alpha^L \leq w_j \leq (w_j)_\alpha^U, j = 1, \ldots, n \tag{2.21}$$

$$(r_{ij})_\alpha^L \leq r_{ij} \leq (r_{ij})_\alpha^U, j = 1, \ldots, n, i = 1, \ldots, m \tag{2.22}$$

RC_i is an interval value based on Eq. (2.20), its upper and lower bounds can be calculated by utilizing the following simplified pair of fractional programming models (see [111] for further details):

$$\begin{aligned} (RC_i)_\alpha^U &= Max \frac{\sqrt{\sum_{j=1}^{n} (w_j (r_{ij})_\alpha^U)^2}}{\sqrt{\sum_{j=1}^{n} (w_j (r_{ij})_\alpha^U)^2} + \sqrt{\sum_{j=1}^{n} (w_j ((r_{ij})_\alpha^U - 1))^2}} \\ s.t. \quad & (w_j)_\alpha^L \leq w_j \leq (w_j)_\alpha^U, j = 1, \ldots, n \end{aligned} \tag{2.23}$$

$$\begin{aligned} (RC_i)_\alpha^L &= Min \frac{\sqrt{\sum_{j=1}^{n} (w_j (r_{ij})_\alpha^L)^2}}{\sqrt{\sum_{j=1}^{n} (w_j (r_{ij})_\alpha^L)^2} + \sqrt{\sum_{j=1}^{n} (w_j ((r_{ij})_\alpha^L - 1))^2}} \\ s.t. \quad & (w_j)_\alpha^L \leq w_j \leq (w_j)_\alpha^U, j = 1, \ldots, n \end{aligned} \tag{2.24}$$

When different alpha levels are set, then, $(RC_i)_\alpha = [(RC_i)^L_\alpha, (RC_i)^U_\alpha]$ can be obtained by solving Eqs. (2.23) and (2.24) respectively. According to Zadeh's extension principle ($\tilde{A} = \bigcup_\alpha \alpha A_\alpha, 0 \leq \alpha \leq 1$, $\tilde{A}$ is a fuzzy number) [141], $\tilde{RC_i}$ can be expressed as:

$$\begin{aligned} \tilde{RC_i} &= \bigcup_\alpha \alpha \cdot (RC_i)_\alpha \\ &= \bigcup_\alpha \alpha[(RC_i)^L_\alpha, (RC_i)^U_\alpha], 0 \leq \alpha \leq 1 \end{aligned} \tag{2.25}$$

where $\tilde{RC_i}$ represents the fuzzy RC of alternative a_i based on corresponding alpha levels from 0 to 1.

For n alternatives, there are usually n fuzzy relative closenesses, which are all expressed by their alpha-level sets. In order to select a best alternative, these fuzzy RC need to be defuzzified. The averaging level cuts [77] is used in this memory because of its the simplest defuzzification method based on alpha-level sets.

Let $\alpha_1, \ldots, \alpha_K$ be different alpha levels, the static rating, $m(\tilde{RC_i})$, of alternative a_i can be determined by [77]

$$m(\tilde{RC_i}) = \frac{1}{K} \sum_{k=1}^{K} (\frac{(RC_i)^L_{\alpha_k} + (RC_i)^U_{\alpha_k}}{2}), i = 1, \ldots, m \tag{2.26}$$

where K is the number of alpha levels.

Finally, the alternatives can be ranked according to the defuzzified value $m(\tilde{RC_i})$, of each alternative.

2.5.4 Hesitant fuzzy sets and hesitant fuzzy linguistic term sets

As mentioned previously, hesitation is quite common in mankind daily life, particularly, when people are in complex or time restriction environment, they are usually under pressure and hesitate about their choices or decisions. Such an issue must be handled to make the decision process close to the real world situations. On such a background, hesitant fuzzy sets (HFS) [98] and hesitant fuzzy linguistic term sets (HFLTS) [88], as the most two popular fuzzy extended theories, were proposed to deal with the hesitant information in quantitative and qualitative context, respectively. Due to the fact that DMs or experts might hesitate when they provide their assessments/opinions in real world EDM problems. Therefore, it seems necessary to deal with such practical and inevitable issue. Thus, in our research, HFS and HFLTS will be employed to handle expert' hesitation for quantitative and qualitative information respectively, which will be reviewed in short as follows.

2.5.4.1 Hesitant fuzzy sets

Hesitant fuzzy sets was introduced by Torra [98] that is the extension of fuzzy sets to model the hesitancy in quantitative contexts reviewed in depth [86, 90]. It is defined as below:

Definition 2 *[98] Let* $M = \{\mu_1, \ldots, \mu_n\}$ *be a set of* n *membership functions. The HFS associated to* M*,* h_M*, is defined as:*

$$h_M : X \to \wp([0,1]) \tag{2.27}$$

$$h_M(x) \to \bigcup_{\mu \in M} \{\mu(x)\} \tag{2.28}$$

where X is a reference set, $x \in X$.

This definition was extended and formalized with the concept of hesitant fuzzy element (HFE) by Xia and Xu [121]. In their proposal, the HFS was expressed by following mathematical representation, i.e.,

$$E = \{\langle x, h_E(x)\rangle : x \in X\} \tag{2.29}$$

where $h_E(x)$ is a set of values in [0,1], denoting the possible membership degrees of the element $x \in E$ to the set E. For convenience, they defined $h_E(x)$ as the HFE and $H = \cup h(x)$ as the HFSs, a HFE is a subset of HFSs (see [121] for further details).

Torra introduced in [98] the concept of the envelop of a HFE and proved that is an intuitionistic fuzzy value (IFV) according to the following definition:

Definition 3 *[98] Let* h *be a HFE, the IFV* $A_{env}(h)$ *is the envelop of* h*, in which* $A_{env}(h)$ *can be represented as* $(h^-, 1 - h^+)$ *being* $h^- = \min\{\sigma | \sigma \in h\}$ *and* $h^+ = \max\{\sigma | \sigma \in h\}$.

Different operations and properties has been defined for HFSs [98] such operations together the managing of intuitionistic fuzzy sets and intervals [90] allow us to interpret HFEs like an interval.

2.5.4.2 Hesitant fuzzy linguistic term sets

Different studies [87, 88, 89] show that when people hesitate under uncertain or pressure environment, they prefer to use natural language to express such hesitant information. To model the hesitant information in qualitative contexts, the concept of hesitant fuzzy linguistic term sets (HFLTS) [88] was introduced and it has drawn great attention recently [65, 104, 106, 115, 116]

The basic concepts and knowledge of HFLTS are introduced in short as follows in order that not-familiar readers understand it easily.

Definition 4 [88] Let $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set, a HFLTS, H_S, on S is an ordered finite subset:

$$H_S = \{s_i, s_{i+1}, \ldots, s_\varsigma\}, s_\varsigma \in S, \varsigma \in \{i, \ldots, j\} \tag{2.30}$$

Example 1 *Let S={absolute weak, very weak, weak, medium, good, very good, excellent} be a linguistic term set and δ be a linguistic variable, then, $H_S^1(\delta)$={good, very good} and $H_S^2(\delta)$={very weak, weak, medium} are two HFLTSs on S.*

HFLTS is a powerful and useful tool to model the experts' hesitation and the use of context-free grammars [88] allows to generate complex linguistic expressions close to the natural language utilized by human beings in real world [88, 89], which can be modelled by HFLTS. This approach has been widely applied to deal with different decision problems.

Definition 5 *[88] Let $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set and G_H be a context-free grammar. The elements of $G_H = (V_N, V_T, I, P)$ are defined as below:*

V_N = {⟨primary term⟩, ⟨composite term⟩, ⟨unary relation⟩, ⟨binary relation⟩, ⟨conjunction⟩}

V_T = {lower than, greater than, at least, at most, between, and, $s_0, s_1, \ldots, s_g$}

$I \in V_N$

P = {I:: = ⟨primary term⟩|⟨composite term⟩
⟨composite term⟩ :: = ⟨unary relation⟩ ⟨primary term⟩|⟨binary relation⟩
⟨primary term⟩ ⟨conjunction⟩ ⟨primary term⟩
⟨primary term⟩ :: = s_0 |s_1| ... |s_g
⟨unary relation⟩ :: = lower than |greater than |at least |at most
⟨binary relation⟩ :: = between
⟨conjunction⟩ :: = and}

S_{ll} denotes the comparative linguistic expressions generated by G_H, which might be either complex linguistic expressions or single linguistic terms.

Example 2 *Considering the context-free grammar, G_H, introduced in Definition 5 and the linguistic term set S from Example 1, the following complex linguistic expressions might be obtained:*

S_{ll_1} = *at least good*
S_{ll_2} = *at most medium*
S_{ll_3} = *between good and very good*

In order to deal with the comparative linguistic expressions, S_{ll}, provided by DMs or experts, S_{ll} are first transformed into H_S by utilizing the transformation function E_{G_H}.

Definition 6 *[89] Let E_{G_H} be a transformation function that transforms S_{ll} into H_S.*

$$E_{G_H} : S_{ll} \to H_S \tag{2.31}$$

Based on E_{G_H}, different expressions, S_{ll}, can be transformed into HFLTSs in different ways according to their meaning:

$E_{G_H}(s_i) = \{s_i | s_i \in S\}$,

E_{G_H}(at most s_i)=$\{s_j | s_j \leq s_i$ and $s_i \in S\}$,

E_{G_H}(at least s_i)=$\{s_j | s_j \geq s_i$ and $s_i \in S\}$,

E_{G_H}(between s_i and s_j)=$\{s_k | s_i \leq s_k \leq s_j$ and $s_k \in S\}$,

In this research, to carry out computations with complex linguistic expressions and avoid losing the initial information, fuzzy numbers are used as the transformed information from H_S.

Once the expressions are represented by H_S, their fuzzy envelop can be obtained [65].

Definition 7 *[65] Let $env_F(\cdot)$ be a fuzzy envelop function that transforms H_S into its fuzzy membership function.*

$$env_F(H_S) = T(a, b, c, d) \tag{2.32}$$

$T(a, b, c, d)$ being a trapezoidal fuzzy membership function (see [65] for further details).

In order to facilitate to not-familiar readers the understanding of the process to obtain the fuzzy envelop based on H_S easily, the process will be briefly reviewed as follows. According to [65], there are four steps (see Figure 2.9) to obtain the fuzzy envelop of the HFLTS:

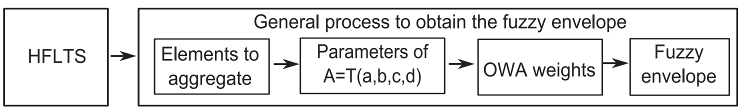

Figure 2.9: General process to obtain the fuzzy envelop

(1) *Obtain the elements to aggregate*: Assume that all linguistic terms $s_k \in S$ are defined by trapezoidal (triangular) membership functions $A^k = T(a_L^k, a_M^k, a_M^k, a_R^k)$, $k = 0, 1, \ldots, g$, therefore, the set of points of all membership functions of the linguistic terms in the HFLTS $H_s = \{s_i, s_{i+1}, \ldots, s_j\}$,

$T = \{a_L^i, a_M^i, a_L^{i+1}, a_R^i, a_M^{i+1}, a_L^{i+2}, a_R^{i+1}, \ldots, a_L^j, a_R^{j-1}, a_M^j, a_R^j\}$

can be regarded as the set of elements to aggregate.

According to the fuzzy partitions [92], it obtains $a_R^{k-1} = a_M^k = a_L^{k+1}$, $k = 1, 2, \ldots, g-1$, based on this, the elements to aggregate can be expressed as:

$$T = \{a_L^i, a_M^i, a_M^{i+1}, \ldots, a_M^j, a_R^j\}.$$

(2) *Compute the parameters of the trapezoidal fuzzy membership function*: Due to $A = T(a, b, c, d)$ is a trapezoidal fuzzy membership and $T = \{a_L^i, a_M^i, a_M^{i+1}, \ldots, a_M^j, a_R^j\}$ is an ordered set, therefore, a and d can be computed by using min and max operator respectively, i.e.,

$$a = min\{a_L^i, a_M^i, a_M^{i+1}, \ldots, a_M^j, a_R^j\} = a_L^i,$$
$$d = min\{a_L^i, a_M^i, a_M^{i+1}, \ldots, a_M^j, a_R^j\} = a_R^j.$$

The parameters b and c can be computed by using OWA operator, i.e.,

$$b = OWA_{w^s}(a_M^i, a_M^{i+1}, \ldots, a_M^j),$$
$$c = OWA_{w^t}(a_M^i, a_M^{i+1}, \ldots, a_M^j).$$

where $s, t = 1, 2$, $s \neq t$ or $s = t$.

(3) *Obtain the OWA weights*: There are different ways to determine the OWA weights, the following approach defined in Definition 8 is employed (see [37, 65] for further details)

Definition 8 *[37] Let α be a parameter belonging to the unit interval [0,1], the first kind of OWA weights $W^1 = (w_1^1, w_2^1, \ldots, w_n^1)^T$ is defined as:*

$w_1^1 = \alpha$, $w_2^1 = \alpha(1-\alpha)$, $w_3^1 = \alpha(1-\alpha)^2, \ldots,$ $w_{n-1}^1 = \alpha(1-\alpha)^{n-2}$, $w_n^1 = (1-\alpha)^{n-1}$.

The second type of OWA weights $W^2 = (w_1^2, w_2^2, \ldots, w_n^2)^T$ is defined as:

$w_1^2 = \alpha^{n-1}$, $w_2^2 = (1-\alpha)\alpha^{n-2}$, $w_3^2 = (1-\alpha)\alpha^{n-3}, \ldots,$ $w_{n-1}^2 = (1-\alpha)\alpha$, $w_n^2 = (1-\alpha)$.

(4) *Obtain the fuzzy envelop*: According to previous steps, for a HFLTS H_s, its fuzzy envelop $env_F(H_s)$ can be obtained as the trapezoidal fuzzy membership function $T(a, b, c, d)$, (see Figure 2.10), i.e.,

$$env_F(H_S) = T(a, b, c, d)$$

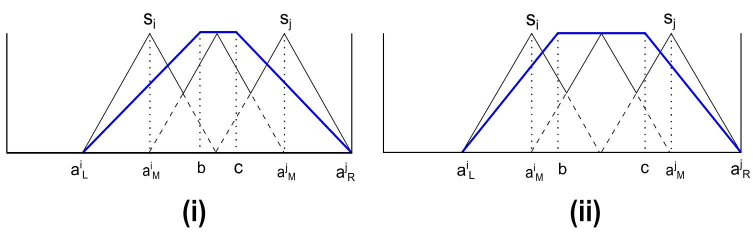

Figure 2.10: The fuzzy envelop of H_s

Chapter 3

Research Results

This chapter provides a summary of the main proposals developed in this research memory. Research findings and results will be discussed for each proposal in short. There are four proposals which are related with the different objectives presented in the Introduction chapter:

1. A group decision method based on prospect theory for emergency situations.
2. A dynamic multi-attribute group emergency decision making method considering experts' hesitation.
3. Managing non-homogeneous information and experts' psychological behavior in group emergency decision making.
4. A hesitant group emergency decision making method based on prospect theory.

3.1 Prospect theory for emergency situations under group decision context

In order to achieve the first objective pointed out in Section 1.2, we highlight the importance of experts' psychological behavior in EDM process, and then analyse the limitations in current GEDM approaches neglecting its consideration. Afterwards, a group decision method based on prospect theory is proposed to improve current GEDM approaches and tested on a real-world problem.

3.1.1 Experts' psychological behavior in EDM

As mentioned previously, due to humans' psychological behavior affects their decision making performance, therefore, it plays a very important role in dealing successfully with decision problems under uncertainty [55, 103]. Consequently, several EDM studies [33, 34, 107] have considered DM's psychological behavior achieving successful results [105, 109, 136].

Nevertheless, the increasing complexity of EDM problems makes that single individuals cannot cope with such processes in many real world EEs [70]. Therefore, a group wisdom view makes easier and more effective the decision processes on real-world complex EEs. However, experts' psychological behavior in these situations has been neglected in extant GEDM problems [39, 60, 129] so far, and it seems necessary to consider it in a similar way to EDM approaches.

To address such a limitation in GEDM problems, we have introduced a new proposal that aims to develop a GEDM method based on prospect theory that considers experts' psychological behavior in the group decision process.

3.1.2 Group emergency decision making method based on prospect theory

This new GEDM method includes several novelties in addition to the use of PT, all of them are enumerated and briefly explained below (further detail can be seen in Chapter 4):

1. This proposal considers a suitable and reasonable group decision scheme for dealing with the complex EEs that hardly would be successfully managed by a single DM. Such a kind of decision scheme collects and fuses each experts' wisdom into a group one. The decision results will be more reliable and reasonable by using a group decision scheme, especially high complex EE.

2. To manage uncertain, vague and imperfect information that is inherent to expert's judgements in these complex GEDM problems, interval-valued modelling is used.

3. Experts' psychological behavior is considered in the GEDM process by using prospect theory. The use of PT in GEDM implies the necessity of defining the Group Reference Point (GRP) concept that reflects the RP of the group. On top of this concept, the GRP model is defined and calculation of gains and losses are determined.

In addition, for carrying out fair comparisons with current studies, we have described an experimental process on a real case study about a barrier lake emergency problem. Eventually, several comparisons are carried out to highlight the validity, feasibility and advantages of this proposal, which not only outstands its importance, but also enriches the current GEDM approaches.

The article associated to this proposal is the following one:

L. Wang, Y. M. Wang, L. Martínez. A group decision method based on prospect theory for emergency situations. Information Sciences, 2017, 418, 119-135.

3.2 A dynamic multi-attribute group emergency decision making method considering experts' hesitation

The dynamic evolution of EEs is fact that has not been comprehensively addressed so far. Therefore, it has been necessary to analyse which are the main elements that affect in a relevant way to the successful management of EEs. From this analysis, different elements can evolve dynamically and change across time in the decision process of the EE, there are two distinct features among them, i.e., dynamic and uncertainty. On such basis, it has been introduced and developed a novel dynamic multi-attribute group emergency decision making (MAGEDM) method under uncertainty that considers all those features for a better managing of the EDM.

3.2.1 Analysis on the features of EEs and related limitations in current studies

Different features have diverse impacts on the EEs, key features are usually playing a determinant factor in dealing successfully with the EEs. Therefore, it is necessary to analyse comprehensively these key features of EEs in order to make the emergency response more effectively and pertinently. The main results of the analysis and some related outcomes obtained are briefly enumerated:

1. There are several characteristics of EEs, such as destructiveness, abruptness, complexity, diffuseness, dynamic evolution, inadequacy, uncertainty and so on, which from a comprehensive analysis on real world EE problems results in dynamic evolution and uncertainty that are the most crucial ones in the process of emergency response and they should be considered with a higher priority.

2. Regarding the dynamic evolution, current EDM studies have discussed about it from the perspective of the time variable, however, the EE time evolution may imply other changes in the features of the EDM problem such as alternatives, criteria and so forth. Nevertheless, current dynamic EDM proposals consider that such features remain unchanged across time. It is an obvious defect in current studies that should be overcome with a novel proposal that can deal with the changes of all these features.

3. Finally, the uncertainty modelling has been discussed in EDM literature by using interval values for quantitative contexts, and fuzzy linguistic term sets for qualitative contexts. However, other types of uncertainty can appear in EDM because of lack of information and time pressure, sometimes it is hard for experts to elicit their knowledge with single assessments because they may hesitate among multiple values for assessing alternatives and criteria. Consequently, the modelling of experts' hesitancy should be provided by EDM models when such a type of uncertainty appears, though it is not any model so far that can manage it.

3.2.2 Dynamic MAGEDM method considering experts' hesitation

To overcome previous barriers, we have proposed a new dynamic MAGEDM method that deals with the dynamic evolution of EEs considering both the time changeableness and elements of the decision framework (alternatives, criteria, and experts). At the same time, our proposal deals with different types of uncertainty that are modelled by using interval values, linguistic term sets, and linguistic expressions based on HFLTS, so imprecision, vagueness, and hesitancy can be managed with this MAGEDM method that has the following novelties:

1. Dynamic evolution is considered in this proposal from a new perspective, this proposal enlarges the concept of dynamic scope. Additionally, the modelling of uncertainty based on experts' hesitancy is first proposed for EDM in this proposal. The new perspectives of dynamic evolution and the inclusion of new uncertainties make closer the EDM method to the real world situation, easy to be accepted and understood.

2. Transformation functions for unifying different uncertain information are defined, which provide the convenience for managing different types of uncertainties. Hence, a simple, correct and flexible way for aggregating experts' opinions is provided, which keeps as much information as possible during the decision process.

3. A novel method for computing criteria weights by using experts' assessments on the importance of each criteria is presented, which facilitates this process for the problems with high uncertainty.

4. A fuzzy TOPSIS method based on alpha-level sets is employed due to its advantages of keeping uncertain information in the decision process.

5. A new selection rule for choosing the best alternative is defined, the new selection rule not only considers each alternative's performance at current decision

moment, but also considers its performance in previous decision moments, obtaining a comprehensive result of each alternative, which provides a different view from current dynamic EDM studies.

To highlight the performance, feasibility and validity of our proposal, we have conducted several comparisons with different previous methods that are carried out from different perspectives.

The article associated to this proposal is the following one:

L. Wang, R. M. Rodríguez, Y. M. Wang. A dynamic multi-attribute group emergency decision making method considering experts' hesitation. International Journal of Computational Intelligence Systems, 2018, 11(1): 163-182.

3.3 Managing non-homogeneous information and experts' psychological behavior in group emergency decision making

During the research developed for previous proposals regarding GEDM, it was detected that there are still several issues that have not been successfully addressed yet, such as the following ones:

(1) In real world emergency problems, there are different types of information regarding the EEs, however, none previous proposals considers different types information at the same time, therefore, it seems necessary and convenient to propose a new decision model to deal with such an issue.

(2) Experts' psychological behavior has been proven that is crucial in the GEDM process, however, there is no proposal considering such an important issue in fuzzy emergency decision environment so far, therefore, it will be included and managed in this proposal.

(3) The CRPs used in GEDM approaches deal just with numerical values, and are not suitable for fuzzy information, additionally, they have a high time cost because of the supervised feedback mechanism that should be softening in GEDM problems due to time restrictions.

(4) The criteria weights determined in existing EDM approaches [33, 68, 107, 109] are provided by DMs, however, it is a big challenge for DM to provide reasonable and scientific criteria weights, especially, when he/she is under pressure in time restriction decision environment, therefore, a novel way for determining criteria weights seems necessary to be explored.

In order to address previous issues, a new GEDM proposal will be developed with the aim of filling these gaps in existing GEDM studies. Therefore, the third objective mentioned in Section 1.2 can be reached. Such a new method is composed by the different phases depicted in Figure 3.1 and briefly explained below.

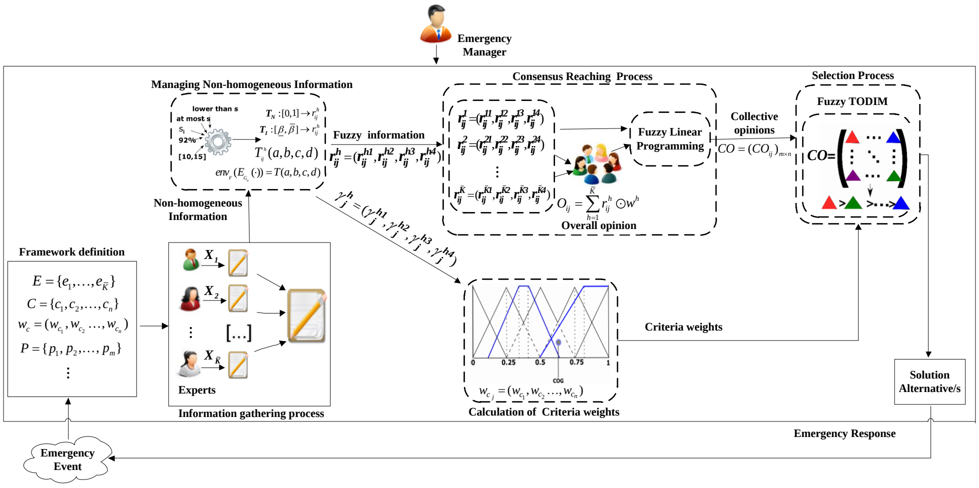

Figure 3.1: Scheme of the processes of our proposal for GEDM

3.3.1 Managing non-homogeneous information

To enrich and enlarge the types of information that can be used in GEDM studies, four types of information (numerical values (N), interval values (I), linguistic terms (S), comparative linguistic expressions (S_{ll})) are considered in this proposal. To operate with such a non-homogeneous information, an unification process will be applied and corresponding transformation functions used as defined if needed.

In this proposal, different types of information are unified by trapezoidal fuzzy numbers, in order to keep the uncertain information as much as possible during the decision process. The transformation functions for different types of information are shown in Figure 3.2.

Several transformation functions have been already introduced in previous research [106], but they are adopted for our needs in this proposal as follows:

1. For numerical values N, they are first normalized into the interval [0,1] and then a transformation function T_N is utilized to transform them into trapezoidal fuzzy numbers.

   Let R be the domain of the numerical values, N_{ij}^h be the numerical value provided by the *h-th* expert over the *i-th* alternative concerning the *j-th* criterion, N_{ij}^h is normalized into the interval [0,1], as follows:

$$\vartheta = \frac{N_{ij}^h}{N^*}$$

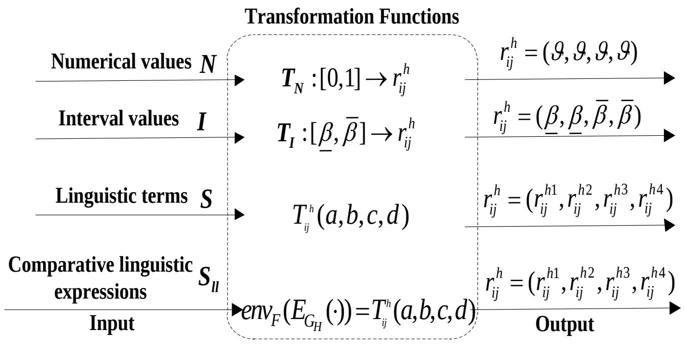

Figure 3.2: Transformation functions for different types of information

where $\vartheta \in [0, 1]$, $N^* = \max_{h=1,2,\ldots,\bar{K}} \{N_{ij}^h\}$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$.

Definition 9 *[105] A numerical value is transformed into a trapezoidal fuzzy number by utilizing a transformation function T_N:*

$$T_N : [0, 1] \rightarrow r_{ij}^h$$

$$T_N(\vartheta) = r_{ij}^h = (\vartheta, \vartheta, \vartheta, \vartheta)$$

2. The interval values I are first normalized into [0,1] and then a transformation function T_I is utilized to transform them into trapezoidal fuzzy numbers.

   Let $[\xi^L, \xi^U]$ be the domain of the interval values, let $[d^L, d^U]_{ij}^h$ be the interval values provided by the *h-th* expert over the *i-th* alternative concerning the *j-th* criterion, where $[d^L, d^U]_{ij}^h \in [\xi^L, \xi^U]$. The interval values $[d^L, d^U]_{ij}^h$ are normalized into $[\underline{\beta}, \bar{\beta}]$ as follows:

$$\underline{\beta} = \tfrac{d^L - \xi^L}{\xi^U - \xi^L} \quad and \quad \bar{\beta} = \tfrac{d^U - \xi^L}{\xi^U - \xi^L}$$

   The transformation function T_I is defined as follows:

Definition 10 *[105] An interval value is transformed into a trapezoidal fuzzy number by utilizing a transformation function T_I :*

$$T_I : [\underline{\beta}, \bar{\beta}] \rightarrow r_{ij}^h$$

$$T_I(\underline{\beta}, \bar{\beta}) = r_{ij}^h = (\underline{\beta}, \underline{\beta}, \bar{\beta}, \bar{\beta})$$

where $\underline{\beta}, \bar{\beta} \in [0, 1]$ and $\underline{\beta} \leq \bar{\beta}$.

3. The linguistic terms $s_k \in S = \{s_0, s_1, \ldots, s_g\}$ are represented by trapezoidal fuzzy numbers that we assume that are normalized in [0,1]. Therefore, the expert e_h provides his/her opinions over the *i-th* alternative concerning the *j-th* criterion as a linguistic term s_k that is represented by a trapezoidal fuzzy number $r_{ij}^h = (r_{ij}^{h1}, r_{ij}^{h2}, r_{ij}^{h3}, r_{ij}^{h4})$.

4. The comparative linguistic expressions, $x_{ij}^h \in S_{ll}$ are transformed into HFLTS by $E_{G_H}(\cdot)$ and its fuzzy envelop $env_F(\cdot)$ obtained by [65],

$$env_F(E_{G_H}(x_{ij}^h)) = T_{ij}^h(a, b, c, d) = r_{ij}^h$$

   E_{G_H} is a function that transforms the linguistic expressions obtained by using G_H, into HFLTS [89]. $T_{ij}^h(a, b, c, d)$ is a trapezoidal fuzzy membership function (see Section 2.5.4.2) corresponding to the trapezoidal fuzzy number $r_{ij}^h = (r_{ij}^{h1}, r_{ij}^{h2}, r_{ij}^{h3}, r_{ij}^{h4})$.

3.3.2 Fuzzy linear programming-based consensus model

It has been already pointed out the lack of GEDM approaches applying low-cost consensus reaching processes and able to deal with fuzzy information. Therefore, to achieve agreed solutions in our GEDM method, we propose a CRP before the application of the TODIM able to deal with fuzzy information and that guarantees minimum cost to achieve the agreement. Hence, we will use a fuzzy linear programming based consensus model presented in [66].

The fuzzy linear programming-based consensus model introduced in [66] was defined as:

$$\begin{cases} \min \sum\limits_{h=1}^{\bar{K}} (w^h)^{\chi}(c - S_p(\tilde{A}_h, \tilde{O})) \\ s.t.\;\; d_p(\tilde{A}_h, \tilde{O}) \leqslant \varepsilon_h,\;\; h = 1, 2, ..., \bar{K} \end{cases} \tag{3.1}$$

where χ is an integer ≥ 1, w^h denotes the *h-th* experts' importance. ε_h denotes a threshold that means the maximum change that the *h-th* expert can make, where $\tilde{A}_h = (a_{h1}, a_{h2}, a_{h3}, a_{h4})$ be the *h-th* experts' individual opinion and $\tilde{O}$ be the overall opinion obtained by aggregating experts' individual opinions. $d_p(\tilde{A}_h, \tilde{O})$ denotes the distance between $\tilde{A}_h$ and $\tilde{O}$, $S_p(\tilde{A}_h, \tilde{O})$ is the similarity between $\tilde{A}_h$ and $\tilde{O}$, c is a constant > 1 (see [66, 105] for further information). The scheme of the CRP model in this proposal is shown in Figure 3.3.

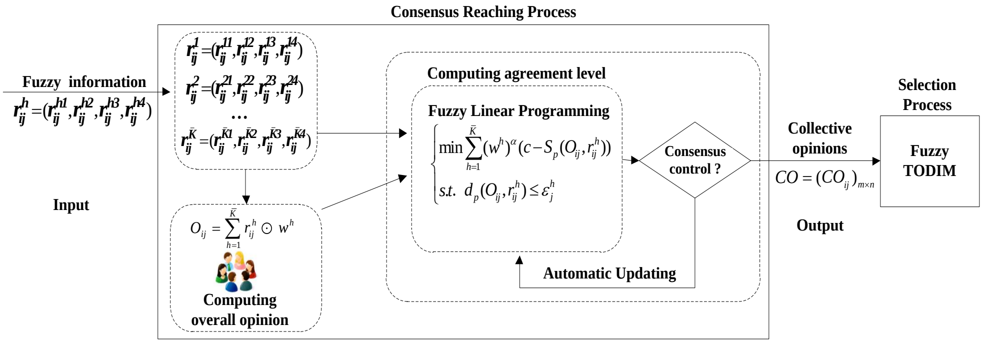

Figure 3.3: CRP model of fuzzy linear programming-based employed in our proposal

3.3.3 Experts' psychological behavior in fuzzy emergency decision environment

We have already seen that experts' psychological behavior could be critical for EDM and prospect theory has been widely used for dealing with experts' psychological behavior. In Section 3.1 we have already developed a GEDM method with prospect theory and interval values. However, such a model cannot deal with the trapezoidal fuzzy numbers that is the input information of this proposal.

Nevertheless, there exists a MCDM method based on PT able to deal with fuzzy information namely, fuzzy TODIM (revised in Section 2.5.2.2) that will be the method used in our proposal to deal with the fuzzy unified information and take into account experts' psychological behavior.

3.3.4 Determination of criteria weights

Determination of criteria weights is a complex task in MCDM and consequently in EDM too. In spite of the existence of multiple proposals for computing criteria weights they are not fully satisfactory. So, we have proposed a new three-step process to determine criteria weights based on fuzzy experts' opinions [105].

Figure 3.4 depicts this process:

(1) *Global fuzzy weights.* The fuzzy weights obtained for the criterion c_j are aggregated by using a max-min composition [132, 133]:

$$\mu_{\tilde{T}_j^h}(\sigma) = \sup_{\sigma=\max\ (t_1,t_2,\ldots,t_{\bar{K}})} \min\ (\mu_{\tilde{T}_j^1}(t_1), \mu_{\tilde{T}_j^2}(t_2), \ldots, \mu_{\tilde{T}_j^{\bar{K}}}(t_{\bar{K}})) \tag{3.2}$$

where $\tilde{T}_j^h$ is the fuzzy membership function of w_j^h, $j = 1, 2, \ldots, n$, and Γ is the universe of discourse.

Suppose that three experts provide their opinions w_1^1, w_1^2 and w_1^3 concerning the criterion c_1, the corresponding fuzzy membership functions are $\tilde{T}_1^1$, $\tilde{T}_1^2$, and $\tilde{T}_1^3$ respectively. According to Eq. (3.2), $\mu_{\tilde{T}_j^h}(\sigma)$ is the area under the bold black line shown in Figure (3.4a).

(2) *Defuzzification.* The center of gravity (COG) method [20] is utilized to calculate the weighting value of the global fuzzy weights:

$$COG_j = \frac{\int t * \mu_{\tilde{T}_j^h}(t)dt}{\int \mu_{\tilde{T}_j^h}(t)dt}, t \in \Gamma \tag{3.3}$$

where Γ is the universe of discourse.

For criterion c_1, Eq. (3.3) means that the center of gravity for each small trapezoid (see Figure 3.4b) is computed and the COG_1 can be obtained by the arithmetic mean of the sum of center of gravity of all small trapezoids.

(3) *Normalization.* When COG_j of all criteria are obtained, the criteria weights w_{c_j} are calculated by using the following equation:

$$w_{c_j} = \frac{COG_j}{\sum_{j=1}^{n} COG_j} \tag{3.4}$$

where $\sum_{j=1}^{n} w_{c_j} = 1$, $w_{c_j} \in [0,1]$, $j = 1, 2, \ldots, n$.

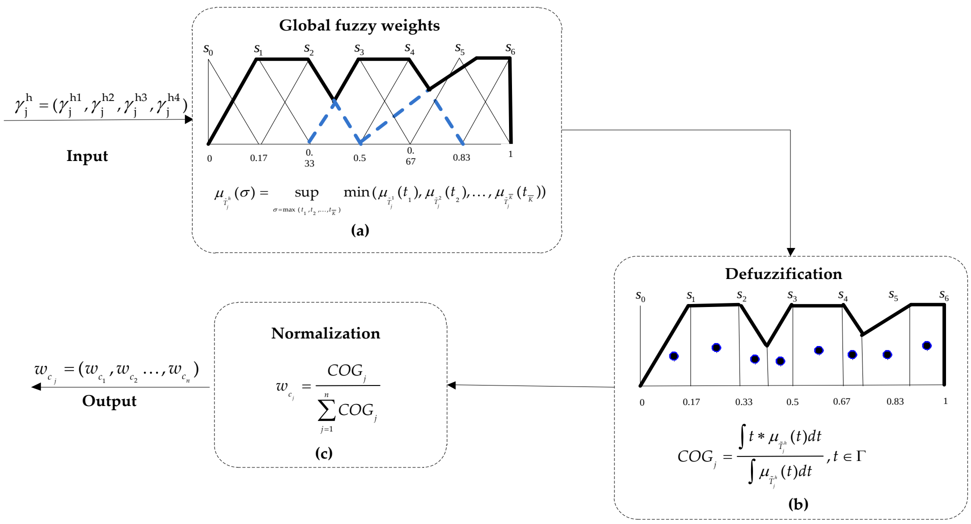

Figure 3.4: Process of determination criteria weights in our proposal

Finally, once the GEDM method was developed, a case study was provided to illustrate the feasibility and validity of this proposal, in addition, sensitivity analysis is carried out to show the robustness of this proposal.

The article associated to this proposal is the following one:

L. Wang, Á. Labella, R. M. Rodríguez, Y. M. Wang, L. Martínez. Managing non-homogeneous information and experts' psychological behavior in group emergency decision making. Symmetry-Basel, 2017, 9(10), 234.

3.4 A hesitant group emergency decision making method based on prospect theory

The complexity and constraints under which experts should manage EDM, makes that experts may hesitate when providing their knowledge about the alternatives and criteria of the problem. For experts' hesitation, it has been previously considered in this memory in qualitative contexts by using HFLTS. Whilst, hesitation in quantitative contexts together with experts' psychological behavior have not been studied in current GEDM studies yet. It seems convenient the development of a novel GEDM method able to deal with experts' psychological behavior and hesitation in quantitative contexts too. Therefore, this proposal provides a new GEDM method to facilitate the modelling of hesitancy in quantitative contexts by means of hesitant fuzzy sets (HFSs) together with experts' psychological behavior by using prospect theory. The fourth objective presented in Section 1.2 can be then reached. For sake of clarity, just remind that related information about prospect theory and HFS used in this proposal have been revised in Section 2.5.1 and Section 2.5.4.1, respectively.

3.4.1 Hesitant group emergency decision making method considering experts' psychological behavior

The aim of this proposal is to keep as much information as possible in the early stages of the decision process by employing HFS that will represent the group hesitation by HFE built by fusing experts' assessments. Such HFSs will be integrated as inputs of the GEDM process that also considers the experts' psychological behavior using prospect theory. A graphical scheme of this proposal is shown in Figure 3.5 and the novel fusion process is further detailed below.

The fusion processes are given as follows:

Step 1: Expert e_h provides his/her assessments, $c_j^h(a_i)$, on emergency alternative a_i concerning criterion c_j.

Step 2: Based on $c_j^h(a_i)$ provided by experts, the experts' preference, $\bar{c}_j^h(a_i)$, on the effective control scope of alternative a_i concerning criterion c_j is determined as

$$\bar{c}_j^h(a_i) = \frac{c_j^h(a_i)}{\max\left\{\max\limits_h c_j^h(a_i)\right\}}, i = 1, 2, \ldots, m; j = 1, 2, \ldots n \tag{3.5}$$

Step 3: Based on $\bar{c}_j^h(a_i)$, the HFEs $\bar{c}_j(a_i)$ for the *j-th* criterion with respect to the *i-th* alternative and the HFS $h_M(a_i)$ can be formed and managed according to their envelops as interval values.

Step 4: Based on Step 3, the lower bound $E_{ij}^L(a_i)$ and upper bound $E_{ij}^U(a_i)$ of the effective control scope E_{ij} can be determined by following two equations:

$$E_{ij}^L = \min_h \left\{\bar{c}_j^1(a_i), \ldots, \bar{c}_j^h(a_i), \ldots, \bar{c}_j^H(a_i)\right\} \tag{3.6}$$

$$E_{ij}^U = \max_h \left\{\bar{c}_j^1(a_i), \ldots, \bar{c}_j^h(a_i), \ldots, \bar{c}_j^H(a_i)\right\} \tag{3.7}$$

The interval value $E_{ij} = [E_{ij}^L, E_{ij}^U]$ is the result of fusion information that means group hesitation in which all experts' assessments are included. Afterwards, the GEDM method proposed in Section 3.1 will be then employed in this proposal to consider experts' psychological behavior based on prospect theory with interval values.

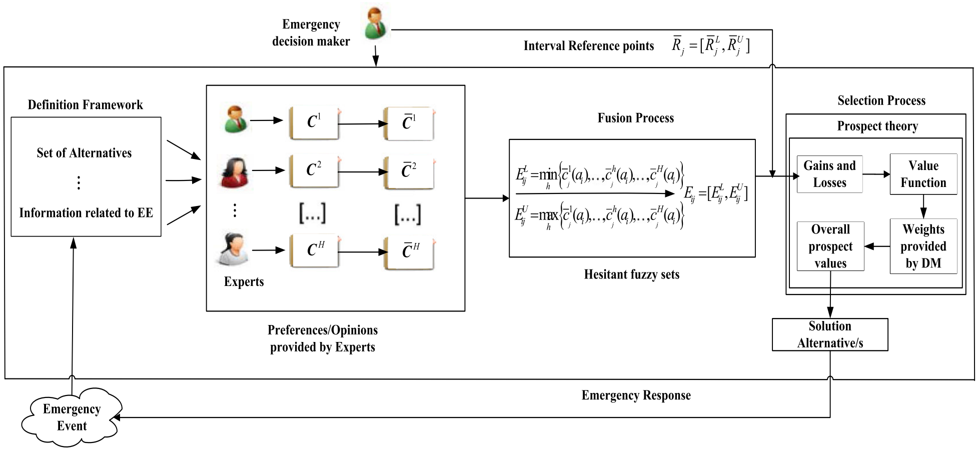

Figure 3.5: Scheme of the processes of GEDM method coping quantitative hesitancy and experts' psychological behavior

A case study and relevant comparisons with previous GEDM studies have been conducted to illustrate the superiority, feasibility and validity of our proposal, consequently, the fourth objective of our researches is reached.

The article associated to this proposal is the following one:

Z. X. Zhang, L. Wang, R. M. Rodríguez, Y. M. Wang, L. Martínez. A hesitant group emergency decision making method based on prospect theory. Complex & Intelligent Systems, 2017, 3, 177-187.

Chapter 4

Publications

4.1 A group decision method based on prospect theory for emergency situations

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A group decision method based on prospect theory for emergency situations

Liang Wang[a,b], Ying-Ming Wang[a,*], Luis Martínez[b]

[a] Decision Sciences Institute, Fuzhou University, Fuzhou 350116, PR China
[b] Department of Computer Sciences, University of Jaén, Jaén 23071, Spain

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ABSTRACT

Urgent and critical situations or so-called emergency events, such as terrorist attacks and natural disasters, often require crucial decisions. When an emergency event occurs, emergency decision making plays an important role in dealing with it, and hence, its importance nowadays is increasing. In the real world, it is difficult for only one decision maker to take a comprehensive decision for coping with an emergency event. Consequently, many practical emergency problems are often characterized by a group emergency decision making (GEDM) scheme. Different studies show that human beings are usually bounded rational under risk and uncertainty, and their psychological behavior is very important in the decision-making process. However, such behavior is neglected in current GEDM studies. Therefore, this study proposes a novel GEDM method that considers experts' psychological behavior in the GEDM process. The method is then applied to a case study and compared with other related approaches. Finally, discussions are presented to illustrate the novelty, feasibility, and validity of the proposed GEDM method, showing the importance of experts' psychological behavior in GEDM.

1. Introduction

In recent years, various emergency events (EEs), such as earthquakes, the missing Malaysian Air flight "MH370," hurricanes, and terrorist attacks, have exerted severely negative impacts on human life and socio-economic development. When a devastating incident occurs, emergency decision making (EDM) is typically characterized by at least time pressure and lack of information, resulting in potentially serious consequences [12,21]. Since EDM plays a crucial role in mitigating the losses of properties and lives caused by EEs, it has received increasing attention from both government and academia, becoming a very active and important research field [12,17,21].

Most EDM models [23,40] assume only one emergency decision maker (DM) and several studies [15,41] have taken the DM's psychological behavior into account, because of its influence on the final decision [40,41]. However, real-world EDM usually requires multiple perspectives of different experts, and just one DM might not be enough to deal effectively with the decision problem. This is particularly true when the decision environment becomes more complex and uncertain [24]. Thus, EDM approaches have evolved to a scheme that considers multiple experts with diverse professional backgrounds (e.g., hydrological, geological, meteorological, sociological, and demographic) who act as a think tank supporting the DM in the decision process, which leads to a group emergency decision-making (GEDM) problem.

* Corresponding author.
E-mail addresses: wangliangg322@hotmail.com (L. Wang), msymwang@hotmail.com, ymwang@fzu.edu.cn (Y.-M. Wang), martin@ujaen.es (L. Martínez).

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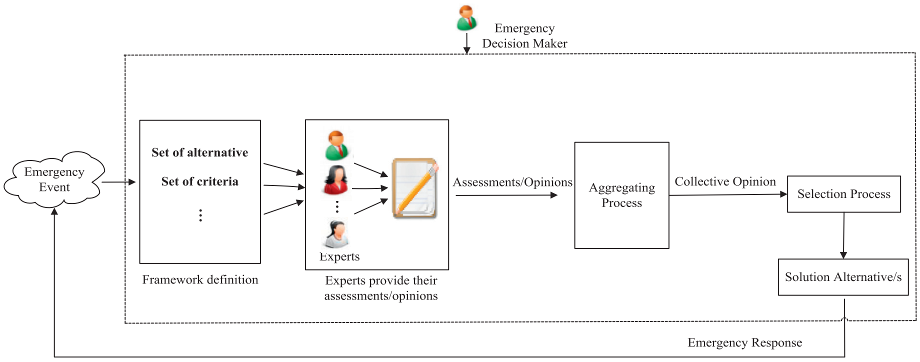


Fig. 1. General scheme of a GEDM process.

Unlike other group decision-making (GDM) problems, such as supplier selection [11,29], supply-chain risk management [5,7], and large construction projects [20], GEDM problems are always defined under strong time constraints [12], which implies high risk and uncertainty [24], and their outcomes might result in extremely serious social consequences in the form of loss of lives and property [21]. Therefore, a GEDM process whose decisions are made among all experts should avoid conflicts and useless solutions [24] in order to obtain higher-quality decisions with timely response [9,33]. Usually, in a two-step decision solving process for GEDM, experts play the role of think tank in supporting the DM who is in charge of the EE, and the process consists of [27] (see Fig. 1): (i) an aggregation process, in which individual information provided by experts is aggregated, and (ii) a selection process, in which alternatives are obtained as solutions to the problems.

Even though existing studies on GEDM [17,21,42,43] have made significant contributions to emergency management, there is one key issue that has not been well addressed so far, namely, *experts' psychological behavior*. Current GEDM approaches [17,21,42,43] neglect this topic. Because experts are always bounded rational under risk and uncertainty, their psychological behavior plays an important role in the GEDM process and must be considered.

To address such a key issue, this study aims to develop a new GEDM method based on prospect theory (PT) to include experts' psychological behavior in the group decision process. Thereafter, this study aims to compare this new method with other GEDM and EDM approaches to show its validity and usefulness.

The remainder of this paper is organized as follows. Section 2 briefly introduces different theories and methods used in our proposed method together with related works. Section 3 presents the proposed GEDM method based on PT. In Section 4, a case study is provided and a comparison with other approaches and related discussions are presented. The conclusions and future works are presented in Section 5.

2. Background

This section provides a brief introduction about concepts related to PT and GDM that are used in our proposed method, and reviews some related works to illustrate the importance of this research.

2.1. Prospect theory

Experts' psychological behavior in GEDM is dealt with in this study using PT. It was introduced in 1979 [19] and extended in 1992 [39] by Daniel Kahneman and Amos Tversky as behavior economic theory, which describes the way in which people choose between probabilistic alternatives that involve risk when the probabilities of outcomes are known. According to the theory, people make decisions based on the potential value of losses and gains rather than the final outcome. PT is regarded as the most influential theory; it has been studied [34,38,39] and widely applied to solve various decision-making problems, such as asset allocation [6], health domains [3,4], portfolio insurance [13], traffic management [22,47], multi-attribute decision making [16], and emergency decision making [23,41], considering humans' psychological behavior under uncertainty.

The first key concept in PT is the *reference point* (RP), which is defined as a neutral position asset or expectation value of people who want to gain an amount or not to lose it. RP determines the feeling of gains or losses based on the difference between expectation and outcomes; the value of the RP is affected by the expectations of people [19].

In multi-attribute decision-making problems, the attributes can be classified into two types: benefits and costs [25]. The higher a benefit attribute is, the better the situation is, while the higher a cost attribute is, the worse the situation is.

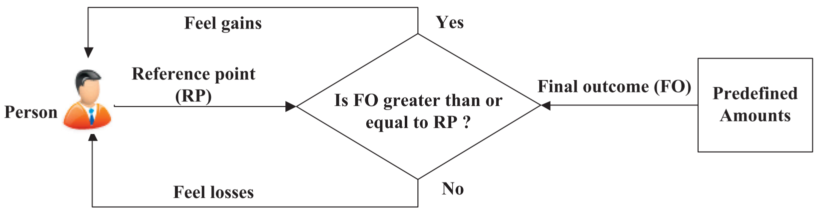


Fig. 2. Gains and losses based on reference point and predefined amounts.

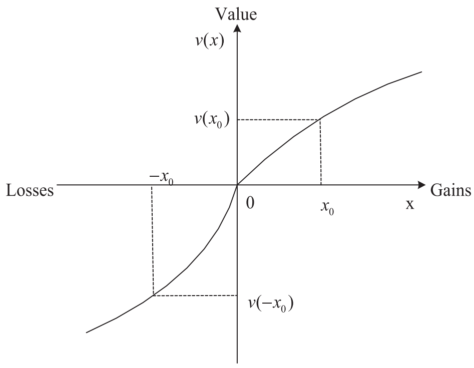


Fig. 3. S-shaped value function of prospect theory.

According to different types of attributes, RP changes with people's expectations with respect to the predefined amounts to gain or lose. For a better understanding of RP in PT, see Fig. 2 (benefit attributes versus cost attributes).

According to Fig. 2, RP is analogously defined for cost attributes.

In Kahneman and Tversky [19] and Tversky and Kahneman [39], it was shown that people's psychological behavior exhibits a risk-averse tendency for gains and a risk-seeking tendency for losses. Therefore, PT describes the decision process in the following three stages.

(i) In the *editing phase*, people decide which outcomes they consider are equivalent, set an RP, consider lesser outcomes as losses and greater outcomes as gains for the benefit attribute, and consider lesser outcomes as gains and greater outcomes as losses for the cost attribute.
(ii) In the *evaluation phase*, people behave as if they would compute a prospect value by using *value function* based on the potential outcomes and then calculate the overall prospect values.
(iii) In the *selection phase*, the alternative of having a higher overall prospect value is finally selected.

The PT involves the following three important principles [19].

- *Reference dependence.* Experts perceive gains and losses according to an RP. Thus, the prospect value function can be divided into a gain domain and a loss domain regarding the RP.
- *Diminishing sensitivity.* Experts exhibit a risk-averse tendency for gains and a risk-seeking tendency for losses. According to the principle of diminishing sensitivity, the prospect value function is concave in the loss domain and convex in the gain domain, that is, the marginal value of both gains and losses is decreasing with size.
- *Loss aversion.* The experts are more sensitive to losses than to equal gains [1]. In accordance with the principle of loss aversion, the prospect value function is steeper in the loss domain than in the gain domain.

According to these three principles, an S-shaped value function is proposed in PT (see Fig. 3), which shows a prospect value function with a convex S-shape for losses and a concave S-shape for gains. Prospect values are calculated for measuring the magnitude of gains and losses by using a value function in PT, which is defined on deviations from the RP, and is expressed in the form of a power law according to the following expression [39].

$$v(x) = \begin{cases} x^{\alpha}, & x \geq 0 \\ -\lambda(-x)^{\beta}, & x < 0 \end{cases} \tag{1}$$

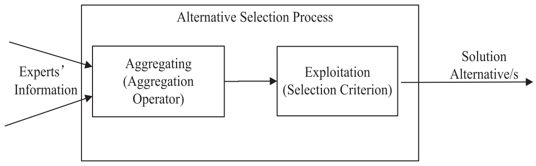


Fig. 4. Classical process for the resolution of group decision-making problem.

where α and β are power parameters related to gains and losses, respectively, $0 \le \alpha,\ \beta \le 1$, where x denotes the gains or losses with $x \ge 0$ or $x < 0$, respectively, and λ is the risk-aversion parameter, which has the characteristic of being steeper for losses than for gains, $\lambda > 1$. For parameters α, β, and λ in Eq. (1), previous studies [1,8,23,39] have determined their values. For example, Abdellaoui et al. [1] suggest $\alpha = 0.725$, $\beta = 0.717$, and $\lambda = 2.04$; Liu et al. [23] suggest $\alpha = 0.85$, $\beta = 0.85$, and $\lambda = 4.1$; and Tversky and Kahneman [39] suggest $\alpha = 0.89$, $\beta = 0.92$, and $\lambda = 2.25$.

2.2. Group decision making

GDM has been defined variously, including as "a decision situation in which more than one individual is involved, each with their own attitudes and viewpoints, recognizing the existence of a common problem and attempting to make a common decision together" [24].

GDM problems are frequently utilized in many complex real-life decision situations, such as supplier selection [11,29], supply-chain risk management [5,7], and humanitarian logistics [2,36]. The solution to a GDM problem can be obtained by applying either a direct approach or an indirect approach in the selection process [18]. A direct approach obtains the solution directly from experts' information, whereas in an indirect approach, collective information is computed before the solution is determined. Regardless of the approach considered, the selection process to solve GDM problems consists of two phases (see Fig. 4) [27]: (1) *an aggregation phase*, in which individual information is aggregated, and (2) *an exploitation phase*, in which an alternative or subset of alternatives is obtained as the solution to the problem.

2.3. Related works

In order to show the importance of GEDM in the real world, this subsection highlights several important studies in the literature that are related to our research [10,21,43,45,46].

These studies have approached GEDM problems from different angles. For example, Levy and Taji [21] utilized a group analytic network process to construct a group decision support system to support hazard planning and emergency management under incomplete information. Yu and Lai [46] proposed a distance-based multi-criteria group decision-making method to support multi-person emergency decision problems. Xu et al. [45] proposed a new conflict-eliminating model for emergency group decision to overcome the drawbacks of existing conflict-eliminating models. Chen et al. [10] proposed a belief structure to represent partially ordered preferences with belief degrees, and used evidential reasoning to combine the partially ordered preferences provided by experts under uncertainty. Xu et al. [43] proposed a consensus model that considers non-cooperative behavior and minority opinions, and proposed a dynamic consensus method for large group emergency decision making in Xu et al. [44].

Experts' psychological behavior is neglected in current GEDM studies [17,21,30,43], and thus, while experts' psychological behavior plays an important role in the group decision process under risk and uncertainty, our proposed method aims to overcome this limitation and shows the importance of this research.

3. Group emergency decision-making method based on prospect theory

The need to make good decisions in GEDM situations drives us to improve the limitation of previous GEDM problems in which experts' behavior is neglected. Therefore, we aim to address such a problem using PT to consider experts' behavior. Here, we introduce a novel GEDM framework to deal with this issue.

To achieve our objectives, the proposed method consists of the following seven phases, depicted graphically in Fig. 5.

(a) *Framework definition*: this defines the notations and structure of the proposed GEDM problem (experts, alternatives, and criteria) as well as the expression domains in which assessments are elicited.
(b) *Information-gathering process*: individual RPs over the alternatives concerning different attributes provided by experts are gathered.
(c) *Aggregation process*: this aggregates the gathered individual RPs are aggregated to obtain the group reference points (GRPs).
(d) *Calculation of gains and losses*: gains and losses are calculated with respect to the GRPs of the different alternatives.

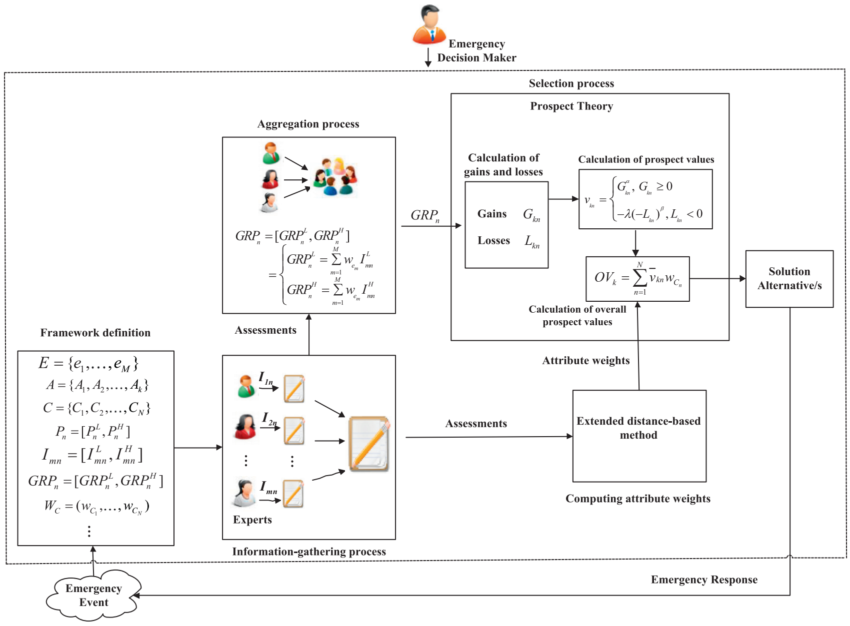


Fig. 5. General scheme of proposed group emergency decision-making method based on prospect theory.

(e) *Calculation of prospect values*: prospect values denote the magnitudes of gains and losses, reflecting the different feelings of experts.
(f) *Computing attribute weights*: attribute weights are computed to weight the importance of each attribute.
(g) *Overall prospect values*: overall prospect value of each alternative is calculated to obtain the solution set of alternatives of the GEDM.

The details of these phases are provided in the following subsections.

3.1. Framework definition of group emergency decision-making problem

The following elements and notations are used in the proposed GEDM method:

- $E = \{e_1, \dots, e_M\}$: the emergency panel of experts, where e_m denotes the mth expert, $m = 1, 2, \dots, M$.
- $W_E = (w_{e_1}, w_{e_2} \dots, w_{e_M})$: the weighting vector of relative importance of each expert in the emergency panel, where w_{e_m} denotes the relative weight of the mth expert, satisfying $w_{e_m} \in [0, 1]$, $m = 1, 2, \dots, M$, and $\sum_{m=1}^{M} w_{e_m} = 1$.
- $A = \{A_1, A_2, \dots, A_K\}$: the set of emergency alternatives, where A_k is the kth emergency alternative, $k = 1, 2, \dots, K$.
- $C = \{C_1, C_2, \dots, C_N\}$: set of criteria/attributes, where C_n denotes the nth criterion/attribute, $n = 1, 2, \dots, N$.
- $W_C = (w_{C_1}, \dots, w_{C_N})$: the weighting vector for attributes, where w_{C_n} denotes the attribute weight of nth attribute, satisfying, $w_{C_n} \in [0, 1]$, $n = 1, 2, \dots, N$ and $\sum_{n=1}^{N} w_{C_n} = 1$.
- $I_{mn} = [I_{mn}^L, I_{mn}^H]$, $I_{mn}^H > I_{mn}^L$: an interval value, where I_{mn} denotes the individual RPs provided by mth expert with respect to the nth criterion/attribute, $m = 1, 2, \dots, M$,$n = 1, 2, \dots, N$ (see Remark 1).
- $P_n = [P_n^L, P_n^H]$, $P_n^H > P_n^L$: the predefined *effective control scope* [41] of alternatives with respect to attribute C_n, which means that the alternative can prevent losses from EE regarding C_n, $n = 1, 2, \dots, N$.
- $GRP_n = [GRP_n^L, GRP_n^H]$, $GRP_n^H > GRP_n^L$: the GRP, where GRP_n denotes the GRP with respect to the nth criterion/attribute, $n = 1, 2, \dots, N$. GRP is a similar concept to RP in PT, which is used in a group decision process. Without loss of generality, we assume $GRP_n^L \geq 0$, $I_{mn}^L \geq 0$, $P_n^L \geq 0$.

Remark 1. In the real world, owing to inadequate or incomplete information, especially in the early stage of emergency event occurrence, and the complexity of emergency situations, it is difficult for experts to estimate damages, losses, or costs

Table 1
Individual RPs over attribute C_n provided by experts.

Experts	Assessments
e_1	$\{I_{11}, ..., I_{1N}\}$
e_2	$\{I_{21}, ..., I_{2N}\}$
......	......
e_M	$\{I_{M1}, ..., I_{MN}\}$

of emergency alternatives using crisp and precise numbers. Thus, interval values are more suitable for uncertainty modeling [41].

3.2. Information-gathering process

Experts provide their individual RPs with respect to each attribute C_n (see Table 1).

The individual RPs I_{mn} provided by expert e_m with respect to attribute C_n are interval values, according to Remark 1.

3.3. Aggregation process

Section 2.1 points out the importance of RPs in PT. Similarly, in GEDM based on PT, GRP denotes the expectations of all experts and is obtained by aggregating experts' individual RPs.

Our proposed method obtains GRPs by means of a weighted average in which each expert e_m is weighted by w_{e_m} to aggregate the individual RPs, I_{mn}, as follows:

$$GRP_n = [GRP_n^L, GRP_n^H] = \begin{cases} GRP_n^L = \sum_{m=1}^{M} w_{e_m} I_{mn}^L \\ GRP_n^H = \sum_{m=1}^{M} w_{e_m} I_{mn}^H \end{cases}, n = 1, 2, \ldots, N \tag{2}$$

If all experts were equally important, then, Eq. (2) could be rewritten as follows:

$$GRP_n = [GRP_n^L, GRP_n^H] = \begin{cases} GRP_n^L = \frac{1}{M} \sum_{m=1}^{M} I_{mn}^L \\ GRP_n^H = \frac{1}{M} \sum_{m=1}^{M} I_{mn}^H \end{cases}, n = 1, 2, \ldots, N \tag{3}$$

$GRP_n = [GRP_n^L, GRP_n^H]$, which represents the expectations of all experts with respect to the nth attribute, which is the result of the aggregation process. GRP_n not only comprehensively considers all experts' individual RPs but also aggregates experts' individual RPs in a simple way with low time cost.

3.4. Calculation of gains and losses

According to PT, gains or losses depend on experts' psychological behavior, like risk aversion or risk seeking [19,39]. To evaluate different alternatives according to the obtained GRP_n and the predefined effective control scopes, P_n, of alternatives, it is necessary to determine the relationship between GRP_n and P_n. Because we are dealing with interval values, there are six possible cases of positional relationship, as shown in Table 2.

For the different cases presented in Table 2, Tables 3 and 4 present the equations for calculating the gains and losses for cost and benefit attributes, respectively, which are further detailed in the Appendix.

Based on Tables 3 and 4, the gain–loss matrix (GLM) can be constructed for the calculation of prospect values of emergency alternatives in the following subsection.

3.5. Calculation of prospect values

As stated in Section 2.1, prospect values are measured by using a value function, reflecting experts' behavior. When a prospect value is equal to or greater than zero, the expert is satisfied with his/her judgment; otherwise, the expert feels regret about his/her judgment. By using PT, the psychological behavior of experts can be described clearly and easily.

Let GLM $= (x_{kn})_{k\times n}$ be the GLM, where x_{kn} denotes G_{kn} or L_{kn}. The prospect value, v_{kn}, of each attribute, C_n, with respect to each alternative, A_k, is obtained as

$$v_{kn} = \begin{cases} G_{kn}^{\alpha}, & G_{kn} \geq 0 \\ -\lambda(-L_{kn})^{\beta}, & L_{kn} < 0 \end{cases} \tag{4}$$

Accordingly, prospect value matrix, $V = (v_{kn})_{k\times n}$, is obtained.

Table 2
Possible cases of positional relationship between GRP_n and P_n.

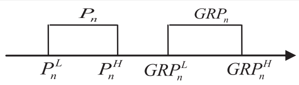
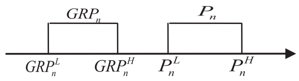
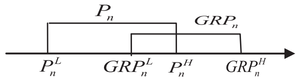
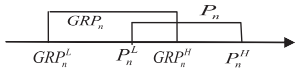
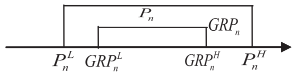
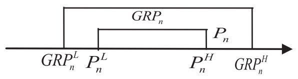
Cases		Positional relationship between GRP_n and P_n
Case 1	$P_n^H < GRP_n^L$	
Case 2	$GRP_n^H < P_n^L$	
Case 3	$P_n^L < GRP_n^L \le P_n^H < GRP_n^H$	
Case 4	$GRP_n^L < P_n^L \le GRP_n^H < P_n^H$	
Case 5	$P_n^L < GRP_n^L < GRP_n^H < P_n^H$	
Case 6	$GRP_n^L \le P_n^L < P_n^H \le GRP_n^H$	

Table 3
Gains and losses for all possible cases (cost attribute).

Cases		Gain G_{kn}	Loss L_{kn}
Case 1	$P_n^H < GRP_n^L$	$GRP_n^L - 0.5(P_n^L + P_n^H)$	0
Case 2	$GRP_n^H < P_n^L$	0	$GRP_n^H - 0.5(P_n^L + P_n^H)$
Case 3	$P_n^L < GRP_n^L \le P_n^H < GRP_n^H$	$0.5(GRP_n^L - P_n^L)$	0
Case 4	$GRP_n^L < P_n^L \le GRP_n^H < P_n^H$	0	$0.5(GRP_n^H - P_n^H)$
Case 5	$P_n^L < GRP_n^L < GRP_n^H < P_n^H$	$0.5(GRP_n^L - P_n^L)$	$0.5(GRP_n^H - P_n^H)$
Case 6	$GRP_n^L \le P_n^L < P_n^H \le GRP_n^H$	0	0

Table 4
Gains and losses for all possible cases (benefit attribute).

Cases		Gain G_{kn}	Loss L_{kn}
Case 1	$P_n^H < GRP_n^L$	0	$0.5(P_n^L + P_n^H) - GRP_n^L$
Case 2	$GRP_n^H < P_n^L$	$0.5(P_n^L + P_n^H) - GRP_n^H$	0
Case 3	$P_n^L < GRP_n^L \le P_n^H < GRP_n^H$	0	$0.5(P_n^L - GRP_n^L)$
Case 4	$GRP_n^L < P_n^L \le GRP_n^H < P_n^H$	$0.5(P_n^H - GRP_n^H)$	0
Case 5	$P_n^L < GRP_n^L < GRP_n^H < P_n^H$	$0.5(P_n^H - GRP_n^H)$	$0.5(P_n^L - GRP_n^L)$
Case 6	$GRP_n^L \le P_n^L < P_n^H \le GRP_n^H$	0	0

Since gains and losses of different attributes are usually incommensurate, V needs to be normalized into comparable values. This is achieved by normalizing each element in V into a corresponding element in matrix $\overline{V} = (\overline{v}_{kn})_{k\times n}$ by using

$$\overline{v}_{kn} = \frac{v_{kn}}{v^*}, k = 1, 2, \ldots, K, n = 1, 2, \ldots, N, \tag{5}$$

where $v^* = \max_{n\in N}\{|v_{kn}|\}$.

3.6. Computing attribute weights

Determination of attribute weights is an essential step in the GEDM process. There are different methods to determine the attribute weights, such as the analytic hierarchy process method [37], entropy-based method [35], and distance-based method [46].

In our proposed method, in order to undertake a fair comparison with the current distance-based GEDM method [46] in Section 4.2, the attribute weights are computed by using the distance-based method in [46]. Because the distance-based method deals with crisp values for computing attribute weights, which is not suitable for our method coping with the interval values, a transformation is provided for interval values.

With respect to the individual RPs, I_{mn}, provided by each expert, the following definition is provided.

Definition 1. [14] For interval value I_{mn}, let σ be an arbitrary value in interval number $[I_{mn}^L, I_{mn}^H]$, regarded as a random variable with uniform distribution. The probability density function of σ is

$$f(\sigma) = \begin{cases} \frac{1}{I_{mn}^H - I_{mn}^L}, I_{mn}^L \le \sigma \le I_{mn}^H, & \\ 0, & otherwise \end{cases} \quad m = 1, \ldots, M, n = 1, \ldots, N \tag{6}$$

where $\int_{I_{mn}^L}^{I_{mn}^H} f(\sigma) d\sigma = 1$ and $f(\sigma) \ge 0$ for all $\sigma \in [I_{mn}^L, I_{mn}^H]$.

According to Definition 1, an information matrix $Y = [y_{mn}]_{m\times n}$ can be obtained by Eq. (7), that is,

$$y_{mn} = \int_{I_{mn}^L}^{I_{mn}^H} \sigma f(\sigma) d\sigma, m = 1, \ldots, M, n = 1, \ldots, N \tag{7}$$

Therefore, an extended distance-based method to calculate the attribute weights is introduced as follows.

(a) Based on I_{mn}, we obtain information matrix, $Y = [y_{mn}]_{m\times n}$ according to Eqs. (6) and (7).

(b) $Y = [y_{mn}]_{m\times n}$ is normalized. For each attribute C_n, all the values are divided by $\sum_{m=1}^{M} y_{mn}$, that is,

$$\bar{y}_{mn} = \frac{y_{mn}}{\sum_{m=1}^{M} y_{mn}}, n = 1, \ldots, N \tag{8}$$

In this way, all values are normalized into the interval [0,1]. The purpose of normalization is to remove the effect of magnitude of data.

(c) Positive and negative values for each attribute, C_n, are determined. For each attribute, C_n, positive and negative values are defined as follows.

$$\text{Positivevalues: } y^+ = (y_1^+, y_2^+, \ldots, y_N^+) \tag{9}$$

$$\text{Negativevalues: } y^- = (y_1^-, y_2^-, \ldots, y_N^-), \tag{10}$$

where

$$y_n^+ = \begin{cases} \max\limits_{1\le m\le M} \{\bar{y}_{mn}\}, n \in N_1 \\ \min\limits_{1\le m\le M} \{\bar{y}_{mn}\}, n \in N_2 \end{cases} \tag{11}$$

$$y_n^- = \begin{cases} \min\limits_{1\le m\le M} \{\bar{y}_{mn}\}, n \in N_1 \\ \max\limits_{1\le m\le M} \{\bar{y}_{mn}\}, n \in N_2 \end{cases} \tag{12}$$

respectively, where N_1 and N_2 represent the benefit and cost attributes, respectively.

(d) Distance between $\bar{y}_{mn}$ and y^+/y^- is computed. The distance between $\bar{y}_{mn}$ and y^+/y^- can be obtained by

$$d_n^+ = \sqrt{\sum_{m=1}^{M} (\bar{y}_{mn} - y_n^+)^2} \tag{13}$$

$$d_n^- = \sqrt{\sum_{m=1}^{M} (\bar{y}_{mn} - y_n^-)^2} \tag{14}$$

(e) Dispersion for each attribute is measured. In the distance-based method, the dispersion measurement for each attribute, C_n, is expressed as

$$\xi_n = \frac{d_n^+}{d_n^+ + d_n^-}. \tag{15}$$

According to Eq. (13), the larger is the value of ξ_n, the larger is the dispersion measurement and accordingly, the more important is attribute C_n, which is consistent with the generic rule of attribute weight determination [35].

(f) Attribute weight w_{C_n} is determined. For each attribute C_n, the weight can be determined based on the dispersion measurement as

$$w_{C_n} = \frac{\xi_n}{\sum_{n=1}^{N} \xi_n}, n = 1, 2, \ldots, N. \tag{16}$$

Table 5
Description of alternatives.

Alternative	Description
A_1	Open small sluice to meet the requirements of the barrier lake floods discharged.
A_2	Open half of the number of sluices, and increase the joint scheduling of the barrier lake and hydropower station in the upstream and downstream areas to reduce the pressure of the barrier lake.
A_3	Open all the sluices, mobilize large, heavy machinery and implement small-scale blasting to reduce the water level of the barrier lake as much as possible to lower the risk of dam break.
A_4	Open all the sluices, and increase the joint scheduling of the barrier lake and hydropower station in the upstream and downstream areas. Meanwhile, mobilize large, heavy machinery and implement large-scale blasting to reduce the water level of the barrier lake as much as possible to lower the risk of dam break.

3.7. Calculation of overall prospect values

Once the attribute weights w_{C_n} and normalized matrix $\overline{V} = (\overline{v}_{kn})_{k\times n}$ are obtained, the overall prospect value of each emergency alternative can be calculated using the simple additive weighting method, that is,

$$OV_k = \sum_{n=1}^{N} \overline{v}_{kn} w_{C_n}, \quad k = 1, 2, \ldots, K, n = 1, 2, \ldots, N \tag{17}$$

The greater OV_k is, the better alternative A_k. Based on the values of OV_k, the ranking of alternatives can be obtained. According to the ranking of alternatives, the DM can select the best alternative to cope with the EE.

In summary, the procedures for the GEDM method based on PT are as follows.

Step 1. Define the framework of the GEDM problem.
Step 2. Each expert involved in EE provides his/her individual RP values I_{mn}.
Step 3. GRP_n can be calculated by aggregating I_{mn} using Eqs. (2) or (3).
Step 4. Gains G_{kn} and losses L_{kn} are calculated for cost/benefit attributes based on Tables 3 and 4, respectively, obtaining the GLM matrix.
Step 5. Based on the GLM matrix, the prospect values $V = (v_{kn})_{k\times n}$ are calculated by using Eq. (4) and are normalized as $\overline{V} = (\overline{v}_{kn})_{k\times n}$ by using Eq. (5).
Step 6. From I_{mn}, an information matrix $Y = [y_{mn}]_{m\times n}$ is calculated by using Eqs. (6)–(7) and the attribute weights w_{C_n} are obtained by using Eqs. (8)–(16).
Step 7. Eventually, an overall prospect value OV_k for each alternative is calculated by using Eq. (17) and is used for ranking the alternatives.

4. Case study and comparison with other approaches

To demonstrate the applicability of the proposed method for solving the GEDM problem, this section presents a case study that is taken from Wang et al. [41]. Comparisons with other approaches and related discussions are provided.

4.1. Case study

To undertake a fair comparison and show the performance of our proposal, a previous case study involving a "*barrier lake emergency*" published in [41] is reconsidered, because the problem framework is close to the decision framework of our proposed approach (see [41] for further details). Based on such an EE, it is assumed that five experts are invited to participate in the GEDM process.

4.1.1. Framework definition

In the case study, four potential emergency states of the barrier lake were possible in the coming 72 hours:

(1) the dam body of the barrier lake would not break;
(2) 1/3 of the dam body of the barrier lake would break;
(3) 1/2 of the dam body of the barrier lake would break; and
(4) the entire dam body of the barrier lake would break.

For such an emergency situation, the local government organized people in the most dangerous areas upstream and downstream of the barrier lake to evacuate to safe areas, and informed people in potentially dangerous areas to prepare for evacuation.

Four emergency alternatives were proposed with three attributes considered in [41], which are described in Tables 5 and 6.

The three attributes considered in this EE are described in Table 6.

The predefined effective control scopes P_n of each alternative with respect to different attributes and its costs are shown in Table 7 following Wang et al. [41].

Table 6
Description of attributes.

Attribute	Description
People affected (C_1)	EE might have impacts on the number of people.
Property loss (C_2)	EE might cause direct and indirect property losses.
Cost of alternative (C_3)	If one alternative is taken, its related cost needs to be considered.

Table 7
Predefined effective control scopes P_n and cost of each alternative from Wang et al. [41].

Alternatives	Criteria/attributes		
	C_1	C_2	C_3
A_1	[3000,3500]	[2500,3500]	[300,350]
A_2	[3500,4000]	[3500,4500]	[350,450]
A_3	[4000,4500]	[4500,5500]	[450,550]
A_4	[5000,5500]	[5500,6500]	[550,650]

Table 8
I_{mn} provided by different experts.

Attribute	Experts				
	e_1	e_2	e_3	e_4	e_5
C_1	[3500,8000]	[4000,6000]	[6000,7000]	[4000,7000]	[5000,8000]
C_2	[3000,4000]	[5000,5500]	[2000,3750]	[3000,3750]	[4000,5250]
C_3	[550,650]	[500,550]	[450,500]	[500,550]	[500,570]

Table 9
The values of GRP_n for different attributes.

GRP	Criteria/attributes		
	C_1	C_2	C_3
GRP_n	[4500,7200]	[3400,4450]	[500,564]

4.1.2. Information-gathering process

The experts provide their individual RPs, I_{mn}, based on their professional knowledge and experience of the possible losses caused by EE with respect to different attributes (see Table 8).

4.1.3. Aggregation process

GRP_n are obtained by aggregating individual RPs, I_{mn}. Without loss of generality, we assume equal weight w_{e_m} for each expert (see Table 9).

4.1.4. Calculation of gains and losses

Based on the positional relationship between GRP_n and P_n shown in Table 2, the gains and losses can be calculated according to Tables 3 and 4 for cost and benefit attributes, respectively. The GLM, obtained for this case is

$$GLM = \begin{bmatrix} -1250 & -450 & 175 \\ -750 & 25 & 100 \\ -250 & 550 & 25 \\ 0 & 1550 & -43 \end{bmatrix}$$

For the sake of clarity, the computation of G_{13} is detailed as follows:

$GRP_3 = [500, 564]$ the cost of A_1 is $[300, 350]$,

Based on Table 2, their positional relationship is Case 1. Then, according to Table 3,

$$GRP_n^L - 0.5(P_n^L + P_n^H) \Rightarrow G_{13} = 500 - 0.5(300 + 350) = 175.$$

4.1.5. Calculation of prospect values

As Section 2.1 points out, the values of α, β, and λ have been studied in the existing literature. In order to undertake a fair comparison with existing studies in the following Section 4.2, the following values are set in this case study in accordance with Wang et al. [41]: $\alpha = 0.89$, $\beta = 0.92$, and $\lambda = 2.25$.

Table 10
Overall prospect values OV_k and corresponding ranking.

	Alternatives			
	A_1	A_2	A_3	A_4
OV_k	−0.3316	−0.0283	**0.0976**	0.0842
Rank	4	3	**1**	2

Based on GLM, the prospect value matrix, $V = (v_{kn})_{k\times n}$, is computed using Eq. (4):

$$V = \begin{bmatrix} -1589.7899 & -621.0664 & 99.1525 \\ -993.6625 & 17.5455 & 60.2560 \\ -361.6490 & 274.7406 & 17.5455 \\ 0 & 690.8682 & -71.6100 \end{bmatrix}$$

Following the details of computation for v_{13}, given that $G_{13} = 175$ ($175 > 0$), based on Eq. (4), we obtain

$$v_{13} = G_{13}^{0.89} = 175^{0.89} = 99.1525.$$

The corresponding normalized matrix $\overline{V} = (\overline{v}_{kn})_{k\times n}$ is:

$$\overline{V} = \begin{bmatrix} -1.0000 & -0.8990 & 1.0000 \\ -0.6250 & 0.0254 & 0.6077 \\ -0.2275 & 0.3977 & 0.1770 \\ 0 & 1.0000 & -0.7222 \end{bmatrix}$$

4.1.6. Computing attribute weights

Based on I_{mn} (Table 8), the information matrix $Y = [y_{mn}]_{m\times n}$ is computed as follows:

$$Y = \begin{bmatrix} 5750 & 3500 & 600 \\ 5000 & 5250 & 525 \\ 6500 & 2875 & 475 \\ 5500 & 3375 & 525 \\ 6500 & 4625 & 535 \end{bmatrix}$$

Using Eqs. (8)–(16), the attribute weights w_{C_n} are obtained as
$w_{C_1} = 0.3676$, $w_{C_2} = 0.3141$, and $w_{C_3} = 0.3183$.

4.1.7. Calculation of overall prospect values

The overall prospect value OV_k of each emergency alternative is calculated by Eq. (17). The results and ranking obtained are shown in Table 10.

Table 10 shows that the overall prospect values, OV_k, vary from negative to positive values. If OV_k are greater than or equal to zero, then the experts feel gains and are satisfied with their assessments of the alternatives. Otherwise, the experts feel losses and regret. According to the principle of PT, the greater is the value OV_k, the better is alternative A_k. Thus, in this case study, alternative A_3, with the greatest overall prospect value, is the best alternative for coping with the EE. Such a selection is fully consistent with the actual emergency response alternative of the barrier lake, which verifies to some extent the validity and feasibility of the proposed approach.

4.2. Comparison with other approaches

To further illustrate the novelty, validity, and feasibility of the proposed method, the following two comparisons are performed.

(1) The proposed method (PT-GEDM) is compared with the distance-based group emergency decision-making (DB-GEDM) method [46], which neglects experts' psychological behavior in the decision process.
(2) The proposed method (PT-GEDM) is compared with the PT-based emergency decision-making (PT-EDM) method [41], which did not apply a group decision scheme to the emergency problem.

Different aspects are outlined in further detail in the following subsections.

Table 11
Comparison results between the prospect theory-based group emergency decision-making method and the distance-based group emergency decision-making method.

			DB-GEDM method					
	PT-GEDM		Lower bound		Upper bound		Middle point	
Alternative	OV_k	Rank	Score	Rank	Score	Rank	Score	Rank
A_1	−0.3316	4	0.5254	4	0.5204	4	0.3295	4
A_2	−0.0283	3	0.6488	3	0.6418	3	0.4216	3
A_3	**0.0976**	**1**	0.8045	2	0.7633	2	0.5270	2
A_4	0.0842	2	**0.9898**	**1**	**0.9120**	**1**	**0.6324**	**1**

Table 12
Comparison results between the prospect theory-based group emergency decision-making method and the prospect theory-based emergency decision-making method.

	PT-GEDM method	PT-EDM method Experts				
Alternative	OV_k (Rank)	e_1 OV_k (Rank)	e_2 OV_k (Rank)	e_3 OV_k (Rank)	e_4 OV_k (Rank)	e_5 OV_k (Rank)
A_1	−0.3316 (4)	−0.1799 (4)	−0.6436 (4)	−0.3676 (4)	−0.3634 (4)	−0.3634 (4)
A_2	−0.0283(3)	0.4095 (2)	−0.2544 (3)	**0.1308 (1)**	0.1741 (2)	−0.1640 (3)
A_3	**0.0976 (1)**	**0.6205 (1)**	**−0.0314 (1)**	−0.0946 (3)	**0.2367 (1)**	**−0.0944 (1)**
A_4	0.0842 (2)	0.3141 (3)	−0.0784 (2)	0.0660 (2)	0.0402 (3)	−0.1273 (2)

4.2.1. Comparison with group emergency decision-making method without experts' psychological behavior

In order to show the performance of the proposed PT-GEDM method, this subsection compares it with the DB-GEDM method [46], which does not consider experts' psychological behavior.

Because the DB-GEDM method in [46] dealt with crisp numbers, in order to undertake a fair comparison, we use the lower bound, upper bound, and middle point of the interval values in Tables 7 and 8 for computing the decision results and the corresponding attribute weights, respectively. The results are shown in Table 11.

Table 11 shows that both methods, PT-GEDM and DB-GEDM, provide similar but different results for this case, and *the best* alternatives are A_3 and A_4, respectively, while the 3rd- and 4th-best alternatives remain consistent.

Therefore, the use of PT to reflect the experts' psychological behavior (positive values: gains; negative values: losses) makes the decision different, which shows the importance of considering the experts' psychological behavior in GEDM. Doing so can lead to decisions that better reflect humans' way of thinking in real–world situations. However, the DB-GEDM method does not consider such information, because it does not consider the experts' psychological behavior in the GEDM process.

4.2.2. Comparison with a prospect theory-based emergency decision-making method

To illustrate the importance of the group decision process in EDM, the proposed PT-GEDM method is compared with the PT-EDM method in [41], which uses the individual RPs, I_{mn}, provided by each expert in Table 8. The comparison results are shown in Table 12.

Table 12 shows that the results obtained by the PT-EDM method vary from one expert to another and are different from those obtained by the PT-GEDM method. Since the behavior of one expert can bias the decision about the EDM situation, the use of a group view that integrates individual information and different psychological behavior can soften extreme bias and produce more balanced decision results.

Therefore, the inclusion of experts' behavior in GEDM problems leads to more reliable decisions than not including them does, and can avoid extreme bias with respect to EDM approaches.

To further illustrate the reliability and validity of the result obtained using the PT-GEDM method, if we were to apply the majority-based method [26,28] to the results obtained by the PT-EDM method shown in Table 12, we would obtain the same solution as our proposal, which is in accordance with our intuition. Even though an expert's individual subjectivity is inevitable during the decision process, the use of group information can soften individual bias and subjectivity, and make the decision more reasonable and reliable. This is the dominant advantage of group decision making, which is an indirect way to demonstrate the advantage and validity of our proposed PT-GEDM method.

4.3. Discussions

From the comparative analysis conducted in Section 4.2, the main novelty and advantages of our proposed PT-GEDM method over current studies can be summarized as follows.

(1) The PT-GEDM method is the first to consider and address experts' psychological behavior under risk and uncertainty using PT in the GEDM problems, compared with current GEDM versions that do not consider experts' psychological behavior. This is a significant difference between our proposal and current GEDM versions.
(2) The PT-GEDM method fully considers experts' subjectivity and bounded rationality on judgments. Each expert's judgments (individual RPs) can be assessed with interval values, owing to their useful and simple technique for representing uncertainty, which is suitable and adequate for emergency situations.
(3) The PT-GEDM method utilizes group information (GRPs) in a more complete way, considering each expert's opinion in the GEDM process and producing a more reasonable and reliable result than the PT-EDM method in [41]. In GEDM problems, the decision becomes more robust, since the information is assessed by multiple experts. Generally, the use of multiple experts in the GEDM process leads to better decisions.
(4) The PT-GEDM method is easy to understand, acceptable to experts, and closer to the real-world situation. In addition, its computation process is simple and fast.

5. Conclusions and future works

This study proposes a new GEDM method based on PT for emergency situations. The method is based on *bounded rationality* using PT and takes into account experts' psychological behavior in decision processes, which overcomes the limitations of previous GEDM methods that have not considered experts' psychological behavior, despite its influence in real-world decision-making processes. Therefore, the proposed PT-GEDM method significantly differs from the previous GEDM approaches and can provide better decision results. In our proposed method, interval numbers are utilized by experts to provide their individual RPs regarding their expectations about potential damages, losses, or costs of alternatives, which are more suitable for uncertainty modeling than crisp and precise numbers are, owing to incomplete information and the complexity of emergency situations. Compared with the PT-EDM method, the use of group information can lead to a more reasonable and reliable decision result, because the viewpoints of multiple experts are considered. An extended distance-based method for determining the attribute weights is provided, and is used to undertake a fair comparison with the previous DB-GEDM method in the case study to demonstrate the novelty, feasibility, and validity of the proposed method. The proposed GEDM method is expected to have more potential applications in the near future.

For future works, a promising research direction is the use of different types of information modeling [31,32] to flexibly assess the features of an emergency event that are usually elicited by experts under a GEDM situation. In addition, the management of time constraints in GEDM seems a promising and fruitful research line.

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Appendix

In order to ensure that the equations for calculating the gains and losses are easily understood in Tables 3 and 4, different cases in Table 2 are discussed.

As stated in Section 3.4, we deal with interval values GRP_n and P_n. Similar to Definition 1, the following definition is provided.

Definition 2. [14] For the predefined effective control scope P_n of alternatives, let x be an arbitrary value in interval value $[P_n^L, P_n^H]$, regarded as a random variable with uniform distribution. The probability density function of x is

$$f(x) = \begin{cases} \frac{1}{P_n^H - P_n^L}, & P_n^L \le x \le P_n^H, \\ 0, & otherwise \end{cases} \quad n = 1, 2, \ldots, N \tag{18}$$

where $\int_{P_n^L}^{P_n^H} f(x)dx = 1$ and $f(x) \ge 0$ for all $x \in [P_n^L, P_n^H]$.

Let L_{kn} and G_{kn} denote the loss and gain, respectively, of the kth alternative over the nth attribute. The following discussion is for cost attribute only.

Case 1: since $P_n^H < GRP_n^L$, there is no loss to experts, that is,

$$L_{kn} = 0, k = 1, 2, \ldots, K, n = 1, 2, \ldots, N \tag{19}$$

According to Definition 2, x is an arbitrary value in interval value $[P_n^L, P_n^H]$, and the perceived gains to experts between $[P_n^L, P_n^H]$ and $[GRP_n^L, GRP_n^H]$ can be given as the following Eq. (20)

$$G_{kn} = (GRP_n^L - P_n^H) + \int_{P_n^L}^{P_n^H} (P_n^H - x) f(x) dx \tag{20}$$

which can be rewritten by Eq. (18) as

$$\begin{aligned}
G_{kn} &= (GRP_n^L - P_n^H) + \int_{P_n^L}^{P_n^H} (P_n^H - x) f(x) dx \\
&= (GRP_n^L - P_n^H) + \int_{P_n^L}^{P_n^H} (P_n^H - x) \frac{1}{P_n^H - P_n^L} dx \\
&= (GRP_n^L - P_n^H) + \frac{1}{P_n^H - P_n^L} (P_n^H * x - \frac{x^2}{2}) \Big|_{P_n^L}^{P_n^H} \\
&= (GRP_n^L - P_n^H) + \frac{1}{P_n^H - P_n^L} \left[(P_n^H * P_n^H - \frac{(P_n^H)^2}{2}) - (P_n^H * P_n^L - \frac{(P_n^L)^2}{2}) \right] \\
&= (GRP_n^L - P_n^H) + \frac{1}{P_n^H - P_n^L} \left[\frac{(P_n^H)^2}{2} - P_n^H * P_n^L + \frac{(P_n^L)^2}{2} \right] \\
&= (GRP_n^L - P_n^H) + \frac{2}{2(P_n^H - P_n^L)} \left[\frac{(P_n^H)^2}{2} - P_n^H * P_n^L + \frac{(P_n^L)^2}{2} \right] \\
&= (GRP_n^L - P_n^H) + \frac{1}{2(P_n^H - P_n^L)} \left[(P_n^H)^2 - 2P_n^H * P_n^L + (P_n^L)^2 \right] \\
&= (GRP_n^L - P_n^H) + \frac{1}{2(P_n^H - P_n^L)} (P_n^H - P_n^L)^2 \\
&= (GRP_n^L - P_n^H) + \frac{(P_n^H - P_n^L)}{2} \\
&= GRP_n^L - 0.5(P_n^L + P_n^H), k = 1, 2, \ldots, K, n = 1, 2, \ldots, N
\end{aligned} \tag{21}$$

Case 2: since $GRP_n^H < P_n^L$, there is no gain to experts, that is,

$$G_{kn} = 0, k = 1, 2, \ldots, K, n = 1, 2, \ldots, N \tag{22}$$

The perceived losses to experts between $[P_n^L, P_n^H]$ and $[GRP_n^L, GRP_n^H]$ can be given as the following Eq. (23):

$$\begin{aligned}
L_{kn} &= (GRP_n^H - P_n^L) + \int_{P_n^L}^{P_n^H} (P_n^L - x) f(x) dx \\
&= (GRP_n^H - P_n^L) + \int_{P_n^L}^{P_n^H} (P_n^L - x) \frac{1}{P_n^H - P_n^L} dx \\
&= (GRP_n^H - P_n^L) + \frac{1}{P_n^H - P_n^L} (P_n^L * x - \frac{x^2}{2}) \Big|_{P_n^L}^{P_n^H} \\
&= (GRP_n^H - P_n^L) + \frac{1}{P_n^H - P_n^L} \left[(P_n^L * P_n^H - \frac{(P_n^H)^2}{2}) - (P_n^L * P_n^L - \frac{(P_n^L)^2}{2}) \right] \\
&= (GRP_n^H - P_n^L) + \frac{1}{P_n^H - P_n^L} \left[P_n^L * P_n^H - \frac{(P_n^H)^2}{2} - \frac{(P_n^L)^2}{2} \right] \\
&= (GRP_n^H - P_n^L) + \frac{2}{2(P_n^H - P_n^L)} \left[P_n^L * P_n^H - \frac{(P_n^H)^2}{2} - \frac{(P_n^L)^2}{2} \right] \\
&= (GRP_n^H - P_n^L) + \frac{1}{2(P_n^H - P_n^L)} \left[2P_n^H * P_n^L - (P_n^H)^2 - (P_n^L)^2 \right] \\
&= (GRP_n^H - P_n^L) - \frac{1}{2(P_n^H - P_n^L)} (P_n^H - P_n^L)^2 \\
&= (GRP_n^H - P_n^L) - \frac{(P_n^H - P_n^L)}{2} \\
&= GRP_n^H - 0.5(P_n^H + P_n^L), k = 1, 2, \ldots, K, n = 1, 2, \ldots, N
\end{aligned} \tag{23}$$

Case 3: since the interval $[GRP_n^L, P_n^H] \subset [GRP_n^L, GRP_n^H]$, there are no losses to experts, that is,

$$L_{kn} = 0, k = 1, 2, \ldots, K, n = 1, 2, \ldots, N \tag{24}$$

Let y be an arbitrary value in interval value $[GRP_n^L, P_n^H]$, and any possible value of y in interval value $[GRP_n^L, P_n^H]$ is equally acceptable to experts [14], which means that the perceived gains or losses to experts are equal to 0; in other words, experts feel neither gains nor losses. Thus, we need to consider only the perceived gains between the interval value $[P_n^L, GRP_n^L]$. Let t be an arbitrary value in interval value $[P_n^L, GRP_n^L]$. According to Definition 2, the gains to experts are given as the following Eq. (25):

$$\begin{aligned}
G_{kn} &= \int_{P_n^L}^{GRP_n^L} (GRP_n^L - t) f(t) dt \\
&= \int_{P_n^L}^{GRP_n^L} (GRP_n^L - t) \frac{1}{GRP_n^L - P_n^L} dt \\
&= \frac{1}{GRP_n^L - P_n^L} (GRP_n^L * t - \frac{t^2}{2}) \Big|_{P_n^L}^{GRP_n^L} \\
&= \frac{1}{GRP_n^L - P_n^L} \left[(GRP_n^L * GRP_n^L - \frac{(GRP_n^L)^2}{2}) - (GRP_n^L * P_n^L - \frac{(P_n^L)^2}{2}) \right] \\
&= \frac{1}{GRP_n^L - P_n^L} \left[\frac{(GRP_n^L)^2}{2} - GRP_n^L * P_n^L + \frac{(P_n^L)^2}{2} \right] \\
&= \frac{1}{2(GRP_n^L - P_n^L)} (GRP_n^L - P_n^L)^2 \\
&= 0.5(GRP_n^L - P_n^L), k = 1, 2, \ldots, K, n = 1, 2, \ldots, N
\end{aligned} \tag{25}$$

When $GRP_n^L = P_n^H$, the interval number $[GRP_n^L, P_n^H]$ is a crisp number, that is, $y = GRP_n^L = P_n^H$. Then, Eq. (25) is a special case of Eq. (21).

Case 4: since $GRP_n^L < P_n^L \le GRP_n^H < P_n^H$, there is no gain to experts, that is,

$$G_{kn} = 0, k = 1, 2, \ldots, K, n = 1, 2, \ldots, N \tag{26}$$

Since $[P_n^L, GRP_n^H] \subset [GRP_n^L, GRP_n^H]$, similar to **Case 3**, let δ be an arbitrary value in interval value $[GRP_n^H, P_n^H]$. According to Definition 2, the losses to experts are given as the following Eq. (27):

$$\begin{aligned}
L_{kn} &= \int_{GRP_n^H}^{P_n^H} (\delta - P_n^H) f(\delta) d\delta \\
&= \frac{1}{P_n^H - GRP_n^H} (\frac{\delta^2}{2} - P_n^H * \delta) \Big|_{GRP_n^H}^{P_n^H} \\
&= \frac{1}{P_n^H - GRP_n^H} \left[(\frac{(P_n^H)^2}{2} - P_n^H * P_n^H) - (\frac{(GRP_n^H)^2}{2} - P_n^H * GRP_n^H) \right] \\
&= \frac{1}{P_n^H - GRP_n^H} \left[P_n^H * GRP_n^H - \frac{(GRP_n^H)^2}{2} - \frac{(P_n^H)^2}{2} \right] \\
&= \frac{-1}{2(P_n^H - GRP_n^H)} (P_n^H - GRP_n^H)^2 \\
&= -0.5(P_n^H - GRP_n^H) \\
&= 0.5(GRP_n^H - P_n^H), k = 1, 2, \ldots, K, n = 1, 2, \ldots, N
\end{aligned} \tag{27}$$

Case 5: since $P_n^L < GRP_n^L < GRP_n^H < P_n^H$, let φ be an arbitrary value in interval value $[P_n^L, GRP_n^L]$. Similar to Case 3, the perceived gains to experts are given as the following Eq. (28):

$$\begin{aligned}
G_{kn} &= \int_{GRP_n^H}^{P_n^H} (GRP_n^L - \varphi) f(\varphi) d\varphi \\
&= 0.5(GRP_n^L - P_n^L), k = 1, 2, \ldots, K, n = 1, 2, \ldots, N
\end{aligned} \tag{28}$$

Let ξ be an arbitrary value in interval value $[GRP_n^H, P_n^H]$. Similar to Case 4, the perceived losses to experts can be given as the following Eq. (29):

$$L_{kn} = \int_{GRP_n^H}^{P_n^H} (\xi - P_n^H) f(\xi) d\xi = 0.5(GRP_n^H - P_n^H), k = 1, 2, \ldots, K, n = 1, 2, \ldots, N \tag{29}$$

When the possible positional relationship between GRP_n and P_n is similar to Case 5, the final outcomes are gains or losses, which depends on the summation of Eqs. (28) and (29).

Case 6: since $GRP_n^L \leq P_n^L < P_n^H \leq GRP_n^H$, that is, $[P_n^L, P_n^H] \subseteq [GRP_n^L, GRP_n^H]$, any arbitrary value in interval $[P_n^L, P_n^H]$ is equally acceptable to experts, which means that the experts feel neither gains nor losses. Thus, the gains and losses are given by the following Eqs. (30) and (31), respectively:

$$G_{kn} = 0, k = 1, 2, \ldots, K, n = 1, 2, \ldots, N \tag{30}$$

$$L_{kn} = 0,\ k = 1, 2, \ldots, K,\ n = 1, 2, \ldots, N \tag{31}$$

According to Eqs. (18)–(31), the equations for calculating the gains and losses for the "cost attribute" of different possible cases are summarized in Table 3. Similarly, Table 4 shows the equations for calculating the gains and losses for the "benefit attribute."

References

[1] M. Abdellaoui, H. Bleichrodt, C. Paraschiv, Loss aversion under prospect theory: a parameter-free measurement, Manage. Sci. 53 (10) (2007) 1659–1674.
[2] M. Ahmadi, A. Seifi, B. Tootooni, A humanitarian logistics model for disaster relief operation considering network failure and standard relief time: a case study on San Francisco district, Trans. Res. Part E 75 (2015) 145–163.
[3] A.E. Attema, W.B. Brouwer, O. Haridon, Prospect theory in the health domain: a quantitative assessment, J. Health Econ. 32 (6) (2013) 1057–1065.
[4] A.E. Attema, W.B. Brouwer, O. Haridon, et al., An elicitation of utility for quality of life under prospect theory, J. Health Econ. 48 (2016) 121–134.
[5] D. Bandaly, A. Satir, Y. Kahyaoglu, et al., Supply chain risk management–I: conceptualization, framework and planning process, Risk Manage. 14 (4) (2012) 249–271.
[6] M.J. Best, R.R. Grauer, Prospect theory and portfolio selection, J. Behav. Exp. Finance 11 (2016) 13–17.
[7] M.F. Blos, M. Quaddus, H.M. Wee, et al., Supply chain risk management (SCRM): a case study on the automotive and electronic industries in Brazil, Supply Chain Manage. 14 (4) (2009) 247–252.
[8] A. Booij, S. Van, G. Kuilen, A parameter-free analysis of the utility of money for the general population under prospect theory, J. Econ. Psychol. 30 (4) (2009) 651–666.
[9] C.T.L. Butler, A. Rothstein, On Conflict and Consensus: A Handbook on Formal Consensus Decision Making, Food Not Bombs Press, 2007.
[10] S.W. Chen, J. Liu, H. Wang, et al., A group decision making model for partially ordered preference under uncertainty, Inf. Fusion 25 (2015) 32–41.
[11] J.F. Chu, X.W. Liu, Y.M. Wang, et al., A group decision making model considering both the additive consistency and group consensus of intuitionistic fuzzy preference relations, Comput. Ind. Eng. 101 (2016) 227–242.
[12] J. Cosgrave, Decision making in emergencies, Disaster. Prev. Manage. Int. J. 5 (4) (1996) 28–35.
[13] H. Dichtl, W. Drobetz, Portfolio insurance and prospect theory investors: popularity and optimal design of capital protected financial products, J. Bank Finance 35 (7) (2011) 1683–1697.
[14] Z.P. Fan, T. Liu, An approach to solve group-decision-making problems with ordinal interval numbers, IEEE Trans. Syst. Man Cybern. Part B 40 (5) (2010) 1413–1423.
[15] Z.P. Fan, Y. Liu, R.J. Shen, Risk decision analysis method for emergency response based on prospect theory, Syst. Eng. Theory Prac. 32 (5) (2012) 977–984.
[16] Z.P. Fan, X. Zhang, F.D. Chen, et al., Multiple attribute decision making considering aspiration-levels: a method based on prospect theory, Comput. Ind. Eng. 65 (2) (2013) 341–350.
[17] P.P. Fu, C. Wu, J. Tang, Unconventional emergency management based on intelligent group decision-making methodology, Adv. Inf. Sci. Serv. Sci. 4 (7) (2012) 208–216.
[18] F. Herrera, E. Herrera-Viedma, J. Verdegay, A sequential selection process in group decision making with linguistic assessments, Inf. Sci. 85 (1995) 223–239.
[19] D. Kahneman, A. Tversky, Prospect theory: an analysis of decision under risk, Econometrica 47 (2) (1979) 263–291.
[20] A. Kandil, K. El-Rayes, Parallel genetic algorithms for optimizing resource utilization in large-scale construction projects, J. Constr. Eng. Manage. 132 (5) (2006) 491–498.
[21] J.K. Levy, K. Taji, Group decision support for hazards planning and emergency management: a group analytic network process (GANP) approach, Math. Comput. Model. Int. J. 46 (7) (2007) 906–917.
[22] X.W. Li., Decision-making method of highway network planning based on prospect theory, Procedia. Soc. Behav. Sci. 96 (2013) 2042–2050.
[23] Y. Liu, Z.P. Fan, Y. Zhang, Risk decision analysis in emergency response: a method based on cumulative prospect theory, Comput. Oper. Res. 42 (2014) 75–82.
[24] J. Lu, G. Zhang, D. Ruan, et al., Multi-Objective Group Decision Making, Imperial College Press, London, 2006.
[25] J. Ma, Z.P. Fan, L.H. Huang, A subjective and objective integrated approach to determine attribute weights, Eur. J. Oper. Res. 112 (2) (1999) 397–404.
[26] K.O. May, A set of independent necessary and sufficient conditions for simple majority decision, Econometrica 20 (4) (1952) 680–684.
[27] I. Palomares, L. Martínez, Low-dimensional visualization of experts' preferences in urgent group decision making under uncertainty, Procedia. Comput. Sci. 29 (2014) 2090–2101.
[28] P. Pérez-Asurmendi, F. Chiclana, Linguistic majorities with difference in support, Appl. Soft Comput. 18 (2014) 196–208.
[29] J.D. Qin, X.W. Liu, W. Pedrycz, An extended TODIM multi-criteria group decision making method for green supplier selection in interval type-2 fuzzy environment, Eur. J. Oper. Res. 258 (2) (2017) 626–638.
[30] J. Qu, X. Meng, H. You, Multi-stage ranking of emergency technology alternatives for water source pollution accidents using a fuzzy group decision making tool, J. Hazard. Mater. 310 (2016) 68–81.
[31] R.M. Rodríguez, Á. Labella, L. Martínez, An overview on fuzzy modelling of complex linguistic preferences in decision making, Int. J. Comput. Intell. Syst. 9 (2016) 81–94.
[32] R.M. Rodríguez, L. Martínez, An analysis of symbolic linguistic computing models in decision making, Int. J. Gen. Syst. 42 (1) (2013) 121–136.
[33] S. Saint, J.R. Lawson. Rules for reaching consensus: Aa modern approach to decision making, Jossey-Bass, San Francisco 1994.
[34] U. Schmidt, C. Starmer, R. Sugden, Third-generation prospect theory, J. Risk Uncertainty 36 (3) (2008) 203–223.
[35] R.K. Singh, A.K. Choudhury, M.K. Tiwari, Improved decision neural network based on consensus method to solve a multi-objective group decision making problem, Adv. Eng. Inf. 21 (2007) 335–348.
[36] S. Tofighi, S.A. Torabi, S.A. Mansouri, Humanitarian logistics network design under mixed uncertainty, Eur. J. Oper. Res. 250 (1) (2016) 239–250.
[37] F. Torfi, R.Z. Farahani, S. Rezapour, Fuzzy AHP to determine the relative weights of evaluation criteria and fuzzy TOPSIS to rank the alternatives, Appl. Soft Comput. 10 (2) (2010) 520–528.
[38] A. Tversky, D. Kahneman, Loss aversion in riskless choice: a reference dependent model, Q. J. Econ. 106 (4) (1991) 1039–1061.
[39] A. Tversky, D. Kahneman, Advances in prospect theory: cumulative representation of uncertainty, J. Risk Uncertainty 5 (4) (1992) 297–323.
[40] L. Wang, Y.M. Wang, B.X. Hu., Dynamic adjusting method of emergency alternatives based on prospect theory, Control. Decis. 31 (1) (2016) 99–104.

[41] L. Wang, Z.X. Zhang, Y.M. Wang, A prospect theory-based interval dynamic reference point method for emergency decision making, Expert Syst. Appl. 42 (23) (2015) 9379–9388.
[42] X. Wei, Q.Y. Su, J.L. Li., The group decision-making rules based on rough sets on large scale engineering emergency, Syst. Eng. Procedia 4 (2012) 331–337.
[43] X.H. Xu, Z.J. Du, X.H. Chen, Consensus model for multi-criteria large-group emergency decision making considering non-cooperative behaviors and minority opinions, Decis. Support Syst. 79 (2015) 150–160.
[44] X.H. Xu, X.Y. Zhong, X.H. Chen, et al., A dynamical consensus method based on exit–delegation mechanism for large group emergency decision making, Knowl. Based Syst. 86 (2015) 237–249.
[45] Y.J. Xu, W.C. Zhang, H.M. Wang, A conflict-eliminating approach for emergency group decision of unconventional incidents, Knowl. Based Syst. 83 (2015) 92–104.
[46] L. Yu, K.K. Lai, A distance-based group decision-making methodology for multi-person multi-criteria emergency decision support, Decis. Support Syst. 51 (2011) 307–315.
[47] L.Z. Zhou, S.Q. Zhong, S.F. Ma, et al., Prospect theory based estimation of driver risk attitudes in route choice behaviors, Accid. Anal. Prev. 73 (2014) 1–11.

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A dynamic multi-attribute group emergency decision making method considering experts' hesitation

Liang Wang [1,2] **, Rosa M. Rodríguez** [3] **, Ying-Ming Wang** [1, *]

[1] *Decision Sciences Institute, Fuzhou University, No. 2, Xueyuan Road, University Town, Fuzhou 350116, PR China*

E-mail: msymwang@hotmail.com

[2] *Department of Computer Science, University of Jaén, Campus Las Lagunillas, s/n, Jaén, 23071, Spain*

E-mail: wangliangg322@hotmail.com

[3] *Department of Computer Science and A.I., University of Granada, Periodista Daniel Saucedo Aranda, s/n, Granada, 18071, Spain*

E-mail: rosam.rodriguez@decsai.ugr.es

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Abstract

Multi-attribute group emergency decision making (MAGEDM) has become a valuable research topic in the last few years due to its effectiveness and reliability in dealing with real-world emergency events (EEs). Dynamic evolution and uncertain information are remarkable features of EEs. The former means that information related to EEs is usually changing with time and the development of EEs. To make an effective and appropriate decision, such an important feature should be addressed during the emergency decision process; however, it has not yet been discussed in current MAGEDM problems. Uncertain information is a distinct feature of EEs, particularly in their early stage; hence, experts involved in a MAGEDM problem might hesitate when they provide their assessments on different alternatives concerning different criteria. Their hesitancy is a practical and inevitable issue, which plays an important role in dealing with EEs successfully, and should be also considered in real world MAGEDM problems. Nevertheless, it has been neglected in existing MAGEDM approaches. To manage such limitations, this study intends to propose a novel MAGEDM method that deals with not only the dynamic evolution of MAGEDM problems, but also takes into account uncertain information, including experts' hesitation. A case study is provided and comparisons with current approaches and related discussions are presented to illustrate the feasibility and validity of the proposed method.

Keywords: Multi-attribute group decision making, Emergency situation, Dynamic evolution, Experts' hesitation

* Corresponding author. Tel.:+86 591 22866677; E-mail:msymwang@hotmail.com

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1. Introduction

With the increasing occurrence of various emergency events (EEs)—such as production accidents, natural disasters, and terrorist attacks—emergency decision making (EDM) has drawn wide attention across the world in the past few years, and especially due to its prominent part in reducing the property loss and casualties in different EEs. Hence, it has become a pressing and important research topic [10,18,29,31].

When an EE occurs, the information related to it changes across time, leading to dynamic evolution. Furthermore, its information is usually uncertain, especially in the early stages. Therefore, EE information plays an important role in the EDM process; it is necessary to take into account both its dynamic evolution and its uncertainty [10,29] to deal with it satisfactorily.

For executing effective emergency responses using updated information to control the situation and mitigate losses caused by EEs, the dynamic evolution [13,29] and uncertain information [14,31] features have been already discussed in current EDM approaches. Nevertheless, these studies [13,29] examines dynamic evolution considering only time changes; the information regarding the alternatives and criteria [13,29] remain unchanged, even though the EE information changes along with the time. Discrete and dynamic decisions with the latest information might make the EDM more effective and appropriate. On the other hand, current EDM approaches deal with the uncertain information using interval values [31] for quantitative contexts, and linguistic term sets [14] for qualitative contexts. However, due to lack of information and time pressure in EDM, decision makers might hesitate when they have to assess the alternatives and criteria. Thus, hesitant information should be considered in these types of problems [27].

Usually, in classical EDM approaches [10,13,14,18,29,31], only one emergency decision maker (DM) is in charge of the EE. However, it is highly challenging for an individual DM [19] to deal with these complicated emergency situations in real world problems. Consequently, multi-attribute group emergency decision making (MAGEDM) might be a powerful and effective way to cope with complex and damaging EEs. A general scheme of a MAGEDM problem is shown in Fig. 1.

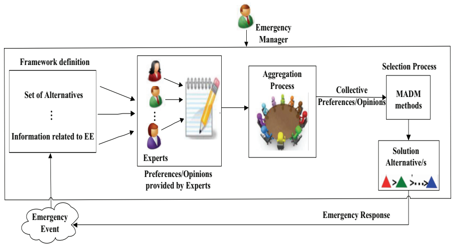

Fig. 1. General scheme of a MAGEDM problem

MAGEDM is a vital decision activity for dealing with real world EEs [11,16,30], wherein experts play the role of think tanks to provide their opinions or assessments of different alternatives regarding different criteria; experts' individual wisdoms are aggregated into a group to help the DM make a final decision.

As far as we know, until now, no proposal in current MAGEDM approaches [35,36,37,38] considers the dynamic evolution of EEs dealing with both the updated information about alternatives and criteria along with the time and the experts' changes (quit or invited to join in the decision process), in addition to the modelling of experts' hesitancy due to uncertain information. Therefore, it is practically significant to address these issues in order to make satisfactory and reasonable decisions in real world MAGEDM problems.

This study aims to develop a new dynamic MAGEDM method that deals with the dynamic evolution of EEs considering both the time changeableness and updated information (alternatives, criteria, and experts). At the same time, it deals with uncertain information by using interval values, linguistic term sets, and linguistic expressions based on hesitant fuzzy linguistic term sets (HFLTS) [27], which are able to model experts' hesitancy.

In dynamic MAGEDM problems, the alternatives are ranked according to the dynamic rating of each alternative at different decision moments. Dynamic rating of each alternative is usually deter-

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mined by the static rating of the alternative at the current decision moment and its dynamic rating in previous one [4]. Therefore, the ranking obtained by using the dynamic ratings could be different from the static ratings. Static ratings are usually obtained by using different multi-attribute decision making methods (MADM) [3,42]. In order to retain uncertain information as much as possible and generate more reasonable decision results, fuzzy TOPSIS method based on alpha-level sets is regarded as the static MADM method in the proposal to obtain the static rating of alternatives at each decision moment because of its capacity and advantages of using uncertain information across the decision process.

The rest of this paper is organized as follows: Section 2 briefly introduces different concepts that will be used in the proposed method. Section 3 presents a novel dynamic MAGEDM method considering experts' hesitancy. In section 4, a case study is introduced, and comparisons with current approaches and related discussions are presented. The conclusions and prospective research areas are offered in section 5.

2. Preliminaries

This section briefly revises basic concepts regarding imprecise and hesitant information and dynamic decision making to understand the proposed dynamic MAGEDM method easily. It also introduces the fuzzy TOPSIS method based on alpha-level sets, which will be utilized as the static MADM in the *computing static rating* process in our proposal to obtain the static rating of alternatives at each decision moment.

2.1. *Dealing with imprecise and hesitant information*

Uncertain information is one of the remarkable features of EEs. It is very important to deal with such type of information to cope with EEs successfully. Therefore, information domains utilized by experts to provide their opinions/assessments in quantitative and qualitative contexts are revised.

(1) *Information domain for quantitative contexts*

In real world problems, it is difficult for experts to provide their assessments using numerical values, when the EE information is uncertain, such as people affected, property losses, or costs of alternatives. However, in such situations, interval values [15,22,31] are suitable for experts to provide their assessments due to their useful and simple technique for representing uncertainty. Thus, interval values are utilized as the information domain for quantitative contexts in our proposal.

Definition 1. [23] *Let $[\eta^L, \eta^U]$ be a domain of the interval value; an interval value I belongs to $[\eta^L, \eta^U]$:*

$$I \in [\eta^L, \eta^U] \tag{1}$$

where η^L and η^U are the lower and upper bounds of the domain, respectively.

(2) *Information domain for qualitative contexts*

A fairly common approach to model qualitative information is the fuzzy linguistic approach [39] based on the fuzzy set theory. Different linguistic models have been discussed in different approaches [20,21,26]. In our proposal, linguistic term sets are utilized to model the uncertain information in qualitative contexts (see Fig. 2).

Definition 2. [27] *Let $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set; a linguistic term, s_i, belongs to S:*

$$s_i \in S = \{s_0, s_1, \ldots, s_g\}, i = 0, 1, \ldots, g \tag{2}$$

where $g+1$ is the granularity of S.

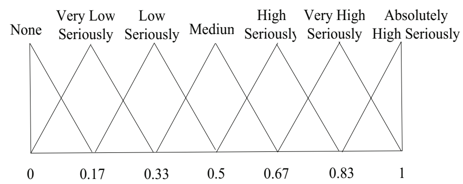

Fig. 2. Linguistic term set

Usually the information of MAGEDM problems is uncertain; experts involved in such problems are bounded by cognition [2] and under pressure because of the urgent time constraints in an emergency response. Moreover, their decision might provoke potentially serious results [16]. Hence, in such situations,

ATLANTIS PRESS International Journal of Computational Intelligence Systems, Vol. 11 (2018) 163–182

it is common for experts to hesitate when they provide their assessments. Therefore, it seems necessary to deal with experts' hesitation in MAGEDM problems.

To model the hesitant information in qualitative contexts, the concept of HFLTS [27] was introduced, drawing increased attention recently [25,27].

Definition 3. [27] *Let $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set; a HFLTS, H_S, on S is an ordered finite subset:*

$$H_S = \{s_i, s_{i+1}, \ldots, s_\varsigma\}, s_\varsigma \in S, \varsigma \in \{i, \ldots, j\} \quad (3)$$

Example 1. *Let S={absolute weak, very weak, weak, medium, good, very good, excellent} be a linguistic term set and δ be a linguistic variable; then, $H_S^1(\delta)$={good, very good} and $H_S^2(\delta)$={very weak, weak, medium} are two HFLTSs on S.*

HFLTS is a powerful and useful tool to model experts' hesitation; the use of context-free grammars [27] allows generation of complex linguistic expressions close to the natural language utilized by human beings in the real world [27,28], which can be modeled by HFLTS. This approach has been widely applied to deal with different decision problems [1,33,34].

Definition 4. [27] *Let $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set and G_H be a context-free grammar. The elements of $G_H = (V_N, V_T, I, P)$ are defined as below:*

V_N ={ $\langle$primary term$\rangle$,$\langle$composite term$\rangle$,$\langle$unary relation$\rangle$,$\langle$binary relation$\rangle$,$\langle$conjunction$\rangle$}

V_T = {lower than, greater than, at least, at most, between, and, $s_0, s_1, \ldots, s_g$}

$I \in V_N$

P = {I:: = $\langle$primary term$\rangle$|$\langle$composite term$\rangle$
$\langle$composite term$\rangle$ :: = $\langle$unary relation$\rangle$ $\langle$primary term$\rangle$|$\langle$binary relation$\rangle$
$\langle$primary term$\rangle$ $\langle$conjunction$\rangle$ $\langle$primary term$\rangle$
$\langle$primary term$\rangle$ :: = s_0 $|s_1| \ldots |s_g$
$\langle$unary relation$\rangle$ :: = lower than |greater than |at least |at most
$\langle$binary relation$\rangle$ :: = between
$\langle$conjunction$\rangle$ :: = and}

S_{ll} denotes the expression domain generated by G_H, which might be either complex linguistic expressions or single linguistic terms.

Example 2. *Considering the context-free grammar, G_H, introduced in Definition 4 and the linguistic term set S from example 1, the following complex linguistic expressions might be obtained:*

S_{ll_1} = at least good
S_{ll_2} = at most medium
S_{ll_3} = between good and very good

Taking into account that experts can provide their assessments by utilizing quantitative and qualitative information in order to make computations with different types of information, it is necessary to unify them into a unique domain. The process of unifying different types of information is presented in section 3.3.

2.2. Dynamic decision making

Some existing dynamic MADM methods [3,4], which have the following remarkable features, are revised:

(i) The alternatives are changeable because they might be deemed non-available or removed; meanwhile new alternatives might be considered and added.
(ii) The criteria are not immobilized, since their values might change along with time, and also, the current criteria might be removed or new criteria might be taken into account.
(iii) The temporal profile of an alternative matters for comparison with other alternatives. This point is referred as the notion of feedback [3,42].

According to these three features, dynamic MADM methods should be capable of managing interdependent decisions in a changing environment, wherein not only alternatives, but criteria might also change (non-available, removed or added new ones, etc.) and the final decisions at each decision moment must consider the feedback from previous ones. Due to the dynamic evolution of EEs, a reasonable and effective MAGEDM method should consider not only the three aforementioned features, but also the changes of experts because they might give up the decision process or new experts might be invited to join the decision process in real world situations.

To make the proposed MAGEDM method understandable, some necessary concepts are first given,

ATLANTIS PRESS International Journal of Computational Intelligence Systems, Vol. 11 (2018) 163–182

and then the dynamic MADM method [3,4,42] is briefly revised.

Definition 5. [3] (**Historical set**) *The historical set of alternatives as decision moment, $t \in T$, is a subset of all alternatives that have ever been available up to and including that decision moment,*

$$H_t \subseteq \bigcup_{s \leqslant t} P_s, \quad s,t \in T \tag{4}$$

Remark 1. [3] *In practical applications, the historical set is updated incrementally. Let $H_0 = \phi$, at each decision moment, $t \in T$. Then, the historical set can be defined as*

$$H_t \subseteq P_t \bigcup H_{t-1}, \quad t \in T \tag{5}$$

Let $T = \{1,2,\ldots\}$ be a set of discrete decision moments (possibly infinite), and P_t be the set of alternatives that are usable at each decision moment, t, $t \in T$. Suppose that a static MADM method is being utilized at each decision moment, $t \in T$, to compute ratings for each available alternative, $p \in P_t$, concerning the assessments of all criteria, $C_t = \{c_1, c_2, \ldots, c_m\}$. The ratings obtained by the static MADM method are called static ratings or non-dynamic ratings, denoted by $R_t(p)$. The dynamic rating of alternatives is computed based on its static rating obtained in the previous stages to which it belonged.

The dynamic decision process deals with a feedback mechanism from previous ones. For any alternative, p, its dynamic rating function, $E_t(p)$, is defined as [3,4,42]:

$$E_t(p) = \begin{cases} R_t(p), & p \in P_t \backslash H_{t-1} \\ \Phi(R_t(p), E_{t-1}(p)), & p \in P_t \cap H_{t-1} \\ E_{t-1}(p), & p \in H_{t-1} \backslash P_t \end{cases} \tag{6}$$

where Φ is an aggregation function (operator).

For each alternative, p, either belonging to the existing set of alternatives, P_t, or carried over from the previous one by means of the historical set, H_{t-1}, there are three different situations.

(i) if the alternative, p, belongs only to the current set of alternatives, P_t, but not to the historical set, H_{t-1}, that is, $p \in P_t \backslash H_{t-1}$, its dynamic rating, $E_t(p)$, is equal to its static rating, $R_t(p)$;

(ii) if the alternative, p, belongs not only to the current, but also the historical set of alternatives, that is, $p \in P_t \bigcap H_{t-1}$, its dynamic rating, $E_t(p)$, is calculated by aggregating its static rating, $R_t(p)$, with its dynamic rating, $E_{t-1}(p)$, at the former decision moment; and

(iii) if the alternative, p, belongs to the historical set of alternatives only, that is, $p \in H_{t-1} \backslash P_t$, its dynamic rating, $E_t(p)$, is equal to $E_{t-1}(p)$.

The dynamic decision process can be conducted for several decision moments. The moment wherein the process is stopped depends on the problem and the DM's assessments.

2.3. Fuzzy TOPSIS method based on alpha-level sets

TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) method was first proposed by Huwang and Yoon [12]; it is a popular MADM method been widely applied to solve different decision problems [5,6,12,32]. To cope with complex problems and uncertain information in the real world, the TOPSIS method has been extended to deal with fuzzy MADM problems [5,6,32].

The fuzzy TOPSIS method based on alpha-level sets [32] is a distinctive and powerful approach among other fuzzy TOPSIS versions [5,6,8,9] due to its prominent advantages of keeping uncertain information in a better way. This is the significant difference between the fuzzy TOPSIS method based on alpha-level sets and other versions. Due to such advantages, the fuzzy TOPSIS method based on alpha-level sets will be used as the static MADM method in order to calculate the static rating of each alternative at different decision moments in our proposal.

In fuzzy MADM problems, criteria/attribute values and the relative weights are usually characterized by fuzzy numbers [6,32]. The most commonly used fuzzy numbers are trapezoidal fuzzy numbers, $\widetilde{A} = (a,b,c,d)$, or triangular fuzzy numbers, $\widetilde{A} = (a,b,d)$, with a degree of membership between 0 and 1. When $b = c$, the triangular fuzzy number is a special case of a trapezoidal fuzzy number.

According to Zadeh's extension principle [41], a fuzzy number/set, $\widetilde{A}$, can be also expressed by its in-

ATLANTIS PRESS International Journal of Computational Intelligence Systems, Vol. 11 (2018) 163–182

tervals, that is,

$$\widetilde{A} = \bigcup_{\alpha} \alpha A_\alpha, 0 \leqslant \alpha \leqslant 1 \tag{7}$$

where

$$\begin{aligned} A_\alpha &= \{x \in X \mid \mu_{\widetilde{A}}(x) \geqslant \alpha\} \\ &= [min\{x \in X \mid \mu_{\widetilde{A}}(x) \geqslant \alpha\}, max\{x \in X \mid \mu_{\widetilde{A}}(x) \geqslant \alpha\}] \end{aligned} \tag{8}$$

α-level sets or α-cuts of $\widetilde{A}$ denoted as A_α. $\mu_{\widetilde{A}}(x)$ is the membership function of fuzzy number $\widetilde{A}$ [32].

Based on the short revision of fuzzy numbers aforementioned, the fuzzy TOPSIS method based on alpha-level sets [32] is briefly introduced.

Let $\widetilde{X} = (\widetilde{x}_{ij})_{n\times m}$ be a fuzzy decision matrix characterized by membership functions, $\mu_{\widetilde{x}_{ij}}(x)$ $(i = 1,\ldots,n,\ j = 1,\ldots,m)$, and $\widetilde{W} = (\widetilde{w}_1,\ldots,\widetilde{w}_m)$ be the fuzzy weights characterized by $\mu_{\widetilde{w}_j}(x)$ $(j = 1,\ldots,m)$. If all the criteria/attributes, $\{c_1,\ldots,c_m\}$, are assessed by using linguistic term sets with the same syntax and semantics, then the fuzzy decision matrix, $\widetilde{X}$, has the same dimension, and therefore, it is not necessary any normalization. Otherwise, $\widetilde{X}$ has to be normalized.

If $\widetilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij}, d_{ij})$ $(i = 1,\ldots,n,\ j = 1,\ldots,m)$ are trapezoidal fuzzy numbers, then the normalization process can be carried out by (the same normalization process for triangular fuzzy numbers)

$$\widetilde{r}_{ij} = (\frac{a_{ij}}{d_j^*}, \frac{b_{ij}}{d_j^*}, \frac{c_{ij}}{d_j^*}, \frac{d_{ij}}{d_j^*}), i = 1,\ldots,n; j \in \Omega_b \tag{9}$$

$$\widetilde{r}_{ij} = (\frac{a_j^-}{d_{ij}}, \frac{a_j^-}{c_{ij}}, \frac{a_j^-}{b_{ij}}, \frac{a_j^-}{a_{ij}}), i = 1,\ldots,n; j \in \Omega_c \tag{10}$$

where

$$d_j^* = \max_i d_{ij}, j \in \Omega_b, \tag{11}$$

$$a_j^- = \min_i a_{ij}, j \in \Omega_c \tag{12}$$

where Ω_b and Ω_c denote the sets of benefit and cost criteria/attributes, respectively.

It can be seen that $\widetilde{r}_{ij}$ belong to [0,1]; thus, positive and negative ideal solutions can be defined as $P^* = \{1,\ldots,1\}$ and $P^- = \{0,\ldots,0\}$, respectively. For a fuzzy decision matrix, $\widetilde{X} = (\widetilde{x}_{ij})_{n\times m}$, without normalization, the positive and negative ideal solutions can be obtained as follows:

$$\begin{aligned} P^* &= \{x_1^*,\ldots,x_m^*\} \\ &= \{(\max_j d_{ij}, j \in \Omega_b), (\min_j a_{ij}, j \in \Omega_c)\} \end{aligned} \tag{13}$$

$$\begin{aligned} P^- &= \{x_1^-,\ldots,x_m^-\} \\ &= \{(\min_j a_{ij}, j \in \Omega_b), (\max_j d_{ij}, j \in \Omega_c)\} \end{aligned} \tag{14}$$

Let $(r_{ij})_\alpha = [(r_{ij})_\alpha^L, (r_{ij})_\alpha^U]$ and $(w_j)_\alpha = [(w_j)_\alpha^L, (w_j)_\alpha^U]$ be alpha-level sets of $\widetilde{r}_{ij}$ and $\widetilde{w}_j$, respectively. Then, the relative closeness (RC), RC_i, of the alternative, p_i, with respect to P^* can be written as:

$$RC_i = \frac{\sqrt{\sum_{j=1}^{m} (w_j r_{ij})^2}}{\sqrt{\sum_{j=1}^{m} (w_j r_{ij})^2} + \sqrt{\sum_{j=1}^{m} (w_j (r_{ij} - 1))^2}} \tag{15}$$

where

$$(w_j)_\alpha^L \leqslant w_j \leqslant (w_j)_\alpha^U, j = 1,\ldots,m \tag{16}$$

$$(r_{ij})_\alpha^L \leqslant r_{ij} \leqslant (r_{ij})_\alpha^U, j = 1,\ldots,m, i = 1,\ldots,n \tag{17}$$

RC_i is an interval value based on Eq. (15); its upper and lower bounds can be calculated by utilizing the following simplified pair of fractional programming models (see [32] for further details):

$$\begin{aligned} (RC_i)_\alpha^U &= Max \frac{\sqrt{\sum_{j=1}^{m} (w_j (r_{ij})_\alpha^U)^2}}{\sqrt{\sum_{j=1}^{m} (w_j (r_{ij})_\alpha^U)^2} + \sqrt{\sum_{j=1}^{m} (w_j ((r_{ij})_\alpha^U - 1))^2}} \\ s.t. &\quad (w_j)_\alpha^L \leqslant w_j \leqslant (w_j)_\alpha^U, j = 1,\ldots,m \end{aligned} \tag{18}$$

$$\begin{aligned} (RC_i)_\alpha^L &= Min \frac{\sqrt{\sum_{j=1}^{m} (w_j (r_{ij})_\alpha^L)^2}}{\sqrt{\sum_{j=1}^{m} (w_j (r_{ij})_\alpha^L)^2} + \sqrt{\sum_{j=1}^{m} (w_j ((r_{ij})_\alpha^L - 1))^2}} \\ s.t. &\quad (w_j)_\alpha^L \leqslant w_j \leqslant (w_j)_\alpha^U, j = 1,\ldots,m \end{aligned} \tag{19}$$

When different alpha levels are set, then $(RC_i)_\alpha = [(RC_i)_\alpha^L, (RC_i)_\alpha^U]$ can be obtained by solving Eqs. (18) and (19), respectively. According to Eq. (7), $\widetilde{RC}_i$ can be expressed as:

$$\begin{aligned} \widetilde{RC}_i &= \bigcup_\alpha \alpha \cdot (RC_i)_\alpha \\ &= \bigcup_\alpha \alpha [(RC_i)_\alpha^L, (RC_i)_\alpha^U], 0 \leqslant \alpha \leqslant 1 \end{aligned} \tag{20}$$

ATLANTIS PRESS International Journal of Computational Intelligence Systems, Vol. 11 (2018) 163–182

where $\widetilde{RC_i}$ represents the fuzzy RC of alternative, p_i, based on corresponding alpha levels from 0 to 1.

According to Eq. (6), the dynamic ratings of alternatives are related not only to their static ones, but also their performance in previous stages if it has one. In order to calculate the dynamic ratings of alternatives, it is firstly necessary to compute the static ratings of alternatives. The averaging level cuts [24] are used in this paper for sake of simplicity to obtain the static ratings of alternatives.

Let $\alpha_1,\ldots,\alpha_K$ be different alpha levels; the static rating, $m(\widetilde{RC_i})$, of alternative, p_i, can be determined by [24]

$$m(\widetilde{RC_i}) = \frac{1}{K}\sum_{k=1}^{K}\left(\frac{(RC_i)^L_{\alpha_k}+(RC_i)^U_{\alpha_k}}{2}\right), i=1,\ldots,n \quad (21)$$

where K is the number of alpha levels.

3. Dynamic MAGEDM method considering experts' hesitation

This section introduces a novel dynamic MAGEDM method that is able to: (a) consider the dynamic evolution feature of EEs in MAGEDM problems; and (b) deal with uncertain information using interval values in quantitative contexts, linguistic terms in qualitative contexts, and model experts' hesitation by means of complex linguistic expressions based on HFLTS.

This proposal extends the general scheme of the MAGEDM process shown in Fig. 1 by adding two new phases to unify the information provided by experts (*unification process*), and then, compute the dynamic rating (*computing dynamic rating*). The aggregation process has been modified, and the selection process is replaced by a new phase adapted to dynamic MAGEDM problem (*computing static rating*). These phases are highlighted in Fig. 3 by using dash lines.

The proposed dynamic MAGEDM method consists of six main phases:

(a) *Framework definition*. It defines the structure of the dynamic MAGEDM problem (notions for decision moments, experts, alternatives, etc.) and the expression domains for quantitative and qualitative contexts wherein assessments can be elicited by involved experts.

(b) *Gathering information*. Assessments of or opinions on different alternatives concerning different criteria and criteria importance are provided by experts at each decision moment.

(c) *Unification process*. The information provided by experts at each decision moment is unified into a fuzzy domain to carry out the computations.

(d) *Aggregation process*. In this process, the unified fuzzy information about the opinions, and criteria importance provided by experts are aggregated.

(e) *Computing static rating*. Fuzzy TOPSIS method based on alpha-level sets is utilized as the static MADM method to calculate the static rating of each alternative at each decision moment.

(f) *Computing dynamic rating*. Dynamic rating for each alternative at each decision moment takes into account not only its static rating in the current stage, but also its performance in previous ones.

These phases are further detailed in the following subsections.

3.1. Framework definition

The following notions and terminology will be used in the proposed dynamic MAGEDM method.

- $T=\{1,2,3,\ldots\}$: the set of discrete decision moments (possible infinite), for each decision moment, $t\in T$.
- $P_t=\{p_1,p_2,\ldots,p_n\}$: the set of available alternatives at decision moment, t, where p_i denotes the *i-th* alternative, $i=1,2,\ldots,n$.
- $C_t=\{c_1,c_2,\ldots,c_m\}$: the set of criteria/attributes at decision moment, t, where c_j denotes the *j-th* criterion/attribute, $j=1,2,\ldots,m$.
- $E_t=\{e_1,e_2,\ldots,e_H\}$: the set of experts at decision moment, t, where e_h denotes the *h-th* expert, $h=1,2,\ldots,H$. In dynamic MAGEDM problems, the experts might leave or be added during the decision process according to expert's willingness or decision problems.

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International Journal of Computational Intelligence Systems, Vol. 11 (2018) 163–182

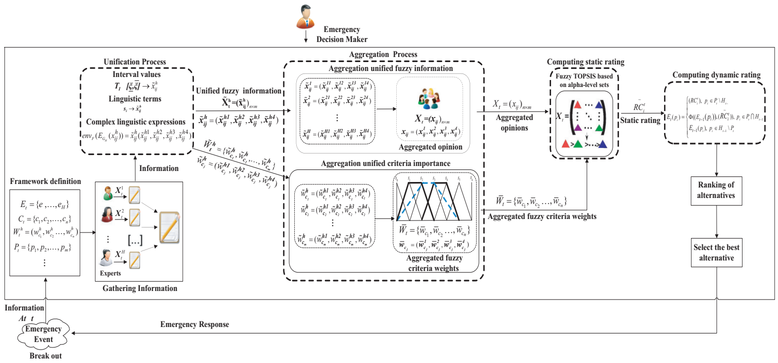

Fig. 3. Dynamic MAGEDM method considering experts' hesitation

- $X_t^h = (x_{ij}^h)_{n\times m}$: the information matrix provided by the expert, e_h, at decision moment, t, where x_{ij}^h denotes the opinions/assessments provided by the h-th expert over the i-th alternative regarding j-th criterion, $h = 1,2,\dots,H$; $i = 1,2,\dots,n$; $j = 1,2,\dots,m$ (see **Remark** 2).
- $\widetilde{X}_t^h = (\widetilde{x}_{ij}^h)_{n\times m}$: the unified information matrix with respect to X_t^h, where $\widetilde{x}_{ij}^h$ denotes the unified fuzzy information corresponding to x_{ij}^h, $h = 1,2,\dots,H$; $i = 1,2,\dots,n$; $j = 1,2,\dots,m$.
- $X_t = (x_{ij})_{n\times m}$: denotes the aggregated information matrix regarding $\widetilde{X}_t^h$, at decision moment, t, $i = 1,2,\dots,n$; $j = 1,2,\dots,m$.
- $W_t^h = (w_{c_1}^h, w_{c_2}^h, \dots, w_{c_m}^h)$: the assessments vector regarding the criteria importance provided by the h-th expert at decision moment, t, where $w_{c_j}^h$ denotes the h-th expert's assessments on the criterion c_j, $h = 1,2,\dots,H$, $j = 1,2,\dots,m$ (see **Remark** 3).
- $\widetilde{W}_t^h = (\widetilde{w}_{c_1}^h, \widetilde{w}_{c_2}^h, \dots, \widetilde{w}_{c_m}^h)$: the unified fuzzy information vector with respect to W_t^h, where $\widetilde{w}_{c_j}^h$ denotes the unified fuzzy information corresponding to $w_{c_j}^h$, $h = 1,2,\dots,H$, $j = 1,2,\dots,m$.
- $\overline{W}_t = (\overline{w}_{c_1}, \overline{w}_{c_2}, \dots, \overline{w}_{c_m})$: the aggregated information vector regarding $\widetilde{W}_t^h$, at decision moment, t, $j = 1,2,\dots,m$.

Remark 2. The expression domains used by experts to express their assessments, x_{ij}^h, will be interval values (I) for quantitative contexts and linguistic terms and complex linguistic expressions for qualitative contexts, which have been revised in section 2.1.

$$x_{ij}^h \in \begin{cases} I \in [\eta^L, \eta^U] \\ s_i \in S = \{s_0, s_1, \dots, s_g\} \\ S_{ll} \end{cases} \tag{22}$$

Remark 3. The expression domains for the criteria importance are either single linguistic terms, $s_i \in S$, or complex linguistic expressions, S_{ll}, because they are close to the natural language employed by people in real world.

$$w_{c_j}^h \in \begin{cases} s_i \in S = \{s_0, s_1, \dots, s_g\} \\ S_{ll} \end{cases} \tag{23}$$

3.2. *Gathering information*

The opinions/assessments, x_{ij}^h, over the alternatives, p_i, regarding criteria, c_j, and the assessments over the criteria importance, $w_{c_j}^h$, provided by the expert, e_h, at decision moment, t, are gathered below.

$$X_t^h = \begin{array}{c} \\ p_1 \\ p_2 \\ \vdots \\ p_n \end{array} \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & \dots & c_m \end{array} \\ \begin{bmatrix} x_{11}^1 & x_{12}^1 & \cdots & x_{1m}^1 \\ x_{21}^1 & x_{22}^1 & \cdots & x_{2m}^1 \\ \vdots & \vdots & \dots & \vdots \\ x_{n1}^1 & x_{n2}^1 & \cdots & x_{nm}^1 \end{bmatrix} \end{array}$$

ATLANTIS PRESS International Journal of Computational Intelligence Systems, Vol. 11 (2018) 163–182

where $x_{ij}^h \in \begin{cases} I \in [\eta^L, \eta^U] \\ S = \{s_0, s_1, \ldots, s_g\} \\ S_{ll} \end{cases}$, $i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, m$; $h = 1, 2, \ldots, H$.

$$W_t^h = \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & \cdots & c_m \end{array} \\ \left[\begin{array}{cccc} w_{c_1}^h & w_{c_2}^h & \cdots & w_{c_m}^h \end{array} \right] \end{array}$$

where $w_{c_j}^h \in \begin{cases} S = \{s_0, s_1, \ldots, s_g\} \\ S_{ll} \end{cases}$, $j = 1, 2, \ldots, m$; $h = 1, 2, \ldots, H$.

3.3. *Unification process*

In this proposal, the expression domains used by experts can be interval values (I), linguistic terms (s_i), or complex linguistic expressions (S_{ll}).

- *Interval values*. Assessments represented by interval values, I, belong to a special domain, $[\eta^L, \eta^U]$, that is, $I \in [\eta^L, \eta^U]$.
- *Linguistic terms*. Assessments represented by linguistic terms s_i, belong to a linguistic term set $S = \{s_0, s_1, \ldots, s_g\}$, that is, $s_i \in S$, where $g + 1$ is the granularity of S.
- *Complex linguistic expressions*. Assessments represented by S_{ll}, generated by G_H (see Definition 4).

As mentioned in section 2.1, to deal with quantitative and qualitative information, a unification process is needed to facilitate the computations.

In order to retain uncertain information, including experts' hesitation, and obtain more reliable results, the assessments, $X_t^h = (x_{ij}^h)_{n\times m}$, and criteria importance, $W_t^h = (w_{c_1}^h, w_{c_2}^h, \ldots, w_{c_m}^h)$, are transformed into its corresponding fuzzy domains, $\widetilde{X}_t^h = (\widetilde{x}_{ij}^h)_{n\times m}$ and $\widetilde{W}_t^h = (\widetilde{w}_{c_1}^h, \widetilde{w}_{c_2}^h, \ldots, \widetilde{w}_{c_m}^h)$, by using transformation functions (see Fig. 4).

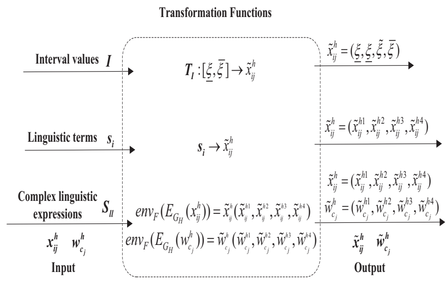

Fig. 4. Transformation functions

The transformation functions are detailed below:

1) Interval values, I, are first normalized into $[0, 1]$, and then, transformed into trapezoidal fuzzy numbers by using a transformation function, T_I. Let $[\eta^L, \eta^U]$ be the domain of the interval values for quantitative contexts; let $x_{ij}^h = [d^L, d^U]$ be the information provided by the expert, e_h, over the *i-th* alternative concerning the *j-th* criterion at decision moment, t, where $x_{ij}^h = [d^L, d^U] \in [\eta^L, \eta^U]$. The interval values, $[d^L, d^U]$, are normalized into $[\underline{\xi}, \overline{\xi}] \in [0, 1]$ as follows:

$$\underline{\xi} = \frac{d^L - \eta^L}{\eta^U - \eta^L} \ and \ \overline{\xi} = \frac{d^U - \eta^L}{\eta^U - \eta^L} \tag{24}$$

The transformation function, T_I, is defined as follows.

Definition 6. Transformation function, T_I, transforms an interval value into a trapezoidal fuzzy number:

$$\begin{array}{l} T_I : [\underline{\xi}, \overline{\xi}] \to \widetilde{x}_{ij}^h \\ T_I(\underline{\xi}, \overline{\xi}) = \widetilde{x}_{ij}^h(\underline{\xi}, \underline{\xi}, \overline{\xi}, \overline{\xi}) \end{array} \tag{25}$$

where $\underline{\xi} \leqslant \overline{\xi}$, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$, $h = 1, 2, \ldots, H$.

2) Linguistic terms, x_{ij}^h and $w_{c_j}^h$, belonging to $S = \{s_0, s_1, \ldots, s_g\}$, are represented by trapezoidal fuzzy numbers. Therefore, their corresponding fuzzy domains are $\widetilde{x}_{ij}^h(\widetilde{x}_{ij}^{h1}, \widetilde{x}_{ij}^{h2}, \widetilde{x}_{ij}^{h3}, \widetilde{x}_{ij}^{h4})$ and $\widetilde{w}_{c_j}^h(\widetilde{w}_{c_j}^{h1}, \widetilde{w}_{c_j}^{h2}, \widetilde{w}_{c_j}^{h3}, \widetilde{w}_{c_j}^{h4})$, respectively.

ATLANTIS PRESS International Journal of Computational Intelligence Systems, Vol. 11 (2018) 163–182

3) Complex linguistic expressions, x_{ij}^h and $w_{c_j}^h$, belonging to S_{ll}, are transformed into HFLTS using the transformation function, $E_{G_H}(\cdot)$ [27], and its fuzzy envelop, $env_F(E_{G_H}(\cdot))$, is obtained by [17]:

$$\begin{array}{l} env_F(E_{G_H}(x_{ij}^h)) = \widetilde{x}_{ij}^h(\widetilde{x}_{ij}^{h1}, \widetilde{x}_{ij}^{h2}, \widetilde{x}_{ij}^{h3}, \widetilde{x}_{ij}^{h4}) \\ env_F(E_{G_H}(w_{c_j}^h)) = \widetilde{w}_{c_j}^h(\widetilde{w}_{c_j}^{h1}, \widetilde{w}_{c_j}^{h2}, \widetilde{w}_{c_j}^{h3}, \widetilde{w}_{c_j}^{h4}) \end{array} \tag{26}$$

According to Eqs. (24)–(26), the gathered information, $X_t^h = (x_{ij}^h)_{n\times m}$ and $W_t^h = (w_{c_1}^h, w_{c_2}^h, \ldots, w_{c_m}^h)$, can be transformed into its corresponding fuzzy domain $\widetilde{X}_t^h = (\widetilde{x}_{ij}^h)_{n\times m}$ and $\widetilde{W}_t^h = (\widetilde{w}_{c_1}^h, \widetilde{w}_{c_2}^h, \ldots, \widetilde{w}_{c_m}^h)$, respectively.

3.4. Aggregation process

The aggregation process is the process wherein experts' opinions are aggregated to obtain collective values for each alternative and criteria weights.

The unified information, $\widetilde{X}_t^h$ and $\widetilde{W}_t^h$, are aggregated to calculate the static rating of alternatives at each decision moment, t. This phase consists of two sub-aggregation processes (see Fig.5): 1) *aggregation of unified fuzzy information* and 2) *aggregation of unified criteria importance*, which are explained as:

1) *Aggregation of unified fuzzy information*.

The aggregated fuzzy information matrix at decision moment, t, $X_t = (x_{ij})_{n\times m}$, where $x_{ij} = (x_{ij}^1, x_{ij}^2, x_{ij}^3, x_{ij}^4)$ is obtained by means of the unified fuzzy information, $\widetilde{X}_t^h = (\widetilde{x}_{ij}^h)_{n\times m}$, where $\widetilde{x}_{ij}^h = (\widetilde{x}_{ij}^{h1}, \widetilde{x}_{ij}^{h2}, \widetilde{x}_{ij}^{h3}, \widetilde{x}_{ij}^{h4})$ is given by:

$$\begin{array}{ll} x_{ij}^1 = \min\limits_h\{\widetilde{x}_{ij}^{h1}\}, & x_{ij}^2 = \frac{1}{H}\sum_{h=1}^H \widetilde{x}_{ij}^{h2} \\ x_{ij}^3 = \frac{1}{H}\sum_{h=1}^H \widetilde{x}_{ij}^{h3}, & x_{ij}^4 = \max\limits_h\{\widetilde{x}_{ij}^{h4}\} \end{array} \tag{27}$$

where $h = 1,2,\ldots,H$; $i = 1,2,\ldots,n$, $j = 1,2,\ldots,m$.

2) *Aggregation of unified criteria importance*.

The aggregated fuzzy criteria weights at decision moment, t, $\overline{W}_t = \{\overline{w}_{c_1}, \overline{w}_{c_2}, \ldots, \overline{w}_{c_m}\}$, where $\overline{w}_{c_j} = (\overline{w}_{c_j}^1, \overline{w}_{c_j}^2, \overline{w}_{c_j}^3, \overline{w}_{c_j}^4)$ can be obtained according to $\widetilde{W}_t^h = \{\widetilde{w}_{c_1}^h, \widetilde{w}_{c_2}^h, \ldots, \widetilde{w}_{c_m}^h\}$, where $\widetilde{w}_{c_j}^h = (\widetilde{w}_{c_j}^{h1}, \widetilde{w}_{c_j}^{h2}, \widetilde{w}_{c_j}^{h3}, \widetilde{w}_{c_j}^{h4})$, utilizing similar equations to Eq. (27):

$$\begin{array}{ll} \overline{w}_{c_j}^1 = \min\limits_h\{\widetilde{w}_{c_j}^{h1}\}, & \overline{w}_{c_j}^2 = \frac{1}{H}\sum_{h=1}^H \widetilde{w}_{c_j}^{h2} \\ \overline{w}_{c_j}^3 = \frac{1}{H}\sum_{h=1}^H \widetilde{w}_{c_j}^{h3}, & \overline{w}_{c_j}^4 = \max\limits_h\{\widetilde{w}_{c_j}^{h4}\} \end{array} \tag{28}$$

where $h = 1,2,\ldots,H$, $j = 1,2,\ldots,m$.

The advantages of the aggregation equations above are not only to retain uncertain information as much as possible and take into account all involved experts' opinions in the dynamic MAGEDM process, but also to ease computation.

3.5. Computing static rating

As noted earlier, the fuzzy TOPSIS method based on alpha-level sets is utilized as the static MADM method to obtain the static ratings of alternatives at each decision moment, t, in our proposal. Since the aggregated fuzzy information matrix, $X_t = (x_{ij})_{n\times m}$ and $\overline{W}_t$, have been already normalized in the *unification process*, it is not necessary to normalize $X_t = (x_{ij})_{n\times m}$ and $\overline{W}_t$ again. Thus, the positive and negative ideal solutions are $P^* = \{1,\ldots,1\}$, and $P^- = \{0,\ldots,0\}$, respectively.

Let $(y_{ij})_\alpha = [(y_{ij})_\alpha^L, (y_{ij})_\alpha^U]$ and $(w_{c_j})_\alpha = [(w_{c_j})_\alpha^L, (w_{c_j})_\alpha^U]$ be the alpha-level sets of x_{ij} and $\overline{w}_{c_j}$, respectively, at decision moment, t. The RC of the alternative, p_i, based on different alpha levels, $(RC_i^t)_\alpha^U$, and $(RC_i^t)_\alpha^L$ can be obtained by using Eqs. (29) and (30), respectively.

$$\begin{array}{l} (RC_i^t)_\alpha^U = Max \dfrac{\sqrt{\sum\limits_{j=1}^m (w_{c_j}(y_{ij})_\alpha^U)^2}}{\sqrt{\sum\limits_{j=1}^m (w_{c_j}(y_{ij})_\alpha^U)^2} + \sqrt{\sum\limits_{j=1}^m (w_{c_j}((y_{ij})_\alpha^U - 1))^2}} \\ s.t. \quad (w_{c_j})_\alpha^L \leqslant w_{c_j} \leqslant (w_{c_j})_\alpha^U, j = 1,\ldots,m \end{array} \tag{29}$$

$$\begin{array}{l} (RC_i^t)_\alpha^L = Min \dfrac{\sqrt{\sum\limits_{j=1}^m (w_{c_j}(y_{ij})_\alpha^L)^2}}{\sqrt{\sum\limits_{j=1}^m (w_{c_j}(y_{ij})_\alpha^L)^2} + \sqrt{\sum\limits_{j=1}^m (w_{c_j}((y_{ij})_\alpha^L - 1))^2}} \\ s.t. \quad (w_{c_j})_\alpha^L \leqslant w_{c_j} \leqslant (w_{c_j})_\alpha^U, j = 1,\ldots,m \end{array} \tag{30}$$

Similar to Eqs. (20) and (21), the fuzzy RC of the alternative, p_i, with different alpha-level sets at decision moment, t, with our notation can be expressed as follows:

$$\begin{array}{l} \widetilde{RC}_i^t = \bigcup_\alpha \alpha \cdot (RC_i^t)_\alpha \\ \qquad = \bigcup_\alpha \alpha[(RC_i^t)_\alpha^L, (RC_i^t)_\alpha^U], 0 \leqslant \alpha \leqslant 1 \end{array} \tag{31}$$

ATLANTIS PRESS International Journal of Computational Intelligence Systems, Vol. 11 (2018) 163–182

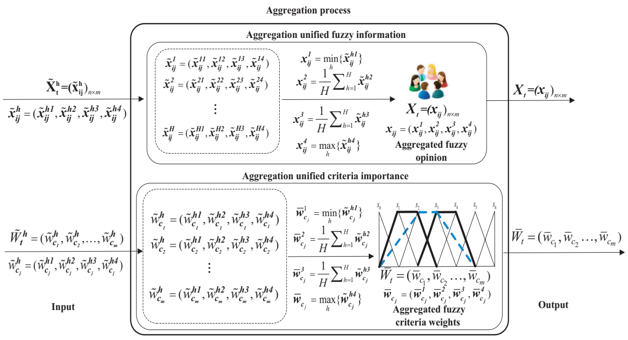

Fig. 5. Aggregation process

According to Eq. (21), the static ratings of alternatives in this study can be obtained as follows:

$$m(\widetilde{RC}_i^t) = \frac{1}{K}\sum_{k=1}^{K}\left(\frac{(RC_i^t)^L_{\alpha_k} + (RC_i^t)^U_{\alpha_k}}{2}\right), i = 1,\ldots,n \quad (32)$$

3.6. Computing dynamic rating

Since EE information changes along with time (alternatives, criteria, and experts), leading to dynamic evolution, it seems necessary to consider the dynamic rating of each alternative. This is a comprehensive factor that indicates the performance of the alternative not only in its current stage, but also in the previous one. In this proposal, the dynamic rating of alternatives based on Eq. (6) is as follows:

$$E_t(p_i) = \begin{cases} m(\widetilde{RC}_i^t), & p_i \in P_t \backslash H_{t-1} \\ \Phi((E_{t-1}(p_i), m(\widetilde{RC}_i^t)), & p_i \in P_t \cap H_{t-1} \\ E_{t-1}(p_i), & p_i \in H_{t-1} \backslash P_t \end{cases} \quad (33)$$

where Φ is an associative aggregation operator that can apply different types of reinforcements (such as, downward reinforcement, upward reinforcement, and full reinforcement) for enhancing different performances in the dynamic context. (see Ref. 4 for details).

The operator selection and reinforcement depend on the characteristics of the problem.

Definition 7. [42] A probabilistic sum function, Φ, is defined as:

$\Phi((E_{t-1}(p_i), m(\widetilde{RC}_i^t))) = E_{t-1}(p_i) + m(\widetilde{RC}_i^t) - E_{t-1}(p_i) \times m(\widetilde{RC}_i^t)$

The ranking of the alternatives is obtained according to the dynamic ratings, $E_t(p_i)$; the higher dynamic rating the better alternative.

4. Case study, comparison with other approaches and discussions

In order to illustrate the feasibility and validity of the proposed dynamic MAGEDM method, a case study adapted from a big explosion[†] that occurred in China is provided, followed by comparisons with other approaches and related discussions.

4.1. Case study

A big explosion took place at a container storage station at the Port of Tianjin, which contained hazardous and flammable chemicals, including sodium nitrate, calcium carbide, and ammonium nitrate, among others. The local government organized relevant departments (fire department, traffic management department, hygiene department, etc.) to collaborate in order to address the emergency situation. Short messages were sent to inform citizens within

[†] Background Information. http://www.safehoo.com/Case/Case/Blow/201602/428723.shtml

ATLANTIS PRESS International Journal of Computational Intelligence Systems, Vol. 11 (2018) 163–182

one kilometer to evacuate to safe areas. In this example, when the explosion occurred, the decision moment, $t = 1$.

4.1.1. Decision moment t=1

Step 1. Framework definition

Assume that three experts $E_1 = \{e_1, e_2, e_3\}$ are invited to join in the MAGEDM process to help the DM to make a decision. Three available alternatives, $P_1 = \{p_1, p_2, p_3\}$, were put forward concerning three criteria, $C_1 = \{c_1, c_2, c_3\}$, which are given in Tables 1 and 2, respectively.

Table 1. Description of available alternatives at $t = 1$

Alternatives	Description
p_1	Inform and evacuate citizens, and meanwhile assign 10 fire squadrons, 300 fire fighters, and 40 fire engines to deal with the explosion.
p_2	Increase to 30 fire squadrons, 900 fire fighters, and 55 fire engines; at the same time, report the latest news to the citizens to avoid panic and riots.
p_3	Ask the professional emergency rescue military for emergency rescue with more than 300 soldiers carrying specific equipment join in the rescue.

Table 2. Description of criteria at $t = 1$

Criteria	Expression domain	Description
People affected (c_1)	I	It means the alternative, p_i, can protect the number of people from the effects caused by EE in domain [0,1000].
Environment affected (c_2)	S_1, S_{II}	It is evaluated by experts by using $s_i \in S_1$={None (N), Very Low Seriously (VLS), Low Seriously (LS), Medium (M), High Seriously (HS), Very High Seriously (VHS), Absolutely High Seriously (AHS)} and S_{II} generated by G_H on S_1 (see Fig. 2).
Property loss (c_3)	I	It means that the alternative, p_i, can protect the direct and indirect property losses caused by EE in domain [0,10] (in billion RMB).

Step 2. Gathering information

The criteria importance, W_1^h, provided by the three experts using linguistic terms $s_i \in S_2$={None (N), Very Low Importance (VLI), Low Importance (LI), Medium Importance (MI), High Importance (HI), Very High Importance (VHI), Absolutely High Importance (AHI)}, and S_{II} generated by G_H on the S_2, are shown in Table 3 ("bt" stands for "between").

Table 3. Criteria importance W_1^h provided by experts at $t = 1$

W_1^h	Criteria		
	c_1	c_2	c_3
W_1^1	VHI	HI	LI
W_1^2	VHI	HI	LI
W_1^3	VHI	bt MI and HI	VLI

The assessments, X_1^h, provided by the three experts over three available alternatives concerning the three criteria at $t = 1$ are shown in Table 4.

Table 4. Assessments X_1^h provided by experts at $t = 1$

X_1^h		Criteria		
		c_1	c_2	c_3
X_1^1	x_{1j}^1	[50,80]	VLS	[0.3,0.5]
	x_{2j}^1	[80,100]	M	[0.4,0.5]
	x_{3j}^1	[45,55]	M	[0.25,0.35]
X_1^2	x_{1j}^2	[40,60]	LS	[0.2,0.3]
	x_{2j}^2	[80,110]	M	[0.3,0.5]
	x_{3j}^2	[30,40]	HS	[0.2,0.25]
X_1^3	x_{1j}^3	[50,60]	LS	[0.18,0.25]
	x_{2j}^3	[70,120]	M	[0.45,0.6]
	x_{3j}^3	[35,45]	At most HS	[0.2,0.3]

Step 3. Unification process

The experts' assessments, X_1^h and W_1^h, at $t = 1$ are transformed into a fuzzy domain by means of the transformation functions defined in section 3.3. The unified results are shown in Tables 5 and 6, respectively.

Table 5. Unified results $\widetilde{X}_1^h$ regarding X_1^h at $t = 1$

$\widetilde{X}_1^h$		Criteria		
		c_1	c_2	c_3
$\widetilde{X}_1^1$	$\widetilde{x}_{1j}^1$	(0.05,0.05,0.08,0,08)	(0,0.17,0.17,0.33)	(0.03,0.03,0.05,0.05)
	$\widetilde{x}_{2j}^1$	(0.08,0.08,0.1,0.1)	(0.33,0.5,0.5,0.67)	(0.04,0.04,0.05,0.05)
	$\widetilde{x}_{3j}^1$	(0.045,0.045,0.055,0.055)	(0.33,0.5,0.5,0.67)	(0.025,0.025,0.035,0.035)
$\widetilde{X}_1^2$	$\widetilde{x}_{1j}^2$	(0.04,0.04,0.06,0.06)	(0.17,0.33,0.33,0.5)	(0.02,,0.02,0.03,0.03)
	$\widetilde{x}_{2j}^2$	(0.08,0.08,0.11,0.11)	(0.33,0.5,0.5,0.67)	(0.03,0.03,0.05,0.05)
	$\widetilde{x}_{3j}^2$	(0.03,0.03,0.04,0.04)	(0.5,0.67,0.67,0.83)	(0.02,0.02,0.025,0.025)
$\widetilde{X}_1^3$	$\widetilde{x}_{1j}^3$	(0.05,0.05,0.06,0.06)	(0.17,0.33,0.33,0.5)	(0.018,0.018,0.025,0.025)
	$\widetilde{x}_{2j}^3$	(0.07,0.07,0.12,0.12)	(0.33,0.5,0.5,0.67)	(0.045,0.045,0.06,0.06)
	$\widetilde{x}_{3j}^3$	(0.035,0.035,0.045,0.045)	(0,0,0.59,0.84)	(0.02,0.02,0.03,0.03)

Table 6. Unified results $\widetilde{W}_1^h$ regarding W_1^h at $t = 1$

$\widetilde{W}_1^h$	Criteria		
	c_1	c_2	c_3
$\widetilde{W}_1^1$	(0.67,0.83,0.83,1)	(0.5,0.67,0.67,0.83)	(0.17,0.33,0.33,0.5)
$\widetilde{W}_1^2$	(0.67,0.83,0.83,1)	(0.5,0.67,0.67,0.83)	(0.17,0.33,0.33,0.5)
$\widetilde{W}_1^3$	(0.67,0.83,0.83,1)	(0.34,0.5,0.67,0.84)	(0,0.17,0.17,0.33)

Step 4. Aggregation process

Based on Tables 5 and 6, the aggregated results, X_1 and $\overline{W}_1$, at $t = 1$ are shown in Table 7 by using the Eqs. (27) and (28), respectively.

Table 7. Aggregated results X_1 and $\overline{W}_1$ regarding $\widetilde{X}_1^h$ and $\widetilde{W}_1^h$ at $t = 1$

Aggregated results		Criteria		
		c_1	c_2	c_3
X_1	x_{1j}	(0.040,0.047,0.067,0,080)	(0,0.277,0.277,0.500)	(0.018,0.023,0.035,0.050)
	x_{2j}	(0.070,0.077,0.110,0.120)	(0.330,0.500,0.500,0.670)	(0.030,0.038,0.053,0.060)
	x_{3j}	(0.030,0.037,0.047,0.055)	(0,0.390,0.587,0.840)	(0.020,0.022,0.030,0.035)
$\overline{W}_1$	$\overline{w}_{c_j}$	(0.670,0.830,0.830,1)	(0.340,0.613,0.670,0.840)	(0,0.277,0.277,0.500)

ATLANTIS PRESS International Journal of Computational Intelligence Systems, Vol. 11 (2018) 163–182

Step 5. Computing static rating

In this case study, 11 alpha-levels are set for computing the fuzzy RC of each alternative [32], that is, $\alpha = \{0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0\}$. According to Eqs. (29)–(32), the results $\widetilde{RC}_i^1$ and $m(\widetilde{RC}_i^1)$ are shown in Table 8 and the fuzzy RC of alternatives graphically shown in Fig. 6.

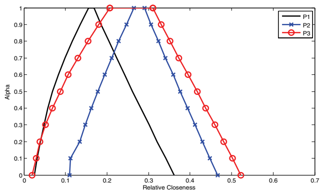

Figure 6: The fuzzy RC of p_i at $t = 1$

Table 8. Alpha-level sets of fuzzy relative closenesses of the three alternatives at $t = 1$

Alpha	Alternatives		
	p_1	p_2	p_3
0	[0.025,0.362]	[0.110,0.466]	[0.019,0.522]
0.1	[0.032,0.342]	[0.112,0.448]	[0.030,0.502]
0.2	[0.040,0.321]	[0.134,0.430]	[0.039,0.481]
0.3	[0.048,0.301]	[0.148,0.412]	[0.052,0.460]
0.4	[0.057,0.281]	[0.162,0.394]	[0.068,0.438]
0.5	[0.069,0.261]	[0.178,0.376]	[0.087,0.417]
0.6	[0.083,0.242]	[0.194,0.359]	[0.107,0.395]
0.7	[0.099,0.223]	[0.211,0.341]	[0.130,0.373]
0.8	[0.117,0.204]	[0.228,0.324]	[0.154,0.352]
0.9	[0.135,0.186]	[0.247,0.306]	[0.180,0.331]
1	[0.155,0.169]	[0.265,0.290]	[0.207,0.310]
Static rating $m(\widetilde{RC}_i^1)$	0.171	**0.279**	0.257
Static ranking	3	**1**	2
Dynamic rating $E_1(p_i)$	0.171	**0.279**	0.257
Dynamic ranking	3	**1**	2

Step 6. Computing dynamic rating

Since $t = 1$ and $p_i \in P_1 \backslash H_0 (i = 1, 2, 3)$, there is no historical available alternative. According to Eq. (33), the dynamic rating of each alternative, $E_1(p_i)$, is equal to its corresponding static rating, $m(\widetilde{RC}_i^1)$. Therefore, the dynamic ranking of alternatives is the same as the static ranking of alternatives. The results are shown from rows 14 to 17 of Table 8, respectively.

Since the dynamic rating, $E_1(p_i)$, is equal to its corresponding static ratings, $m(\widetilde{RC}_i^1)$, according to the static ranking of alternatives in Table 8, the DM can select the best alternative, p_2, with the greatest rating among $P_1 = \{p_1, p_2, p_3\}$ at decision moment, $t = 1$, to cope with the EE.

While the alternative, p_2, is selected and implemented to cope with the explosion for a while, the information related to the explosion is simultaneously changing because of its dynamic evolution. Hence, in order to make the emergency response pertinent and effective, the latest information about the explosion should be considered in the MAGEDM process. This is regarded as decision moment $t = 2$ in this case study.

4.1.2. Decision moment at t=2

Step 1. Framework definition

At decision moment, $t = 2$, one more expert, e_4, is invited to participate in the decision process, that is, $E_2 = \{e_1, e_2, e_3, e_4\}$. Furthermore, a new alternative, p_4, and criterion, c_4, are added, that is, $P_2 = \{p_1, p_2, p_3, p_4\}$ and $C_2 = \{c_1, c_2, c_3, c_4\}$, which are given in Tables 9 and 10, respectively.

Table 9. Description of alternatives at $t = 2$

Alternatives	Relationship with H_1	Description
p_1	$p_1 \in P_2 \cap H_1$	Inform and evacuate citizens; meanwhile assign 10 fire squadrons, 300 fire fighters, and 40 fire engines to deal with the EE.
p_2	$p_2 \in P_2 \cap H_1$	Increase to 30 fire squadrons, 900 fire fighters, and 55 fire engines; at the same time, report the latest news to the citizens to avoid panic and riots.
p_3	$p_3 \in P_2 \cap H_1$	Ask the professional emergency rescue military for emergency rescue with more than 300 soldiers carrying specific equipment join in the rescue.
p_4	$p_4 \in P_2 \backslash H_1$	Ask neighboring cities for their fire police to provide support; at the same time, fire police and military must collaborate to deal with the problems.

Table 10. Description of the added criterion c_4 at $t = 2$

Criteria	Expression domain	Description
Social impacts (c_4)	S_3, S_{ll}	It means the impacts on social development or people's daily life, and so on, which are evaluated by experts by using linguistic terms $s_i \in S_3$={None (N), Very Low (VL), Low (L), Medium (M), High (H), Very High (VH), Absolutely High (AH)}, and S_{ll} generated by G_H on the S_3 (Same granularity with criterion c_2).

Step 2. Gathering information

The assessments, X_2^h, provided by the four experts over the four alternatives concerning the four criteria, and criteria importance, W_2^h, at $t = 2$ are

ATLANTIS PRESS International Journal of Computational Intelligence Systems, Vol. 11 (2018) 163–182

given in Tables 11 and 12, respectively.

Table 11. Assessments X_2^h provided by experts at $t = 2$

X_2^h		Criteria c_1	c_2	c_3	c_4
X_2^1	x_{1j}^1	[30,40]	VLS	[0.2,0.25]	VL
	x_{2j}^1	[50,60]	LS	[0.2,0.3]	VL
	x_{3j}^1	[40,60]	LS	[0.3,0.35]	L
	x_{4j}^1	[90,120]	At least HS	[0.55,0.65]	H
X_2^2	x_{1j}^2	[40,50]	VLS	[0.25,0.35]	VL
	x_{2j}^2	[60,70]	LS	[0.3,0.35]	VL
	x_{3j}^2	[30,50]	M	[0.2,0.3]	L
	x_{4j}^2	[100,140]	VHS	[0.6,0.7]	VH
X_2^3	x_{1j}^3	[30,50]	LS	[0.2,0.3]	VL
	x_{2j}^3	[40,50]	LS	[0.25,0.3]	L
	x_{3j}^3	[40,60]	M	[0.15,0.25]	L
	x_{4j}^3	[90,130]	HS	[0.5,0.7]	VH
X_2^4	x_{1j}^4	[40,50]	VLS	[0.2,0.35]	VL
	x_{2j}^4	[60,70]	VLS	[0.2,0.3]	VL
	x_{3j}^4	[50,60]	M	[0.3,0.45]	L
	x_{4j}^4	[100,150]	At least HS	[0.65,0.8]	VH

Table 12. Criteria importance W_2^h provided by experts at $t = 2$

W_2^h	Criteria c_1	c_2	c_3	c_4
W_2^1	HI	MI	LI	HI
W_2^2	VHI	HI	LI	bt MI and HI
W_2^3	HI	MI	LI	HI
W_2^4	VHI	bt MI and HI	LI	HI

Step 4. Aggregation process

Similar to decision moment, $t = 1$, to save space, only the aggregated results, X_2 and $\overline{W}_2$, at $t = 2$, are given in Table 13.

Step 5. Computing static rating

Based on 11 alpha-levels, the results, $\widetilde{RC}_i^2$; static rating, $m(\widetilde{RC}_i^2)$; and static ranking of alternatives are given in Table 14 according to Eqs. (29)–(32), and the fuzzy RC of alternatives is graphically shown in Fig. 7.

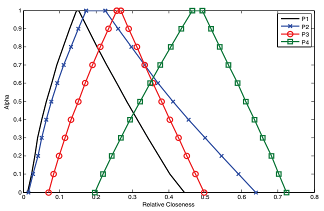

Figure 7: The fuzzy RC of p_i at $t = 2$

Table 14. Alpha-level sets of the fuzzy relative closenesses of the four alternatives at $t = 2$

Alpha	Alternatives p_1	p_2	p_3	p_4
0	[0.012,0.443]	[0.015,0.639]	[0.070,0.498]	[0.197,0.724]
0.1	[0.023,0.403]	[0.027,0.592]	[0.084,0.474]	[0.219,0.703]
0.2	[0.032,0.373]	[0.041,0.545]	[0.099,0.450]	[0.243,0.682]
0.3	[0.040,0.343]	[0.051,0.500]	[0.115,0.427]	[0.268,0.659]
0.4	[0.051,0.314]	[0.063,0.455]	[0.132,0.403]	[0.294,0.636]
0.5	[0.063,0.285]	[0.077,0.412]	[0.151,0.380]	[0.321,0.612]
0.6	[0.078,0.257]	[0.094,0.371]	[0.170,0.357]	[0.349,0.589]
0.7	[0.093,0.230]	[0.111,0.334]	[0.191,0.334]	[0.377,0.564]
0.8	[0.110,0.203]	[0.131,0.297]	[0.212,0.312]	[0.406,0.540]
0.9	[0.128,0.178]	[0.151,0.261]	[0.234,0.290]	[0.435,0.516]
1	[0.146,0.153]	[0.173,0.226]	[0.257,0.269]	[0.465,0.492]
Static rating $m(\widetilde{RC}_i^2)$	0.179	0.253	0.269	0.468
Static ranking	4	3	2	1
Dynamic rating $E_2(p_i)$	0.319	0.461	0.457	**0.468**
Dynamic ranking	4	2	3	**1**

Step 6. Computing dynamic rating

Due to $p_i \in P_2 \cap H_1 (i = 1,2,3)$, their dynamic ratings, $E_2(p_i)(i = 1,2,3)$, should be calculated according to Eq. (33). In this case study, the associative aggregation operator utilized is the *probabilistic sum function* (a t-conorm exhibiting upward reinforcement, see Ref. 42 for details).

According to **Definition** 7, $E_2(p_1)$ is computed as follows:

Since $E_1(p_1) = 0.171$ and $m(\widetilde{RC}_1^2) = 0.179$

$$E_2(p_1) = E_1(p_1) + m(\widetilde{RC}_1^2) - E_1(p_1) \times m(\widetilde{RC}_1^2)$$
$$= 0.171 + 0.179 - 0.171 \times 0.179 = 0.319$$

The dynamic rating, $E_2(p_i)$, of each available alternative and the dynamic ranking of alternatives at $t = 2$ are given in Table 14 from rows 16 to 17, respectively.

According to Table 14, it can be seen that the dynamic ranking is different from the static one because the dynamic method considers the alternative behavior across the time. Therefore, based on the dynamic ranking, the DM can select the best alternative, p_4, with the highest dynamic rating among $P_2 = \{p_1, p_2, p_3, p_4\}$ at decision moment, $t = 2$, to deal with the explosion. It can be seen that the best alternative has changed at $t = 2$ because the latest information about the explosion has been considered in the decision process.

While, the alternative, p_4, is being carried out to deal with the explosion for a period, more information related to the explosion is collected along the time. The latest collected information should be also considered in the MAGEDM process. It is regarded as decision moment, $t = 3$.

ATLANTIS PRESS International Journal of Computational Intelligence Systems, Vol. 11 (2018) 163–182

Table 13. Aggregated results X_2 and $\overline{W}_2$ at $t = 2$

Aggregated results		Criteria c_1	c_2	c_3	c_4
X_2	x_{1j}	(0.030,0.035,0.048,0.050)	(0,0.210,0.210,0.500)	(0.020,0.021,0.031,0.035)	(0,0.170,0.170,0.330)
	x_{2j}	(0.040,0.053,0.220,0.700)	(0,0.290,0.290,0.500)	(0.020,0.024,0.031,0.035)	(0,0.210,0.210,0.500)
	x_{3j}	(0.030,0.040,0.058,0.060)	(0.170,0.458,0.458,0.670)	(0.015,0.024,0.034,0.045)	(0.170,0.330,0.330,0.500)
	x_{4j}	(0.090,0.095,0.135,0.150)	(0.500,0.805,0.805,1)	(0.050,0.058,0.071,0.080)	(0.500,0.790,0.790,1)
$\overline{W}_2$	$\overline{w}_{c_j}$	(0.500,0.750,0.750,1)	(0.330,0.543,0.585,0.840)	(0.170,0.330,0.330,0.500)	(0.350,0.628,0.670,0.840)

4.1.3. Decision moment at t=3

Step 1. Framework definition

At decision moment, $t = 3$, alternative, p_1, is removed due to its ineffectiveness; meanwhile a new criterion, c_5, and one new alternative, p_5, are added, that is, $C_3 = \{c_1, c_2, c_3, c_4, c_5\}$, $P_3 = \{p_2, p_3, p_4, p_5\}$, which are given in Tables 15 and 16, respectively.

Table 15. Description of the added criterion c_5 at $t = 3$

Criteria	Expression domain	Description
Cost of alternative (c_5)	I	It means the cost of alternative, p_i, ($i = 2,3,4,5$), including all the direct and indirect expenses in domain [0,100] (in million RMB).

Table 16. Description of alternatives at $t = 3$

Alternatives	Relationship with H_2	Description
p_2	$p_2 \in P_3 \cap H_2$	Increase to 30 fire squadrons, 900 fire fighters, and 55 fire engines; at the same time, report the latest news to the citizens to avoid panic and riots.
p_3	$p_3 \in P_3 \cap H_2$	Ask the professional emergency rescue military for emergency rescue with more than 300 soldiers carryinh specific equipment join in the rescue.
p_4	$p_4 \in P_3 \cap H_2$	Ask neighbor cities for their fire police in order to provide support; at the same time, fire police and military must collaborate to deal with the problems.
p_5	$p_5 \in P_3 \setminus H_2$	Block the boundary of the explosion areas; let the material in the explosion areas burn down.

Step 2. Gathering information

The criteria importance, W_3^h, and the assessments, X_3^h, provided by experts at $t = 3$ are given in Tables 17 and 18, respectively.

Table 17. Criteria importance W_3^h provided by experts at $t = 3$

W_3^h	Criteria c_1	c_2	c_3	c_4	c_5
W_3^1	HI	MI	LI	HI	MI
W_3^2	VHI	HI	LI	HI	MI
W_3^3	VHI	LI	VLI	MI	VLI
W_3^4	HI	MI	LI	MI	VLI

Table 18. Assessments X_3^h provided by experts at $t = 3$

X_3^h		Criteria c_1	c_2	c_3	c_4	c_5
X_3^1	x_{2j}^1	[80,90]	M	[0.3,0.4]	L	[30,50]
	x_{3j}^1	[50,70]	M	[0.25,0.35]	M	[40,60]
	x_{4j}^1	[90,120]	bt M and HS	[0.35,0.45]	H	[70,80]
	x_{5j}^1	[70,100]	VHS	[0.4,0.5]	VH	[25,45]
X_3^2	x_{2j}^2	[60,80]	LS	[0.15,0.25]	VL	[50,60]
	x_{3j}^2	[70,90]	LS	[0.3,0.4]	L	[40,55]
	x_{4j}^2	[90,110]	At most M	[0.4,0.5]	M	[60,80]
	x_{5j}^2	[50,70]	HS	[0.25,0.4]	VH	[35,50]
X_3^3	x_{2j}^3	[40,50]	VLS	[0.2,0.25]	L	[40,60]
	x_{3j}^3	[60,75]	M	[0.15,0.2]	L	[30,50]
	x_{4j}^3	[80,100]	M	[0.4,0.45]	H	[70,90]
	x_{5j}^3	[30,45]	VHS	[0.1,0.25]	VH	[25,45]
X_3^4	x_{2j}^4	[45,65]	LS	[0.35,0.4]	VL	[35,55]
	x_{3j}^4	[40,60]	bt LS and M	[0.5,0.55]	L	[30,45]
	x_{4j}^4	[70,80]	M	[0.6,0.7]	M	[60,75]
	x_{5j}^4	[30,50]	HS	[0.3,0.5]	H	[30,35]

Step 4. Aggregation process

To save space, similar to $t = 2$, only the aggregated results, X_3 and $\overline{W}_3$, at $t = 3$ are given in Table 19.

Step 5. Computing static rating

Based on 11 alpha-levels, the fuzzy RC, $\widetilde{RC}_i^3$; static rating, $m(\widetilde{RC}_i^3)$; and static ranking of alternatives are given in Table 20 according to Eqs. (29)–(32), and the fuzzy RC of alternatives is graphically shown in Fig. 8.

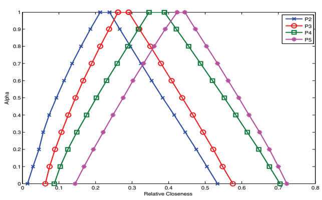

Fig. 8. The fuzzy relative closeness of p_i at $t = 3$

ATLANTIS PRESS International Journal of Computational Intelligence Systems, Vol. 11 (2018) 163–182

Table 19. Aggregated results X_3 and $\overline{W}_3$ at $t=3$

Aggregated results		Criteria c_1	c_2	c_3	c_4	c_5
X_3	x_{2j}	(0.040,0.056,0.071,0.090)	(0,0.333,0.333,0.670)	(0.015,0.025,0.033,0.040)	(0,0.250,0.250,0.500)	(0.300,0.388,0.563,0.600)
	x_{3j}	(0.040,0.055,0.074,0.090)	(0.170,0.418,0.458,0.670)	(0.015,0.030,0.038,0.055)	(0.170,0.373,0.373,0.670)	(0.300,0.350,0.525,0.600)
	x_{4j}	(0.070,0.083,0.103,0.120)	(0,0.375,0.508,0.840)	(0.035,0.044,0.053,0.070)	(0.330,0.585,0.585,0.830)	(0.600,0.650,0.813,0.900)
	x_{5j}	(0.030,0.045,0.066,0.100)	(0.500,0.750,0.750,1)	(0.010,0.026,0.041,0.050)	(0.500,0.790,0.833,1)	(0.250,0.288,0.438,0.500)
$\overline{W}_3$	$\overline{w}_{c_j}$	(0.500,0.750,0.750,1)	(0.170,0.500,0.500,0.830)	(0,0.290,0.290,0.500)	(0.330,0.585,0.585,0.830)	(0,0.335,0.335,0.670)

Table 20. Alpha-level sets of the fuzzy relative closenesses of the four alternatives at $t=3$

Alpha	Alternatives p_2	p_3	p_4	p_5
0	[0.015,0.534]	[0.063,0.575]	[0.088,0.704]	[0.145,0.723]
0.1	[0.030,0.505]	[0.075,0.548]	[0.104,0.675]	[0.169,0.700]
0.2	[0.046,0.475]	[0.090,0.522]	[0.125,0.646]	[0.194,0.675]
0.3	[0.058,0.445]	[0.107,0.494]	[0.149,0.615]	[0.221,0.648]
0.4	[0.074,0.415]	[0.126,0.466]	[0.176,0.584]	[0.248,0.620]
0.5	[0.093,0.384]	[0.146,0.437]	[0.203,0.552]	[0.276,0.592]
0.6	[0.114,0.354]	[0.167,0.408]	[0.231,0.520]	[0.303,0.562]
0.7	[0.137,0.324]	[0.190,0.379]	[0.260,0.487]	[0.331,0.533]
0.8	[0.161,0.295]	[0.213,0.350]	[0.288,0.454]	[0.361,0.503]
0.9	[0.186,0.267]	[0.237,0.320]	[0.317,0.421]	[0.391,0.473]
1	[0.213,0.239]	[0.262,0.291]	[0.347,0.388]	[0.423,0.444]
Static rating $m(\widetilde{RC}_i^3)$	0.244	0.294	0.379	0.433
Static ranking	4	3	2	1
Dynamic rating $E_3(p_i)$	0.593	0.617	**0.670**	0.433
Dynamic ranking	3	2	**1**	4

Step 6. Computing dynamic rating

Similar to $t=2$, the dynamic rating, $E_3(p_i)$, and the dynamic ranking of alternatives at $t=3$ are given in Table 20 from rows 16 to 17, respectively. Again, dynamic and static rankings are different. Therefore, based on the dynamic ranking of the four alternatives in Table 20, p_4 is the best one with the highest dynamic rating among $P_3=\{p_2,p_3,p_4,p_5\}$ at $t=3$ to cope with the explosion.

It can be seen that the best alternative, p_4, at $t=3$, is consistent with the best one at $t=2$. This interesting phenomenon can be explained by the fact that the dynamic rating here consists of not only each alternative's performance at current stage, but also at previous stage.

To save space, only three different decision moments have been conducted in this case study. In real world problems, the proposed dynamic MAGEDM method can be applied for more than three decision moments until the problems are solved.

4.2. *Comparison with other approaches*

To further demonstrate the feasibility and validity of the proposed dynamic MAGEDM method, a comparison with the approach introduced by Campanella et al. [4] is carried out, along with their discussions.

1) A brief summary of current dynamic EDM methods is provided to highlight the advantages of our proposal.

2) A current dynamic MADM approach [4] is utilized for the comparison with our proposed method.

4.2.1. *Comparison with current dynamic EDM methods*

Due to the fact that there is no any existing MAGEDM approach to deal with dynamic evolution of EEs considering updated information (alternative, criteria and experts) and experts' hesitation, some characteristics have been studied to highlight the advantages of our proposal in comparison with other approaches [13,29,31,40] (see Table 21).

Table 21. Comparison with current dynamic EDM methods

Literature	Type of decision	Perspective of dynamic	Hesitant information
Refs. 13,29	Individual DM	Time changes and executive effect of alternative, without updated information (alternative, criteria)	No
Refs. 31	Individual DM	Time changes and dynamic reference points, without updated information (alternative, criteria)	No
Refs. 40	Group decision	Time changes and similarity between predicated scenario and historical scenario, without updated information (alternative, criteria, expert)	No
Our proposal	Group decision	Time changes with updated information (alternative, criteria, experts)	Yes

According to Table 21, we can see that current dynamic EDM methods are mainly focused on the perspective of time changes. However, our proposal deals with the dynamic evolution of EEs not only from the perspective of time, but also considering the updated information (alternative, criteria, and experts) along with the time and development of EEs. Therefore, the decision processes are more close to real world situations than the current dynamic EDM methods.

ATLANTIS PRESS International Journal of Computational Intelligence Systems, Vol. 11 (2018) 163–182

Furthermore, our proposal considers experts' hesitation due to lack of information and time pressure, which is inevitable in EDM problems.

4.2.2. *Comparison with a current dynamic MADM method*

To make a comparison with the recent dynamic MADM method proposed by Campanella et al. [4], the aggregated results, X_t and $\overline{W}_t$ ($t = 1,2,3$), in Tables 7, 13, and 19 are defuzzied into crisp numbers using the equation $\frac{a+2b+2c+d}{6}$ because it is an easy defuzzification [7] method, wherein a fuzzy number, $\widetilde{A} = (a,b,c,d)$. The results are shown in Table 22.

Table 22. Defuzzied values of X_t and $\overline{W}_t$

Decision moment		Criteria c_1	c_2	c_3	c_4	c_5
$t=1$	p_1	0.058	0.268	0.031	-	-
	p_2	0.094	0.500	0.046	-	-
	p_3	0.042	0.466	0.026	-	-
	weights	0.832	0.624	0.268	-	-
$t=2$	p_1	0.041	0.223	0.027	0.168	-
	p_2	0.214	0.277	0.028	0.223	-
	p_3	0.048	0.445	0.029	0.332	-
	p_4	0.117	0.787	0.065	0.777	-
	weights	0.750	0.571	0.332	0.631	-
$t=3$	p_2	0.064	0.333	0.028	0.250	0.467
	p_3	0.065	0.432	0.034	0.388	0.442
	p_4	0.093	0.434	0.050	0.583	0.738
	p_5	0.059	0.750	0.033	0.791	0.367
	weights	0.750	0.500	0.277	0.583	0.335

- means the criteria unavailable in specifical decision moment

As the sum of defuzzied criteria weights in Table 22 at each decision moment is greater than 1, and it must be equal to 1, it is necessary to normalize the weights. The normalized criteria weights at each decision moment are given in Table 23.

Table 23. Normalized criteria weights at each decision moment

Decision moment	Normalized criteria weights c_1	c_2	c_3	c_4	c_5
$t=1$	0.483	0.362	0.155	-	-
$t=2$	0.329	0.250	0.145	0.276	-
$t=3$	0.307	0.204	0.113	0.239	0.137

- means the criteria unavailable in specifical decision moment

Based on Tables 22 and 23, static and dynamic ratings for each alternative at different decision moments are computed with the *weighted mean operator* and *probabilistic sum operator* (e.g., associative) according to the method presented in Ref. 4. The results are given in Table 24.

Table 24. The results obtained by the method in Ref. 4

Decision moment		Alternatives p_1	p_2	p_3	p_4	p_5
$t=1$	Static rating	0.130	0.144	0.237	-	-
	Static ranking	3	2	1		
	Dynamic rating	0.130	0.144	**0.237**	-	-
	Dynamic ranking	3	2	**1**	-	-
$t=2$	Static rating	0.120	0.205	0.223	0.459	-
	Static ranking	4	3	2	1	
	Dynamic rating	0.234	0.319	0.407	0.459	-
	Dynamic ranking	4	3	2	**1**	-
$t=3$	Static rating	-	0.215	0.265	0.363	0.414
	Static ranking	-	4	3	2	1
	Dynamic rating	-	0.466	0.564	0.655	0.414
	Dynamic ranking	-	3	2	**1**	4

- means the alternative unavailable in specifical decision moment

For the sake of clarity, an example, the static rating of p_1 at $t = 1$ in Table 24, can be computed as below:

$static\ \ rating\ \ p_1 = 0.058 \times 0.483 + 0.268 \times 0.362 + 0.031 \times 0.155 = 0.130.$

The dynamic rating of p_1 at $t = 2$ can be calculated based on its static rating (0.120) at $t = 2$, and its dynamic rating (0.130) at $t = 1$, as shown below:

dynamic rating $p_1 = (0.120 + 0.130) - 0.120 \times 0.130 = 0.234$

From Table 24, it can be seen that, although the method in Ref. 4 leads to the same best alternatives at different decision moments ($t = 2,3$), it is obvious that the values obtained by the method in Ref. 4 are significantly lower than those obtained by our proposed method at each decision moment. This is because our proposal deals with uncertain information, including experts' hesitation. Additionally, the computation process retains as much information as possible. Therefore, the proposed method shows its validity and feasibility through the comparison.

4.2.3. **Discussions**

To overcome the limitations pointed out in section 1, this paper proposes a dynamic MAGEDM method to deal with the dynamic evolution of EEs and uncertain information including experts' hesitation. A case study and comparisons with current approaches have been conducted to demonstrate the novelty and validity of the proposed dynamic MAGEDM method.

Compared to existing MAGEDM approaches [11,16,30,35,36,37,38], the advantages of the proposed dynamic MAGEDM method are as follows:

1) The proposed dynamic MAGEDM method considers the dynamic evolution feature of EEs, which is a crucial factor in real world problems;

ATLANTIS PRESS International Journal of Computational Intelligence Systems, Vol. 11 (2018) 163–182

it fully takes into account the updated information across the time and the development of EEs. The proposed method is close to the real-world situations and easy to understand. This is the significant difference between our proposal and other versions [13,29], wherein the alternatives and criteria are fixed without considering the updated information along the time.

2) Hesitancy is a quite normal behavior in human beings daily life particularly in uncertain environment. Experts involved in MAGEDM problems featured by lack of information and time pressure might hesitate among several values when they provide their opinions, however, such a practical issue is neglected in existing MAGEDM approaches [11,16,30,35,36,37,38]. To fill the gap in current MAGEDM approaches, the proposed method has taken into account the experts' hesitation by using complex linguistic expressions based on HFLTS.

3) To keep the uncertain and hesitant information provided by experts as much as possible, a fuzzy TOPSIS method based on alpha-level sets is utilized to obtain the static ratings of alternatives at each decision moment, which can provide much more information for each alternative and is suitable for the problems defined in fuzzy environment.

5. Conclusion and future works

Dynamic evolution and uncertain information are the outstanding features of EEs, they are the key factors in the process of dealing with the EEs successfully. Information plays a crucial part in all different types of decision problems no exception for MAGEDM problems. Due to the dynamic evolution of EEs, the information is updating along with the time and the development of EEs. However, the dynamic methods in current EDM approaches are mainly focused on changeable time; they neglect information changes along with the evolution of EEs. The information is usually uncertain in MAGEDM problems—particularly in the early occurrence stage—in such a fuzzy environment that experts might hesitate about their assessments. However, this important issue is not considered in current MAGEDM problems. Thus, this paper proposes a dynamic MAGEDM method that considers not only the dynamic evolution of EEs, including the updated information (alternatives, criteria, and experts), but also the experts' hesitation. A fuzzy TOPSIS method based on alpha-level sets is applied to obtain the static ratings of available alternatives, which deals with fuzzy information across the decision process, and is suitable for the problems defined in fuzzy environments. Comparisons with other approaches and related discussions have been provided to illustrate the novelty and advantages of our proposal.

Future research could investigate use of decision support systems with big data based on computer science and the Internet.

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References

1. I. Beg and T. Rashid. TOPSIS for hesitant fuzzy linguistic term sets. *International Journal of Intelligent Systems*, 28(12):1162–1171, 2013.
2. C. Camerer. Bounded rationality in individual decision making. *Experimental economics*, 1(2):163–183, 1998.
3. G. Campanella, A. Pereira, R. A Ribeiro, and M. R. Varela. Collaborative dynamic decision making: A case study from B2B supplier selection. In *Euro Working Group Workshop on Decision Support Systems*, pages 88–102. Springer, 2011.
4. G. Campanella and R. A. Ribeiro. A framework for dynamic multiple-criteria decision making. *Decision Support Systems*, 52(1):52–60, 2011.
5. C. T. Chen. Extensions of the TOPSIS for group decision-making under fuzzy environment. *Fuzzy sets and systems*, 114(1):1–9, 2000.
6. C. T. Chen, C. T. Lin, and S. F. Huang. A fuzzy approach for supplier evaluation and selection in supply

ATLANTIS PRESS International Journal of Computational Intelligence Systems, Vol. 11 (2018) 163–182

chain management. *International journal of production economics*, 102(2):289–301, 2006.
7. S. J. Chen and S. M. Chen. Fuzzy risk analysis based on similarity measures of generalized fuzzy numbers. *IEEE Transactions on fuzzy systems*, 11(1):45–56, 2003.
8. T. C. Chu. Facility location selection using fuzzy TOPSIS under group decisions. *International journal of uncertainty, fuzziness and knowledge-based systems*, 10(06):687–701, 2002.
9. T. C. Chu and Y. C. Lin. A fuzzy TOPSIS method for robot selection. *The International Journal of Advanced Manufacturing Technology*, 21(4):284–290, 2003.
10. Z. P. Fan, Y. Liu, and R. J. Shen. Risk decision analysis method for emergency response based on prospect theory. *Systems Engineering-Theory & Practice*, 5:011, 2012.
11. P. P. Fu, W. Chong, and T. Jie. Unconventional emergency management based on intelligent group decision-making methodology. *Advances in Information Sciences & Service Sciences*, 4(7), 2012.
12. C. L. Hwang and K. Yoon. *Multiple attribute decision making: methods and applications*. Berlin: Springer, 1981.
13. Y. P. Jiang, Z. P. Fan, and M. M. Su. Study on the dynamic adjusting method for emergency decision. *Chinese Journal of Management Science*, 19(5):104–108, 2011.
14. Y. B. Ju, A. H. Wang, and X. Y. Liu. Evaluating emergency response capacity by fuzzy AHP and 2-tuple fuzzy linguistic approach. *Expert Systems with Applications*, 39(8):6972–6981, 2012.
15. D. Kuchta. Fuzzy capital budgeting. *Fuzzy Sets and Systems*, 111(3):367–385, 2000.
16. J. K. Levy and T. Kouichi. Group decision support for hazards planning and emergency management: A group analytic network process (GANP) approach. *Mathematical and Computer Modelling*, 46(7):906–917, 2007.
17. H. B. Liu and R. M. Rodríguez. A fuzzy envelope for hesitant fuzzy linguistic term set and its application to multicriteria decision making. *Information Sciences*, 258:220–238, 2014.
18. Y. Liu, Z. P. Fan, and Y. Zhang. Risk decision analysis in emergency response: A method based on cumulative prospect theory. *Computers & Operations Research*, 42:75–82, 2014.
19. J. Lu and D. Ruan. *Multi-objective group decision making: methods, software and applications with fuzzy set techniques*, volume 6. Imperial College Press, 2007.
20. L. Martínez, F. Herrera, et al. An overview on the 2-tuple linguistic model for computing with words in decision making: Extensions, applications and challenges. *Information Sciences*, 207:1–18, 2012.
21. L. Martínez, D. Ruan, and F. Herrera. Computing with words in decision support systems: an overview on models and applications. *International Journal of Computational Intelligence Systems*, 3(4):382–395, 2010.
22. J. M. Merigó, M. Casanovas, and J. B. Yang. Group decision making with expertons and uncertain generalized probabilistic weighted aggregation operators. *European Journal of Operational Research*, 235(1):215–224, 2014.
23. R. E. Moore, R. B. Kearfott, and M. J. Cloud. *Introduction to interval analysis*. SIAM, 2009.
24. Mourad Oussalah. On the compatibility between defuzzification and fuzzy arithmetic operations. *Fuzzy Sets and Systems*, 128(2):247–260, 2002.
25. R. M. Rodríguez, A. Labella, and L. Martínez. An overview on fuzzy modelling of complex linguistic preferences in decision making. *International Journal of Computational Intelligence Systems*, 9:81–94, 2016.
26. R. M. Rodríguez and L. Martínez. An analysis of symbolic linguistic computing models in decision making. *International Journal of General Systems*, 42(1):121–136, 2013.
27. R. M. Rodríguez, L. Martínez, and F. Herrera. Hesitant fuzzy linguistic term sets for decision making. *IEEE Transactions on Fuzzy Systems*, 20(1):109–119, 2012.
28. R. M. Rodríguez, L. Martínez, and F. Herrera. A group decision making model dealing with comparative linguistic expressions based on hesitant fuzzy linguistic term sets. *Information Sciences*, 241:28–42, 2013.
29. L. Wang, Y. M. Wang, and B. X. Hu. Dynamic adjusting method of emergency alternatives based on prospect theory. *Control and Decision*, 1:99–104, 2016.
30. L. Wang, Y. M. Wang, and L. Martínez. A group decision method based on prospect theory for emergency situations. *Information Sciences*, 418-419:119–135, 2017.
31. L. Wang, Z. X. Zhang, and Y. M. Wang. A prospect theory-based interval dynamic reference point method for emergency decision making. *Expert Systems with Applications*, 42(23):9379–9388, 2015.
32. Y. M. Wang and T. M.S. Elhag. Fuzzy TOPSIS method based on alpha level sets with an application to bridge risk assessment. *Expert systems with applications*, 31(2):309–319, 2006.
33. C. P. Wei, Z. L. Ren, and R. M. Rodríguez. A hesitant fuzzy linguistic TODIM method based on a score function. *International Journal of Computational Intelligence Systems*, 8(4):701–712, 2015.
34. C. P. Wei, N. Zhao, and X. J. Tang. Operators and comparisons of hesitant fuzzy linguistic term sets.

ATLANTIS PRESS International Journal of Computational Intelligence Systems, Vol. 11 (2018) 163–182

IEEE Transactions on Fuzzy Systems, 22(3):575–585, 2014.

35. X. H. Xu, Z. J. Du, and X. H. Chen. Consensus model for multi-criteria large-group emergency decision making considering non-cooperative behaviors and minority opinions. *Decision Support Systems*, 79:150–160, 2015.
36. X. H. Xu, X. Y. Zhong, X. H. Chen, and Y. J. Zhou. A dynamical consensus method based on exit–delegation mechanism for large group emergency decision making. *Knowledge-Based Systems*, 86:237–249, 2015.
37. Y. J. Xu, W. C. Zhang, and H. M. Wang. A conflict-eliminating approach for emergency group decision of unconventional incidents. *Knowledge-Based Systems*, 83:92–104, 2015.
38. L. Yu and K. K. Lai. A distance-based group decision-making methodology for multi-person multi-criteria emergency decision support. *Decision Support Systems*, 51(2):307–315, 2011.
39. L. Zadeh. The concept of a linguistic variable and its applications to approximate reasoning. *Information Sciences, Part I, II, III*, (8,9):199–249, 301–357, 43–80, 1975.
40. K. Zhang. Research on dynamic emergency decision method based on case-based reasoning. *Computer Engineering & Applications*, 50(23):56–60, 2014.
41. H. J. Zimmennann. Fuzzy set theory and its applications, 1991.
42. Y. Zulueta, J. Martinez-Moreno, R. B. Pérez, and L. Martinez. A discrete time variable index for supporting dynamic multi-criteria decision making. *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, 22(01):1–22, 2014.

4.3 Managing non-homogeneous information and experts' psychological behavior in group emergency decision making

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Managing Non-Homogeneous Information and Experts' Psychological Behavior in Group Emergency Decision Making

Liang Wang [1,2], Álvaro Labella [1], Rosa M. Rodríguez [3,*], Ying-Ming Wang [2] and Luis Martínez [1]

1 Department of Computer Science, University of Jaén, 23071 Jaén, Spain; wangliangg322@hotmail.com (L.W.); alabella@ujaen.es (Á.L.); martin@ujaen.es (L.M.)
2 Decision Sciences Institute, Fuzhou University, Fuzhou 350116, China; msymwang@hotmail.com
3 Department of Computer Science and A.I., University of Granada, 18071 Granada, Spain
* Correspondence: rosam.rodriguez@decsai.ugr.es

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Abstract: After an emergency event (EE) happens, emergency decision making (EDM) is a common and effective way to deal with the emergency situation, which plays an important role in mitigating its level of harm. In the real world, it is a big challenge for an individual emergency manager (EM) to make a proper and comprehensive decision for coping with an EE. Consequently, many practical EDM problems drive group emergency decision making (GEDM) problems whose main limitations are related to the lack of flexibility in knowledge elicitation, disagreements in the group and the consideration of experts' psychological behavior in the decision process. Hence, this paper proposes a novel GEDM approach that allows more flexibility for preference elicitation under uncertainty, provides a consensus process to avoid disagreements and considers experts' psychological behavior by using the fuzzy TODIM method based on prospect theory. Eventually, a group decision support system (GDSS) is developed to support the whole GEDM process defined in the proposed method demonstrating its novelty, validity and feasibility.

Keywords: group emergency decision making; non-homogeneous information; psychological behavior; group decision support system

1. Introduction

Emergencies are defined as events that suddenly take place, causing or having the possibility of provoking intense death and injury, property loss, ecological damage and social hazards. In recent years, various emergency events, such as earthquakes, floods, hurricanes, terrorist attacks, etc., have exerted severely negative impacts on human life and socio-economic development. When an emergency event (EE) occurs, Emergency Decision Making (EDM) is typically characterized by at least uncertainty, time pressure, and lack of information, resulting in potentially serious consequences [1]. Since EDM plays a crucial role in alleviating the losses of properties and lives caused by EEs, it has received increasing attention from both government and academia because of the frequent occurrence of EEs, becoming a very active and important research field in recent years [1–5].

When an EE occurs, it is hard to collect the information related to the event and predict its evolution particularly in the early stage because of the inadequate and uncertain information. Consequently, it is too complex for just one emergency manager (EM) to make comprehensive judgments under emergency situations. Therefore, EDM requires multiple experts from diverse professional backgrounds (such as hydrological, geological, meteorological, sociological, demographic, etc.) to help the EM make a decision. This leads to Group EDM (GEDM) problems. Figure 1 shows a graphical general scheme for GEDM problems, in which experts play a role of think tank in supporting the EM who is in charge of the EE.

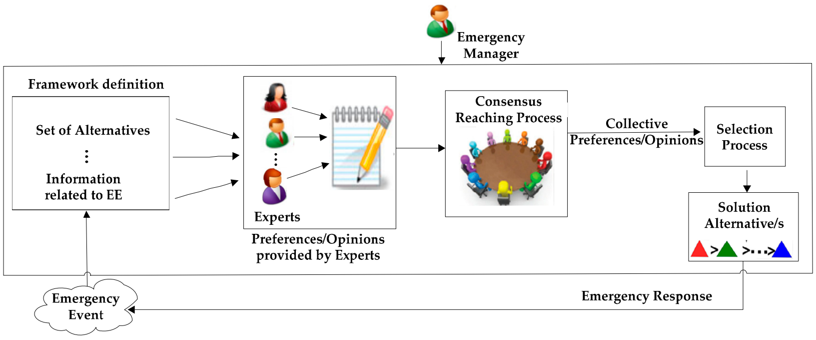

Figure 1. The general scheme of GEDM process.

In the real world, it is common that experts with different background and knowledge might have different attitudes or opinions over different alternatives concerning different criteria. Moreover, criteria defined in a GEDM problem might have different nature, qualitative or quantitative. Therefore, experts might hesitate and express their opinions or assessments by using different types of information according to their knowledge and criteria nature. The complexity of GEDM problems could imply not only the use of a non-homogeneous context in which multiple information types can be utilized by experts to elicit their knowledge and expertise, but also the modeling of uncertain assessments including hesitancy. However, current EDM approaches deal with the information using only one expression domain: numerical values [4], interval values [3] or linguistic information [6].

Traditionally, group decision making (GDM) approaches have shown that a solution can be obtained under disagreement among experts [7,8], however several experts may not accept the decision made because they might consider that their individual opinions have not been taken into account sufficiently [9,10]. Such a situation could be very serious in GEDM driving either to deadlock in the decision or in a harmful decision. Hence, it seems necessary and reasonable to achieve a consensus among all experts involved in the GEDM problem before making the decision. The Consensus Reaching Process (CRP) is a way to integrate group wisdom into one and then reach an agreement among all experts in the GEDM problem. There are already different approaches [1,4,11] focused on how to reach as much agreement as possible among all experts participating in the problem. However, they have strict expression domains [1,11]; or time cost [4,5]. However, time is extremely valuable, because it means lives and chances, thus emergency responses cannot afford a time-consuming consensus model.

Different behavioral experiments [12–14] show that human beings are usually bounded rationally in decision-making processes under risk and uncertainty. Therefore, psychological behavior plays a crucial role in the decision processes. Nevertheless, as far as we know, experts' psychological behavior is neglected in current GEDM [1,4,5,11,15] approaches.

According to the previous limitations presented in current GEDM methods, the aim of this paper is to propose a new GEDM method that overcomes them. Such a method is able:

1. To allow more flexibility for eliciting information by dealing with non-homogeneous information including hesitancy.
2. To include a consensus model with low time cost to achieve an agreement among experts involved in the GEDM problem.
3. To take into account experts' psychological behavior by means of the fuzzy TODIM method [16–18] based on prospect theory [14].

Furthermore, the proposed method is implemented into a Group Decision Support System (GDSS) named GENESIS (Group EmergeNcy dEcision SupportIng System) based on FLINTSTONES (Fuzzy LINguisTic DeciSion Tools eNhacemEnt Suite) [19,20] that supports the whole GEDM process effectively and in a timely way, as shown in an illustrative example.

The remainder of this paper is organized as follows: Section 2 revises briefly different concepts about non-homogeneous information, CRPs and the fuzzy TODIM, which will be used in our proposal together with some related works. Section 3 presents the new GEDM method that integrates the novelties pointed out previously. Section 4 introduces the structure and components of the GDSS, GENESIS, and shows an example to illustrate the feasibility and validity of the proposed method. A sensitive analysis is also presented to study the robustness of the proposal. Section 5 presents some conclusions and future works.

2. Preliminaries

In this section, some basic concepts about non-homogeneous information and CRPs are revised in short in order that readers can understand easily the proposed GEDM model. It also reviews the fuzzy TODIM method that is used in the selection process of the proposal to obtain the ranking of alternatives considering experts' psychological behavior. Eventually, some related works to illustrate the importance of this research are reviewed.

2.1. Non-Homogeneous Information in Decision Making

Nowadays, real-world decision-making problems are more diversified and complex because of rapid socio-economic development, such as EDM problems [2,3], GEDM problems [1,15], and Intelligent GEDM problems [11]. Those problems are usually defined under uncertainty because of inadequate and uncertain information. The complexity of these problems implies multiple experts with different backgrounds and knowledge participating in the decision process.

To model the uncertainty and non-homogeneous information, such as numerical values, interval values and linguistic terms elicited by experts, several approaches have been discussed in current GDM approaches. Some of them [21–25] make the computations using directly the non-homogeneous information [26] and others unify the information into one domain [24,27], being the most common one the linguistic information. Recently, the inclusion of hesitancy is becoming more important [28,29].

The concept of hesitant fuzzy linguistic term sets (HFLTS) [30] has been introduced to model experts' hesitation in qualitative settings and it has been applied in decision making problems obtaining successful results. It is defined as follows.

Definition 1 [30]**.** *Let $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set, a HFLTS H_S, is defined as an ordered finite subset of consecutive linguistic terms of S*:

$$H_S = \{s_i, s_{i+1}, \ldots, s_j\}, s_k \in S, \; k \in \{i, \ldots, j\}$$

Nevertheless, when experts provide their opinions and they feel hesitation among several linguistic terms, they do not use multiple linguistic terms, but linguistic expressions close to the natural language used by human beings. Hence, Rodríguez et al. [30] proposed the use of context-free grammars G_H to build complex linguistic expressions more flexible and richer than single linguistic terms [29,30]. The expressions produced by the context-free grammar G_H, may be either a single linguistic term $s_i \in S$, or comparative linguistic expressions S_{ll} (see [29,30] for further detail).

In our proposal, the non-homogeneous information including experts' hesitancy will be transformed into a unified fuzzy domain to facilitate the computations (see Section 3.3).

2.2. Consensus Reaching Processes

GDM problems are usually solved by a selection process that obtains the best alternative as a solution to the problem. However, sometimes the goal of the problem is not to obtain the best solution, but an accepted one for all involved experts in the problem. In such a situation, it seems necessary to apply a CRP. Consensus can be defined as [9] "a state of mutual agreement among members of a group in which the decision made satisfies all of them". Therefore, a consensus process requires that experts modify their opinions making them closer to each other and this way to obtain a collective opinion that is satisfactory for all of them [10,31–34].

In GEDM process, experts play a role of think tank in supporting EM to make a decision, recently several proposals [1,4,5,11,15,34] integrate CRP into GEDM to deal with experts' opinions in order to achieve an agreement among all experts involved and make a right decision. However, these approaches deal just with numerical values [1,5,25] and are not suitable for other types of information, additionally, they have a high time cost [4,5] because of the supervised feedback mechanism that should be avoided in GEDM problems.

Due to these reasons and the type of information used in our proposal, a fuzzy linear programming-based consensus model [34] with low time cost will be utilized to achieve consensus in our proposal. Before introducing the fuzzy linear programming model, it is necessary to revise the definition of the distance between fuzzy numbers, which will be used.

Definition 2 [34]**.** *Let $A = (a_1, a_2, a_3, a_4)$ and $B = (b_1, b_2, b_3, b_4)$ be two trapezoidal fuzzy numbers. The distance between A and B can be obtained as follows, the measure of d_p can also be called as l_p metric:*

$$d_p(A, B) = \left(\sum_{i=1}^{4} (|a_i - b_i|)^p \right)^{1/p} \tag{1}$$

where p is an integer ≥ 1. Let U be the universe of discourse and $u = \max(U) - \min(U)$. The similarity between A and B can be defined as [34,35]:

$$S_p(A, B) = 1 - \frac{1}{4u^p} (d_p(A, B))^p \tag{2}$$

The dissimilarity is defined as $c - S_p(A, B)$, where c is a constant >1. The selection of c will influence in the final result of the aggregation.

Let $\widetilde{A}_h = (a_{h1}, a_{h2}, a_{h3}, a_{h4})$ be the h-th expert's individual opinion and $\widetilde{O}$ be the overall opinion obtained by aggregating experts' individual opinions.

The fuzzy linear programming model is [34]:

$$\begin{cases} \min \sum_{h=1}^{\overline{K}} (w^h)^\alpha (c - S_p(\widetilde{A}_h, \widetilde{O})) \\ s.t.\ d_p(\widetilde{A}_h, \widetilde{O}) \leq \varepsilon_h,\ h = 1, 2, \ldots, \overline{K} \end{cases} \tag{3}$$

where α is an integer ≥ 1, w^h denotes the h-th experts' importance. ε_h denotes a threshold that means the maximum change that the h-th expert can make. $d_p(\widetilde{A}_h, \widetilde{O})$ denotes the distance between $\widetilde{A}_h$ and $\widetilde{O}$, which can be obtained according to Equation (1).

2.3. Fuzzy TODIM Method

Some studies [12–14] have shown that human beings are bounded rationally especially in risk and uncertain decision processes and their psychological behavior is very important in the decision process. Therefore, it seems necessary to consider experts' psychological behavior in GEDM problem.

TODIM method was proposed by Gomes and Lima [36,37]; it is a popular multi-criteria decision making (MCDM) method based on prospect theory [13] considering humans psychological behavior. It has been widely applied to solve different decision problems [38,39]. To cope with complex problems and uncertain information in the real world, the TODIM method has been extended to deal with fuzzy MCDM problems [16,17].

In our proposal, we will use fuzzy TODIM method [16–18] based on prospect theory [14] because of its advantage and capability of capturing the experts' psychological behavior under fuzzy environment.

The fuzzy TODIM was introduced in [18] and briefly summarized below:

Let $P = \{p_1, p_2, \ldots, p_m\}$ be a set of alternatives, $C = \{c_1, c_2, \ldots, c_n\}$ be a set of criteria and $w_c = (w_{c_1}, w_{c_2}, \ldots, w_{c_n})$ be a weighting vector for criteria, where w_{c_j} denotes the weight of criterion c_j. Let $A = (a_{ij})_{m\times n}$ be a fuzzy decision matrix, where $a_{ij} = (a_{ij}^1, a_{ij}^2, a_{ij}^3, a_{ij}^4)$ denotes the rating of the alternative p_i with respect to criterion c_j.

Step 1: To normalize the fuzzy decision matrix $A = (a_{ij})_{m\times n}$ into the correspondent normalized fuzzy decision matrix $G = (g_{ij})_{m\times n}$, according to the cost and benefit criteria.

Step 2: To determine the reference criterion c_r and calculate the relative weight w_{jr} of criterion c_j $(j = 1, 2, \ldots, n)$, i.e.,

$$w_{jr} = w_{c_j}/w_r \tag{4}$$

where $w_r = \max\left\{w_{c_j}\middle| j = 1, 2, \ldots, n\right\}$.

Step 3: To calculate the dominance degree, $\Phi_j(p_i, p_k)$, of alternative p_i, $(i = 1, 2, \ldots, m)$ over the remaining alternatives p_k $(k = 1, 2, \ldots, m)$ concerning criterion c_j $(j = 1, 2, \ldots, n)$, i.e.,

$$\Phi_j(p_i, p_k) = \begin{cases} \sqrt{w_{jr}/(\sum_{j=1}^{n} w_{jr})d(g_{ij}, g_{kj})}, & \mathbb{F}(g_{ij}) - \mathbb{F}(g_{kj}) \geq 0 \\ -\frac{1}{\theta}\sqrt{(\sum_{j=1}^{n} w_{jr})/w_{jr}d(g_{ij}, g_{kj})}, & \mathbb{F}(g_{ij}) - \mathbb{F}(g_{kj}) < 0 \end{cases} \tag{5}$$

where θ is the attenuation factor of the losses, $\theta > 0$. $d(g_{ij}, g_{kj})$ denotes the distance between two fuzzy numbers g_{ij} and g_{kj} and $\mathbb{F}(*)$ is a defuzzification function [18].

Step 4: To calculate the dominance degree, $\delta(p_i, p_k)$, of alternative p_i, $(i = 1, 2, \ldots, m)$ over the remaining alternatives p_k $(k = 1, 2, \ldots, m)$, i.e.,

$$\delta(p_i, p_k) = \sum_{j=1}^{n} \Phi_j(p_i, p_k) \tag{6}$$

Step 5: To calculate the overall dominance degree, $\eta(p_i)$, of alternative p_i, $(i = 1, 2, \ldots, m)$, i.e.,

$$\eta(p_i) = \frac{\sum_{k=1}^{m} \delta(p_i, p_k) - \min_i\{\sum_{k=1}^{m} \delta(p_i, p_k)\}}{\max_i\{\sum_{k=1}^{m} \delta(p_i, p_k)\} - \min_i\{\sum_{k=1}^{m} \delta(p_i, p_k)\}} \tag{7}$$

Step 6: According to the overall dominance degrees of each alternative, the corresponding ranking can be determined such that the bigger $\eta(p_i)$, the better alternative p_i.

2.4. Related Works

In order to show the importance of GEDM in the real world, this subsection reviews several important studies in the literature that are related to our research [1,4–6,40].

These studies have approached GEDM problems from different aspects. For example, Wang et al. [40] proposed a group emergency decision method based on prospect theory by using interval values. Xu et al. [4] proposed a consensus model for multi-criteria large group emergency decision making considering non-cooperative behaviors and minority opinions, wherein numerical value is employed to represent experts' assessments. Ju et al. [6] presented a model to evaluate emergency response capacity by using 2-tuple fuzzy linguistic information. Xu et al. [5] proposed a conflict-eliminating approach for GEDM problem. Levy and Taji [1] utilized a group analytic network process to construct a group decision support system to support hazard planning and emergency management under incomplete information.

So far, there is not any proposal in previous GEDM approaches [1,4,5,11,40] that considers the non-homogeneous information together with the experts' hesitation due to uncertain information. In addition, those GEDM approaches [1,4,5,11] dealing with the consensus process; just make use of it with strict expression domains or high time cost. However, time is extremely valuable in EDM process, which means life and opportunity. Furthermore, experts' psychological behavior is neglected in current GEDM approaches [1,4,5] that plays an important role in the GEDM process under risk and uncertainty.

As pointed out in Introduction, our proposed method aims to overcome such limitations and shows the relevance of this research.

3. Managing Non-Homogeneous Information and Experts' Psychological Behavior in GEDM

This section introduces a new GEDM method to overcome the limitations pointed out in the Introduction regarding the current GEDM methods. This proposal is able: (i) to manage non-homogeneous information, including hesitant information (ii) to achieve consensus with low time cost, (iii) to take into account the experts' psychological behavior in the GEDM process.

Our proposal extends the general scheme of a GEDM process shown in Figure 1 by adding two new phases to deal with non-homogeneous information and calculate the criteria weights, and modifying another two phases (CRP and selection process), they are highlighted in Figure 2 by using dashed lines.

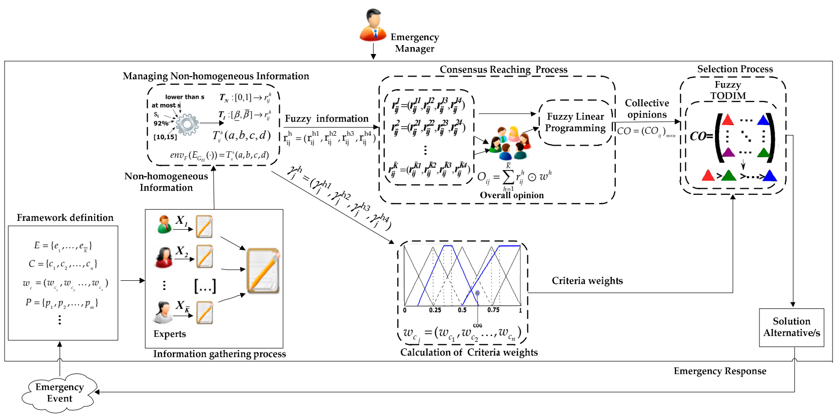

Figure 2. Scheme of proposed GEDM method.

It consists of six main phases:

1. Definition framework. The main features, terminology and expression domains utilized in the proposed GEDM problem are defined.
2. Information gathering process. Opinions or assessments over different alternatives concerning different criteria and importance of criteria provided by experts using multiple types of information are gathered.

3. Managing non-homogeneous information. The non-homogeneous information gathered is unified into a fuzzy domain to deal with the decision computations.
4. Consensus reaching process. A fuzzy linear programming-based consensus model [34] is utilized to deal with fuzzy information and achieve an agreement among all the experts involved in the GEDM problem.
5. Calculation of criteria weights. Criteria weights are calculated by using experts' opinions.
6. Selection process-fuzzy TODIM method. Fuzzy TODIM method is applied to manage experts' psychological behavior in GEDM processes and obtain the ranking of alternatives.

According to the ranking of alternatives, the EM can select the best or more suitable alternative to cope with the EE. These phases are further detailed in the following subsections.

3.1. Definition Framework

The framework for GEDM problem is established by defining its main features and terminology.

- $P = \{p_1, p_2, \ldots, p_m\}$: the set of emergency alternatives, where p_i is the i-th emergency alternative, $i = 1, 2, \ldots, m$.
- $C = \{c_1, c_2, \ldots, c_n\}$: the set of criteria/attributes, where c_j denotes the j-th criterion/attribute, $j = 1, 2, \ldots, n$.
- $w_c = (w_{c_1}, w_{c_2} \ldots, w_{c_n})$: the weighting vector for the criteria, where w_{c_j} denotes the criterion weight of the j-th criterion/attribute, satisfying $\sum_{j=1}^{n} w_{c_j} = 1$, $w_{c_j} \in [0,1]$ $j = 1, 2, \ldots, n$.
- $E = \{e_1, \ldots, e_{\overline{K}}\}$: the set of experts, where e_h denotes the h-th expert, $h = 1, 2, \ldots, \overline{K}$.
- $X_h = (x_{ij}^h)_{m \times n}$: the information matrix provided by the h-th expert, where x_{ij}^h represents the assessments/opinions provided by the h-th expert over the i-th alternative concerning the j-th criterion, $h = 1, 2, \ldots, \overline{K}$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$ (see Remark 1).
- $w_h = (w_1^h, w_2^h \ldots, w_n^h)$: the assessment vector of criteria importance provided by the expert e_h, where w_j^h represents the importance provided by the h-th expert on the importance of criterion c_j, $h = 1, 2, \ldots, \overline{K}$, $j = 1, 2, \ldots, n$ (see Remark 2).
- r_{ij}^h: denotes the experts' assessments, x_{ij}^h, unified in a fuzzy domain, $h = 1, 2, \ldots, \overline{K}$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$.
- γ_j^h: denotes the experts' opinions regarding the criteria importance, w_j^h, unified in a fuzzy domain, $h = 1, 2, \ldots, \overline{K}$, $j = 1, 2, \ldots, n$.

Remark 1. *In our method, experts can provide their opinions/assessments by utilizing multiple expression domains (numerical values (N), interval values (I), linguistic terms (S) and comparative linguistic expressions (S_{ll})) according to their background, degree of knowledge, hesitancy and criteria nature.*

$$x_{ij}^h \in \begin{cases} N \in R \\ I \in [\xi^L, \xi^U] \\ S = \{s_0, s_1, \ldots, s_g\} \\ S_{ll} \end{cases} \tag{8}$$

Remark 2. *In GEDM problems, the criteria need to be weighted. However, due to the complexity of EEs, it is not easy to collect the related information about the criteria, especially at the early stage of EE. In such situation, a possible way is to calculate the criteria weights from experts' knowledge and experience. In this proposal, experts can express their opinions about the criteria importance by utilizing either S_{ll} or S, because S_{ll} and S are* *more flexible and similar to the natural language utilized by human beings in real-world EE situations, and they are suitable for GEDM problems defined in uncertain contexts.*

$$w_j^h \in \begin{cases} S = \{s_0, s_1, \ldots, s_g\} \\ S_{ll} \end{cases} \tag{9}$$

3.2. Information Gathering Process

Once the framework of GEDM problem is defined, experts can provide their judgments over the emergency alternatives p_i concerning each criterion c_j and the importance over different criteria (see Tables 1 and 2) by using the expression domains defined previously.

Table 1. Assessments over alternative p_i concerning criterion c_j.

Experts	Assessments
e_1	$\{x_{ij}^1, \ldots, x_{mn}^1\}$
e_2	$\{x_{ij}^2, \ldots, x_{mn}^2\}$
$\cdots\cdots$	$\cdots\cdots$
$e_{\overline{K}}$	$\{x_{ij}^{\overline{K}}, \ldots, x_{mn}^{\overline{K}}\}$

Table 2. Importance over criteria c_j.

Experts	Assessments
e_1	$\{w_1^1, \ldots, w_n^1\}$
e_2	$\{w_1^2, \ldots, w_n^2\}$
$\cdots\cdots$	$\cdots\cdots$
$e_{\overline{K}}$	$\{w_1^{\overline{K}}, \ldots, w_n^{\overline{K}}\}$

For example, the information on alternatives with respect to criteria provided by expert e_1 can be expressed as:

$$X_1 = \begin{array}{c} \\ p_1 \\ p_2 \\ \vdots \\ p_m \end{array} \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & \cdots & c_n \end{array} \\ \begin{bmatrix} x_{11}^1 & x_{12}^1 & \cdots & x_{1n}^1 \\ x_{21}^1 & x_{22}^1 & \cdots & x_{2n}^1 \\ \vdots & \vdots & \cdots & \vdots \\ x_{m1}^1 & x_{m2}^1 & \cdots & x_{mn}^1 \end{bmatrix} \end{array}$$

where $x_{ij}^1 \in \begin{cases} N \in R \\ I \in [\xi^L, \xi^U] \\ S = \{s_0, s_1, \ldots, s_g\} \\ S_{ll} \end{cases}, i = 1, 2, \ldots, m, j = 1, 2, \ldots, n.$

The information on the importance of criterion c_j provided by expert e_1 can be expressed as:

$$w_1 = \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & \cdots & c_n \end{array} \\ \begin{bmatrix} w_1^1 & w_2^1 & \cdots & w_n^1 \end{bmatrix} \end{array}$$

where $w_j^1 \in \begin{cases} S = \{s_0, s_1, \ldots, s_g\} \\ S_{ll} \end{cases}, j = 1, 2, \ldots, n.$

3.3. Managing Non-Homogeneous Information

As it was stated in Section 2.1, our proposal deals with non-homogeneous information including hesitant information. Therefore, the expression domains used by experts to provide their assessments in this proposal are the following ones:

- *Numerical value*. Assessments represented as numerical values N belonging to a specific numerical scale R, i.e., $N \in R$.
- *Interval value*. Assessments represented as interval values I, belonging to a specific domain $[\xi^L, \xi^U]$, i.e., $I \in [\xi^L, \xi^U]$.
- *Linguistic terms*. Assessments represented as linguistic terms $s_k \in S = \{s_0, s_1, \ldots, s_g\}$, $k \in \{0, \ldots, g\}$, with granularity $g + 1$.
- *Comparative linguistic expressions*. Assessments represented as comparative linguistic expressions S_{II} generated by a context-free grammar G_H [29,30].

In order to make computations with non-homogeneous information elicited by experts, it is necessary to conduct the different types of information into a unique expression domain. Most approaches unify the non-homogeneous information into linguistic information [23,24]. Nevertheless, in order to keep the uncertainty provided by experts involved in a GEDM problem, we unify the information into a fuzzy domain r_{ij}^h, by introducing some transformation functions (see Figure 3).

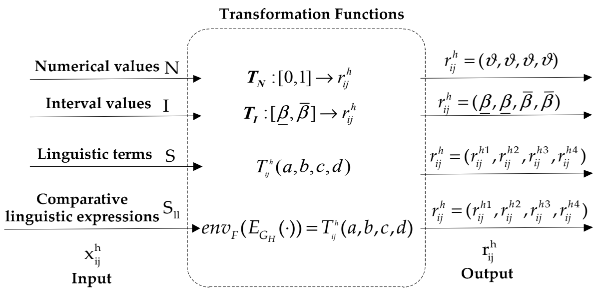

Figure 3. Unification process for non-homogeneous information.

The following transformation functions are defined to unify the information into a fuzzy domain.

1. For numerical values N, they are first normalized into the interval $[0,1]$ and then a transformation function T_N is utilized to transform them into trapezoidal fuzzy numbers. Let R be the domain of the numerical values, N_{ij}^h be the numerical value provided by the h-th expert over the i-th alternative concerning the j-th criterion, N_{ij}^h is normalized into the interval $[0,1]$, as follows:

$$\vartheta = \frac{N_{ij}^h}{N^*}$$

where $\vartheta \in [0,1]$, $N^* = \max_{h=1,2,\ldots,\overline{K}} \{N_{ij}^h\}$, $i = 1,2,\ldots,m$, $j = 1,2,\ldots,n$.

Definition 3. *A numerical value is transformed into a trapezoidal fuzzy number by utilizing a transformation function T_N:*

$$T_N : [0,1] \rightarrow r_{ij}^h \tag{10}$$

$$T_N(\vartheta) = r_{ij}^h = (\vartheta, \vartheta, \vartheta, \vartheta)$$

2. The interval values I are first normalized into $[0,1]$ and then a transformation function T_I is utilized to transform them into trapezoidal fuzzy numbers. Let $[\xi^L, \xi^U]$ be the domain of the interval values, let $[d^L, d^U]_{ij}^h$ be the interval values provided by the h-th expert over the i-th alternative concerning the j-th criterion, where $[d^L, d^U]_{ij}^h \in [\xi^L, \xi^U]$. The interval values $[d^L, d^U]_{ij}^h$ are normalized into $[\,\underline{\beta}, \overline{\beta}\,]$ as follows:

$$\underline{\beta} = \frac{d^L - \xi^L}{\xi^U - \xi^L} \text{ and } \overline{\beta} = \frac{d^U - \xi^L}{\xi^U - \xi^L} \tag{11}$$

The transformation function T_I is defined as follows.

Definition 4. *An interval value is transformed into a trapezoidal fuzzy number by utilizing a transformation function* T_I:

$$T_I : [\,\underline{\beta}, \overline{\beta}] \rightarrow r_{ij}^h \tag{12}$$

$$T_I(\,\underline{\beta}, \overline{\beta}) = r_{ij}^h = (\underline{\beta}, \underline{\beta}, \overline{\beta}, \overline{\beta})$$

where $\underline{\beta}, \overline{\beta} \in [0,1]$ *and* $\underline{\beta} \leq \overline{\beta}$.

3. The linguistic terms $s_k \in S = \{s_0, s_1, \ldots, s_g\}$, are represented by trapezoidal fuzzy numbers. Therefore, the expert e_h provides his/her opinions over the i-th alternative concerning the j-th criterion as a linguistic term s_k that is represented by a trapezoidal fuzzy number $r_{ij}^h = (r_{ij}^{h1}, r_{ij}^{h2}, r_{ij}^{h3}, r_{ij}^{h4})$.
4. The comparative linguistic expressions, $x_{ij}^h \in S_{II}$, are transformed into HFLTS by $E_{G_H}(\cdot)$ and its fuzzy envelop $env_F(\cdot)$ obtained by [41],

$$env_F(E_{G_H}(x_{ij}^h)) = T_{ij}^h(a,b,c,d) = r_{ij}^h \tag{13}$$

E_{G_H} is a function that transforms the linguistic expressions obtained by using G_H, into HFLTS [30]. $T_{ij}^h(a,b,c,d)$ is a trapezoidal fuzzy membership function corresponding to the trapezoidal fuzzy number $r_{ij}^h = (r_{ij}^{h1}, r_{ij}^{h2}, r_{ij}^{h3}, r_{ij}^{h4})$.

3.4. Consensus Reaching Process

As stated in Section 2.2, a fuzzy linear programming-based consensus model [34] is used in our proposal to achieve an agreement among all the experts involved in the problem. This model is able to deal with fuzzy information and update experts' opinions automatically without a supervised feedback mechanism [33], which is adequate for GEDM problems defined in fuzzy environment (see Figure 4).

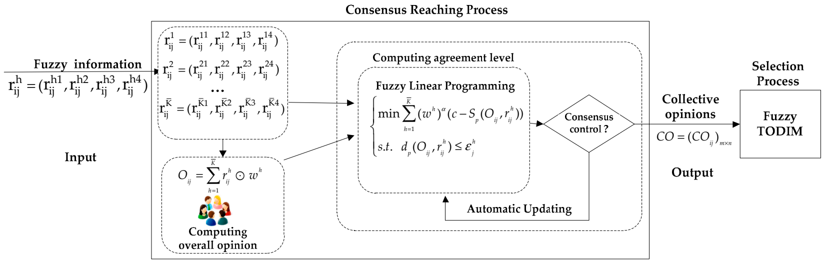

Figure 4. The process of fuzzy linear programming-based consensus model.

The fuzzy linear programming-based consensus model is given by,

$$\begin{cases} \min \sum\limits_{h=1}^{\overline{K}} (w^h)^{\alpha} (c - S_p(O_{ij}, r_{ij}^h)) \\ s.t.\ d_p(O_{ij}, r_{ij}^h) \le \varepsilon_j^h,\ h = 1, 2, ..., \overline{K};\ j = 1, 2, \ldots, n,\ i = 1, 2, \ldots, m. \end{cases} \tag{14}$$

According to Figure 4, the input information is represented in a fuzzy domain, which is obtained from the previous phase. It consists of three steps that are further detailed as follows:

1. *Computing overall opinion*. As introduced in Section 2.2, before applying fuzzy linear programming model, the overall opinions, O_{ij}, are obtained by aggregating the individual expert opinions, r_{ij}^h. Let O_{ij} be the overall opinion over the i-th alternative concerning the j-th criterion. It can be obtained as follows:

$$O_{ij} = \sum_{h=1}^{\overline{K}} r_{ij}^h \odot w^h,\ h = 1, 2, \ldots, \overline{K},\ j = 1, 2, \ldots, n,\ i = 1, 2, \ldots, m. \tag{15}$$

where $\odot$ is an aggregation operator. For example, suppose that $r_{12}^1 = (0.17, 0.34, 0.5, 0.67)$, $r_{12}^2 = (0, 0.17, 0.34, 0.5)$ and $(w^1, w^2) = (0.6, 0.4)$, then O_{12} could be computed by a weighted average operator:

$$\begin{aligned} O_{12} &= 0.6 \odot (0.17, 0.34, 0.5, 0.67) + 0.4 \odot (0, 0.17, 0.34, 0.5) \\ &= (0.102, 0.272, 0.436, 0.602) \end{aligned}$$

2. *Computing agreement level*. In this step, there are two processes:

   (i) *Computing the distance and similarity*. The distance, $d_p(O_{ij}, r_{ij}^h)$, between the overall opinion, O_{ij}, and the individual opinion, r_{ij}^h, and its similarity, $S_p(O_{ij}, r_{ij}^h)$, can be computed according to Equations (1) and (2) respectively.

   (ii) *Determining the threshold values.* The threshold value, ε_j^h, is an important factor in the fuzzy linear programming model, which means the maximum change that the expert e_h can make concerning the j-th criterion. There are different ways to determine the threshold value ε_j^h [34,35]. In this paper, ε_j^h will be calculated by the h-th experts' familiarity degree concerning the j-th criterion using a linguistic term set $S = \{s_0, s_1, \ldots, s_g\}$, because the linguistic terms are flexible and able to deal with uncertain and vague information. The more familiar the expert is with the criterion, the less change he/she will make. Therefore, a negative operator is applied to the familiarity degree to obtain the threshold, which is defined as follows:

Definition 5. *Let $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set, a negative operator:*

$$Neg(s_k) = \widetilde{s}_q, \text{ such that } q = g - k,\ k = \{0, \ldots, g\}. \tag{16}$$

where $g + 1$ is the cardinality of S.

Thus, the ε_j^h can be computed by using the center of gravity (COG) method [42], i.e., $\varepsilon_j^h = COG(\widetilde{s}_q)$ (see Equation (18)).

3. *Control consensus.* When all constraints meet the conditions in Equation (14), it means that the consensus has been reached, and the final overall opinion, O_{ij}, is the aggregated collective opinion denoted as $CO = (CO_{ij})_{m \times n}$ Which will be used as input in the selection process.

3.5. Calculation of Criteria Weights

In this phase, the weights of criteria, w_{c_j}, are calculated by utilizing the experts' assessments provided over the criteria importance which were unified into a fuzzy domain. Figure 5 shows the process of computing criteria weights.

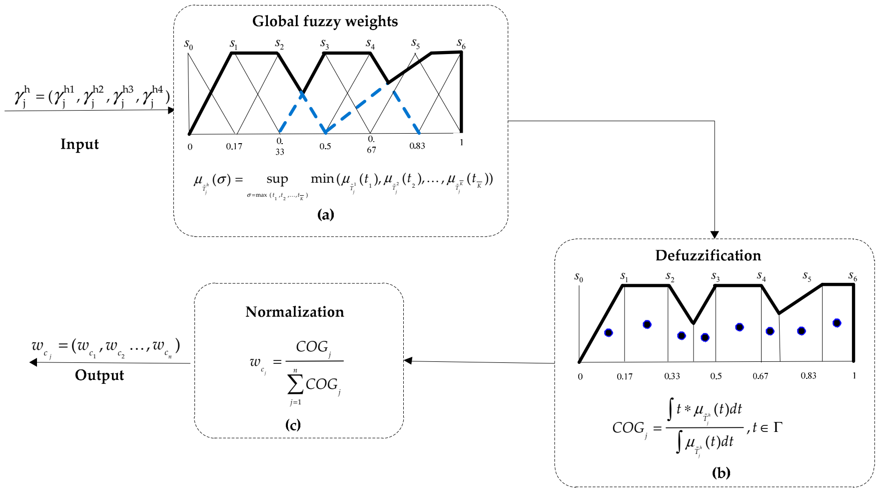

Figure 5. Computing criteria weights.

Three steps are comprised:

1. *Global fuzzy weights.* The fuzzy weights obtained for the criterion c_j are aggregated by using a max-min composition [43,44]:

$$\mu_{\widetilde{T}_j^h}(\sigma) = \sup_{\sigma=\max(t_1,t_2,\ldots,t_{\overline{K}})} \min\left(\mu_{\widetilde{T}_j^1}(t_1), \mu_{\widetilde{T}_j^2}(t_2), \ldots, \mu_{\widetilde{T}_j^{\overline{K}}}(t_{\overline{K}})\right), t_h \in \Gamma, h \in \{1, 2, \ldots, \overline{K}\} \tag{17}$$

where $\widetilde{T}_j^h$ is the fuzzy membership function of w_j^h, $j = 1, 2, \ldots, n$, and Γ is the universe of discourse.

Suppose that three experts provide their opinions w_1^1, w_1^2 and w_1^3 concerning the criterion c_1, the corresponding fuzzy membership functions are $\widetilde{T}_1^1$, $\widetilde{T}_1^2$ and $\widetilde{T}_1^3$ respectively. According to Equation (17), $\mu_{\widetilde{T}_j^h}(\sigma)$ is the area under the bold black line shown in Figure 5a.

2. *Defuzzification.* The COG method [42] is utilized to calculate the weighting value of the global fuzzy weights:

$$COG_j = \frac{\int t * \mu_{\widetilde{T}_j^h}(t)dt}{\int \mu_{\widetilde{T}_j^h}(t)dt}, t \in \Gamma \tag{18}$$

where Γ is the universe of discourse.

For criterion c_1, Equation (18) means that the center of gravity for each small trapezoid (see Figure 5b) is computed and the COG_1 can be obtained by the arithmetic mean of the sum of center of gravity of all small trapezoids.

3. *Normalization.* When COG_j of all criteria are obtained, the criteria weights w_{c_j} are calculated by using the following equation:

$$w_{c_j} = \frac{COG_j}{\sum_{j=1}^{n} COG_j} \tag{19}$$

where $\sum\limits_{j=1}^{n} w_{c_j} = 1, w_{c_j} \in [0,1]\ j = 1,2,\ldots,n.$

3.6. Selection Process—Fuzzy TODIM Method

As it was pointed out in Introduction, the experts' psychological behavior are neglected in current GEDM approaches. However, our proposal takes into account experts' psychological behavior by means of fuzzy TODIM based on prospect theory dealing with the problem defined in a fuzzy environment.

Once the criteria weights w_{c_j} and the aggregated collective opinions $CO = (CO_{ij})_{m\times n}$ are obtained, the fuzzy TODIM method is applied to obtain a ranking of alternatives and select the best one. To do so, the fuzzy TODIM method introduced in Section 2.3 is used. The step 1 is not necessary to do it, because the collective opinion matrix $CO = (CO_{ij})_{m\times n}$, is already normalized and the step 3 has been modified to adapted it to GEDM problem as it is shown below:

Step 3: To calculate the dominance degree, $\Phi_j(p_i, p_k)$, of alternative p_i $(i = 1,2,\ldots,m)$ over the remaining alternatives p_k $(k = 1,2,\ldots,m)$ concerning criterion c_j $(j = 1,2,\ldots,n)$, i.e.,

$$\Phi_j(p_i,p_k) = \begin{cases} \sqrt{d(CO_{ij},CO_{kj})w_{jr}/(\sum_{j=1}^{n} w_{jr})}, & \widetilde{m}(CO_{ij}) - \widetilde{m}(CO_{kj}) \geq 0 \\ -\frac{1}{\theta}\sqrt{d(CO_{ij},CO_{kj})(\sum_{j=1}^{n} w_{jr})/w_{jr}}, & \widetilde{m}(CO_{ij}) - \widetilde{m}(CO_{kj}) < 0 \end{cases} \tag{20}$$

CO_{ij} denotes the trapezoidal fuzzy number $CO_{ij} = (CO_{ij}^1, CO_{ij}^2, CO_{ij}^3, CO_{ij}^4)$ that represents the information about the i-th alternative concerning the j-th criterion. $\widetilde{m}(CO_{ij})$ and $\widetilde{m}(CO_{kj})$ denotes the defuzzified value of the fuzzy number CO_{ij} and CO_{kj}, respectively, where $\widetilde{m}(CO_{ij}) = \frac{CO_{ij}^1+2CO_{ij}^2+2CO_{ij}^3+CO_{ij}^4}{6}$ [42]. $d(CO_{ij}, CO_{kj})$ denotes the gains or losses of the alternative p_i over p_k concerning the criterion c_j, where $d(CO_{ij}, CO_{kj}) = \sqrt{\sum_{\ell=1}^{4}(CO_{ij}^{\ell} - CO_{kj}^{\ell})^2}$ [45].

For benefit criteria, $d(CO_{ij}, CO_{kj})$ denotes the gains with $\widetilde{m}(CO_{ij}) - \widetilde{m}(CO_{kj}) \geq 0$ or losses with $\widetilde{m}(CO_{ij}) - \widetilde{m}(CO_{kj}) < 0$, respectively. $\Phi_j(p_i, p_k)$ can be expressed as:

$$\Phi_j(p_i,p_k) = \begin{cases} \sqrt{d(CO_{ij},CO_{kj})w_{jr}/(\sum_{j=1}^{n} w_{jr})}, & \widetilde{m}(CO_{ij}) - \widetilde{m}(CO_{kj}) \geq 0 \\ -\frac{1}{\theta}\sqrt{d(CO_{ij},CO_{kj})(\sum_{j=1}^{n} w_{jr})/w_{jr}}, & \widetilde{m}(CO_{ij}) - \widetilde{m}(CO_{kj}) < 0 \end{cases} \tag{21}$$

For cost criteria, $d(CO_{ij}, CO_{kj})$ denotes the gains with $\widetilde{m}(CO_{ij}) - \widetilde{m}(CO_{kj}) \leq 0$ or losses with $\widetilde{m}(CO_{ij}) - \widetilde{m}(CO_{kj}) > 0$, respectively, $\Phi_j(p_i, p_k)$ can be expressed as:

$$\Phi_j(p_i, p_k) = \begin{cases} \sqrt{d(CO_{ij}, CO_{kj})w_{jr}/(\sum_{j=1}^{n} w_{jr})}, \ \widetilde{m}(CO_{ij}) - \widetilde{m}(CO_{kj}) \leq 0 \\ -\frac{1}{\theta}\sqrt{d(CO_{ij}, CO_{kj})(\sum_{j=1}^{n} w_{jr})/w_{jr}}, \ \widetilde{m}(CO_{ij}) - \widetilde{m}(CO_{kj}) > 0 \end{cases} \tag{22}$$

Finally, the ranking of alternatives can be determined according to their overall dominance degree.

4. Group Decision Support System for GEDM Based on GENESIS: Case Study

EEs are always characterized by complexity, risk and uncertainty, and a delayed or wrong decision may result in extremely serious consequences. Thus, it is necessary to make a decision in short time, taking into account the opinions of multiple experts involved in the problem.

In order to deal properly with real-world GEDM problems and make timely and effective decisions, we have implemented a GDSS named GENESIS to support the proposed GEDM method. This section introduces the structure and components of GENESIS (see Figure 6); and shows a case study to illustrate the applicability and robustness of the proposed method by using GENESIS.

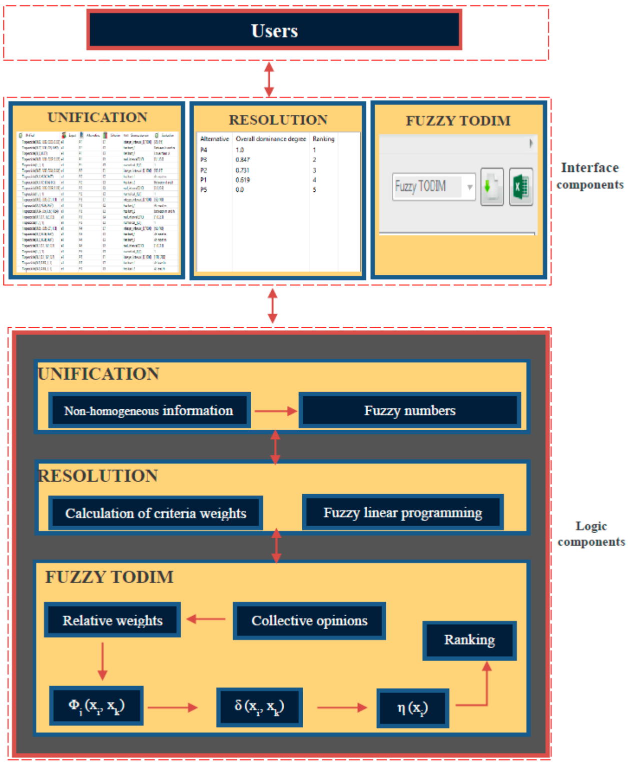

Figure 6. Structure of GENESIS.

4.1. GENESIS: (Group EmergeNcy dEcision SupportIng System)

Since our proposal deals with non-homogeneous and fuzzy information, in order to facilitate the transformation of non-homogeneous information and the decision process of the proposed method in a simple and fast manner, GENESIS has been implemented to use different components and specific functions based on FLINTSTONES [19,20] developed by using Eclipse Rich Client Platform (Eclipse RCP), which is a component-based application [46], a platform that builds and deploys rich client applications.

GENESIS consists of six components (see Figure 6):

(1) Two components taken from FLINTSTONES are adapted to define different transformation functions to unify non-homogeneous information into a fuzzy domain and show its user interface respectively.
(2) Two new components are defined for the resolution processes and show their interface to compute the criteria weights and obtain the consensus opinion based on fuzzy linear programming-based consensus model.
(3) Two new components are introduced to carry out the steps defined in the fuzzy TODIM method such as the computation of the relative weights, dominance degree etc., and show its user interface.

4.2. Case Study

In order to demonstrate the applicability of the proposed GEDM method, this section presents an example adapted from a big explosion of Tianjin Port that occurred in the north of China (Background Information Source. http://www.safehoo.com/Case/Case/Blow/201602/428723.shtml).

The blasts took place at a warehouse at the port that contained hazardous and flammable chemicals, including calcium carbide, sodium cyanide, potassium nitrate, ammonium nitrate and sodium nitrate, etc.

In this problem, we assume that six experts are invited to participate in the EDM process to support the EM to make the final decision. In order to solve this GEDM problem, we have used the proposed method by means of GENESIS.

4.2.1. Framework Definition

When the explosion occurred, the local government organized people located within two kilometers of the explosion area, evacuated them to safety areas and sent short messages to inform people in potentially dangerous areas to prepare for evacuation and keep distances from the dangerous area. Five emergency alternatives $\{p_1, p_2, \ldots, p_5\}$ were put forward taking into account five criteria $\{c_1, c_2, \ldots, c_5\}$, which are described in Tables 3 and 4, respectively.

For the criteria importance, the linguistic term set is S_1 = {*absolutely low importance (ali), very low importance (vli), low importance (li), medium importance (mi), high importance (hi), very high importance (vhi), absolutely high importance (ahi)*}. (see Figure 7 "syntax for S_1")

For criteria C_2 and C_3, the experts provide their opinions using linguistic term sets S_2 = {*none (n), very low seriously (vls), low seriously (ls), medium (m), high seriously (hs), very high seriously (vhs), absolutely seriously (as)*} and S_3 = {*none (n), very low (vl), low (l), medium (m), high (h), very high (vh), absolutely high (ah)*} (see Figure 7 "syntax for C_2" and "syntax for C_3"), respectively.

Table 3. Description of alternatives.

Alternative	Description
Evacuate people (p_1)	Evacuate and inform people, and at same time, assign 9 fire squadrons and 35 fire engines to deal with the emergency event.
Increase help and report (p_2)	Increase to 23 fire squadrons, 93 fire engines and more than 600 fire fighters for participating in dealing with the emergency event; at the same time, the local government report the latest news to the masses in order to avoid causing panic and riot.
Rescue military (p_3)	Local government asks the Chinese professional emergency rescue military for emergency rescue. More than 300 soldiers with professional equipment join the rescue action.
Joint rescue (p_4)	Fire squadrons and the military work together dealing with the problems, at the same time, local government asks neighbor cities for fire police to provide support.
Block boundary of explosion areas (p_5)	Block the boundary of the explosion areas; let the material in the explosion areas burn down.

Table 4. Description of criteria.

Criteria	Expression Domain	Description
People affected (C_1)	Interval values	It means that alternative p_i can protect the number of people from the effects caused by EE in [0,1000].
Negative effect on the environment (C_2)	Linguistic	It is evaluated by experts on linguistic expressions.
Social impacts (C_3)	Linguistic	It means the impacts on social development or people's daily life etc. that are evaluated by experts on linguistic expressions.
Property loss (C_4)	Interval values	It means that the alternative p_i can protect the direct and indirect property losses that are caused by the EE in [0,10]. (in billion RMB).
Cost of alternative (C_5)	Numerical values	The numerical values are 0 and 1. 0 means that expert e_h does not care about the cost; 1 means that he/she cares about it.

Note: assume that above criteria are independent.

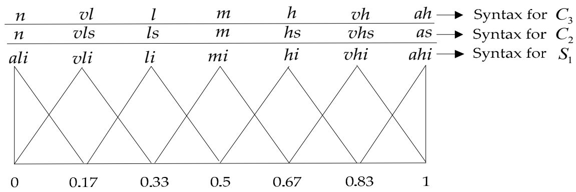

Figure 7. Linguistic term set for S_1, C_2 and C_3.

4.2.2. Information Gathering Process

The assessments provided by experts over the alternatives concerning criteria, and their opinions regarding the criteria importance and the familiarity degree for each criterion are shown in Tables 5–7 respectively. This phase is supported by GENESIS to facilitate the information gathering process (see Figure 8).

Table 5. Assessments provided by all experts on different alternatives concerning each criterion.

Expert	Alternative	Criteria				
		C_1	C_2	C_3	C_4	C_5
		Interval Values [0,1000]	**Linguistic**	**Linguistic**	**Interval Values [0,10]**	**Numerical Values (0,1)**
e_1	P_1	[20,25]	*ls*	*l*	[0.2,0.3]	1
	P_2	[30,35]	*ls*	*m*	[0.2,0.35]	1
	P_3	[50,80]	*m*	*h*	[0.5,0.8]	1
	P_4	[100,150]	*hs*	*bt m* and *h*	[1.0,2.0]	1
	P_5	[60,70]	*vhs*	*vh*	[0.1,0.2]	1
e_2	P_1	[30,50]	*vls*	*At* most *vl*	[0.25,0.4]	1
	P_2	[40,50]	*vls*	*vl*	[0.3,0.5]	1
	P_3	[100,150]	*ls*	*m*	[0.6,1.5]	1
	P_4	[150,250]	*m*	*l*	[2.0,2.5]	1
	P_5	[80,100]	*hs*	*vh*	[0.1,0.25]	1
e_3	P_1	[20,30]	*vls*	*l*	[0.1,0.15]	1
	P_2	[30,60]	*ls*	*l*	[0.15,0.25]	1
	P_3	[60,100]	*bt ls* and *m*	*h*	[0.2,0.3]	1
	P_4	[200,300]	*ls*	*m*	[1.5,2.5]	1
	P_5	[50,80]	*hs*	*bt h* and *vh*	[0.2,0.25]	1
e_4	P_1	[25,40]	*vls*	*vl*	[0.2,0.25]	1
	P_2	[30,45]	*vls*	At most *l*	[0.4,0.5]	1
	P_3	[80,150]	*ls*	*m*	[0.6,1.0]	1
	P_4	[200,250]	*bt ls* and *m*	*l*	[1.5,3.0]	1
	P_5	[50,70]	*vhs*	*vh*	[0.3,0.6]	1
e_5	P_1	[20,30]	*vls*	*l*	[0.25,0.3]	1
	P_2	[30,40]	*ls*	*vl*	[0.3,0.4]	1
	P_3	[50,80]	*At* most *m*	*m*	[0.5,1.0]	1
	P_4	[150,300]	*vls*	*l*	[2.0,2.5]	1
	P_5	[40,70]	*bt hs* and *vhs*	*vh*	[0.35,0.5]	1
e_6	P_1	[30,40]	*ls*	*vl*	[0.2,0.3]	1
	P_2	[20,50]	*vls*	*vl*	[0.5,0.6]	1
	P_3	[40,70]	*ls*	*l*	[0.4,0.6]	1
	P_4	[200,300]	*m*	*bt vl* and *l*	[2.5,3.5]	1
	P_5	[50,60]	*hs*	*h*	[0.3,0.5]	1

Table 6. The importance of each criterion provided by each expert.

Experts	Criteria				
	C_1	C_2	C_3	C_4	C_5
e_1	*vhi*	*hi*	*hi*	*li*	*mi*
e_2	*bt hi and vhi*	*hi*	*hi*	*mi*	*li*
e_3	*hi*	*mi*	*hi*	*li*	*vli*
e_4	*vhi*	*mi*	*mi*	*li*	*vli*
e_5	*hi*	*mi*	*hi*	*mi*	*li*
e_6	At least *hi*	*hi*	*hi*	*mi*	*li*

Note: "bt" means between in Tables 5 and 6.

Table 7. The familiarity degree provided by all experts for each criterion.

Experts	Criteria				
	C_1	C_2	C_3	C_4	C_5
e_1	*vs*	*s*	*vs*	*m*	*m*
e_2	*s*	*m*	*s*	*vs*	*m*
e_3	*m*	*vs*	*vs*	*m*	*s*
e_4	*vs*	*m*	*s*	*s*	*m*
e_5	*m*	*vs*	*vs*	*s*	*u*
e_6	*s*	*s*	*s*	*m*	*m*

Symmetry **2017**, *9*, 234

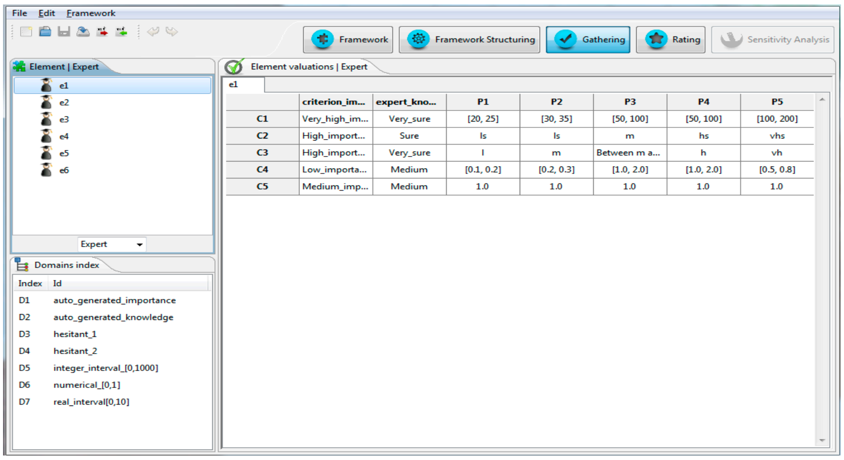

Figure 8. Gathered information in GENESIS.

4.2.3. Managing Non-Homogeneous Information

All experts' assessments are transformed into trapezoidal fuzzy numbers by utilizing the transformation functions defined in Section 3.2. Therefore, GENESIS makes all the necessary computations to unify the non-homogeneous information into a fuzzy domain in a simple and fast way. Figure 9 shows the interface of such a process.

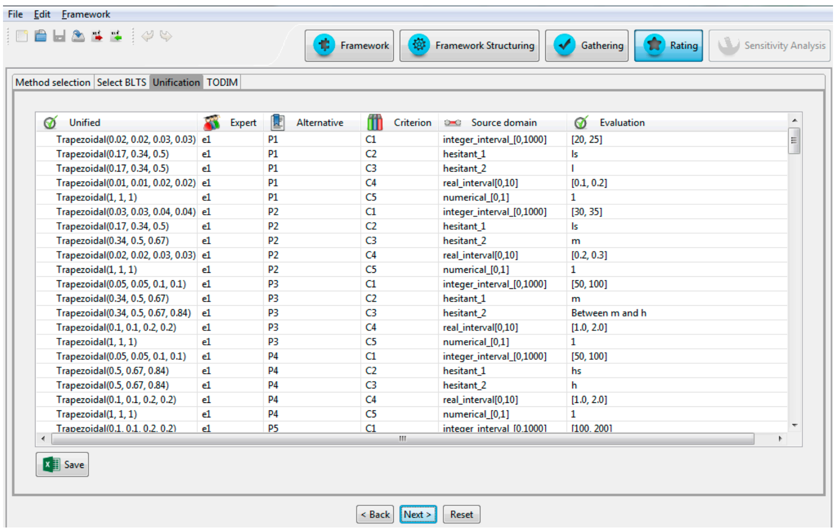

Figure 9. Unification results of non-homogeneous information.

4.2.4. Consensus Reaching Process

The fuzzy linear programming-based consensus model is utilized to achieve the consensus among all experts involved in the GEDM problem and obtain the collective opinion that will be used in the selection process. Before applying the CRP, the threshold values in Equation (14) should be determined.

Let S_4 = {s_0: *none (n)*, s_1*: very unsure (vu)*, s_2*: unsure (u)*, s_3*: medium (m)*, s_4*: sure (s)*, s_5*: very sure (vs)*, s_6*: absolutely sure (as)*} be the linguistic term set (see Figure 10) used by experts to express their familiarity degree for each criterion.

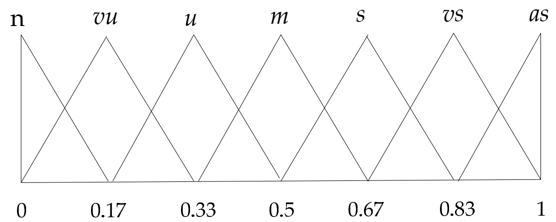

Figure 10. Linguistic term set S_4.

Expert e_h provides his/her familiarity degree for the criterion c_j by using a linguistic term $s_k \in S_4$. According to Equation (16), $\widetilde{s}_q = s_{6-k}$, then, the COG of $\widetilde{s}_q$ is regarded as the threshold value for the expert e_h about the criterion c_j, shown in Table 8. Table 7 is the familiarity degree provided by all experts for each criterion.

Table 8. Threshold values for $\widetilde{s}_q$ transformed by negative operator.

$\widetilde{s}_q$	Threshold Value
$\widetilde{s}_0$	0
$\widetilde{s}_1$	0.17
$\widetilde{s}_2$	0.33
$\widetilde{s}_3$	0.5
$\widetilde{s}_4$	0.67
$\widetilde{s}_5$	0.83
$\widetilde{s}_6$	1

For example, expert e_1 provides his/her familiarity degree for the criterion c_1, $s_5 = vs$, then according to Equation (16), $\widetilde{s}_1 = vu$, and, the COG of $\widetilde{s}_1$ is 0.17, i.e., $\varepsilon_1^1 = COG(vu) = 0.17$, it means that the maximum change that expert e_1 can make is 0.17 for the criterion c_1.

In this GEDM problem, experts' weights w^h have the same importance. The parameters p, α and c used in Equation (13) are set, $p = 2$, $\alpha = 2$ and $c = 1.5$ respectively [36].

When all constraints meet the conditions in Equation (14), the aggregated collective opinion, $CO = (CO_{ij})_{5\times5}$, is obtained.

$$CO = \begin{bmatrix} (0.02,0.02,0.04,0.04) & (0.11,0.22,0.22,0.41) & (0.11,0.22,0.22,0.49) & (0.02,0.02,0.03,0.03) & (1,1,1,1) \\ (0.03,0.03,0.08,0.08) & (0.14,0.19,0.19,0.60) & (0.11,0.17,0.17,0.44) & (0.04,0.04,0.06,0.06) & (1,1,1,1) \\ (0.07,0.07,0.28,0.28) & (0.18,0.30,0.32,0.53) & (0.21,0.37,0.37,0.79) & (0.06,0.06,0.25,0.25) & (1,1,1,1) \\ (0.17,0.17,0.50,0.50) & (0.20,0.38,0.38,0.69) & (0.15,0.30,0.33,0.62) & (0.23,0.23,0.40,0.40) & (1,1,1,1) \\ (0.05,0.05,0.09,0.09) & (0.53,0.70,0.70,0.90) & (0.58,0.73,0.73,0.95) & (0.02,0.02,0.05,0.05) & (1,1,1,1) \end{bmatrix}$$

4.2.5. Calculation of Criteria Weights

Using Table 6, the criteria weights are calculated by GENESIS (see Figure 11).

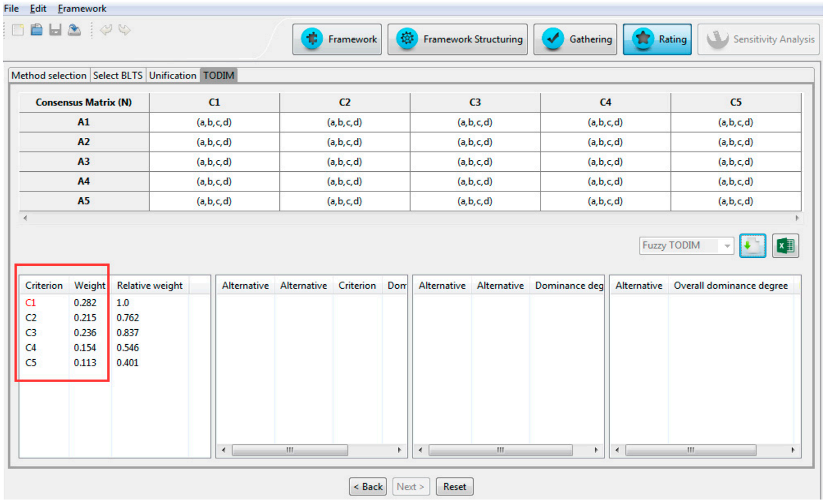

Figure 11. Criteria weights w_{c_j} obtained by GENESIS.

4.2.6. Selection Process-Fuzzy TODIM Method

Once the criteria weights w_{c_j} and the aggregated collective opinion $CO = (CO_{ij})_{m \times n}$ are obtained, fuzzy TODIM method is applied to calculate the overall dominance degree for each alternative and then the ranking of the alternative is obtained. Figure 12 shows the results obtained for each step of the fuzzy TODIM method.

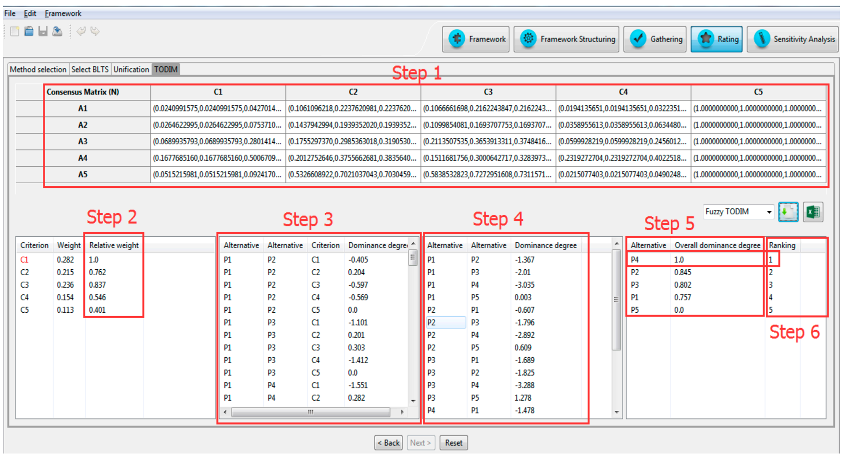

Figure 12. The results of different steps based on fuzzy TODIM by using GENESIS.

The ranking of alternatives is obtained according to the overall dominance degree for each alternative:

$$p_4 \succ p_2 \succ p_3 \succ p_1 \succ p_5$$

Finally, the EM can select p_4, "joint rescue" as the best alternative for the emergency response.

4.2.7. Sensitivity Analysis

To illustrate the feasibility and validity of the proposed method, sensitivity analysis is carried out in a similar way to other TODIM-based proposals in literature [38].

In this case, two aspects of sensitivity analysis are conducted: (i) the analysis about the weight evolution of the most important criterion and (ii) the evolution of attenuation factor θ.

For the weight evolution of the most important criterion, in this case study, it is C_1. First, let the weight of criterion C_1 be equal to the second most important criterion, i.e., $C_1 = 0.236$, then changing the weight of C_1 from 0.236 to 1. The reason for doing this is that the most important criterion is always the same and never changes, hence the relative weights are always calculated according to the same criterion. Applying these changes, the ranking of alternatives does not change.

The attenuation factor θ evolution, is changed from 1 to 15. When these alterations are carried out, there is no any change in the ranking of alternatives.

From the sensitivity analysis, it is easy to see that the ranking of alternatives is consistent with each other. It shows the feasibility, validity and the robustness of the proposed method.

5. Conclusions and Future Works

The non-homogeneous information including experts' hesitancy is not available in current GEDM approaches. To fill such a gap, this paper has taken into account the non-homogeneous information including experts' hesitancy, which extends the scope of non-homogeneous information defined in previous approaches. In order to make computations with non-homogeneous information defined in our proposal, different transformation functions have been presented to unify it into fuzzy numbers. A fuzzy linear programming-based consensus model with a new way for determining the threshold values has been applied to obtain the collective opinion, which is suitable for dealing with the fuzzy information. Experts' psychological behavior is very important in decision processes under risk and uncertainty; however, it is neglected in current GEDM approaches. To address such an important issue, fuzzy TODIM method has been utilized in our proposal due to its advantage of capturing human beings psychological behavior. Furthermore, a case study has been provided to illustrate the feasibility and validity of the proposed method by using GENESIS supporting the whole decision process.

Future research could be the use of computer science and Internet technology for supporting the EDM based on big data, which will lead to more reliable decisions. Furthermore, game theory [47,48] can be applied to deal with the emergency problems under uncertainty.

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References

1. Levy, J.K.; Taji, K. Group decision support for hazards planning and emergency management: A group analytic network process (GANP) approach. *Math. Comput. Model. Int. J.* **2007**, *46*, 906–917. [CrossRef]
2. Liu, Y.; Fan, Z.P.; Zhang, Y. Risk decision analysis in emergency response: A method based on cumulative prospect theory. *Comput. Oper. Res.* **2014**, *42*, 75–82. [CrossRef]

3. Wang, L.; Zhang, Z.X.; Wang, Y.M. A prospect theory-based interval dynamic reference point method for emergency decision making. *Expert Syst. Appl.* **2015**, *42*, 9379–9388. [CrossRef]
4. Xu, X.H.; Du, Z.J.; Chen, X.H. Consensus model for multi-criteria large-group emergency decision making considering non-cooperative behaviors and minority opinions. *Decis. Support Syst.* **2015**, *79*, 150–160. [CrossRef]
5. Xu, Y.; Zhang, W.; Wang, H. A conflict-eliminating approach for emergency group decision of unconventional incidents. *Knowl. Based Syst.* **2015**, *83*, 92–104. [CrossRef]
6. Ju, Y.; Wang, A.; Liu, X. Evaluating emergency response capacity by fuzzy AHP and 2-tuple fuzzy linguistic approach. *Expert Syst. Appl.* **2012**, *39*, 6972–6981. [CrossRef]
7. Herrera, F.; Herrera-Viedma, E.; Verdegay, J.L. A sequential selection process in group decision making with a linguistic assessment approach. *Inf. Sci.* **1995**, *85*, 223–239. [CrossRef]
8. Roubens, M. Fuzzy sets and decision analysis. *Fuzzy Sets Syst.* **1997**, *90*, 199–206. [CrossRef]
9. Butler, C.L.; Rothstein, A. *On Conflict and Consensus: A Handbook on Formal Consensus Decision Making*, 2nd ed.; Citeseer: Food Not Bombs Publishing: Portland, ME, USA, 2007; pp. 21–22.
10. Saint, S.; Lawson, J.R. *Rules for Reaching Consensus: A Modern Approach to Decision Making*; Jossey-Bass: San Francisco, CA, USA, 1994; pp. 27–49.
11. Ping, F.; Chong, W.; Jie, T. Unconventional emergency management based on intelligent group decision-making methodology. *Adv. Inf. Sci. Serv. Sci.* **2012**, *4*, 208–216.
12. Camerer, C. Bounded rationality in individual decision making. *Exp. Econ.* **1998**, *1*, 163–183. [CrossRef]
13. Tversky, A.; Kahneman, D. Advances in prospect theory: Cumulative representation of uncertainty. *J. Risk Uncertain.* **1992**, *5*, 297–323. [CrossRef]
14. Kahneman, D.; Tversky, A. Prospect theory: An analysis of decision under risk. *Econometrica* **1979**, *47*, 263–291. [CrossRef]
15. Yu, L.; Lai, K.K. A distance-based group decision-making methodology for multi-person multi-criteria emergency decision support. *Decis. Support Syst.* **2011**, *51*, 307–315. [CrossRef]
16. Wei, C.; Ren, Z.; Rodriguez, R.M. A hesitant fuzzy linguistic TODIM method based on a score function. *Int. J. Comput. Intell. Syst.* **2015**, *8*, 701–712. [CrossRef]
17. Tosun, Ö.; Akyüz, G. A fuzzy TODIM approach for the supplier selection problem. *Int. J. Comput. Intell. Syst.* **2015**, *8*, 317–329. [CrossRef]
18. Krohling, R.A.; Souza, T.T. Combining prospect theory and fuzzy numbers to multi-criteria decision making. *Expert Syst. Appl.* **2012**, *39*, 11487–11493. [CrossRef]
19. Estrella, F.J.; Espinilla, M.; Herrera, F.; Martínez, L. Flintstones: A fuzzy linguistic decision tools enhancement suite based on the 2-tuple linguistic model and extensions. *Inf. Sci.* **2014**, *280*, 152–170. [CrossRef]
20. Martínez, L.; Rodriguez, R.M.; Herrera, F. Flintstones: A Fuzzy Linguistic Decision Tools Enhancement Suite. In *The 2-Tuple Linguistic Model*; Springer: Cham, Switzerland, 2015; pp. 145–168.
21. Palomares, I.; Rodríguez, R.M.; Martínez, L. An attitude-driven web consensus support system for heterogeneous group decision making. *Expert Syst. Appl.* **2013**, *40*, 139–149. [CrossRef]
22. Cabrerizo, F.J.; Herrera-Viedma, E.; Pedrycz, W. A method based on PSO and granular computing of linguistic information to solve group decision making problems defined in heterogeneous contexts. *Eur. J. Oper. Res.* **2013**, *230*, 624–633. [CrossRef]
23. Herrera, F.; Martínez, L.; Sánchez, P.J. Managing non-homogeneous information in group decision making. *Eur. J. Oper. Res.* **2005**, *166*, 115–132. [CrossRef]
24. Peng, D.H.; Gao, C.Y.; Zhai, L.L. Multi-criteria group decision making with heterogeneous information based on ideal points concept. *Int. J. Comput. Intell. Syst.* **2013**, *6*, 616–625. [CrossRef]
25. Li, D.F.; Huang, Z.G.; Chen, G.H. A systematic approach to heterogeneous multiattribute group decision making. *Comput. Ind. Eng.* **2010**, *59*, 561–572. [CrossRef]
26. Zhang, G.; Lu, J. An integrated group decision-making method dealing with fuzzy preferences for alternatives and individual judgments for selection criteria. *Group Decis. Negot.* **2003**, *12*, 501–515. [CrossRef]
27. Estrella, F.J.; Cevik Onar, S.; Rodríguez, R.M.; Basar, O.; Luis, M.; Cengiz, K. Selecting firms in university technoparks: A hesitant linguistic fuzzy TOPSIS model for heterogeneous contexts. *J. Intell. Fuzzy Syst.* **2017**, *2*, 1155–1172. [CrossRef]
28. Rodríguez, R.M.; Labella, A.; Martínez, L. An overview on fuzzy modelling of complex linguistic preferences in decision making. *Int. J. Comput. Intell. Syst.* **2016**, *9*, 81–94. [CrossRef]

29. Rodríguez, R.M.; Martínez, L.; Herrera, F. Hesitant fuzzy linguistic term sets for decision making. *IEEE Trans. Fuzzy Syst.* **2012**, *20*, 109–119. [CrossRef]
30. Rodríguez, R.M.; Martínez, L.; Herrera, F. A group decision making model dealing with comparative linguistic expressions based on hesitant fuzzy linguistic term sets. *Inf. Sci.* **2013**, *241*, 28–42. [CrossRef]
31. Quesada, F.J.; Palomares, I.; Martínez, L. Managing experts behavior in large-scale consensus reaching processes with uninorm aggregation operators. *Appl. Soft Comput.* **2015**, *35*, 873–887. [CrossRef]
32. Dong, Y.; Luo, N.; Liang, H. Consensus building in multiperson decision making with heterogeneous preference representation structures: A perspective based on prospect theory. *Appl. Soft Comput.* **2015**, *35*, 898–910. [CrossRef]
33. Palomares, I.; Estrella, F.J.; Martínez, L.; Herrera, F. Consensus under a fuzzy context: Taxonomy, analysis framework afryca and experimental case of study. *Inf. Fusion* **2014**, *20*, 252–271. [CrossRef]
34. Liu, J.; Chan, F.T.; Li, Y.; Zhang, Y.; Deng, Y. A new optimal consensus method with minimum cost in fuzzy group decision. *Knowl. Based Syst.* **2012**, *35*, 357–360. [CrossRef]
35. Lee, H.S. Optimal consensus of fuzzy opinions under group decision making environment. *Fuzzy Sets Syst.* **2002**, *132*, 303–315. [CrossRef]
36. Gomes, L.; Lima, M. From modeling individual preferences to multicriteria ranking of discrete alternatives: A look at prospect theory and the additive difference model. *Found. Comput. Decis. Sci.* **1992**, *17*, 171–184.
37. Gomes, L.; Lima, M. TODIM: Basics and application to multicriteria ranking of projects with environmental impacts. *Found. Comput. Decis. Sci.* **1992**, *16*, 113–127.
38. Gomes, L. An application of the TODIM method to the multicriteria rental evaluation of residential properties. *Eur. J. Oper. Res.* **2009**, *193*, 204–211. [CrossRef]
39. Pereira, J.; Gomes, L.F.A.M.; Paredes, F. Robustness analysis in a TODIM-based multicriteria evaluation model of rental properties. *Technol. Econ. Dev. Ecol.* **2013**, *19*, 176–190. [CrossRef]
40. Wang, L.; Wang, Y.M.; Martinez, L. A group decision method based on prospect theory for emergency situations. *Inf. Sci.* **2017**, *418–419*, 119–135. [CrossRef]
41. Liu, H.; Rodríguez, R.M. A fuzzy envelope for hesitant fuzzy linguistic term set and its application to multicriteria decision making. *Inf. Sci.* **2014**, *258*, 220–238. [CrossRef]
42. Chen, S.J.; Chen, S.M. Fuzzy risk analysis based on similarity measures of generalized fuzzy numbers. *IEEE Trans. Fuzzy Syst.* **2003**, *11*, 45–56. [CrossRef]
43. Zadeh, L.A. Similarity relations and fuzzy orderings. *Inf. Sci.* **1971**, *3*, 177–200. [CrossRef]
44. Zadeh, L.A. Outline of a new approach to the analysis of complex systems and decision processes. *IEEE Trans. Syst. Man Cybern.* **1973**, *SMC-3*, 28–44. [CrossRef]
45. Mahdavi, I.; Mahdavi-Amiri, N.; Heidarzade, A.; Nourifara, R. Designing a model of fuzzy TOPSIS in multiple criteria decision making. *Appl. Math. Comput.* **2008**, *206*, 607–617. [CrossRef]
46. Eclipse Foundation. Available online: http://www.eclipse.org (accessed on 18 April 2016).
47. Zheng, X.; Cheng, Y. Modeling cooperative and competitive behaviors in emergency evacuation: A game-theoretical approach. *Comput. Math. Appl.* **2011**, *62*, 4627–4634. [CrossRef]
48. Arena, P.; Fazzino, S.; Fortuna, L.; Maniscalco, P. Game theory and non-liner dynamic: The parrondo paradox case study. *Chaos Solitons Fract.* **2003**, *17*, 545–555. [CrossRef]

4.4 A hesitant group emergency decision making method based on prospect theory

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ORIGINAL PAPER

A hesitant group emergency decision making method based on prospect theory

Zi-Xin Zhang[1,2] · Liang Wang[1,2] · Rosa M. Rodríguez[3] · Ying-Ming Wang[1] · Luis Martínez[2]

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Abstract Group emergency decision-making (GEDM) problems have drawn great attention in past few years due to its advantages of dealing with the emergency events (EEs) effectively. Due to the fact that EEs are usually featured by lack of information and time pressure, decision makers (DMs) are often bound rational and their psychological behaviors are very crucial to the GEDM process. However, DM's psychological behaviors are neglected in current GEDM approaches. The assessments representing the individual wisdom provided by each expert are usually aggregated in the GEDM process. Nevertheless, the aggregation process always implies summarization of data that can result in loss of information. To overcome these limitations pointed out previously, this paper proposes a new GEDM method that considers the DM's psychological behaviors in the decision process using prospect theory and replaces the aggregation process by a fusion method with hesitant fuzzy set, which keeps the experts' information as much as possible. A case study is provided to illustrate the validity and feasibility of the proposed method.

Keywords Group emergency decision making · Hesitant fuzzy sets · Prospect theory

Ying-Ming Wang
ymwang@fzu.edu.cn

Zi-Xin Zhang
zhangzixin768@outlook.com

Liang Wang
wangliangg322@hotmail.com

Rosa M. Rodríguez
rosam.rodriguez@decsai.ugr.es

Luis Martínez
martin@ujaen.es

1 Decision Science Institute, Fuzhou University, Fuzhou 350116, People's Republic of China

2 Department of Computer Sciences, University of Jaén, Jaén 23071, Spain

3 Department of Computer Science and A.I., University of Granada, Granada 18071, Spain

Introduction

Emergency event (EE) is defined as "events which suddenly take place, causing or having the possibility to cause intense death and injury, property loss, ecological damage and social hazards" [18], such as earthquakes, air crash, hurricanes, terrorist attacks, etc. When an EE occurs, it must be dealt with some measures to mitigate the losses of properties and lives, the process of selecting the measures is defined emergency decision making (EDM). EDM has received increasing attention and became a very active and important research field in recent years [12,15,20,35–37] because it plays a crucial role in mitigating the losses of properties and lives caused by EE.

Because EDM is typically characterized by time pressure and lack of information [10,17], it is difficult for a decision maker (DM) to predict its evolution and make comprehensive judgments under emergency situations. Therefore, EDM requires multiple experts from diverse professional backgrounds (such as hydrological, geological, meteorological, sociological, demographic, etc.) who help the DM to make a decision; this leads to GEDM problems. Usually, the GEDM consists of two processes [22,26]: (i) the aggregation process, where the individual information provided by experts is aggregated, and (ii) a selection process, in which an alternative is obtained as the solution to response the EE (see Fig. 1).

Current GEDM studies [41–43] have made significant contributions to emergency management; however, there are still two key issues that have not been well addressed yet:

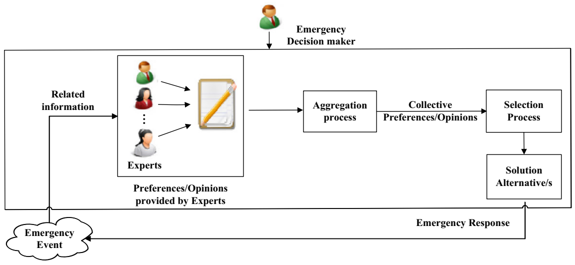


Fig. 1 The general scheme of GEDM process

1. *Losing information in aggregation process*. Current GEDM studies [41–43], use aggregation process that may imply loss of useful information for the decision process from the very beginning. Therefore, an important challenge for GEDM is to keep as much information as possible about the group, avoiding such a lost.
2. *DM's psychological behaviors in selection process*. Different studies [7,8] have shown that the DM is bounded rational under risk and uncertainty and his/her psychological behavior plays an important role in GEDM processes. However, such an important issue has been neglected in current GEDM methods.

Therefore, to overcome such limitations, this paper aims at developing a new GEDM method based on hesitant fuzzy set (HFS) with the following main contributions:

1. It considers different experts' opinions as the group hesitancy and fuses them into HFSs.
2. At the same time, it takes into account the DM's psychological behaviors using prospect theory (PT) in the selection process, because of its advantages of capturing human beings psychological behaviors under risk and uncertainty [16].

The remainder of this paper is organized as follows: Sect. 2 briefly introduces the basic knowledge about HFS and PT that will be used in the proposed method. Section 3 presents a new GEDM method that includes a fusion method by using HFS to keep the experts' information as much as possible and considers the DM's psychological behaviors using PT in the selection process. In Sect. 4, a case study is provided. Section 5 offers the conclusions.

Preliminaries

This section provides a brief review of different concepts about HFS and PT that will be utilized in the proposed method and to make it understood easily.

Hesitant fuzzy sets

HFS was introduced by Torra [30] as an extension of fuzzy sets to model the hesitancy in quantitative contexts reviewed in depth [23,25]. It is defined as below:

Definition 1 [30] Let $M = \{\mu_1, \ldots, \mu_n\}$ be a set of n membership functions. The HFS associated to M, h_M, is defined as:

$$h_M : X \to \wp([0, 1])$$

$$h_M(x) \to \bigcup_{\mu \in M} \{\mu(x)\} \tag{1}$$

where X is a reference set, $x \in X$.

This definition was extended and formalized with the concept of hesitant fuzzy element (HFE) by Xia and Xu [40]. In their proposal, the HFS was expressed by following mathematical representation, i.e.,

$$E = \{\langle x, h_E(x)\rangle : x \in X\}$$

where $h_E(x)$ is a set of values in [0,1], denoting the possible membership degrees of the element $x \in E$ to the set E. For convenience, they defined $h = h_E(x)$ as the HFE and $H = \cup h(x)$ as the HFSs, a HFE is a subset of HFSs (see [40] for further details).

Torra introduced in [30] the concept of envelop of a HFE and proved that is a intuitionistic fuzzy value (IFV) according to the following definition:

Definition 2 [30] Let h be a HFE, the IFV $A_{\text{env}}(h)$ is the envelop of h, in which $A_{\text{env}}(h)$can be represented as $\left(h^-, 1 - h^+\right)$ being $h^- = \min\{\sigma|\sigma \in h\}$ and $h^+ = \max\{\sigma|\sigma \in h\}$.

Different operations and properties has been defined for HFSs [30] such operations together the managing of intuitionistic fuzzy sets and intervals [14,30] allow us to interpret HFEs like an interval.

Table 1 Summary of the related works on hesitant fuzzy set

Authors	Contributions	Year
Torra [30]	Hesitant fuzzy set (HFS)	2010
Bedregal et al. [3]	Typical hesitant fuzzy set (THFS)	2014
Xia and Xu [40]	Hesitant fuzzy element (HFE)	2011
	Hesitant fuzzy weighted average operator	2011
	Hesitant fuzzy power average operator	2011
Yu [45]	Hesitant fuzzy Choquet integral operator	2011
Zhou [48]	Distance measures for hesitant fuzzy set	2012
Wei [38]	Entropy measures for hesitant fuzzy set	2016
Rodriguez et al. [25]	Hesitant fuzzy linguistic term set (HFLTS)	2014
Cevik Onar [9]	Multi-criteria decision making AHP method	2014
Wei [39]	Multi-criteria decision making VIKOR method	2014
Xue [44]	Group decision making	2017
Yu [46]	Personal evaluation	2013
Aliahmadipour [2]	Clustering	2016

Many researchers have paid attention on HFSs because it is a useful approach to model experts' hesitation. Therefore, different proposals have been introduced in the literature. Bedregal et al. [3] presented a special case of HFS named Typical Hesitant Fuzzy Set, that introduces some restrictions, because a HFS should be a finite and nonempty set. Many aggregation operators for HFSs have been defined such as, hesitant fuzzy weighted average [40], hesitant fuzzy power average [40], hesitant fuzzy choquet integral [45] and so on [23,25]. Distance measures are widely used in different fields such as, machine learning and decision making, for this reason some of them have been extended to deal with HFS [48]. Some entropy measures have been also defined for HFS [38]. And there are many applications based on HFS such as, multi-criteria decision making [9,39], group decision making [44], evaluation [46], and clustering approaches [2].

Recently, Rodriguez et al. [27] proposed the concept of Hesitant Fuzzy Linguistic Term Set (HFLTS), which not only keeps the basis on the fuzzy linguistic approach [47], but also extends the idea of HFS to linguistic contexts [24]. It has drawn great attention since it has been applied to solve different decision problems [4,19,26].

For sake of clarity, we make a summary of the related works on hesitant fuzzy set, see the following Table 1.

Prospect theory

PT was firstly proposed by Kahneman and Tversky [16] in 1979, which describes the human beings behavioral characteristics and provides a way to compute gains, losses, and prospect values, which has been widely used to solve various decision making problems considering human beings psychological behaviors [13,28,29,32,33].

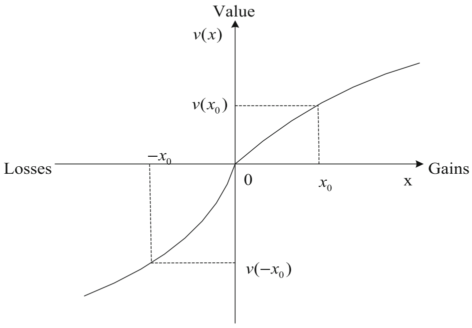


Fig. 2 S-shape value function of PT

Reference point (RP) is one key element in PT, which is defined as a neutral position asset or expectation value of people who wants to obtain or not loss, and decides the feeling of gains or losses based on the actual amounts to people; the location of the RP can be affected by the expectations of the people [16].

Gains and losses are defined with regards to the RP; the DM's psychological behaviors are exhibited risk-averse tendency for gains and risk-seeking tendency for losses. For measuring the magnitude of gains and losses, a value function is used in PT, which is defined on deviations from the RP with a concave and convex S-shape for losses and gains, respectively (see Fig. 2), and it is expressed in form of a power law according to the following expression [16].

$$v(x) = \begin{cases} x^{\alpha}, & x \geq 0 \\ -\lambda(-x)^{\beta}, & x < 0 \end{cases} \tag{2}$$

where x denotes the gains or losses, with $x \geq 0$ or with $x < 0$ respectively. αandβ are power parameters related to gains and losses, respectively, $0 \leq \alpha, \beta \leq 1$. λ is the risk aversion parameter, which represents a characteristic of being steeper for losses than for gains, $\lambda > 1$. The values of α, β and λ are determined through experiments [1,5,6,34].

A hesitant group emergency decision making dealing with DM's behaviors

This section introduces a novel hesitant GEDM method based on PT that aims at keeping experts' information as much as possible during the decision process and taking into account the DM's psychological behaviors during the selection process.

The proposed method consists of three main phases and graphically in Fig. 3:

1. Definition framework;
2. Information fusion based on HFS;
3. Alternative selection based on PT.

These phases are further detailed in the following subsections.

Definition framework

The basic notations that will be used in our proposal are given.

- $A = \{a_1, \ldots, a_i, \ldots, a_I\}$: set of alternatives, where a_i denotes the *i-th* alternative, $i = 1, 2, \ldots, I$.
- $C = \{c_1, \ldots, c_j, \ldots, c_J\}$: set of criteria, where$c_j$denotes the *j-th* criterion, $j = 1, 2, \ldots, J$.
- $S = \{s_1, \ldots, s_m, \ldots, s_M\}$: set of emergency situations, where s_m denotes the *m-th* emergency situation, $m = 1, 2, \ldots, M$.
- $W = \left(w_{c_1}, \ldots, w_{c_j}, \ldots, w_{c_J}\right)$: vector of criteria weights, where w_{c_j}denotes the weight of the *j-th* criterion, $j = 1, 2, \ldots, J$.
- $E = \{e_1, \ldots, e_h, \ldots, e_H\}$: set of experts, where$e_h$denotes the *h-th* expert, $h = 1, 2, \ldots, H$.
- $C^h = \left\{c_j^h(a_i)\right\}$: set of opinions provided by expert e_h, where $c_j^h(a_i) \in R$ denotes the preference over the$i$-th alternative regarding to the$j$-th criterion, $i = 1, 2, \ldots, I$; $h = 1, 2, \ldots, H$; $j = 1, 2, \ldots, J$.
- $\bar{C}^h = \left\{\bar{c}_j^h(a_i)\right\}$: denotes the normalization of C^h, where $\bar{c}_j^h(a_i) \in [0, 1]i = 1, 2, \ldots, I; h = 1, 2, \ldots, H; j = 1, 2, \ldots, J$.
- $h_M(a_i) = \{\bar{c}_1(a_i), \ldots, \bar{c}_J(a_i)\}$: denotes the HFS of experts' preference, where $\bar{c}_j(a_i)$ is the hesitant fuzzy element (HFE) and $\bar{c}_j(a_i) = \left\{\bar{c}_j^1(a_i), \ldots, \bar{c}_j^h(a_i), \ldots, \bar{c}_j^H(a_i)\right\}$, $i = 1, 2, \ldots, I$; $h = 1, 2, \ldots, H$; $j = 1, 2, \ldots, J$.
- $E_{ij} = \left[E_{ij}^L, E_{ij}^U\right]$: be an interval value, where E_{ij} denotes the effective control scope [37] over the *i*-th alternative with respect to the *j*-th criterion.
- $R_j = \left[R_j^L, R_j^U\right]$: be an interval value, where R_j^L, R_j^U are preferences, and $R_j$denotes the RP provided by the DM with respect to the *j*-th criterion.
- $\bar{R}_j = \left[\bar{R}_j^L, \bar{R}_j^U\right]$: denotes the normalization of R_j, where $\bar{R}_j \in [0, 1]j = 1, 2, \ldots, J$.

Information fusion based on HFS

As it was pointed out in the introduction, the aggregation always implies a summarization of original experts' opin-

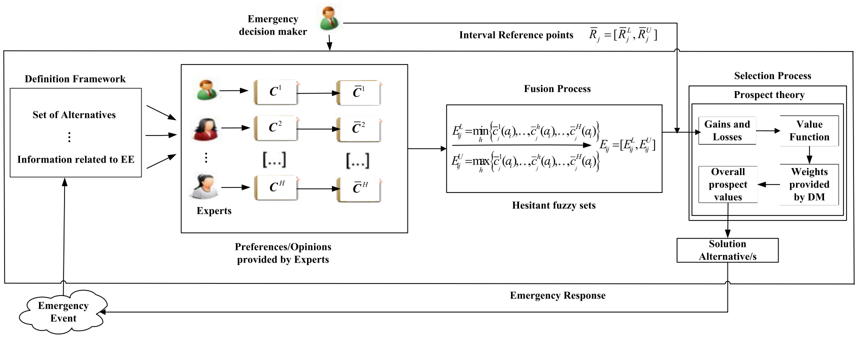


Fig. 3 General framework of proposed method

ions that can imply loss of information from different points of view such as distribution, diversity of data, etc. This loss of information can either bias or lead to wrong decisions regardless the aggregation operator. To overcome such a limitation, the experts' preferences in the GEDM problem are considered as the group hesitation about the alternatives and they will be fused by utilizing a HFS to keep as much information as possible.

Keeping in mind this idea, the effective control scopes of alternatives and the RP must be determined. The effective control scope of alternatives can be obtained by considering the group hesitation about it as a HFS, which is introduced in detail as follows:

Step 1: The experts involved in the GEDM problem provide the related information $c_j^h(a_i)$ about the emergency alternative with regarding to different criteria through analyzing the emergency alternatives

Step 2: Based on information $c_j^h(a_i)$ provided by experts, the preference of experts $\bar{c}_j^h(a_i)$ of effective control scope of the *i-th* alternative with respect to the *j-th* criterion can be calculated by Eq. (3):

$$\bar{c}_j^h(a_i) = \frac{c_j^h(a_i)}{\max\left\{\max\limits_h c_j^h(a_i)\right\}}, \quad j = 1, 2, \ldots J \tag{3}$$

Step 3: From $\bar{c}_j^h(a_i)$ calculated by Eq. (3), the HFEs $\bar{c}_j(a_i)$ for the *j-th* criterion with respect to the *i-th* alternative and the HFS $h_M(a_i)$ can be formed and managed according to their envelops as interval values.

Step 4: Based on step 3, the lower bound E_{ij}^L and upper bound E_{ij}^U of the effective control scope E_{ij} can be calculated by Eqs. (4), (5) [31].

$$E_{ij}^L = \min_h \left\{\bar{c}_j^1(a_i), \ldots, \bar{c}_j^h(a_i), \ldots, \bar{c}_j^H(a_i)\right\} \tag{4}$$

$$E_{ij}^U = \max_h \left\{\bar{c}_j^1(a_i), \ldots, \bar{c}_j^h(a_i), \ldots, \bar{c}_j^H(a_i)\right\} \tag{5}$$

The interval value E_{ij} is the result of fusion information that can avoid the loss of information and keep the experts' opinions as much as possible.

In order to facilitate the computations, the preferences R_j need to be transformed into $\bar{R}_j$ by utilizing the Eqs. (6), (7):

$$\bar{R}_j^L = \frac{R_j^L}{\max\left\{\max\limits_h c_j^h(a_i)\right\}} \tag{6}$$

$$\bar{R}_j^U = \frac{R_j^U}{\max\left\{\max\limits_h c_j^h(a_i)\right\}} \tag{7}$$

Alternative selection based on PT

Due to the fact that DM's psychological behaviors play an important role in GEDM process, this proposal uses PT to address such an important issue, because of its advantages to capture the psychological behaviors.

Calculation of gains and losses

According to the RP $\bar{R}_j$ and the effective control scope E_{ij} of emergency alternatives, gains and losses can be obtained.

Due to the fact that, we are dealing with interval values, before obtaining the gains and losses, the relationship between $\bar{R}_j$ and E_{ij} should be determined. There are six possible cases of positional relationship between $\bar{R}_j$ and E_{ij} as shown in Table 2.

To obtain the gains and losses with respect to each alternative, the following definition is provided.

Definition 3 For the effective control scope E_{ij} of alternatives, let x be an arbitrary value in interval number $[E_{ij}^L, E_{ij}^U]$, regarded as a random variable with uniform distribution [11]. The probability density function of x is

$$f(x) = \begin{cases} \frac{1}{E_{ij}^U - E_{ij}^L}, & E_{ij}^L \le x \le E_{ij}^U \\ 0, & \text{otherwise} \end{cases}, \quad i = 1, 2, \ldots, I; \quad j = 1, 2, \ldots, J \tag{8}$$

where $\int_{E_{ij}^L}^{E_{ij}^U} f(x)\mathrm{d}x = 1$ and $f(x) \ge 0$ for all $x \in [E_{ij}^L, E_{ij}^U]$.

From Table 2, the calculation of gains and losses is discussed. In general, the criteria can be classified into two types: benefit and cost [21]. A benefit criterion means the higher the better while a cost criterion the higher the worse. Note that for cost criteria, if $E_{ij}^U < \bar{R}_j^L$, the expert feels gains, and if $E_{ij}^L > \bar{R}_j^U$, the expert feels losses. The following discussion is for cost criterion only.

Case 1: obviously, there is no loss to the expert, since $E_{ij}^U < \bar{R}_j^L$,

$$L_{ij} = 0, \quad i = 1, 2, \ldots, I; \quad j = 1, 2, \ldots, J \tag{9}$$

According to Definition 3, the gain to the expert is given by

$$G_{ij} = \int_{E_{ij}^L}^{E_{ij}^U} (\bar{R}_j^L - x) f(x)\mathrm{d}x, \quad i = 1, 2, \ldots, I; \quad j = 1, 2, \ldots, J \tag{10}$$

Table 2 Possible cases of positional relationship between $\bar{R}_j$ and E_{ij}

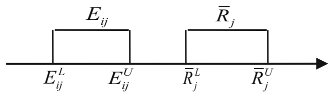
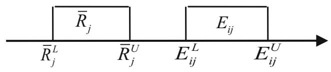
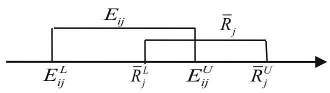
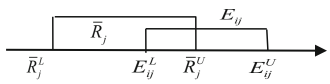
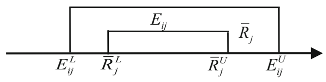
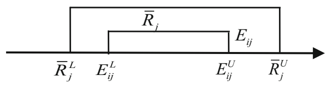
Cases		Positional relationship between $\bar{R}_j$ and E_{ij}
Case 1	$E_{ij}^U < \bar{R}_j^L$	
Case 2	$\bar{R}_j^U < E_{ij}^L$	
Case 3	$E_{ij}^L < \bar{R}_j^L \leq E_{ij}^U < \bar{R}_j^U$	
Case 4	$\bar{R}_j^L < E_{ij}^L \leq \bar{R}_j^U < E_{ij}^U$	
Case 5	$E_{ij}^L < \bar{R}_j^L < \bar{R}_j^U < E_{ij}^U$	
Case 6	$\bar{R}_j^L \leq E_{ij}^L < E_{ij}^U \leq \bar{R}_j^U$	

Table 3 Gains and losses for all possible cases (cost criteria)

Cases		Gain G_{ij}	Loss L_{ij}
Case 1	$E_{ij}^U < \bar{R}_j^L$	$\bar{R}_j^L - 0.5(E_{ij}^L + E_{ij}^U)$	0
Case 2	$\bar{R}_j^U < E_{ij}^L$	0	$\bar{R}_j^U - 0.5(E_{ij}^L + E_{ij}^U)$
Case 3	$E_{ij}^L < \bar{R}_j^L \leq E_{ij}^U < \bar{R}_j^U$	$0.5(\bar{R}_j^L - E_{ij}^L)$	0
Case 4	$\bar{R}_j^L < E_{ij}^L \leq \bar{R}_j^U < E_{ij}^U$	0	$0.5(\bar{R}_j^U - E_{ij}^U)$
Case 5	$E_{ij}^L < \bar{R}_j^L < \bar{R}_j^U < E_{ij}^U$	$0.5(\bar{R}_j^L - E_{ij}^L)$	$0.5(\bar{R}_j^U - E_{ij}^U)$
Case 6	$\bar{R}_j^L \leq E_{ij}^L < E_{ij}^U \leq \bar{R}_j^U$	0	0

Obviously, by Eqs. (9), (10) can be rewritten as:

$$G_{ij} = \bar{R}_j^L - 0.5(E_{ij}^L + E_{ij}^U), \quad i = 1, 2, \ldots, I; \quad j = 1, 2, \ldots, J \tag{11}$$

Similar to Case 1, the rest cases can be calculated respectively. The calculation formulae of gain and loss for all possible cases are summarized in Table 3, which shows the gain and loss for all possible cases for "cost criteria".

Similar to cost criterion, the calculation formulae of gain and loss for all possible cases with respect to the benefit criterion are summarized in Table 4:

Furthermore, based on Tables 3 and 4, the gain and loss matrix GM and LM can be constructed, which are used to calculate prospect values using value function.

Calculation of overall prospect values

Let $\mathrm{GM} = (G_{ij})_{I\times J}$ be the gain matrix, $\mathrm{LM} = (L_{ij})_{I\times J}$ be the loss matrix, $\mathrm{VM} = (v_{ij})_{I\times J}$ be the value matrix, where

$$v_{ij} = (G_{ij})^{\alpha} + \left[-\lambda\left(-L_{ij}\right)^{\beta}\right], i = 1, 2, \ldots, I; \quad j = 1, 2, \ldots, J \tag{12}$$

different values can be used for the parameters of Eq. (12) according to [32] we will use, $\alpha = 0.88, \beta = 0.92, \lambda =$

Table 4 Gains and losses for all possible cases (benefit criteria)

Cases		Gain G_{ij}	Loss L_{ij}
Case 1	$E_{ij}^U < \bar{R}_j^L$	0	$0.5(E_{ij}^L + E_{ij}^U) - \bar{R}_j^L$
Case 2	$\bar{R}_j^U < E_{ij}^L$	$0.5(E_{ij}^L + E_{ij}^U) - \bar{R}_j^U$	0
Case 3	$E_{ij}^L < \bar{R}_j^L \le E_{ij}^U < \bar{R}_j^U$	0	$0.5(E_{ij}^L - \bar{R}_j^L)$
Case 4	$\bar{R}_j^L < E_{ij}^L \le \bar{R}_j^U < E_{ij}^U$	$0.5(E_{ij}^U - \bar{R}_j^U)$	0
Case 5	$E_{ij}^L < \bar{R}_j^L < \bar{R}_j^U < E_{ij}^U$	$0.5(E_{ij}^U - \bar{R}_j^U)$	$0.5(E_{ij}^L - \bar{R}_j^L)$
Case 6	$\bar{R}_j^L \le E_{ij}^L < E_{ij}^U \le \bar{R}_j^U$	0	0

2.25; and v_{ij} denotes the value of the *i-th* alternative with respect to the *j-th* criterion. In PT, magnitude of gains and losses are measured by Eq. (12), the prospect values reflect the different feelings of DM, the higher v_{ij} is, the better DM feels. It means that the DM is satisfied with the decisions what he/she has done; otherwise, the DM feels regret or depressed with the decisions what he/she has done. By using PT, the psychological behaviors of DM can be described clearly and easily understood.

The attribute weights are provided by DM, using the simple additive weighting method, the Overall Prospect Value (OPV) of each alternative can be obtained, i.e.,

$$\mathrm{OPV}_i = \sum_{j=1}^{J} v_{ij} w_{c_j}, \quad i = 1, 2, \ldots, I; \quad j = 1, 2, \ldots, J \tag{13}$$

Obviously, the bigger OPV_i, the better alternative a_i. Based on the OPV_i, the ranking of alternatives can be obtained. According to the ranking of alternatives, the DM can select the best alternative to cope with the EE.

Case study and comparison

Case study

To illustrate the validity and feasibility of the proposed method, this section presents an adapted real case about a barrier lake emergency caused by a huge earthquake that occurred in southwestern China.

A barrier lake, formed by fallen rocks and a landslide after a huge earthquake, threatened the lives and properties of thousands of people both upstream and downstream. When a barrier lake formed, the DM must obey the principles of immediate response, timeously rescue, evacuating and blasting, so as to control the situation effectively and prevent it from further deterioration. The following criteria are concerned in our proposal:

c_1: The cost of alternatives (10,000 RMB which is the acronym of “renminbi”, the official currency of the People’s Republic of China).

c_2: The number of casualties.

c_3: Property loss (10,000 RMB).

The emergency alternatives are described as follows:

a_1: Evacuate people from the most dangerous upstream and downstream areas of the barrier lake to safe areas, and inform people in potentially dangerous areas to prepare for evacuation. At the same time, combine repeated small batch quantities of artificial blasting and excavation of drain grooves to meet the requirements of the discharged barrier lake floods;

a_2: Based on a_1, increase the joint scheduling of the reservoir and hydropower station in the upstream and downstream areas to reduce the pressure of the barrier lake;

a_3: Based on a_2, mobilize large, heavy machinery and implement large-scale blasting to reduce the water level of the barrier lake as much as possible to lower the risk of dam break;

a_4: Based on a_3, increase the joint scheduling of the reservoir and hydropower station in the upstream and downstream areas. Meanwhile, mobilize large, heavy machinery and implement large-scale blasting to reduce the water level of the barrier lake as much as possible to lower the risk of dam break.

Analyzing by professional experts, the barrier lake might be evolve into four possible emergency situations in 72 h, the emergency situations are as follows:

s_1: The dam body of the barrier lake will not break;

s_2: $1/3$ of the dam body of the barrier lake will break;

s_3: $1/2$ of the dam body of the barrier lake will break;

s_4: The entire dam body of the barrier lake will break.

Assume that three experts are invited to participate in the decision process to help DM makes a final decision. First, they are asked to define the effective control scope of the four emergency alternatives mentioned above. Through analyzing these emergency alternatives, the preferences $c_j^h(a_i)$ and $\bar{c}_j^h(a_i)$ of the effective control scopes for alternatives are given (see Table 5), where $\bar{c}_j^h(a_i)$ is calculated by Eq. (3).

Table 5 $c_j^h(a_i)$ and $\bar{c}_j^h(a_i)$ of effective control scope for alternatives

Alternatives	Experts	Criteria (weights)					
		c_1 (0.3)		c_2 (0.4)		c_3 (0.3)	
		$c_1^h(a_i)$	$\bar{c}_1^h(a_i)$	$c_2^h(a_i)$	$\bar{c}_2^h(a_i)$	$c_3^h(a_i)$	$\bar{c}_3^h(a_i)$
	e_1	250	0.42	5000	0.59	3500	0.64
a_1	e_2	280	0.47	5000	0.59	3000	0.55
	e_3	300	0.50	4000	0.47	4000	0.73
	e_1	300	0.50	6500	0.76	4000	0.73
a_2	e_2	300	0.50	5500	0.65	4000	0.73
	e_3	350	0.58	5000	0.59	4500	0.82
	e_1	400	0.67	7000	0.82	4500	0.82
a_3	e_2	350	0.58	7500	0.88	4500	0.82
	e_3	400	0.67	6500	0.76	5000	0.91
	e_1	600	1.00	8000	0.94	5000	0.91
a_4	e_2	500	0.83	8500	1.00	5300	0.96
	e_3	550	0.92	7500	0.88	5500	1.00

Table 6 The HFEs $\bar{c}_j(a_i)$ and the effective control scope E_{ij} for the alternatives

Alternatives	Criteria					
	c_1		c_2		c_3	
	$\bar{c}_1(a_i)$	E_{i1}	$\bar{c}_2(a_i)$	E_{i2}	$\bar{c}_3(a_i)$	E_{i3}
a_1	$\langle \bar{c}_1(a_1), \{0.42, 0.47, 0.50\}\rangle$	[0.42, 0.50]	$\langle \bar{c}_2(a_1), \{0.59, 0.59, 0.47\}\rangle$	[0.47, 0.59]	$\langle \bar{c}_3(a_1), \{0.64, 0.55, 0.73\}\rangle$	[0.55, 0.73]
a_2	$\langle \bar{c}_1(a_2), \{0.50, 0.50, 0.58\}\rangle$	[0.40, 0.58]	$\langle \bar{c}_2(a_2), \{0.76, 0.65, 0.59\}\rangle$	[0.59, 0.76]	$\langle \bar{c}_3(a_2), \{0.73, 0.73, 0.82\}\rangle$	[0.73, 0.82]
a_3	$\langle \bar{c}_1(a_3), \{0.67, 0.58, 0.67\}\rangle$	[0.58, 0.67]	$\langle \bar{c}_2(a_3), \{0.82, 0.88, 0.76\}\rangle$	[0.76, 0.88]	$\langle \bar{c}_3(a_3), \{0.82, 0.82, 0.91\}\rangle$	[0.82, 0.91]
a_4	$\langle \bar{c}_1(a_4), \{1.00, 0.83, 0.92\}\rangle$	[0.83, 1.00]	$\langle \bar{c}_2(a_4), \{0.94, 1.00, 0.88\}\rangle$	[0.88, 1.00]	$\langle \bar{c}_4(a_4), \{0.91, 0.96, 1.00\}\rangle$	[0.91, 1.00]

Based on the data $\bar{c}_j^h(a_i)$ in Table 5, the HFEs $\bar{c}_j(a_i)$ can be obtained and the effective control scope for the alternatives with respect to different criteria can be calculated by Eqs. (4), (5). Table 6 shows the results.

According to the four possible emergency situations of the barrier lake, the DM provided the RP according to his/her professional knowledge and experience by using interval values. The R_j and $\bar{R}_j$ are shown in Table 7, where $\bar{R}_j$ are obtained by Eqs. (6), (7).

According to the effective control scope E_{ij} and the RP $\bar{R}_j$ in Tables 6 and 7, respectively, and the positional relationship between $\bar{R}_j$ and E_{ij} in Table 2, the gain matrix (GM) and loss matrix (LM) can be constructed based on the equations in Tables 3 and 4, respectively, the GM and LM are as follows,

$$\text{GM} = \begin{bmatrix} 0.04 & 0 & 0.09 \\ 0 & 0 & 0.23 \\ 0 & 0.03 & 0.32 \\ 0 & 0.12 & 0.41 \end{bmatrix}, \quad \text{LM} = \begin{bmatrix} 0 & -0.18 & 0 \\ 0 & -0.06 & 0 \\ -0.04 & 0 & 0 \\ -0.33 & 0 & 0 \end{bmatrix}$$

Table 7 The RP R_j and $\bar{R}_j$

RP	c_1	c_2	c_3
R_j	[300, 350]	[6000, 7000]	[2000, 3000]
$\bar{R}_j$	[0.5, 0.58]	[0.71, 0.82]	[0.36, 0.55]

For sake of clarity, the computation of G_{13} is detailed:

$\bar{R}_3 = [0.36, 0.55]$, the effective control scope on property loss of a_1 is [0.55, 0.73], based on Table 2, their positional relationship is case 2, then:

$$0.5(E_{13}^L + E_{13}^H) - \bar{R}_3 \Rightarrow G_{13}$$
$$= 0.5(0.55 + 0.73) - 0.55 = 0.09,$$

according to Table 4.

Based on the GM and LM, the value matrix (VM) can be obtained directly by Eq. (12), just because both the R_j and E_{ij} are dimensionless. The VM is

Table 8 Overall prospect values and the ranking of alternatives

Alternatives	a_1	a_2	a_3	a_4
OPV_i	−0.1278	0.0150	**0.0912**	−0.04823
Ranking	4	2	**1**	3

$$\mathrm{VM} = \begin{bmatrix} 0.0611 & -0.4562 & 0.1212 \\ 0 & -0.1662 & 0.2715 \\ -0.1210 & 0.0450 & 0.3561 \\ -0.8191 & 0.1522 & 0.4554 \end{bmatrix}.$$

Following the details of computation for v_{13}, given that, $G_{13} = 0.09 (0.09 > 0)$, based on Eq. (12):

$$v_{13} = G_{13}^{0.88} = 0.09^{0.88} = 0.1212.$$

The OPV_i of each alternative can be obtained by Eq. (13) and the weighting vector provided by DM (see Table 5), the results of OPV_i and the ranking of alternatives based on OPV_i are shown in Table 8.

Following the details of computation for a_1, based on Eq. (13):

$$\begin{aligned}\mathrm{OPV}_1 &= 0.3 * 0.0611 + (-0.4562) * 0.4 + 0.1212 * 0.3 \\ &= -0.1278.\end{aligned}$$

According to Table 8, the alternative a_3 with the highest OPV is the best one (values are in bold) for coping with the barrier lake emergency situation.

Comparison

To illustrate the validity and feasibility of the proposed method, a comparison between the new information fusion process using HFSs and the aggregation process using weighted average method is performed.

The weighted average method is widely used to aggregate the experts' opinion in the aggregation process of the group decision making problem. In the weighted average method, the weight is assigned to each expert. In this paper, we assume that three experts' opinions are equally important, i.e., $(1/3, 1/3, 1/3)$. Let $\tilde{E}_{ij}$ be the aggregated information of the effective control scope. In order to make a validity comparison between the results of the two different methods, the $\tilde{c}_1^h(a_i)$ will be utilized to generate the effective control scope $\tilde{E}_{ij}$, where $\tilde{E}_{ij} = \frac{1}{3}\sum_{h=1}^{3} \tilde{c}_j^h(a_i)$. The results are shown in Table 9.

According to the RP $\bar{R}_j$ and the effective control scope $\tilde{E}_{ij}$ in Tables 7 and 9, respectively, and the positional relationship between $\bar{R}_j$ and E_{ij} in Table 2, the gain matrix $\widetilde{\mathrm{GM}}$ and loss matrix $\widetilde{\mathrm{LM}}$ can be constructed based on the equations in Tables 3 and 4, respectively, the $\widetilde{\mathrm{GM}}$and $\widetilde{\mathrm{LM}}$ are given as follows,

$$\widetilde{\mathrm{GM}} = \begin{bmatrix} 0.04 & 0 & 0.0909 \\ 0 & 0 & 0.2118 \\ 0 & 0 & 0.3027 \\ 0 & 0.1176 & 0.4116 \end{bmatrix},$$

$$\widetilde{\mathrm{LM}} = \begin{bmatrix} 0 & -0.1565 & 0 \\ 0 & -0.0382 & 0 \\ -0.0558 & 0 & 0 \\ -0.3342 & 0 & 0 \end{bmatrix}$$

Table 9 The aggregated information of the effective control scope $\tilde{E}_{ij}$

Alternatives	Experts (weights)	Criteria (weights)					
		c_1 (0.3)		c_2 (0.4)		c_3 (0.3)	
		$\tilde{c}_1^h(a_i)$	$\tilde{E}_{i1}$	$\tilde{c}_2^h(a_i)$	$\tilde{E}_{i2}$	$\tilde{c}_3^h(a_i)$	$\tilde{E}_{i3}$
	$e_1(1/3)$	0.42		0.59		0.64	
a_1	$e_2(1/3)$	0.47	0.4607	0.59	0.5494	0.55	0.6364
	$e_3(1/3)$	0.50		0.47		0.73	
	$e_1(1/3)$	0.50		0.76		0.73	
a_2	$e_2(1/3)$	0.50	0.5275	0.65	0.6676	0.73	0.7573
	$e_3(1/3)$	0.58		0.59		0.82	
	$e_1(1/3)$	0.67		0.82		0.82	
a_3	$e_2(1/3)$	0.58	0.6392	0.88	0.8235	0.82	0.8482
	$e_3(1/3)$	0.67		0.76		0.91	
	$e_1(1/3)$	1.00		0.94		0.91	
a_4	$e_2(1/3)$	0.83	0.9175	1.00	0.9412	0.96	0.9571
	$e_3(1/3)$	0.92		0.88		1.00	

Table 10 Overall prospect values and the ranking of alternatives

Alternatives	Our proposal		Weighted average method	
	OPV_i	Ranking	$\widetilde{\text{OPV}_i}$	Ranking
a_1	−0.1278	4	−0.1096	4
a_2	0.0150	2	0.0319	2
a_3	**0.0912**	**1**	**0.0573**	**1**
a_4	−0.0482	3	−0.0480	3

The bold values highlight the optimal results of the proposed method and the weighted average method

Based on the $\widetilde{\text{GM}}$ and $\widetilde{\text{LM}}$, the value matrix $\widetilde{\text{VM}}$ can be obtained directly by Eq. (12), i.e.,

$$\widetilde{\text{VM}} = \begin{bmatrix} 0.0580 & -0.4084 & 0.1212 \\ 0 & -0.1117 & 0.2552 \\ -0.1582 & 0 & 0.3494 \\ -0.8208 & 0.1521 & 0.4579 \end{bmatrix}.$$

The $\widetilde{\text{OPV}_i}$ of each alternative can be obtained by Eq. (13), similar to the calculation of OPV_i. The results of $\widetilde{\text{OPV}_i}$ and corresponding ranking of alternatives are shown in Table 10 from column 4–5.

As it can be seen that the ranking of alternatives obtained by different methods is the same, it verifies the validity and feasibility of our proposal.

The value obtained for the best alternative based on our proposal is greater than the value obtained based on the weighted average method because the proposal considers all the information provided by experts avoiding the loss of information.

Conclusion

Current GEDM approaches aggregate experts' individual assessments that may incur in loss of information that bias the decision process. Therefore, to take all the experts' opinions into account and also, DM's psychological behaviors during the GEDM process, this paper has introduced a new GEDM that considers DM's psychological behaviors using PT and the aggregation process is replaced by a fusion process using HFSs. Eventually, a case study and a comparison with the weighted average method about a barrier lake EE that happened in real world is provided to illustrate the validity and feasibility of the proposed method.

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References

1. Abdellaoui M, Bleichrodt H, Paraschiv C (2007) Loss aversion under prospect theory: a parameter-free measurement. Manag Sci 53:1659–1674
2. Aliahmadipour L, Torra V, Eslami E et al (2016) A definition for hesitant fuzzy partitions. Int J Comput Intell Syst 9:497–505
3. Bedregal B, Reiser R, Bustince H et al (2014) Aggregation functions for typical hesitant fuzzy elements and the action of automorphisms. Inf Sci 255:82–99
4. Beg I, Rashid T (2013) Topsis for hesitant fuzzy linguistic term sets. Int J Intell Syst 28:1162–1171
5. Bleichrodt H, Schmidt U, Zank H (2009) Additive utility in prospect theory. Manag Sci 55:863–873
6. Booij AS, Kuilen G (2009) A parameter-free analysis of the utility of money for the general population under prospect theory. J Econ Psychol 30:651–666
7. Camerer C (2003) Behavioral game theory: experiments in strategic interaction. Princeton University Press, Princeton
8. Camerer C (1998) Bounded rationality in individual decision making. Exp Econ 1:163–183
9. Cevik Onar S, Oztaysi B, Kahraman C (2014) Strategic decision selection using hesitant fuzzy topsis and interval type-2 fuzzy AHP: a case study. Int J Comput Intell Syst 7:1002–1021
10. Cosgrave J (1996) Decision making in emergencies. Disaster Prev Manag Int J 5:28–35
11. Fan ZP, Liu Y (2010) An approach to solve group-decision-making problems with ordinal interval numbers. IEEE Trans Syst Man Cybern Part B (Cybern) 40:1413–1423
12. Fan ZP, Liu Y, Shen RJ (2012) Risk decision analysis method for emergency response based on prospect theory. Syst Eng Theory Pract 5:011
13. Fan ZP, Zhang X, Chen FD et al (2013) Multiple attribute decision making considering aspiration-levels: a method based on prospect theory. Comput Ind Eng 65:341–350
14. Herrera F, Martınez L, Sánchez PJ (2005) Managing non-homogeneous information in group decision making. Eur J Oper Res 166:115–132
15. Huang ZF, Zheng PJ, Ma YQ et al (2016) Proactive control for oversaturation mitigation on evacuation network: a multi-agent simulation approach. Int J Comput Intell Syst 9:1055–1067
16. Kahneman D, Tversky A (1979) Prospect theory: an analysis of decision under risk. Econom J Econom Soc 2:263–291
17. Levy JK, Taji K (2007) Group decision support for hazards planning and emergency management: a group analytic network process (GANP) approach. Math Comput Modell 46:906–917
18. Liu BS, Zhao X, Li Y (2016) Review and prospect of studies on emergency management. Procedia Eng 145:1501–1508
19. Liu HB, Rodríguez RM (2014) A fuzzy envelope for hesitant fuzzy linguistic term set and its application to multicriteria decision making. Inf Sci 258:220–238
20. Liu Y, Fan ZP, Zhang Y (2014) Risk decision analysis in emergency response: a method based on cumulative prospect theory. Comput Oper Res 42:75–82
21. Ma J, Fan ZP, Huang LH (1999) A subjective and objective integrated approach to determine attribute weights. Eur J Oper Res 112:397–404

22. Palomares I, Martínez L (2014) Low-dimensional visualization of experts' preferences in urgent group decision making under uncertainty1. Procedia Comput Sci 29:2090–2101
23. Rodríguez RM, Bedregal B, Bustince H, Dong YC, Farhadinia B, Kahraman C, Martínez L, Torra V, Xu YJ, Xu ZS (2016) A position and perspective analysis of hesitant fuzzy sets on information fusion in decision making towards high quality progress. Inf Fusion 29:89–97
24. Rodríguez RM, Labella A, Martínez L (2016) An overview on fuzzy modelling of complex linguistic preferences in decision making. Int J Comput Intell Syst 9:81–94
25. Rodríguez RM, Martínez L, Torra V et al (2014) Hesitant fuzzy sets: state of the art and future directions. Int J Intell Syst 29:495–524
26. Rodríguez RM, Martínez L, Herrera F (2013) A group decision making model dealing with comparative linguistic expressions based on hesitant fuzzy linguistic term sets. Inf Sci 241:28–42
27. Rodriguez RM, Martinez L, Herrera F (2012) Hesitant fuzzy linguistic term sets for decision making. IEEE Trans Fuzzy Syst 20:109–119
28. Schmidt U, Starmer C, Sugden R (2008) Third-generation prospect theory. J Risk Uncertain 36:203–223
29. Schmidt U, Zank H (2012) A genuine foundation for prospect theory. J Risk Uncertain 45:97–113
30. Torra V (2010) Hesitant fuzzy sets. Int J Intell Syst 25:529–539
31. Torra V, Narukawa Y (2009) On hesitant fuzzy sets and decision. In: Fuzzy systems, 2009. FUZZ-IEEE 2009, IEEE international conference on (IEEE, 2009), pp 1378–1382
32. Tversky A, Kahneman D (1992) Advances in prospect theory: cumulative representation of uncertainty. J Risk Uncertain 5:297–323
33. Tversky A, Kahneman D (1991) Loss aversion in riskless choice: a reference-dependent model. Q J Econ 4:1039–1061
34. Wakker PP (2010) Prospect theory: for risk and ambiguity. Cambridge University Press, Cambridge
35. Wang L, Wang YM (2013) Study on the emergency decision method of dynamic reference point based on prospect theory. Chin J Manag Sci 1:132–140
36. Wang L, Wang YM, Hu BX (2016) Dynamic adjusting method of emergency alternatives based on prospect theory. Kongzhi yu Juece/Control Decis 31:99–104
37. Wang L, Zhang ZX, Wang YM (2015) A prospect theory-based interval dynamic reference point method for emergency decision making. Expert Syst Appl 42:9379–9388
38. Wei C, Yan F, Rodríguez RM (2016) Entropy measures for hesitant fuzzy sets and their application in multi-criteria decision-making. J Intell Fuzzy Syst 31:673–685
39. Wei G, Zhang N (2014) A multiple criteria hesitant fuzzy decision making with shapley value-based Vikor method. J Intell Fuzzy Syst 26:1065–1075
40. Xia M, Xu Z (2011) Hesitant fuzzy information aggregation in decision making. Int J Approx Reason 52:395–407
41. Xu X, Du Z, Chen X (2015) Consensus model for multi-criteria large-group emergency decision making considering non-cooperative behaviors and minority opinions. Decis Support Syst 79:150–160
42. Xu X, Zhong X, Chen X et al (2015) A dynamical consensus method based on exit-delegation mechanism for large group emergency decision making. Knowl Based Syst 86:237–249
43. Xu Y, Zhang W, Wang H (2015) A conflict-eliminating approach for emergency group decision of unconventional incidents. Knowl Based Syst 83:92–104
44. Xue M, Du Y (2017) A group decision-making model based on regression method with hesitant fuzzy preference relations. Math Probl Eng 2017:1–8
45. Yu D, Wu Y, Zhou W (2011) Multi-criteria decision making based on choquet integral under hesitant fuzzy environment. J Comput Inf Syst 7:4506–4513
46. Yu D, Zhang W, Xu Y (2013) Group decision making under hesitant fuzzy environment with application to personnel evaluation. Knowl Based Syst 52:1–10
47. Zadeh LA (1975) The concept of a linguistic variable and its application to approximate reasoning–I. Inf Sci 8:199–249
48. Zhou X, Li Q (2012) Some new similarity measures for hesitant fuzzy sets and their applications in multiple attribute decision making. arXiv:1211.4125

Chapter 5

Conclusions and Future Works

Chapter 5 concludes our research memory by revising the conclusions about the main proposals and results obtained, and pointing out possible promising future works.

5.1 Conclusions

Emergency decision making problems have gained special importance, due to the frequently occurrence of EEs in recent years that have caused important losses for mankind activities and social development.

Since the importance of EDM in mitigating and reducing the various losses and damages (property, lives, environment etc.) caused by EEs, many models and approaches for EDM problems have been proposed by different researchers. However, due to the complexity and diversity of nowadays EEs, contemporary EDM problems still face important challenges related to the management of uncertainty and experts' behaviors together with a more comprehensive view of their dynamics among others pointed out in Chapter 1.

Therefore, across our research we have obtained novel, remarkable and relevant results regarding those challenges that not only fulfill the objectives indicated in Section 1.2, but also provide new views in the solving processes of GEDM and new research opportunities for the future.

Consequently, we should conclude from our research results that:

1. The use of prospect theory for considering experts' psychological behavior is a useful tool that may improve GEDM resolution providing more accurate solutions because experts' behavior can modify solutions in an important manner and PT shows us how to manage such an issue in GEDM problems in which pressure on experts could be really tough.

2. EDM usually implies multiple decision situations across time about some EEs, so the management of the dynamics of the decision making problems has been a challenge, it has been just studied from assessments changes during time. But it has been shown in this memory other elements of the EDM can evolve and must be also considered to achieve more accurate and comprehensive results in the dynamic EDM.

3. The complexity of EEs and EDM problems may imply the managing of multiple types of uncertainties. Therefore, the development of EDM frameworks able to deal with multiple types of information makes more flexible and reliable the gathering of experts' information in order to achieve better solutions as it has been shown in our results.

4. Among the different types of uncertainties that could be managed in EDM, the hesitancy should be also considered. Therefore different models and tools have been introduced in our memory to deal with it.

5.2 Future Works

Even though several methods, tools and approaches have been proposed in this research, there are still challenges within EDM and GEDM problems that should be further studied. In near future, we will focus on the extension of the proposals presented and the development of solution for new problems:

1. An initial straight extension of our research is the integration of our models with an emergency decision support system that facilitates the use of our results into real world EDM management.

2. A promising research in this topic might be the study of data-driven decision models that facilitate, automatize and improve current EDM models.

3. Finally, a more ambitious research direction might be the integration of IoT devices stream data, data-driven models and our emergency decision support system into a more comprehensive and hybrid solution for supporting EDM decision makers.

Additional Publications

Regarding the diffusion of our scientific results, besides the publications included in this memory, we highlight the following contributions:

- International Journals

- L. Wang, Z. X. Zhang, Y. M. Wang. A prospect theory-based interval dynamic reference point method for emergency decision making. *Expert Systems with Applications*, vol. 42, issue 23, pp. 9379-9388, 2015.
- Z. X. Zhang, L. Wang, Y. M. Wang. An emergency decision making method based on prospect-theory for different emergency situations. *International Journal of Disaster Risk Science.* vol. 9, issue 3, pp. 407-420, 2018.
- L. Chen, Y. M. Wang, L. Wang. Congestion measurement under different policy objectives: an analysis of Chinese industry. *Journal of Cleaner Production*, vol. 112, issue 1, pp. 2943-2952, 2016.
- J. F. Chu, X. W. Liu, L. Wang, Y. M. Wang. A group decision making approach based on newly defined additively consistent interval-valued intuitionistic preference relations. *International Journal of Fuzzy Systems*, vol. 20, issue 3, pp. 1027-1046, 2018.
- Z. L. Wang, Y. M. Wang, L. Wang. Tri-level multi-attribute group decision making based on regret theory in multi-granular linguistic contexts. Journal of Intelligent and Fuzzy Systems, vol. 35, issue 1, pp. 793-806, 2018.

• International Conferences

- L. Wang, Y. M. Wang, R. M. Rodríguez, L. Martínez. A hesitant fuzzy linguistic model for emergency decision making based on fuzzy TODIM method. 2017 IEEE International Conference on Fuzzy Systems, IEEE-FUZZ 2017, July 9, 2017 - July 12, 2017. Naples, Italy.

Appendix A

Resumen escrito en Español

Título de la tesis: *Toma de decisión en situaciones de emergencia bajo incertidumbre*

Este apéndice incluye el título, índice, introducción, resumen y conclusiones escritas en español, como parte de los requisitos necesarios para obtener el doctorado según el artículo 23.2 del Reglamento de Estudio de Doctorado de la Universidad de Jaén.

En primer lugar, se presenta el índice de la memoria. A continuación, se introduce de forma breve la investigación llevada cabo, indicando motivación, objetivos y la estructura de los capítulos que la componen. Seguidamente, se presenta un resumen de la misma, para finalmente concluir con el apartado de conclusiones obtenidas y trabajos futuros.

Contenido

A.1 Motivación

Las situaciones de emergencia (SEs) se definen como aquellas situaciones que ocurren de forma inesperada y que pueden causar un gran número de pérdidas humanas, materiales, daños medioambientales y a la sociedad [105]. Dichas situaciones normalmente se caracterizan por su capacidad destructiva, carácter repentino, complejidad, variabilidad, incertidumbre etc. Dependiendo de su naturaleza, las SEs se pueden clasificar en 4 categorías [63], desastres naturales, desastres causados por accidentes, incidentes de salud pública y amenazas de carácter social.

En los últimos años, SEs como terremotos, inundaciones, huracanes, ataques terroristas etc., han causado enormes pérdidas y daños que han impactado negativamente en la vida de los seres humanos y en el desarrollo socio-económico de los países que las han sufrido. La Toma de Decisión en situaciones de Emergencia (TDE) es un proceso fundamental a la hora de mitigar y reducir los daños o pérdidas (materiales, vidas, medioambientales etc.) en este tipo de situaciones [136]. Cuando se produce un evento de esta índole, un decisor toma el mando del proceso de TDE, asumiendo la responsabilidad y consecuencias de las decisiones tomadas y jugando un papel crucial en el éxito de la gestión de una SE.

Debido al papel fundamental que desempeñan los decisores y la TDE a la hora de mitigar los daños y pérdidas causadas por SEs, ha surgido una activa e importante rama de investigación orientada al estudio de la gestión de este tipo de situaciones [33, 54, 69, 109, 126, 129]. Este profundo interés en la TDE ha provocado la aparición de numerosas publicaciones de diversos autores en la literatura especializada, que analizan diferentes aspectos a tener en cuenta cuando hablamos de TDE, por ejemplo:

- ◇ Número de expertos involucrados en el proceso de TDE, ya sean problemas clásicos de TDE donde un único decisor participa en el proceso de decisión [27, 54, 57, 68], o, por otra parte, la Toma de Decisión en Grupo en situaciones de Emergencia (TDGE) [39, 60, 96, 117, 129, 137] donde múltiples expertos apoyan al decisor en la toma de decisión. Los esquemas generales para ambos tipos de problemas se muestran en la Figure A.1 y en la Figure A.2, respectivamente.

- ◇ El comportamiento psicológico del decisor en el proceso de TDE [33, 69, 107, 109] y la agregación de las opiniones/valoraciones de los expertos en el proceso de TDGE [123, 124, 125, 126, 127].

- ◇ Elementos relativos a las SEs a tener en cuenta en el proceso de TDE [61], especialmente la incertidumbre, la información incompleta o vaga [54, 64, 91, 109, 122] y la evolución dinámica de las SEs [40, 53, 107].

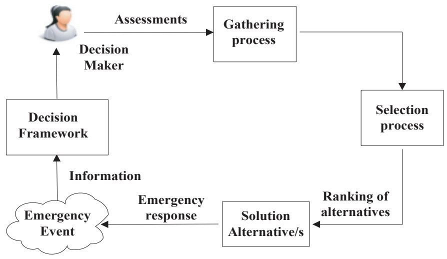

Figure A.1: Esquema general clásico de TDE

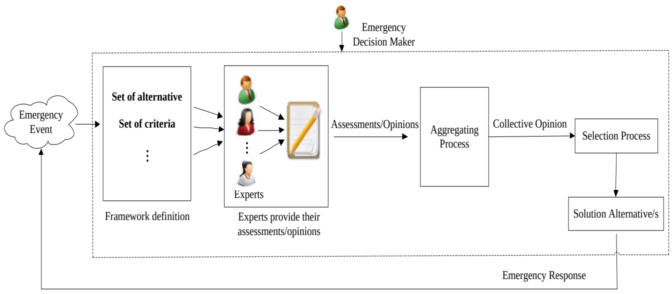

Figure A.2: Esquema general de TDGE

A consecuencia del rápido desarrollo de la tecnología, economía y en general de la sociedad en la última década, las SEs son cada vez más variadas y complejas, lo que supone un enorme reto para el decisor gestionar dichas situaciones con éxito, particularmente cuando el entorno de decisión resulta complejo e incierto [70]. Sin embargo, la participación de un grupo de expertos multidisciplinar con conocimientos en diversos campos (hidrología, meteorología, sociología y demografía) podría servir como un punto de apoyo para el decisor en el proceso de decisión, surgiendo de esta forma los problemas de TDGE, cuyo esquema general es representado en la Figure A.2.

Diferentes experimentos que analizan el comportamiento humano en situaciones de toma de decisión [14, 55, 103] han demostrado que éstos tienen dificultades para tomar decisiones racionales cuando se encuentran bajo presión o en condiciones donde el tiempo juega en su contra. Es evidente que el comportamiento psicológico de las personas afecta directamente a su forma de actuar frente a una SE (asunción del riesgo, aversión al riesgo, neutro) [55], por lo tanto, el comportamiento psicológico también desempeña un rol muy importante en el proceso de toma de decisión y debe ser considerado en cualquier tipo de problema de TDE.

Existe un proverbio en la cultura China que reza "*Conócete a ti mismo y conoce a tu enemigo, ganarás todas las guerras*", este proverbio nos enseña, que para vencer a nuestros enemigos en cualquier guerra, debemos conocernos no sólo a nosotros mismos, sino también a ellos. Esta enseñanza también puede aplicarse a problemas de TDE. Numerosos enfoques para la TDE se han propuesto con el principal objetivo de dar respuesta a las SEs de forma pertinente, efectiva y exitosa, analizando sus peculiares características [61] desde diferentes puntos de vista, como la incertidumbre, la información incompleta [54, 64, 91, 109, 122], registros históricos [91, 137], efectos dominó [138] entre otros.

A pesar de la gran cantidad de modelos y enfoques existentes orientados al tratamiento de problemas de TDE, propuestos por una extensa variedad de autores, y que han contribuido a mejorar la gestión de SEs, hoy en día, aún queda un largo camino por recorrer en la gestión de problemas reales de TDE, cada vez más complejos y que requieren de un estudio más profundo y exhaustivo y una mejora de las propuestas existentes. Algunas de las dificultades y retos que platean este tipo de problemas son descritos a continuación, siendo la principal motivación para desarrollar esta memoria de investigación:

- ◇ *Inclusión del comportamiento psicológico de los expertos en el proceso de TDGE*: Como ya se mencionó anteriormente, el comportamiento psicológico de los expertos desempeña un rol fundamental en el proceso de decisión, especialmente en situaciones de incertidumbre y riesgo donde se ha demostrado que los seres humanos tiene dificultades a la hora de tomar decisiones. Sin embargo, a pesar del evidente papel protagonista del comportamiento psicológico de los decisores en la TDE, ninguno de los enfoques actuales de TDE han considerado esta cuestión [39, 60, 96, 117, 129, 137]. Por lo tanto, resulta necesario proponer un modelo que sea capaz de tratar los problemas de TDGE de una manera más efectiva y que considere este tipo de cuestiones.

- ◇ *Indecisión de los expertos en los procesos de TDGE*: La indecisión es un comportamiento común e inevitable en nuestro día a día, y que aparece habitualmente en problemas de TDE debido a su complejidad y a las restricciones de tiempo y donde las decisiones tomadas pueden causar graves pérdidas humanas y/o materiales. Cuando los expertos no poseen los conocimientos necesarios o no están familiarizados con algún aspecto específico del problema, lo habitual es que éstos duden a la hora de proporcionar sus valoraciones u opiniones. A pesar de la evidente importancia de esta cuestión en los problemas de TDE, hasta ahora no ha sido tratada en los actuales enfoques de TDGE [39, 60, 96, 117, 129, 137] . Por lo tanto, resulta un importante y trascendente reto a considerar.

- ◇ *Agregación de la opinión de los expertos*: Debido a la importancia de la opinión de los expertos para tratar con los problemas de decisión de forma exitosa, es necesario manejar la opinión de los mismos de forma rápida, apropiada y conservando la mayor cantidad de conocimiento posible. Existen diversos enfoques de TDGE [123, 124, 126, 127] que presentan diferentes modelos de consenso y métodos que agregan la opinión de los expertos desde diferentes puntos de vista. Sin embargo, existen limitaciones que dichos modelos y métodos no pueden gestionar, como por ejemplo, la pérdida de información en las etapas iniciales del problema de decisión [126, 127], el coste temporal de los modelos de consenso [123, 124] y dominios de información que resultan inadecuados para manejar información difusa [60, 39, 125, 129]. Por lo tanto, obtener resultados de decisión precisos sin un modelo de agregación adecuado para problemas de TDGE es casi imposible.

◇ *Evolución dinámica en problemas de TDGE*: Despues de llevar a cabo un análisis de los existentes estudios sobre la evolución dinámica en problemas de TDE [53, 107], se detectó que éstos solo consideran los cambios producidos por el paso del tiempo, sin embargo, la evolución de las SEs no solo afecta a los cambios temporales sino también a la información relativa a las propias SEs (alternativas, criterios, etc.). Por lo tanto, la evolución dinámica de las SEs está relacionada con más aspectos que únicamente los cambios temporales, algo habitual en el mundo real y que se debería tener en cuenta en la resolución de estos problemas.

◇ *Tipos de información a tratar en problemas de TDGE*: La información resulta un elemento crucial en cualquier tipo de problema de toma de decisión, y los problemas de TDGE no son una excepción. Los actuales enfoques de TDGE propuestos tratan con información que se representa mediante un único dominio de expresión: valores numéricos [123], valores en intervalos [109] o información lingüística [54]. Sin embargo, la información que rodea a las SEs del mundo real suele ser de diferente naturaleza (numérica, intervalar, lingüística, información dudosa) y diferentes tipos suelen aparecer de forma simultánea. Sin embargo ninguna propuesta actual de TDGE maneja múltiples tipos de información al mismo tiempo.

◇ *Cálculo de la importancia de los criterios en problemas de TDGE*: El cálculo de la importancia de los criterios puede clasificarse en base a tres categorías diferentes, dependiendo del método que se utilice [38, 112]: métodos subjetivos, objetivos e híbridos. Los métodos subjetivos usan las preferencias de un decisor para determinar el peso de los criterios [36, 110]; los métodos objetivos usan una matriz de decisión para determinar los pesos de los criterios [21, 22]; los métodos híbridos combinan las preferencias de un decisor con una matriz de decisión para determinar la importancia de los criterios [72, 114]. Los métodos subjetivos se usan habitualmente en los estudios de TDE [33, 53, 68, 69, 107, 109], en los cuales se emplea diferentes pesos para los criterios. En SEs realmente complejas, es difícil para el decisor proporcionar pesos razonables para los criterios, particularmente, cuando el decisor se encuentra en situaciones bajo presión y/o duda. Por consiguiente, sería un reto definir una forma más efectiva y adecuada de determinar la importancia de los criterios en problemas de TDGE.

Los retos anteriormente expuestos para problemas de TDGE evidencian que los actuales enfoques de TDE no satisfacen las necesidades que demandan los problemas de TDGE del mundo real, como el *comportamiento psicológico y duda de los expertos*, la *fusión de las opiniones de los expertos*, la *evolución dinámica*, *información heterogénea*, *determinación de la importancia de los criterios*. Por esta razón, en esta memoria de investigación se lleva a cabo una investigación que nos permita superar estos desafíos.

A.2 Objetivos

Una vez introducidos los retos que plantean los problemas de TDE en la sección anterior, esta investigación se centrará en la superación de dichos retos. Teniendo en mente dicho propósito, se pretende alcanzar 4 objetivos principales:

1. Desarrollar un nuevo enfoque para la TDGE que considere el comportamiento psicológico de los expertos [108] obviado en los recientes estudios. Para ilustrar las ventajas, validez y viabilidad del nuevo método propuesto, dicho enfoque se aplicará a un caso de estudio y se llevará a cabo un análisis comparativo con otros enfoques de TDE existentes.

2. Definir un nuevo enfoque en cuanto a evolución dinámica para problemas de TDE se refiere que considere, no sólo los cambios temporales, sino también aquellos cambios relativos a la estructura del problema (alternativas, criterios, etc.) y diferentes tipos de incertidumbre. Seguidamente, se propondrá un nuevo enfoque dinámico de TDGE [106] que permita superar las limitaciones en los actuales enfoques de TDGE, incluyendo esta nueva visión de dinamismo e información heterogénea que será aplicada a un problema de toma de decisión basado en una situación de emergencia provocada por una explosión, ilustrando su originalidad, ventajas y validez.

3. Definir un esquema de TDGE que permita el modelado de diferentes tipos de

incertidumbre mediante información intervalar, difusa y dudosa. Adicionalmente, se definirá un nuevo modelo de consenso con un bajo coste temporal, que trate las opiniones de los expertos de forma correcta y sea adecuado para tratar información difusa. También se definirá una nueva forma de determinar la importancia de los criterios. Posteriormente, se propondrá un nuevo método de TDGE [105] que trate información heterogénea y busque una solución consensuada en un intervalo corto de tiempo, teniendo en cuenta el comportamiento psicológico de los expertos. Dicha propuesta será aplicada a un caso de estudio real de TDGE para mostrar su validez y funcionamiento.

4. Mejorar la fusión de la información en TDGE y considerar la duda en el comportamiento de los expertos [136] con el objetivo de construir un método de TDGE capaz de obtener mejores resultados gracias a la conservación de una mayor cantidad de información en el proceso de agregación en comparación con los procesos de agregación clásicos. Dicho método de TDGE se aplicará a un problema real de situación de emergencia.

A.3 Estructura

Para alcanzar los objetivos presentados en la Sección A.2, y teniendo en cuenta el artículo 23, punto 3, de la normativa vigente para los Estudios de Doctorado en la Universidad de Jaén, correspondiente al programa establecido en el RD 99/2011, esta memoria de investigación se presentará como un conjunto de artículos publicados por el estudiante de doctorado.

Tres artículos han sido publicados en revistas internacionales indexadas por la base de datos de JCR (Journal Citation Reports), producida por ISI (Institute for Scientific Information). Otro artículo ha sido publicado en la revista internacional, *Complex Intelligent Systems* indexada en el *Emergency Sources Citation Index*. Por tanto, la memoria se compone de un total de cuatro artículos que han sido publicados en revistas internacionales de reconocido prestigio.

A continuación se hace una breve descripción de la estructura de la memoria:

- ◇ Capítulo 2: Este capítulo revisa conceptos y métodos teóricos (conceptos de toma de decisión, toma de decisión en situaciones de emergencia, teoría de prospectos, el método difuso TODIM y el método difuso TOPSIS, conjuntos difusos dudosos (CDDs), conjuntos de términos lingüísticos difusos dudosos (CTLDD) etc.) que son empleados en nuestras propuestas para alcanzar los diferentes objetivos presentados en la Sección A.2.

- ◇ Capítulo 3: Este capítulo introduce las propuestas publicadas que forman parte de esta memoria de investigación. Cada artículo se introduce de forma breve, además, se presentan una breve discusión de los resultados obtenidos con el objetivo de clarificar los logros alcanzados en nuestra investigación.

◇ Capítulo 4: Este capítulo constituye el núcleo de la memoria de investigación, incluyendo un compendio de las publicaciones desarrolladas como resultado de la investigación realizada. Para cada publicación se indican los índices de calidad de las revistas donde las propuestas han sido publicadas.

◇ Capítulo 5: Este capítulo expone las conclusiones finales extraídas de esta investigación y propuestas para trabajos futuros.

A.4 Resumen

Las cada vez más comunes situaciones de emergencia (SEs) que están aconteciendo en los últimos años, han causado numerosas pérdidas y daños (materiales, vidas, medioambientales etc.) que han impactado negativamente en la vida del ser humano y en su desarrollo socio-económico. La Toma de Decisión en situaciones de Emergencia (TDE) es un proceso fundamental a la hora de mitigar y reducir los daños y pérdidas causadas por SEs y en el que un decisor es el responsable de tomar la decisión final. Debido a la importancia de la TD y la TDE a la hora de gestionar de forma exitosa SEs del mundo real, tanto el ámbito académico como el político han mostrando un gran interés en esta temática, convirtiéndose en una importante rama de investigación. Numerosos modelos y métodos se han propuesto para tratar con problemas de TDE, obteniendo satisfactorios resultados. Sin embargo, es innegable que aún existen limitaciones que deben ser estudiadas y retos que deben de ser afrontados como, por ejemplo, el *comportamiento psicológico y duda de los expertos*, la *agregación de las opiniones de los expertos*, *información heterogénea*, la *evolución dinámica* etc. Para lograr superar estos retos, en esta investigación se propone:

1. Un método de toma de decisión en grupo basado en la teoría de prospecto enfocado a situaciones de emergencia. El uso de la teoría de prospecto para la consideración del comportamiento psicológico de los expertos resulta una herramienta útil que podría mejorar la resolución de problemas de TDGE, proporcionando resultados más precisos, ya que es evidente que el comportamiento psicológico de los expertos puede influir de forma directa en las decisiones.

2. Un método dinámico multi-atributo de toma de decisión en grupo en situaciones de emergencia que considere la vacilación de los expertos a la hora de proporcionar sus opiniones. Este método gestiona la evolución dinámica de SEs

considerando tanto los cambios producidos a raíz del paso del tiempo como elementos del problema (alternativas, criterios y expertos). Además, este método está preparado para tratar con diferentes tipos de incertidumbre que serán modelados mediante el uso de intervalos, conjuntos de términos lingüísticos y expresiones lingüísticas basadas en conjuntos de términos lingüísticos difusos dudosos. De esta manera, la imprecisión y la falta de información pueden ser gestionadas.

3. Un método para el manejo de información no homogénea y el comportamiento psicológico de los expertos. Se consideran múltiples tipos de información, lo que facilita la labor de los expertos a la hora de dar sus valoraciones y permite obtener información más fiable. Además, el uso de múltiples tipos de información, permite aproximarse más a las SEs del mundo real.

4. Un método de toma de decisión en grupo en situaciones de emergencia que considera la duda de los expertos basado en la teoría de prospectos. Además de los diferentes tipos de incertidumbre y comportamientos psicológicos de los expertos, la duda o vacilación es una constante en nuestra vida diaria y debería ser tratada con propiedad. Este método no solamente trata con el comportamiento psicológico de los expertos, sino que también considera la duda de los mismos y que refleja sus verdaderas sensaciones a la hora de proporcionar sus valoraciones haciendo que los resultados sean más precisos y fiables.

A.5 Conclusiones y Trabajos Futuros

Esta sección finaliza la memoria de investigación revisando las diferentes conclusiones obtenidas de las diferentes propuestas que se han realizado en las mismas y exponiendo futuras líneas de investigación que podrían iniciarse partiendo de los resultados obtenidos. Finalmente, se indican las publicaciones adicionales derivadas de la investigación realizada.

A.5.1 Conclusiones

Los problemas de toma de decisión en situaciones de emergencia han visto incrementada su importancia debido a las frecuentes SEs acontecidas en los últimos años que han causado importantes pérdidas humanas y materiales.

Debido a la importancia de la TDE a la hora de mitigar y reducir las posibles pérdidas y daños (materiales, vidas, medioambientales etc.) causadas por las SEs, diferentes investigadores han propuesto numerosos modelos y enfoques orientados a problemas de TDE. Sin embargo, debido a su mayor complejidad y diversidad, los actuales problemas de TDE nos hacen afrontar importantes retos relacionados con el manejo de la incertidumbre y el comportamiento de los expertos, junto con una visión más completa de sus dinámicas, entre otros factores.

Por lo tanto, a lo largo de nuestra investigación, hemos obtenido innovadores, destacables y relevantes resultados que permiten no solo superar los retos planteados en la Sección A.2, sino que también nos han proporcionado una nueva visión en la resolución de problemas de TDGE y nuevas oportunidades de investigación para el futuro.

En consecuencia, concluimos que los resultados de nuestra investigación son:

1. La teoría de prospectos (TP) ha resultado ser una herramienta útil para considerar el comportamiento psicológico de los expertos y que mejora la resolución de problemas de TDGE, proporcionando soluciones más precisas, ya que el comportamiento de los expertos tiene una incidencia directa en las soluciones obtenidas. Además, la TP nos muestra como gestionar situaciones en problemas de TDGE donde la presión ejercida sobre los expertos pueden ser realmente alta.

2. La TDE normalmente implica tomar múltiples decisiones a lo largo del tiempo sobre una SE, por lo que la gestión del dinamismo del problema de toma de decisión ha sido un reto, sobre todo teniendo en cuenta que previamente solamente había sido estudiada en base a los cambios de valoraciones producidos a lo largo del tiempo. Sin embargo, esta memoria ha mostrado que otros elementos de la TDE pueden evolucionar y deben ser considerados para alcanzar resultados más precisos y exhaustivos en la evolución dinámica de la TDE.

3. La complejidad de las SEs y problemas de TDE podrían implicar la gestión de múltiples tipos de incertidumbre. Por lo tanto, el desarrollo de esquemas de TDE capaces de tratar con múltiples tipos de información hace más flexible y fiable la recopilación de información con el objetivo de alcanzar mejores soluciones, como se muestra en los resultados presentados.

4. Entre los diferentes tipos de incertidumbre que se podrían manejar en la TDE, es innegable que la duda o vacilación de los expertos debe de ser considerada. Por lo tanto, se han introducido diferentes modelos y herramientas para tratar este tipo de incertidumbre.

A.5.2 Trabajos Futuros

A pesar de los métodos, herramientas y enfoques que se han propuesto en esta investigación, aún existen retos que afrontar y analizar en problemas de TDE y TDGE.

En un futuro no muy lejano, nos centraremos en la ampliación de las propuestas presentadas y en el estudio de soluciones para nuevos problemas:

1. La integración de nuestros modelos en un sistema de soporte a la decisión que facilite el uso de nuestros resultados en la gestión de la TDG en el mundo real.

2. El estudio de modelos de decisión guiado por datos que faciliten, automaticen y mejoren los actuales modelos de TDE.

3. Finalmente, y como meta más ambiciosa, la integración de dispositivos IoT, modelos guiados por datos y nuestro sistema de soporte a la decisión en situaciones de emergencia en una única solución más completa e híbrida que sirva de apoyo a los decisores en el ámbito de la TDE.

Publicaciones adicionales En relación a la difusión y publicación de los resultados presentados, además de las publicaciones presentadas en esta memoria, destacamos las siguientes aportaciones:

- Revistas Internacionales

  - L. Wang, Z. X. Zhang, Y. M. Wang. A prospect theory-based interval dynamic reference point method for emergency decision making. *Expert Systems with Applications*, vol. 42, issue 23, pp. 9379-9388, 2015.
  - Z. X. Zhang, L. Wang, Y. M. Wang. An emergency decision making method based on prospect-theory for different emergency situations. *International Journal of Disaster Risk Science*, vol. 9, issue 3, pp. 407-420, 2018.
  - L. Chen, Y. M. Wang, L. Wang. Congestion measurement under different policy objectives: an analysis of Chinese industry. *Journal of Cleaner Production*, vol. 112, issue 1, pp. 2943-2952, 2016.
  - J. F. Chu, X. W. Liu, L. Wang, Y. M. Wang. A group decision making approach based on newly defined additively consistent interval-valued intuitionistic preference relations. *International Journal of Fuzzy Systems*, vol. 20, issue 3, pp. 1027-1046, 2018.
  - Z. L. Wang, Y. M. Wang, L. Wang. Tri-level multi-attribute group decision making based on regret theory in multi-granular linguistic contexts. Journal of Intelligent and Fuzzy Systems, vol. 35, issue 1, pp. 793-806, 2018.

- Congresos Internacionales

- L. Wang, Y. M. Wang, R. M. Rodríguez, L. Martínez. A hesitant fuzzy linguistic model for emergency decision making based on fuzzy TODIM method. 2017 IEEE International Conference on Fuzzy Systems, IEEE-FUZZ 2017, July 9, 2017 - July 12, 2017. Naples, Italy.

List of Figures

List of Tables

Bibliography

[1] M. Abdellaoui, H. Bleichrodt, and C. Paraschiv. Loss aversion under prospect theory: A parameter-free measurement. *Management Science*, 53(10):1659–1674, 2007.

[2] A. R. AlBattat and A. P. MatSom. Emergency planning and disaster recovery in malaysian hospitality industry. *Procedia-Social and Behavioral Sciences*, 144:45–53, 2014.

[3] K. T. Atanassov. Intuitionistic fuzzy sets. *Fuzzy sets and Systems*, 20(1):87–96, 1986.

[4] A. E. Attema, W. Brouwer, and O. Haridon. Prospect theory in the health domain: A quantitative assessment. *Journal of health economics*, 32(6):1057–1065, 2013.

[5] A. E. Attema, W. Brouwer, O. Haridon, and J. L. Pinto. An elicitation of utility for quality of life under prospect theory. *Journal of health economics*, 48:121–134, 2016.

[6] D. Bandaly, A. Satir, Y. Kahyaoglu, and L. Shanker. Supply chain risk management–i: Conceptualization, framework and planning process. *Risk Management*, 14(4):249–271, 2012.

[7] D. E. Bell. Regret in decision making under uncertainty. *Operations research*, 30(5):961–981, 1982.

[8] D. E. Bell. Disappointment in decision making under uncertainty. *Operations research*, 33(1):1–27, 1985.

[9] R. E. Bellman and L. A. Zadeh. Decision-making in a fuzzy environment. *Management science*, 17(4):141–164, 1970.

[10] M. J. Best and R. R. Grauer. Prospect theory and portfolio selection. *Journal of Behavioral and Experimental Finance*, 11:13–17, 2016.

[11] M. F. Blos, M. Quaddus, H. M. Wee, and K. Watanabe. Supply chain risk management(SCRM): a case study on the automotive and electronic industries in brazil. *Supply Chain Management: An International Journal*, 14(4):247–252, 2009.

[12] C. T. Butler and A. Rothstein. *On conflict and consensus: A handbook on formal consensus decisionmaking*. Citeseer, 2007.

[13] E. Calixto and E. L. Larouvere. The regional emergency plan requirement: Application of the best practices to the brazilian case. *Safety science*, 48(8):991–999, 2010.

[14] C. Camerer. Bounded rationality in individual decision making. *Experimental economics*, 1(2):163–183, 1998.

[15] J. H. Canós, M. Borges, M. Penadés, A. Gómez, and M. Llavador. Improving emergency plans management with SAGA. *Technological Forecasting and Social Change*, 80(9):1868–1876, 2013.

[16] C. T. Chen. Extensions of the TOPSIS for group decision-making under fuzzy environment. *Fuzzy sets and systems*, 114(1):1–9, 2000.

[17] C. T. Chen, C. T. Lin, and S. F. Huang. A fuzzy approach for supplier evaluation and selection in supply chain management. *International journal of production economics*, 102(2):289–301, 2006.

[18] G. H. Chen and X. M. Zhang. Fuzzy-based methodology for performance assessment of emergency planning and its application. *Journal of Loss Prevention in the Process Industries*, 22(2):125–132, 2009.

[19] R. Q. Chen and R. P Tong. Techniques for the compilation of emergency plan for work accidents in enterprise. *China Safety Science Journal*, 12, 2006.

[20] S. J. Chen and S. M. Chen. Fuzzy risk analysis based on similarity measures of generalized fuzzy numbers. *IEEE Transactions on fuzzy systems*, 11(1):45–56, 2003.

[21] T. Y. Chen and C. H. Li. Determining objective weights with intuitionistic fuzzy entropy measures: a comparative analysis. *Information Sciences*, 180(21):4207–4222, 2010.

[22] T. Y. Chen and C. H. Li. Objective weights with intuitionistic fuzzy entropy measures and computational experiment analysis. *Applied Soft Computing*, 11(8):5411–5423, 2011.

[23] C. Y. Cheng and X. Qian. Evaluation of emergency planning for water pollution incidents in reservoir based on fuzzy comprehensive assessment. *Procedia Environmental Sciences*, 2:566–570, 2010.

[24] J. F. Chu, X. W. Liu, Y. M. Wang, and K. S. Chin. A group decision making model considering both the additive consistency and group consensus of intuitionistic fuzzy preference relations. *Computers & Industrial Engineering*, 101:227–242, 2016.

[25] T. C. Chu. Facility location selection using fuzzy TOPSIS under group decisions. *International journal of uncertainty, fuzziness and knowledge-based systems*, 10(06):687–701, 2002.

[26] T. C. Chu and Y. C. Lin. A fuzzy TOPSIS method for robot selection. *The International Journal of Advanced Manufacturing Technology*, 21(4):284–290, 2003.

[27] J. Cosgrave. Decision making in emergencies. *Disaster Prevention and Management: An International Journal*, 5(4):28–35, 1996.

[28] K. K. Damghani, M. T. Taghavifard, and R. T. Moghaddam. Decision making under uncertain and risky situations. In *Enterprise Risk Management Symposium Monograph Society of Actuaries-Schaumburg, Illinois*, volume 15, 2009.

[29] H. Dichtl and W. Drobetz. Portfolio insurance and prospect theory investors: Popularity and optimal design of capital protected financial products. *Journal of Banking & Finance*, 35(7):1683–1697, 2011.

[30] H. C. Ding, J. J. Tong, Y. F. Wang, and L. G. Zhang. Development of emergency planning zone for high temperature gas-cooled reactor. *Annals of Nuclear Energy*, 111:347–353, 2018.

[31] J. R. Eiser and J. Pligt. *Attitudes and decisions*. Psychology Press, 2015.

[32] D. Ergu, M. S. Zhang, Z. L. Guo, and X. F. Yang. Consistency simulation and optimization for HPIBM model in emergency decision making. *Procedia Computer Science*, 31:558–566, 2014.

[33] Z. P. Fan, Y. Liu, and R. J. Shen. Risk decision analysis method for emergency response based on prospect theory. *Systems Engineering-Theory & Practice*, 5:011, 2012.

[34] Z. P. Fan, X. Zhang, F. D. Chen, and Y. Liu. Multiple attribute decision making considering aspiration-levels: A method based on prospect theory. *Computers & Industrial Engineering*, 65(2):341–350, 2013.

[35] T. M. Ferreira, R. Vicente, J. da Silva, A. Costa, and R. Maio. Urban fire risk: Evaluation and emergency planning. *Journal of Cultural Heritage*, 20:739–745, 2016.

[36] J. Figueira and B. Roy. Determining the weights of criteria in the ELECTRE type methods with a revised simos' procedure. *European Journal of Operational Research*, 139(2):317–326, 2002.

[37] D. Filev and R. R. Yager. On the issue of obtaining OWA operator weights. *Fuzzy sets and systems*, 94(2):157–169, 1998.

[38] C. Fu and Y. M. Wang. An interval difference based evidential reasoning approach with unknown attribute weights and utilities of assessment grades. *Computers & Industrial Engineering*, 81:109–117, 2015.

[39] P. P. Fu, C. Wu, and J. Tang. Unconventional emergency management based on intelligent group decision-making methodology. *Advances in Information Sciences & Service Sciences*, 4(7):208–216, 2012.

[40] C. Girard, P. David, E. Piatyszek, and J. Flaus. Emergency response plan: Model-based assessment with multi-state degradation. *Safety science*, 85:230–240, 2016.

[41] L. Gomes. An application of the TODIM method to the multicriteria rental evaluation of residential properties. *European Journal of Operational Research*, 193(1):204–211, 2009.

[42] L. Gomes and M. Lima. From modeling individual preferences to multicriteria ranking of discrete alternatives: a look at prospect theory and the additive difference model. *Foundations of Computing and Decision Sciences*, 17(3):171–184, 1992.

[43] L. Gomes and M. Lima. TODIM: Basics and application to multicriteria ranking of projects with environmental impacts. *Foundations of Computing and Decision Sciences*, 16(4):113–127, 1992.

[44] F. Herrera, E. Herrera-Viedma, and J. L. Verdegay. A sequential selection process in group decision making with a linguistic assessment approach. *Information Sciences*, 85(4):223–239, 1995.

[45] M. Hoard, J. Homer, W. Manley, P. Furbee, A. Haque, and J. Helmkamp. Systems modeling in support of evidence-based disaster planning for rural areas. *International journal of hygiene and environmental health*, 208(1-2):117–125, 2005.

[46] C. A. Holloway. *Decision making under uncertainty: models and choices*. Prentice Hall, 1979.

[47] B. Hosseinnia, N. Khakzad, and G. Reniers. Multi-plant emergency response for tackling major accidents in chemical industrial areas. *Safety science*, 102:275–289, 2018.

[48] C. L. Hwang and K. Yoon. *Multiple attribute decision making: methods and applications*. Berlin: Springer, 1981.

[49] C. L. Hwang and K. Yoon. *Multiple attribute decision-making: methods and applications-A state of the art survey*. Springer-Verlag, Berlin, 1981.

[50] Prachi J. What is decision making? Website, 2018. `https://www.managementstudyguide.com/what-is-decision-making.htm`.

[51] L. Jenkins. Determining the most informative scenarios of environmental impact from potential major accidents. *Journal of environmental management*, 55(1):15–25, 1999.

[52] L. Jenkins. Selecting scenarios for environmental disaster planning. *European Journal of Operational Research*, 121(2):275–286, 2000.

[53] Y. P. Jiang, Z. P. Fan, and M. M. Su. Study on the dynamic adjusting method for emergency decision. *Chinese Journal of Management Science*, 19(5):104–108, 2011.

[54] Y. B. Ju, A. H. Wang, and X. Y. Liu. Evaluating emergency response capacity by fuzzy AHP and 2-tuple fuzzy linguistic approach. *Expert Systems with Applications*, 39(8):6972–6981, 2012.

[55] D. Kahneman and A. Tversky. Prospect theory: An analysis of decision under risk. *Econometrica*, 47:263–291, 1979.

[56] A. Kandil and K. El-Rayes. Parallel genetic algorithms for optimizing resource utilization in large-scale construction projects. *Journal of Construction engineering and Management*, 132(5):491–498, 2006.

[57] J. Kang, J. X. Zhang, and Y. Q. Bai. Modeling and evaluation of the oil-spill emergency response capability based on linguistic variables. *Marine pollution bulletin*, 113(1-2):293–301, 2016.

[58] G. Klir and B. Yuan. *Fuzzy sets and fuzzy logic*, volume 4. Prentice hall New Jersey, 1995.

[59] Renato A Krohling and T. T. Souza. Combining prospect theory and fuzzy numbers to multi-criteria decision making. *Expert Systems with Applications*, 39(13):11487–11493, 2012.

[60] J. K. Levy and K. Taji. Group decision support for hazards planning and emergency management: A group analytic network process GANP approach. *Mathematical and Computer Modelling*, 46(7-8):906–917, 2007.

[61] H. X. Li, J. Wang, X. F. Yuan, and J. Du. Unconventional emergency system characteristics and emergency decision-making analysis. *Procedia Engineering*, 71:199–206, 2014.

[62] X. W. Li. Decision-making method of highway network planning based on prospect theory. *Procedia-Social and Behavioral Sciences*, 96, 2013.

[63] B. S. Liu, X. Zhao, and Y. Li. Review and prospect of studies on emergency management. *Procedia Engineering*, 145:1501–1508, 2016.

[64] D. Liu, H. W. Wang, C. Qi, P. Zhao, and J. Wang. Hierarchical task network-based emergency task planning with incomplete information, concurrency and uncertain duration. *Knowledge-Based Systems*, 112:67–79, 2016.

[65] H. B. Liu and R. M. Rodríguez. A fuzzy envelope for hesitant fuzzy linguistic term set and its application to multicriteria decision making. *Information Sciences*, 258:220–238, 2014.

[66] J. Liu, F. T. Chan, Y. Li, Y. Zhang, and Y. Deng. A new optimal consensus method with minimum cost in fuzzy group decision. *Knowledge-Based Systems*, 35:357–360, 2012.

[67] T. M. Liu. Design of the emergency plan system's concept. *Journal of Safety Science and Technology*, 8(7), 2011.

[68] Y. Liu, Z. P. Fan, Y. Yuan, and H. Y. Li. A FTA-based method for risk decision-making in emergency response. *Computers & Operations Research*, 42:49–57, 2014.

[69] Y. Liu, Z. P. Fan, and Y. Zhang. Risk decision analysis in emergency response: A method based on cumulative prospect theory. *Computers & Operations Research*, 42:75–82, 2014.

[70] J. Lu, G. Zhang, and D. Ruan. *Multi-objective group decision making: methods, software and applications with fuzzy set techniques*. Imperial College Press, London, 2007.

[71] H. B. Luo, X. F. Peng, and B. T. Zhong. Application of ontology in emergency plan management of metro operation. *Procedia engineering*, 164:158–165, 2016.

[72] J. Ma, Z. P. Fan, and L. H. Huang. A subjective and objective integrated approach to determine attribute weights. *European journal of operational research*, 112(2):397–404, 1999.

[73] L. Martínez and J. Montero. Challenges for improving consensus reaching process in collective decisions. *New Mathematics and Natural Computation*, 3(02):203–217, 2007.

[74] L. Martínez, D. Ruan, F. Herrera, E. Herrera-Viedma, and P. P. Wang. Linguistic decision making: Tools and applications. *Information Sciences*, 179(14):2279–2298, 2009.

[75] R. E. Moore. *Methods and applications of interval analysis*, volume 2. Siam, 1979.

[76] N. Mukherjee, L. V. Dicks, G. E. Shackelford, et al. Comparing groups versus individuals in decision making: a systematic review protocol. *Environmental Evidence*, 5(1):19, 2016.

[77] Mourad Oussalah. On the compatibility between defuzzification and fuzzy arithmetic operations. *Fuzzy Sets and Systems*, 128(2):247–260, 2002.

[78] I. Palomares, F. J. Estrella, L. Martínez, and F. Herrera. Consensus under a fuzzy context: Taxonomy, analysis framework afryca and experimental case of study. *Information Fusion*, 20:252–271, 2014.

[79] I. Palomares and L. Martínez. Low-dimensional visualization of experts's preferences in urgent group decision making under uncertainty. *Procedia Computer Science*, 29:2090–2101, 2014.

[80] J. Pereira, L. Gomes, and F. Paredes. Robustness analysis in a TODIM-based multicriteria evaluation model of rental properties. *Technological and Economic Development of Economy*, 19:S176–S190, 2013.

[81] E. Pilone, P. Mussini, M. Demichela, and G. Camuncoli. Municipal emergency plans in italy: requirements and drawbacks. *Safety science*, 85:163–170, 2016.

[82] C. Qi, D. Wang, H. Munoz-Avila, P. Zhao, and H. W. Wang. Hierarchical task network planning with resources and temporal constraints. *Knowledge-Based Systems*, 133:17–32, 2017.

[83] J. D. Qin, X. W. Liu, and W. Pedrycz. An extended TODIM multi-criteria group decision making method for green supplier selection in interval type-2 fuzzy environment. *European Journal of Operational Research*, 258(2):626–638, 2017.

[84] B. V. Ramabrahmam, B. Sreenivasulu, and M. M. Mallikarjunan. Model on-site emergency plan. case study: toxic gas release from an ammonia storage terminal. *Journal of loss prevention in the process industries*, 9(4):259–265, 1996.

[85] C. G. Ramsay. Protecting your business: from emergency planning to crisis management. *Journal of Hazardous Materials*, 65(1-2):131–149, 1999.

[86] R. M. Rodríguez, B. Bedregal, H. Bustince, et al. A position and perspective analysis of hesitant fuzzy sets on information fusion in decision making. towards high quality progress. *Information Fusion*, 29:89–97, 2016.

[87] R. M. Rodríguez, A. Labella, and L. Martínez. An overview on fuzzy modelling of complex linguistic preferences in decision making. *International Journal of Computational Intelligence Systems*, 9:81–94, 2016.

[88] R. M. Rodríguez, L. Martínez, and F. Herrera. Hesitant fuzzy linguistic term sets for decision making. *IEEE Transactions on Fuzzy Systems*, 20(1):109–119, 2012.

[89] R. M. Rodríguez, L. Martínez, and F. Herrera. A group decision making model dealing with comparative linguistic expressions based on hesitant fuzzy linguistic term sets. *Information Sciences*, 241:28–42, 2013.

[90] R. M. Rodríguez, L. Martínez, V. Torra, et al. Hesitant fuzzy sets: state of the art and future directions. *International Journal of Intelligent Systems*, 29(6):495–524, 2014.

[91] C. Romanowski, R. Raj, J. Schneider, S. Mishra, V. Shivshankar, S. Ayengar, and F. Cueva. Regional response to large-scale emergency events: Building on historical data. *International Journal of Critical Infrastructure Protection*, 11:12–21, 2015.

[92] E. H. Ruspini. A new approach to clustering. *Information and control*, 15(1):22–32, 1969.

[93] N. Sacuta, L. Gauvreau, and S. E. Greenberg. Emergency response planning: an example of international collaboration in CCS community outreach and project development. *Energy Procedia*, 37:7388–7394, 2013.

[94] S. Saint and J. R Lawson. *Rules for reaching consensus: a modern approach to decision making*. Pfeiffer, 1994.

[95] U. Schmidt, C. Starmer, and R. Sugden. Third-generation prospect theory. *Journal of Risk and Uncertainty*, 36(3):203, 2008.

[96] S. G. Shi, J. C. Cao, L. I. Feng, W. Y. Liang, and L. Q. Zhang. Construction of a technique plan repository and evaluation system based on ahp group decision-making for emergency treatment and disposal in chemical pollution accidents. *Journal of hazardous materials*, 276:200–206, 2014.

[97] P. Tang and G. Q. Shen. Decision-making model to generate novel emergency response plans for improving coordination during large-scale emergencies. *Knowledge-based systems*, 90:111–128, 2015.

[98] V. Torra. Hesitant fuzzy sets. *International Journal of Intelligent Systems*, 25(6):529–539, 2010.

[99] Ö. Tosun and G. Akyüz. A fuzzy TODIM approach for the supplier selection problem. *International Journal of Computational Intelligence Systems*, 8(2):317–329, 2015.

[100] Robert L Trewatha. Management; functions and behavior. Technical report, 1979.

[101] J. M. Tseng, M. Y. Liu, R. H. Chang, J. L. Su, and C. M. Shu. Emergency response plan of chlorine gas for process plants in taiwan. *Journal of loss prevention in the process industries*, 21(4):393–399, 2008.

[102] A. Tversky and D. Kahneman. Loss aversion in riskless choice: A reference-dependent model. *The quarterly journal of economics*, 106(4):1039–1061, 1991.

[103] A. Tversky and D. Kahneman. Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and uncertainty*, 5(4):297–323, 1992.

[104] J. Q. Wang, J. T. Wu, J. Wang, H. Y. Zhang, and X. H. Chen. Interval-valued hesitant fuzzy linguistic sets and their applications in multi-criteria decision-making problems. *Information Sciences*, 288:55–72, 2014.

[105] L. Wang, Á. Labella, R. M. Rodríguez, Y. M. Wang, and L. Martínez. Managing non-homogeneous information and experts' psychological behavior in group emergency decision making. *Symmetry*, 9(10):234, 2017.

[106] L. Wang, R. M. Rodríguez, and Y. M. Wang. A dynamic multi-attribute group emergency decision making method considering experts' hesitation. *International Journal of Computational Intelligence Systems*, 11(1):163–182, 2018.

[107] L. Wang, Y. M. Wang, and B. X. Hu. Dynamic adjusting method of emergency alternatives based on prospect theory. *Control & Decision*, 31(1):99–104, 2016.

[108] L. Wang, Y. M. Wang, and L. Martínez. A group decision method based on prospect theory for emergency situations. *Information Sciences*, 418:119–135, 2017.

[109] L. Wang, Z. X. Zhang, and Y. M. Wang. A prospect theory-based interval dynamic reference point method for emergency decision making. *Expert Systems with Applications*, 42(23):9379–9388, 2015.

[110] Y. M. Wang. On fuzzy multiattribute decision-making models and methods with incomplete preference information. *Fuzzy Sets and Systems*, 151(2):285–301, 2005.

[111] Y. M. Wang and T. M.S. Elhag. Fuzzy TOPSIS method based on alpha level sets with an application to bridge risk assessment. *Expert systems with applications*, 31(2):309–319, 2006.

[112] Y. M. Wang and Y. Luo. Integration of correlations with standard deviations for determining attribute weights in multiple attribute decision making. *Mathematical and Computer Modelling*, 51(1-2):1–12, 2010.

[113] Y. M. Wang, Y. Luo, and X. W. Liu. Two new models for determining OWA operator weights. *Computers & Industrial Engineering*, 52(2):203–209, 2007.

[114] Y. M. Wang and C. Parkan. A general multiple attribute decision-making approach for integrating subjective preferences and objective information. *Fuzzy Sets and Systems*, 157(10):1333–1345, 2006.

[115] C. P. Wei, Z. Ren, and R. M. Rodríguez. A hesitant fuzzy linguistic TODIM method based on a score function. *International Journal of Computational Intelligence Systems*, 8(4):701–712, 2015.

[116] C. P. Wei, N. Zhao, and X. J. Tang. Operators and comparisons of hesitant fuzzy linguistic term sets. *IEEE Transactions on Fuzzy Systems*, 22(3):575–585, 2014.

[117] X. Wei, Q. Y. Su, and J. L. Li. The group decision-making rules based on rough sets on large scale engineering emergency. *Systems Engineering Procedia*, 4:331–337, 2012.

[118] G. Williams, S. Batho, and L. Russell. Responding to urban crisis: The emergency planning response to the bombing of manchester city centre. *Cities*, 17(4):293–304, 2000.

[119] W. Winston. *Decision making under uncertainty.* Palisade Corporation, 1999.

[120] Z. B. Wu and J. P. Xu. A consistency and consensus based decision support model for group decision making with multiplicative preference relations. *Decision Support Systems*, 52(3):757–767, 2012.

[121] M. M. Xia and Z. S. Xu. Hesitant fuzzy information aggregation in decision making. *International journal of approximate reasoning*, 52(3):395–407, 2011.

[122] X. H. Xu, C. G. Cai, and X. H. Chen. A multi-stage conflict style large group decision-making emergency method based on interval fuzzy number. *Operations Research and Management Science*, 24(4):9–15, 2015.

[123] X. H. Xu, Z. J. Du, and X. H. Chen. Consensus model for multi-criteria large-group emergency decision making considering non-cooperative behaviors and minority opinions. *Decision Support Systems*, 79:150–160, 2015.

[124] X. H. Xu, D. Liang, X. H. Chen, and Y. J. Zhou. A risk elimination coordination method for large group decision-making in natural disaster emergencies. *Human and Ecological Risk Assessment: An International Journal*, 21(5):1314–1325, 2015.

[125] X. H. Xu, L. Y. Zhang, and Q. F. Wan. A variation coefficient similarity measure and its application in emergency group decision-making. *Systems Engineering Procedia*, 5:119–124, 2012.

[126] X. H. Xu, X. Y. Zhong, X. H. Chen, and Y. J. Zhou. A dynamical consensus method based on exit–delegation mechanism for large group emergency decision making. *Knowledge-Based Systems*, 86:237–249, 2015.

[127] Y. J. Xu, W. C. Zhang, and H. M. Wang. A conflict-eliminating approach for emergency group decision of unconventional incidents. *Knowledge-Based Systems*, 83:92–104, 2015.

[128] J. J. Yang and C. H. Xu. Emergency decision engineering model based on sequential games. *Systems Engineering Procedia*, 5:276–282, 2012.

[129] L. Yu and K. K. Lai. A distance-based group decision-making methodology for multi-person multi-criteria emergency decision support. *Decision Support Systems*, 51(2):307–315, 2011.

[130] L. Zadeh. The concept of a linguistic variable and its applications to approximate reasoning. *Information Sciences, Part I, II, III*, (8,9):199–249, 301–357, 43–80, 1975.

[131] L. A. Zadeh. Fuzzy sets. *Information and Control*, 8(3):199–249, 1965.

[132] L. A. Zadeh. Similarity relations and fuzzy orderings. *Information sciences*, 3(2):177–200, 1971.

[133] Lotfi A Zadeh. Outline of a new approach to the analysis of complex systems and decision processes. *IEEE Transactions on systems, Man, and Cybernetics*, (1):28–44, 1973.

[134] P. Zeniewski and R. Bolado-Lavin. A review of national gas emergency plans in the european union. *Energy policy*, 49:652–662, 2012.

[135] N. Zhang, X. Y. Ni, H. Huang, and M. Duarte. Risk-based personal emergency response plan under hazardous gas leakage: Optimal information dissemination and regional evacuation in metropolises. *Physica A: Statistical Mechanics and its Applications*, 473:237–250, 2017.

[136] Z. X. Zhang, L. Wang, R. M. Rodríguez, Y. M. Wang, and L. Martínez. A hesitant group emergency decision making method based on prospect theory. *Complex & Intelligent Systems*, 3(3):177–187, 2017.

[137] J. D. Zhao, T. Jin, and H. Z. Shen. A case-based evolutionary group decision support method for emergency response. In *Pacific-Asia Workshop on Intelligence and Security Informatics*, pages 94–104. Springer, 2007.

[138] J. F. Zhou and G. Reniers. Petri-net based evaluation of emergency response actions for preventing domino effects triggered by fire. *Journal of Loss Prevention in the Process Industries*, 51:94–101, 2018.

[139] J. X. Zhou, S. Li, G. Z. Nie, X. W. Fan, J. X. Tan, L. B. Meng, C. Xia, and Q. Zhou. Research on earthquake emergency response modes of individuals based on social surveillance video. *International journal of disaster risk reduction*, 28:350–362, 2018.

[140] L. Z. Zhou, S. Q. Zhong, S. F. Ma, and N. Jia. Prospect theory based estimation of drivers' risk attitudes in route choice behaviors. *Accident Analysis & Prevention*, 73:1–11, 2014.

[141] H. J. Zimmennann. Fuzzy set theory and its applications, 1991.