Preference Criterion and Consistency in the Rule-Based System Based on a Lattice-Valued Logic

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Abstract

The paper combines the ideas from preferential logic and many-valued logic to address the consistency in the knowledge-based system. The consistency and simplification theories of the rule-base in a lattice-valued propositional logic system LP(X) are formulated. Then, the verification of the consistency of the rule-base is transformed into a finite and achievable simplification problem. The consistency reflects the preferential interpretation. The principle of uncertainty minimization is in fact a preference criterion among different interpretations of the premise.

Keywords: consistency, preferential logic, lattice-valued logic, rule-base system.

1 Introduction

The ability to reason in a reasonable way with incomplete or inconsistent information is a major challenge, and its significance should be obvious. The consistency of a knowledge-base is an essential issue for the knowledge-based intelligent information processing. However, this cannot be done effectively using classical logic. Reasoning based on classical logic cannot solve the problem because the presence of a single contradiction results in trivialization—anything follows from A $\wedge\neg$A, and so all inconsistencies are treated as equally bad. Hence, faced with an inconsistent set, if we want to perform automated reasoning, we must either remove information until consistency is achieved again, or adopt a non-classical logic. The problem with the former approach is that we may be forced to make premature decisions about which information to discard. Alternatively multi-valued logics allow such reasoning. Multi-valued logics permit some contradictions to be true, without the resulting trivialization of classical logic. For example, multi-valued logics use additional truth values to represent different types of contradiction. Multi-valued logics are useful for merging information from inconsistent viewpoints because they allow us to explicitly represent different levels of agreement. The choice of values to use in the logic depends on how we wish to combine information from individual agents. The values used mostly are not totally ordered, but partially ordered.

In the framework of the lattice-valued first-order logic system LP(X) which is in the attempt of handling fuzziness and incomparability, this paper focuses on how to define and verify the consistency degree of the rule-base in the intelligent information process system. We incorporate a concepts introduced by McCarthy [2] and later considered by Shoham [5], according to which inferences from a given theory are made with respect to a subset of the models of that theory. The principle of uncertainty minimization is in fact a preference criterion among different interpretations of the premise. In our case the idea is to give precedence to those valuations that minimize the amount of uncertain information in the set of premises. The truth values are therefore arranged according to an order relation that reflects differences in the amount of uncertainty that each one of them exhibits. Then we choose those valuations that minimize the amount of uncertainty with respect to this order. The intuition behind this approach is that incomplete or contradictory data corresponds to inadequate information about the real world, and therefore it should be minimized.

Based on the above ideas, some kinds of the rule-bases in LP(X) with truth-value in a lattice-valued logical algebra – lattice implication algebra [6-7] as the generalized clause set forms are presented [8-10]. Then the consistency and simplification theories of the rule-base in LP(X) are formulated. Therefore, the verification of the α-consistency of the rule-base is transformed into a finite and achievable simplification problem. Finally, a simplification search algorithm for verifying the consistency of the rule-base is proposed. The α-consistency reflects the consistency degree of preferential interpretation. The uncertainty minimization is in fact taken as a preference criterion among different interpretations of the premise.

2 Lattice-valued Logic LP(X)

Lattice-valued logics are particularly interesting, as they can handle both inconsistency and incompleteness. In general, lattice structures apply whenever ordinal information must be represented. The question of the appropriate operation and lattice structure has generated much literature. One of most important work is by Goguen [1] who established *L*-fuzzy logic of which truth value set is a complete lattice-ordered monoid, which is also called a complete residuated lattice in Pavelka and Novak′s *L*-fuzzy logic [3-4].

Definition 2.1 [4] A *residuated lattice* (RL) is a structure $<L, \otimes, \rightarrow>$, where

(1) $\mathbf{L}=<L, \leq, \vee, \wedge, O, I>$ is a bounded lattice with the least element O and the greatest element I.

(2) $<\otimes, \rightarrow>$ is an adjoint couple on *L,* i.e.,

(a) $\otimes$ is istone (ordering preserving) on $L\times L$; (b) $\rightarrow$ is antione (order reversing) in the first and isotone in the second variable on $L\times L$; (c) for all $x, y, z\in L$ hold the adjointness condition or Galois correspondence: $x\otimes y\leq c$ iff $x\leq y\rightarrow z$

(3) $<L, \otimes, I>$ is a commutative monoid.

The operation $\otimes$ is called multiplication and $\rightarrow$ is called residuation.

Since this algebraic structure is quite general, it is relevant to ask whether one can specify the structure. In this note, we specify the algebraic structure to lattice implication algebras introduced by Xu [7]. This module of our system is strongly related to previous developments in the theory of lattice-valued logic [6].

Definition 2.2 (LIA) [6] Let $(L, \vee, \wedge, ')$ be a bounded lattice with an order-reversing involution " $'$ " and the universal bounds O, I, $\rightarrow: L\times L\rightarrow L$ be a mapping. $(L, \vee, \wedge, ', \rightarrow)$ is called a *lattice implication algebra* (LIA) if the following axioms hold for all $x, y, z\in L$:

(A_1) $x \rightarrow (y \rightarrow z)=y \rightarrow (x \rightarrow z)$, (exchange property)

(A_2) $x\rightarrow x=$I, (identity)

(A_3) $x \rightarrow y=y' \rightarrow x'$, (contraposition or contrapositive symmetry)

(A_4) $x\rightarrow y=y\rightarrow x=$I implies $x=y$, (equivalency)

(A_5) $(x\rightarrow y)\rightarrow y=(y\rightarrow x)\rightarrow x$,

(A_6) $(x\vee y)\rightarrow z=(x\rightarrow z)\wedge(y\rightarrow z)$,

(A_7) $(x\wedge y)\rightarrow z=(x\rightarrow z)\vee(y\rightarrow z)$.

Some basic concepts and properties of LIAs can be seen in [6].

In the following, we always assume that $(L, \vee, \wedge, ', \rightarrow, O, I)$ is a lattice implication algebra, in short *L*.

Definition 2.3 Let *X* be a set of propositional variables, $T=L\cup\{', \rightarrow\}$ be a type with $ar(')=1$, $ar(\rightarrow)=2$ and $ar(a)=0$ for every $a\in L$. The propositional algebra of the lattice-valued propositional calculus on the set of propositional variables is the free *T* algebra on *X* and is denoted by $LP(X)$.

Proposition 2.1 $LP(X)$ is the minimal set *Y* which satisfies the following conditions:

(1) $X\bigcup L\subseteq Y$,

(2) If $p,q\in Y$, then $p', p\rightarrow q\in Y$.

Note that *L* and $LP(X)$ are the algebras with the same type *T*, where $T=L\bigcup\{',\rightarrow\}$. Moreover, note that $\vee$, $\wedge$, $\otimes$ and $\oplus$ can all be expressed by $'$ and $\rightarrow$, so $p\vee q$, $p\wedge q$, $p\otimes q$, and $p\oplus q\in LP(X)$ if $p, q\in LP(X)$. We denote all *L*-fuzzy sets of $LP(X)$ as $\mathbf{F}_L(LP(X))$.

Definition 2.4 A valuation of $LP(X)$ is a propositional algebra homomorphism γ : $LP(X)\rightarrow L$.

If γ is a valuation of $LP(X)$, we have $\gamma(\alpha) = \alpha$ for every $\alpha \in L$.

Definition 2.5 Let $p \in LP(X)$, $\alpha \in L$. If $\gamma(p) \geq \alpha$ for every valuation γ of $LP(X)$, then p is said to be valid by truth-value level α. If α=I, then we say that p is valid.

Definition 2.6 Let p, $q \in LP(X)$. We say that p and q are equivalent propositions and written as $p=q$, if $\gamma(p)=\gamma(q)$ for every valuation γ of $LP(X)$.

Definition 2.7 Let p, $q \in LP(X)$. If $\gamma(p) \leq \gamma(q)$ for every valuation γ of $LP(X)$, we say that p is always less than q, denoted by $p \leq q$.

Definition 2.8 Let $p \in LP(X)$, $\alpha \in L$. If $\gamma(p) \leq \alpha$ for every valuation γ of $LP(X)$, we say that p is always false by truth-value level α, in short α-false. If α=O, then we say that p is invalid.

Definition 2.9 Let $p \in LP(X)$, $\alpha \in L$. If there exists a valuation γ of $LP(X)$ such that $\gamma(p) \geq \alpha$, then p is said to be α-satisfiable. If α=I, then we say that p is satisfiable.

In the following section, we always suppose that L is a complete lattice implication algebra. And we use $LP(X)$ to express the set of lattice-valued logical formulae based on the lattice implication algebra L.

Suppose that F is the set of all L-type formula in LP(X), and $\boldsymbol{F}_L(F)$ represents the set of all the L-type fuzzy set on F. Let $A \in \boldsymbol{F}_L(F)$, A is called a non-logical fuzzy axiom set. For any logical formula $\varphi \in F$, always associated with a value $A(\varphi) \in L$. In the real-world practices, one may suppose that $A(\varphi)$ is the minimal truth-value degree of a proposition φ or possibility degree, or credibility degree (based on the application context). It is expected that during the reasoning process, every inferred formula $\psi \in LP(X)$, whose associated (truth) value should be larger than $A(\psi)$. Thus, we need to know the minimal value $A(\psi)$ of the involved formula ψ, in addition, the associated truth values may continuously improved during the deduction process. In the following, we always assume that for any L-type formula φ, $A(\varphi) > 0$ or A is said to be regular, and φ is called a non-logical axiom of A.

Definition 2.10 Let $A \in \boldsymbol{F}_L(F)$, $\varphi \in F$, $\alpha \in L$. φ is said to be α-true in A, denoted as $A \models_{\overline{\alpha}} \varphi$, if

$$\alpha = \wedge\{\gamma(\varphi);\ \lambda \text{ is a valuation satisfying } A\}.$$

Set

$$Con(A)(\varphi) = \alpha = \wedge\{\gamma(\varphi);\ \lambda \text{ is a valuation satisfying } A\}$$

Now we consider the syntax in the following part. For arbitrary p, $q \in F$, set $p \otimes q =: (p \rightarrow q')'$.

Definition 2.11 Let $A \in \boldsymbol{F}_L(LP(X))$, $\varphi \in LP(X)$. A formal proof ω from A to φ is a finite sequence as follows：

$$(\varphi_1, \alpha_1), (\varphi_2, \alpha_2), \ldots, (\varphi_n, \alpha_n),$$

where $\varphi_n = \varphi$, and for any i, $1 \leq i \leq n$, $\varphi_i \in LP(X)$, $\alpha_i \in L$, and

(1) $A_L(\varphi_i) = \alpha_i$, or

(2) $A(\varphi_i) = \alpha_i$, or

(3) there exist j, $k < i$, such that $\varphi_j = \varphi_k \rightarrow \varphi_I$ and $\alpha_i = \alpha_j \otimes \alpha_k$, or

(4) there exists $j < i$, and $\alpha \in L$, such that $\varphi_i = \alpha \rightarrow \varphi_j$ and $\alpha_i = \alpha \rightarrow \alpha_j$.

Here n is called a length of the proof ω, denoted as $l(\omega)$. Moreover, α_n is called the value of the proof ω, denoted as $val(\omega)$.

Definition 2.12 Let $A \in \boldsymbol{F}_L(LP(X))$, $\varphi \in LP(X)$, $\alpha \in L$. φ is called an α-theorem of A, denoted as $A \vdash_\alpha \varphi$, if

$$\alpha \leq \vee\{val(\omega);\ \omega \text{ is a proof from } A \text{ to } \varphi\}$$

If $\alpha = \vee\{val(\omega);\ \omega$ is a proof from A to $\varphi\}$, then, denoted as $A \Vdash_\alpha \varphi$.

Set $\overline{Con}(A) \in \boldsymbol{F}_L(LP(X))$ for $A \in \boldsymbol{F}_L(LP(X))$ such that： for any $\varphi \in LP(X)$, if $A \Vdash_\alpha \varphi$, then $\overline{Con}(A)(\varphi) = \alpha$.

Definition 2.13 Let $A \in \boldsymbol{F}_L(LP(X))$. If

$\beta = \vee\{\alpha$; there exist $\alpha_1, \alpha_2 \in L$, $\varphi \in LP(X)$ such that $A \vdash_{\alpha_1} \varphi$, $A \vdash_{\alpha_2} \varphi'$ and $\alpha = \alpha_1 \otimes \alpha_2\}$, then A is said to be β'-consistent or A is said to be β-contradict, also the contradiction degree of A is β, denoted as Cont(A).

If A is I-consistent, then A is called shortly consistent. If A is O-consistent, then A is called shortly contradict.

Corollary 2.1 If A is β'- consistent, then

$\beta = \vee\{ \alpha_1 \otimes \alpha_2 ,\ A \vdash_{\alpha_1} \varphi,\ A \vdash_{\alpha_2} \varphi',\ \varphi \in LP(X)\}$.

The truth values are arranged according to an order relation that reflects differences in the amount of uncertainty that each one of them exhibits. Accordingly we choose those valuations that minimize the amount of uncertainty with respect to this order, i.e., it reflects the consistency of logic system. How to determine the consistency based on the valuation in the rule-based system is provided in the following section.

3 Consistency of Rule-Base in LP(X)

In a rule-based intelligent information processing system, the rule-base $\mathfrak{R}$ has generally the type of n rules:

R_i: If A1i, $A_{2i}, \ldots, A_{mi}$, then $B_{1i}, B_{2i}, \ldots, B_{ki}$, $i=1,\ldots,n$

In the rule-base $\mathfrak{R}$, each rule is actually a first-order logical formula in LP(X). Suppose that the logical relationships among $A_{1i}, A_{2i}, \ldots, A_{mi}$ and those among $B_{1i}, B_{2i}, \ldots, B_{ki}$ are taken as "$\wedge$," and those among *n* rules as "$\vee$," respectively.

Accordingly, the matrix M_i of the formula of the *i*th rule in $\mathfrak{R}$ in LP(X) can be written as:

$$(A_{1i} \wedge A_{2i} \wedge \ldots \wedge A_{mi}) \rightarrow (B_{1i} \wedge B_{2i} \wedge \ldots \wedge B_{ki}) = \bigwedge_{t=1}^{k} \bigvee_{s=1}^{m} (A_{si} \rightarrow B_{ti}) \quad (3.1)$$

Consequently, $\mathfrak{R}$ can be written by the matrixes:

$$M: \bigvee_{i=1}^{n} (\bigwedge_{t=1}^{k} (\bigvee_{s=1}^{m} (A_{si} \rightarrow B_{ti})))$$

$$= \bigwedge_{t_1=1}^{k} \cdots \bigwedge_{t_i=1}^{k} \cdots \bigwedge_{t_n=1}^{k} (\bigvee_{i=1}^{n} \bigvee_{s=1}^{m} (A_{si} \rightarrow B_{t_i i})) \quad (3.2)$$

$$M = \{ \bigvee_{i=1}^{n} \bigvee_{s=1}^{m} (A_{si} \rightarrow B_{t_i i}) \mid t_i = 1, \ldots, k, i=1,\ldots, n\} \quad (3.3)$$

Under many situations in intelligent information process systems, the established rule-base is not only regarded as the formal logical formulae, but also associated with the rich semantic interpretation, which is naturally associated during the knowledge acquisition process from the domain experts. From the logical point of view, the formulae are always associated with a valuation ν_e in LP(X) such that

$$\nu_e(A_{si}) = \sigma_{si},\ \nu_e(B_{ti}) = \omega_{ti},\ s=1, 2,\ldots, m,\ t=1, 2,\ldots, k,\ i=1, 2, \ldots, n. \quad (3.4)$$

Based on Definition 2.13 and Corollary 2.1, in intelligent information process systems, if the valuation in Eq. (3.4) is obtained, then the local consistency degree of the rule-base is defined as follows:

Definition 3.1 Let ν be the valuation of LP(X), M be the generalized clause set of the formula corresponding to the rule-base $\mathfrak{R}$. ν (M) is called the local consistency degree of $\mathfrak{R}$ with respect to ν.

Accordingly, the local consistency degree of $\mathfrak{R}$ corresponding to rule-base case in (3.1) with respect to the valuation ν_e is given as follows respectively:

$$\nu_e(M) = \bigvee_{i=1}^{n} (\bigwedge_{t=1}^{k} (\bigvee_{s=1}^{m} (\sigma_{si} \rightarrow \omega_{ti}))) \quad (3.6)$$

Definition 3.2 Let M be the generalized clause set of the formula corresponding to the rule-base $\mathfrak{R}$. *The* uniform consistency degree *of* $\mathfrak{R}$, denoted as C (M), is defined as

C (M) = $\vee\{\nu(M) \mid$ D is an interpretation in LP(X), ν is the corresponding valuation$\}$

Theorem 3.1 (Existence of the uniform consistency degree of $\mathfrak{R}$) Let L be a complete lattice implication algebra, M be the generalized clause set of the formula corresponding to the rule-base $\mathfrak{R}$. C (M) exists.

Lemma 3.2 Let $\alpha \in$ L, M be the generalized clause set of the formula corresponding to the rule-base $\mathfrak{R}$. Then

M $\leq \alpha$ if and only if C (M) $\leq \alpha$.

Definition 3.3 Let M be the generalized clause set of the formula corresponding to the rule-base $\mathfrak{R}$. If M is simplified into $\alpha - \Box$, the α is called the local simplification degree *of M*, denoted as s (M) $= \alpha$.

Definition 3.4 Let M be the generalized clause set of the formula corresponding to the rule-base $\mathfrak{R}$. T*he* uniform simplification degree of *M*, denoted as S (M), is defined as

S (M) = $\wedge$ { α | there exists a simplification s, s (M) = α }

Theorem 3.3 (Existence of the uniform simplification degree of $\mathfrak{R}$) Let L be a complete lattice implication algebra, M be the generalized clause set of the formula corresponding to the rule-base $\mathfrak{R}$. Then S (M) exists.

Theorem 3.4 Let L be complete lattice implication algebra, M be the generalized clause set of the formula corresponding to the rule-base $\mathfrak{R}$. Then C (M) = S (M).

Corollary 3.1 Let L be a finite lattice implication algebra, M be the generalized clause set of the formula corresponding to the rule-base $\mathfrak{R}$. Then C (M) = S (M).

Theorem 3.5 Let L be a completely distributive lattice implication algebra, M be the generalized clause set of the formula corresponding to the rule-base $\mathfrak{R}$. If the relationship among the rules in the rule-base $\mathfrak{R}$ is taken as "$\wedge$," then

$$S(M) = C(M) = \bigwedge_{i=1}^{n} C(M_i) = \bigwedge_{i=1}^{n} S(M_i).$$

Here M_i (i =1,…, n) is the generalized clause set of the formula corresponding to the *i*th rule in the rule-base $\mathfrak{R}$.

Theorem 3.5 shows that it is possible to increase the consistency degree and the simplification degree while the number of the rules decreases.

Theorem 3.6 L be a completely distributive lattice implication algebra, M_i (i=1,…, n) be the generalized clause set of the formula corresponding to the *i*th rule-base R_i. If the relationship among the rules in the rule-base $\mathfrak{R}$ is taken as "$\wedge$," and there are R_{n_j}, j=1,…, k,

$$\bigwedge_{i=1}^{n} C(M_i) = \bigwedge_{j=1}^{k} C(M_{n_j}),$$

then for any arbitrary $\mathfrak{R}^* \in \wp_k(\mathfrak{R})$, $C(M^*) \leq C(M^0)$ holds for the generalized clause set M^* of the GSSF corresponding to $\mathfrak{R}^*$ and the generalized clause set M^0 for $\mathfrak{R}$ - $\{R_{n_j}$ |j=1,…, k}, where

$\wp_k(\mathfrak{R}) = \{\mathfrak{R}^* | \mathfrak{R}^* \in \wp(\mathfrak{R}), R_{n_j} \in \mathfrak{R}^*$, j=1,…, k}.

4 Determination of Consistency Degrees of Rule-Bases

4.1 Determination of the local consistency of the rule-base

If a valuation ν in LP(X) is given, then the local consistency of the rule-base can be obtained from the valuation ν; Moreover, if the truth-value σ_{si} of A_{si} and the truth-value ω_{ti} of B_{ti} (s=1, 2,…, m, t=1, 2,…, k, i=1, 2,…, n) can be obtained, then a family of valuations **V** in LP(X) can be generated by these values. For each valuation $\nu \in$ **V**, ν can further generate a valuation ν in LP(X) such that $\nu(A_{si})=\sigma_{si}$ and $\nu(B_{ti})=\omega_{ti}$ (s=1, 2,…, m, i=1, 2,…, n, t=1, 2,…, k). Then the local consistency of the rule-based can be calculated from ν(M).

4.2 Determination of the uniform consistency degree of the rule-base

Let $|L| < +\infty$. Generally, since the interpretation set of LP(X) is an infinite set, so is the corresponding valuation set. To determine C (M) directly by all the valuations of LP(X) does not seem feasible. According to Theorem 3.4, the determination of C (M) is transformed into that of S (M) so that an infinity problem is transformed into a finite problem. According to Definition 3.4, however, it is required to obtain many local simplification degrees to finally determine the uniform simplification degree S (M) while the cardinality of L is relatively large.

Let M be the generalized clause set of the formula corresponding to the rule-base $\mathfrak{R}$. Considering Definition 3.4, in the following, we propose an algorithm to determine S (M), i.e., a stepwise search algorithm:

Step 1. A predict initial value α_0 of C (M) is given by the expert;

Step 2. Simplify the M according to α_0, denoted this simplification as s_0

If s_0 (M) =α_0, when there does not exist α_1, then stop s_0 and s_0 (M) = α_0; when there exists α_1 in [O, α_0), go to Step 3;

If $s_0(M) \neq \alpha_0$, set A={ $\alpha \in$ L| α //α_0} $\cup$ {α_0}, when for any $\alpha \in$A, there does not exist such α_1 in (α, I), then S(M) = s_0(M)=I; for any $\theta \in \{\alpha \mid \alpha \in A, (\alpha, I) \neq \phi\}$, select α_1 in (θ, I), then go to Step 3.

Step 3. Simplify M according to α_1. Repeat Step 2.

Step 4. After stopping Step 2, set S (M) = $\wedge\{\alpha \in L|$ there exists a simplification s, s (M) $=\alpha\}$.

Since $|L| < +\infty$, then S (M) can be obtained by circularly operating Step 2 and Step 3 in finite times.

5 Conclusions

The paper investigated the consistency in the knowledge-based system combining a kind of lattice-valued logic and preferential logic. The consisteny degree was defined in lattice-valued logic based on the valuations that minimize the amount of uncertain information in the set of premises. The consistency actually reflects the preferential interpretation. The intuition behind is that uncertainty minimization is in fact a preference criterion among different interpretations of the premise, actually incomplete or contradictory data corresponds to inadequate information about the real world, and therefore it should be minimized. Concretely, the consistency and simplification theories of the rule-base in a lattice-valued propositional logic system LP(X) were formulated. Then, the verification of the consistency of the rule-base was transformed into a finite and achievable simplification problem. These provide certain theoretical support for preference modeling from the logic point of view, specially the partially ordered preference relationship.

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