Satisfiability in a Linguistic-Valued Logic and Its Quasi-horn Clause Inference Framework

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Abstract. In this paper, we focus on the linguistic-valued logic system with truth-values in the lattice-ordered linguistic truth-valued algebra, then investigate its satisfiability problem and its corresponding Quasi-Horn-clause logic framework, while their soundness and completeness theorems are provided. The present framework reflects the symbolic approach acts by direct reasoning on linguistic truth values, i.e., reasoning with words, and provides a theoretical support for natural-language based reasoning and decision making system.

1 Introduction

Resolution principle is a single rule of inference for a test of unsatisfiability of a logical formula. Since its introduction in 1965 [1], automated reasoning based on Robinson's resolution rule has been extensively studied in the context of finding natural and efficient proof systems to support a wide spectrum of computational tasks. As the use of non-classical logics becomes increasingly important in computer science and artificial intelligence, the development of efficient automated reasoning on non-classical logic is currently an active area of research. e.g., on fuzzy logic, among others, see [2]-[7] and on many-valued logic, among others, see [8]-[21].

Lattice-valued logic is an important many-valued logic. In this paper, we extend the resolution principle from two-valued logic into lattice-valued logic with truth-value in a lattice-ordered logical algebraic structure - Residuated Lattice (RL). RL structure is a very popular algebra for inexact concepts as shown by Goguen in [22]. There have been considerable efforts about many-valued logic based on a RL, where Pavelka [23] and Novak [24] systematically discussed propositional and first-order calculi with truth values in an enriched RL. Although there are some important investigations in [17], [19-21] among others, up to now, the study of proving systems and automated reasoning based on RL, has not been extensively reported.

In this paper, we focus on a lattice-valued logic [19-20], [25] with truth-value in a LIA [25-26], which is a kind of RLs established by combining the lattice and

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implication algebra. This kind of lattice-valued logics are an extension of classical logic in several aspects such as connectives, truth-valued field and inference rules, which includes Łukasiewicz logic $L[0, 1]$ with truth-values in [0, 1] as a special case. Especially their implication connectives are more general and not reducible to the other classical connectives (like ~, ∧ and ∨), unlike the Kleene implication ($p\rightarrow q = \sim p\vee q$, ~ is the negation, it implies that the formulae of its logic are syntactically equivalent to those in classical logic). This irreducibility, though semantically justifiable, complicates the calculus.

On the other hand, we all know that as human beings we are bound to express ourselves in a natural language that uses words. The meanings of words are inherently, imprecise, vague, and fuzzy, mostly can be very qualitative in nature. Hence, it is necessary to investigate natural language based reasoning within the realm of AI. A nice feature of linguistic variables is that their values are structured, which makes it possible to compute the representations of composed linguistic values from those of their composing parts. In order for linguistic variables to be useful tools of analysis, one ought to be able to manipulate them through various operations so that one can directly symbolically manipulate the linguistic variables themselves.

Accordingly, in the present work, we characterize the set of linguistic values by a lattice-valued algebraic structure (i.e., LIA) and investigate the corresponding logic systems with linguistic truth-value LIA. Furthermore, we address the satisfiability problems of the logical formula with respect to a certain linguistic truth-value level and the corresponding Quasi-Horn clause logic framework by establishing the resolution principles. A key idea behind these approaches is to directly manipulate the available linguistic information and knowledge, i.e., the symbolic approach acts by direct computation on linguistic truth values.

The paper is organized as follows. In Section 2, we describe and define the linguistic truth-value algebra and recall some basic concepts about LIAs, then define the lattice-valued propositional logic system with truth-valued in a linguistic-valued LIA. In Section 3, the corresponding resolution principle as well as its theorems of soundness and completeness are given on proving the satisfiability of logical formulae with respect to a certain linguistic truth-value level. Section 4 establishes a calculus for linguistic-valued Quasi-Horn clause and claims its soundness and completeness. The conclusion is included in Section 5.

2 Linguistic Truth-Value Logics

2.1 Linguistic-Valued Logical Algebra

2.1.1 Linguistic Assessment Instead of Numerical Assessment

Human beings cannot be seen as a precision mechanism. They usually express their knowledge about the world using linguistic variable in natural language with full of vague and imprecise concepts. The linguistic approach is an approximate technique appropriate for dealing with qualitative aspects of problems. Since words are less precise than numbers, the concept of a linguistic variable serves the purpose of providing a measure for an approximate characterization of the phenomena which are too complex or ill-defined to be amenable to their description by conventional quantitative terms [28].

In order for linguistic variables to be useful tools of analysis, one ought to be able to manipulate them through various operations so that one can directly symbolically manipulate the linguistic variables themselves, i.e., symbolic approach acting by direct reasoning on words, different from approximation approach which uses the associated membership functions, avoid the burden steps implying the investigation of human factor, semantics of the linguistic terms, subjective beliefs etc.

2.1.2 Lattice Structure and Lattice Implication Algebras

A nice feature of linguistic variables is that their values are structured. Symbolic linguistic approach assumes that the linguistic term set is an ordered structure. In general, lattice structures apply whenever ordinal information must be represented. Lattice structures actually provide one possible solution for model the linguistic value structure. The question of the appropriate operation and lattice structure has generated much literature [22-25]. One of most important work is by Goguen [22] who established *L*-fuzzy logic of which truth value set is a complete lattice-ordered monoid, which is also called a complete residuated lattice in Pavelka and Novak´s *L*-fuzzy logic [23-24].

Definition 2.1 [23]**.** A *residuated lattice* (RL) is a structure $<L, \otimes, \rightarrow>$, where

(1) $\mathbf{L}=<L, \leq, \vee, \wedge, O, I>$ is a bounded lattice with the least element O and the greatest element I.

(2) $<\otimes, \rightarrow>$ is an adjoint couple on *L,* i.e.,

(a) $\otimes$ is istone (ordering preserving) on $L\times L$; (b) $\rightarrow$ is antione (order reversing) in the first and isotone in the second variable on $L\times L$; (c) for all $x, y, z\in L$ hold the adjointness condition or Galois correspondence: $x\otimes y\leq c$ iff $x\leq y\rightarrow z$

(3) $<L, \otimes, I>$ is a commutative monoid.

The operation $\otimes$ is called multiplication and $\rightarrow$ is called residuation.

Since this algebraic structure is quite general, it is relevant to ask whether one can specify the structure. In this note, we specify the algebraic structure to lattice implication algebras introduced by Xu [25-26].

Definition 2.2 (LIA)**.** Let $(L, \vee, \wedge, ')$ be a bounded lattice with an order-reversing involution " $'$ " and the universal bounds O, I, $\rightarrow: L\times L\rightarrow L$ be a mapping. $(L, \vee, \wedge, ', \rightarrow)$ is called a *lattice implication algebra* (LIA) if the following axioms hold for all $x, y, z\in L$:

(A_1) $x\rightarrow(y\rightarrow z)=y\rightarrow(x\rightarrow z)$, (exchange property)
(A_2) $x\rightarrow x=I$, (identity)
(A_3) $x\rightarrow y=y'\rightarrow x'$, (contraposition or contrapositive symmetry)
(A_4) $x\rightarrow y=y\rightarrow x=I$ implies $x=y$, (equivalency)
(A_5) $(x\rightarrow y)\rightarrow y=(y\rightarrow x)\rightarrow x$,
(A_6) $(x\vee y)\rightarrow z=(x\rightarrow z)\wedge(y\rightarrow z)$,
(A_7) $(x\wedge y)\rightarrow z=(x\rightarrow z)\vee(y\rightarrow z)$.

Some basic concepts and properties of LIAs can be seen in [25].

2.1.3 Lattice-Valued Linguistic Algebra

A linguistic term differs from a numerical one in that its values are not numbers, but words or sentences in a natural or artificial language. On the other hand, these words, in different natural language, seem difficult to distinguish their boundary sometime, but their meaning of common usage can be understood. Moreover, there are some "vague overlap districts" among some words which cannot be strictly linearly ordered, as given in Fig. 1 of Example 1.1.

Example 1.1. The ordering relationships in some linguistic terms:

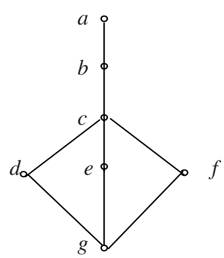

Fig. 1. Linguistic terms in an ordering a=very true, b=more true, c=true, d=approximately true e=possibly true, f=more or less true, g=little true

Note that *d, e*, and *f* are incomparable. One can not collapse that structure into a linearly ordered structure, because then one would impose an ordering on *d*, *e*, and *f* which was originally not present. This means the set of linguistic values may not be strictly linearly ordered. It is shown that linguistic term can be ordered by their meanings in natural language. Naturally, it should be suitable to represent the linguistic values by a partially ordered set or lattice. To attain this goal we characterize the set of linguistic truth-values by a LIA structure, i.e., use the LIA to construct the structure of value sets of linguistic variables.

In general, the value of a linguistic variable can be a linguistic expression involving a set of linguistic terms such as "*high*," "*middle*," and "*low*," modifiers such as "*very*," "*more or less*" (called hedges [27]) and connectives (e.g., "*and*," "*or*"). Let us consider the domain of the linguistic variable "*truth*": domain (*truth*)={*true, false, very true, more or less true, possibly true, very false, possibly false*, ...}, which can be regarded as a partially ordered set whose elements are ordered by their meanings and also regarded as an algebraically generated set from the generators *G*={*true, false*} by means of a set of linguistic modifiers *M*={*very, more or less, possibly*, ...}. The generators *G* can be regarded as the prime term, different prime terms correspond to the different linguistic variables.

Taking into account the above remarks, construction of an appropriate set of linguistic values for an application can be carried out step by step. Consider a set of linguistic hedges, e.g. H^{+}={*very, more or plus*}, H={*approximately, possibly, more or less, little*}, where H^{+} consists of hedges which strengthen the meanings of "true" and the hedges in H weaken it. Put $H=H^{+}\cup H$. H^{+}, H can be ordered by the degree of strengthening or weakening. We say that $a \leq b$ if and only if $a(True) \leq b(True)$ in the natural language, where a and b are linguistic hedges.

Applying the hedges of H to the primary term "*true*" or "*false*" we obtain a partially ordered set or lattice. For example, we can obtain a lattice generated from "*true*" or "*false*" by means of operations in H. We add these three special elements I, M, O called "*absolutely true*," "*medium*," and "*absolutely false*" to the obtaining set so that they have natural ordering relationship with the linguistic truth values. The set of linguistic truth-values obtained by the above procedure is a lattice with the boundary. Moreover, one can define $\wedge$, $\vee$, implication $\rightarrow$ and complement operation $'$ on this lattice according to the LIA structure.

Consider a totally ordered linguistic term set as an example, let n=5, L_n={s_0=*Poor*, s_1=*Low*, s_2=*Average*, s_3=*High*, s_4=*Good*}. Any label, s_i, represents a possible value for a linguistic variable, and has the following characteristics [28]:

1. The set is ordered: $s_i \leq s_j$ if $i \leq j$.
2. There is the negation operator: Neg(s_i) = s_j such that $j = n$-1 -i.
3. There is the maximization operator: Max(s_i, s_j)= s_i if $s_j \leq s_i$.
4. There is the minimization operator: Min(s_i, s_j) = s_i if $s_i \leq s_j$.

Moreover, implication ($\rightarrow$) based on the L_n structure can be further defined.

For example, let $L_n=\{s_i ; 1 \leq i \leq n\}$ is a totally ordered linguistic term set (O=$s_1 < s_2 < \ldots < s_n$=I). If the negation operator $'$: $L_n \longrightarrow L_n$ is defined by $(s_i)' = a_{n+1-i}$, and $\longrightarrow : L \times L \longrightarrow L$ is defined by $s_i \rightarrow s_j = s_{n+j-i}$ ($i, j \in \{1, \ldots, n\}$). Then ($L_n$, $\vee$, $\wedge$, ', $\longrightarrow$) is a residuated lattice, also a LIA, denoted by $\boldsymbol{L_n}$.

2.2 Linguistic-Valued Propositional Logic

In the following sections, L always represents the linguistic truth-value LIA.

Definition 2.3. Let X be the set of propositional variables, $T = L \cup \{', \otimes, \rightarrow\}$ be a type with $ar(') = 1$, $ar(\otimes) = ar(\rightarrow) = 2$ and $ar(a) = 0$ for every $a \in L$. The propositional algebra of the linguistic-valued propositional calculus on the set of propositional variables is the free T algebra on X and is denoted by $LP(X)$.

Proposition 2.1. $LP(X)$ is the minimal set Y which satisfies the following conditions: (1) $X \bigcup L \subseteq Y$; (2) If $p, q \in Y$, then $p \otimes q$, $p', p \rightarrow q \in Y$.

Note that L and $LP(X)$ are the algebras with the same type T, where $T = L \bigcup \{', \otimes, \rightarrow\}$.

Definition 2.4 A valuation of $LP(X)$ is a propositional algebra homomorphism γ: $LP(X) \rightarrow L$.

Definition 2.5. Let $p \in LP(X)$, $\alpha \in L$. If $\gamma(p) \geq \alpha$ for every valuation γ of $LP(X)$, we say that p is valid by truth-value level α. If there exists a valuation γ of $LP(X)$ such that $\gamma(p) \geq \alpha$, then p is called α-satisfiable.

Beginning from the normal form is the usual way to discuss the satisfiability of the formula in classical logic. As a first step towards a variant resolution, it is important to deal with implication connectives and consider the generalized normal form.

Definition 2.6. An L-valued propositional logical formula f is called an extremely simple form, in short ESF, if a logical formula f^* obtained by deleting any constant or literal or implication term appearing in f is not equivalent to f.

Definition 2.7. if f is an ESF containing no connectives other than implication connectives, then f is called an indecomposable extremely simple form, in short IESF.

Definition 2.8. All the constants, literals and IESF´s are called generalized literals.

Definition 2.9. An L-valued propositional logical formula G is called a generalized clause (phrase), if G is a formula of the form:

$$G=g_1 \vee \ldots \vee g_i \vee \ldots \vee g_n \ \ (G=g_1 \wedge \ldots \wedge g_i \wedge \ldots \wedge g_n)$$

where g_i (i=1,..., n) are generalized literals.

A conjunction (disjunction) of finite generalized clauses (phrases) is called a generalized conjunctive (conjunctive) normal form.

3 α-Satisfiability of Linguistic-Valued Propositional Logic

The following concepts and theorems can be obtained based on the work in [19]. Due to the limited space, the proofs are omitted.

Definition 3.1. (α-Resolution) Let $\alpha \in L$, and G_1 and G_2 be two generalized clauses of the form: $G_1=g_1 \vee \ldots \vee g_i \vee \ldots \vee g_m$ and $G_2=h_1 \vee \ldots \vee h_j \vee \ldots \vee h_n$. If $g_i \wedge h_j \leq \alpha$, then $G=g_1 \vee \ldots \vee g_{i-1} \vee g_{i+1} \vee \ldots \vee g_m \vee h_1 \vee \ldots \vee h_{j-1} \vee h_{j+1} \vee \ldots \vee h_n$ is called an α-resolvent of G_1 and G_2, denoted by $G=R_\alpha(G_1, G_2)$, and g_i and h_j form an α-resolution pair, denoted by (g_i, h_j)-α. It can be regarded as the complemented pair in the sense of α-false.

Definition 3.2. Suppose a generalized conjunctive normal form $S=C_1 \wedge C_2 \wedge \ldots \wedge C_n$, $\alpha \in L$, $\omega=\{D_1, D_2,..., D_m\}$ is called an α-resolution deduction from S to generalized clause D_m, if

(1) $D_i \in \{C_1, C_2, \ldots, C_n\}$ or (2) there exist j, $k<i$, such that $D_i = R_\alpha(D_j, D_k)$.

If there exists an α-resolution deduction from S to the empty clause $\varnothing$ (denoted by α-false), then ω is called an α-refutation.

Theorem 3.1. (Soundness) Suppose a generalized conjunctive normal form $S=C_1 \wedge C_2 \wedge \ldots \wedge C_n$, $\alpha \in L$, $\{D_1, D_2,..., D_m\}$ is an α-resolution deduction from S to a generalized clause D_m. If $D_m=\alpha$-false, then $S \leq \alpha$, that is, if $D_m \leq \alpha$, then $S \leq \alpha$

Theorem 3.2. (**Completeness**) Let S be a regular generalized conjunctive normal form, $\alpha \in L$, $\alpha<$I, α be a dual numerator in L. And suppose that there exists $\beta \in L$ such that $\beta \wedge (\beta \to \beta') > \alpha$. If $S \leq \alpha$, then there exists an α-resolution deduction from S to α-false.

4 Linguistic-Valued Quasi-horn Clause Logic

In [20], [25], a lattice-valued first-order logic LF(X) based on LIA is established. In this section, we consider the restrict set $\Im$ of LF(X), called lattice-valued Quasi-Horn clause class as an extension of classical Horn clause. Some concepts about symbols, terms, well-formed formulas, and interpretation can be referred to [20], [25].

Definition 4.1. A linguistic-valued Quasi-Horn clause (in short, *L*-type Q-Horn clause) is a well-formed formula without free variables as the following form:

$$(\forall x_1)\cdots(\forall x_k)(p \to q) \tag{1}$$

or

$$(\forall x_1)\cdots(\forall x_k)(q) \tag{2}$$

where q is an atom formula with the variables $x_1, \cdots, x_k$ only, p is a formula without restriction of qualifiers and including only connective $\vee$ or $\wedge$.

In classical logic, a clause with at most one positive literal is called a Horn clause. Prolog programming problem in knowledge engineering, as the direct application of Horn, generally consists of three clauses as follows:

(1) Facts (or asserts) expressing the related objects and the relations among these objects, P.
(2) Rules defining the relationship among some objects. P : - P_1, P_2, …, P_n.
(3) Problems (or objectives). ? - Q_1, …, Q_m.

Here P in (1) is obviously a Horn clause. (2) can be represented as

$$(P_1 \wedge P_2 \wedge \cdots \wedge P_n) \to P .$$

That is, $\neg P_1 \vee \cdots \vee \neg P_n \vee P$, also a Horn clause. (3) represents if $Q_1 \wedge \cdots \wedge Q_m$ can be inferred from (1) and (2). From the resolution point of view, the negation of conclusion of objective, i.e., $\neg Q_1 \vee \cdots \vee \neg Q_m$, is again a Horn clause. Prolog programming is based on this kind of Horn clause logic.

The *L*-type Q-Horn clause in Definition 3.1 is an extension of classical Horn clause in order to establish an extended linguistic-valued or called a *L*-type Prolog programming. *L*-type Q-Horn clause looks similar as that in classical logic, however, here "$\to$" has been different and been generalized, i.e., not any more a Kleene implication, but the more general implication form in LIA. At the same time, the rules and facts in classical Prolog can be interpreted as the set of axioms which are always true. The *L*-type Prolog which will be established in the following is an extension of this framework, i.e., the rules and facts are extended into L-type fuzzy rules and fuzzy facts, i.e., truth-value is extended from {0, 1} into lattice-ordered linguistic truth-values, and the reasoning process is associated with the change of truth-values.

Suppose that F is the set of all *L*-type Q-Horn clauses in $\Im$, and $\boldsymbol{F}_L(F)$ represents the set of all the *L*-type fuzzy set on F. Let $A \in \boldsymbol{F}_L(F)$, A is called a non-logical fuzzy axiom set. For any logical formula $\varphi \in F$, always associated with a value $A(\varphi) \in L$. In the real-world practices, one may suppose that $A(\varphi)$ is the minimal truth-value degree of a proposition φ or possibility degree, or credibility degree (based on the application context). It is expected that during the reasoning process, every inferred formula $\psi\in LP(X)$, whose associated (truth) value should be larger than $A(\psi)$. Thus, we need to know the minimal value $A(\psi)$ of the involved formula ψ, in addition, the associated truth values may continuously improved during the deduction process.

In the following, we always assume that for any L-type Q-Horn clause φ, $A(\varphi) > 0$ or A is said to be regular, and φ is called a non-logical axiom of A.

Definition 4.2. Let D be an interpretation of the language $\mathfrak{I}$, $A\in \boldsymbol{F}_L$(F). D is called a model of A or D satisfies A, if for arbitrary $\varphi\in F$, $A(\varphi)\leq\gamma(\varphi)_D$ holds, where $\gamma(\varphi)_D$ is the truth-value of φ under the interpretation D.

Definition 4.3. Let $A\in \boldsymbol{F}_L(F)$, $\varphi\in F$, $\alpha\in L$. φ is said to be α-true in A, denoted as $A \|_{\alpha} = \varphi$ if $|\alpha = \wedge\{\gamma(\varphi)_D;\ D \text{ is an interpretation satisfying } A\}$.

Set $Con(A)(\varphi) = \alpha = \wedge\{\gamma(\varphi)_D;\ D \text{ is an interpretation satisfying } A\}$

Now we consider the syntax in the following part. For arbitrary p, $q\in F$, set $p\otimes q =: (p\to q')'$.

Theorem 4.1. For any $p, q, r, \alpha\in L$, $m, n\in N$, the following statements hold:

(1) $\|\!\!= p \to \mathrm{I}$; (2) $\| = p \to p$; (3) $\| = p\wedge q \to p$; (4) $\| = p\wedge q\to q$

(5) $\| = (p')' \to p$; (6) $\| = \forall xp \to p$

Definition 4.4. Axioms in lattice-valued first-order Q-Horn clause logic system are L-type fuzzy set A_L ($A_L\in \boldsymbol{F}_L(F)$) in the following forms:

$$A_L(\varphi) = \begin{cases} \mathrm{I}, & \varphi \text{ is a formula in the form of } (1) \sim (6) \text{ in Theorem 4.1;} \\ \alpha, & \varphi = \alpha, \alpha\in L; \\ \mathrm{O}, & \text{Otherwise.} \end{cases}$$

Definition 4.5. Let $A\in \boldsymbol{F}_L$(F), $\varphi\in F$. A formal proof ω of φ from A is a finite sequence in the following form: $(\varphi_1,\alpha_1),\cdots,(\varphi_n,\alpha_n)$, where $\varphi_n=\varphi$. For any i, $1\leq i\leq n$, $(\varphi_i,\alpha_i)\in F\times L$, and

(i) $A_L(\varphi_i)=\alpha_i$ or (ii) $A(\varphi_i)=\alpha_i$ or (iii) There exists j, $k<i$, $\varphi_j = (\forall x_1)\cdots(\forall x_m)(p)$, $\varphi_k = (\forall x_1)\cdots(\forall x_m)(p\to q)$, $\varphi_i = (\forall x_1)\cdots(\forall x_m)(q)$, and $\alpha_i = [\alpha_j \otimes A[(\forall x_1)\cdots(\forall x_m)(p\to q)]]\vee A[(\forall x_1)\cdots(\forall x_m)(q)]$.

(iv) There exist $k<i$, such that $\varphi_k = (\forall x_1)\cdots(\forall x_k)(q)$ and there exists a term $t_{i_1},\cdots,t_{i_r}$ ($i_1,\cdots,i_r\in\{1,\ldots,k\}$) without free variables, where φ_k is obtained from φ_i by replacing the variables x_{i_j} ($j=1,\cdots,r$) in q as t_{i_j}, the qualifiers only bound to the remaining variables. In addition, $\alpha_i = A[(\forall x_1)\cdots(\forall x_k)(q)]\vee A(\varphi_i)$.

Here n is called the length of the proof ω, denoted as $l(\omega)$; α_n is called the value of the proof ω, denoted as $val(\omega)$.

Definition 4.6. Let $A \in \boldsymbol{F}_L(F)$, $\varphi \in F$, $\alpha \in L$. φ is called an α-theorem of A, denoted as $A \Vdash_\alpha \varphi$, if $\alpha = \vee\{val(\omega);\ \omega \text{ is a proof of } \varphi \text{ from A}\}$.

Denote $Ded(A): F \to L$ as $Ded(A)(\varphi) = \vee\{val(\omega);\ \omega \text{ is a proof of } \varphi \text{ from A}\}$.

More generally, since it is reasonable that the uncertain premises will infer the uncertain conclusion, so starting from L-type fuzzy premise set $A \in \boldsymbol{F}_L(F)$, the inferred output would be also L-type fuzzy set A, $B \in F_L(F)$. We can generalize Definition 3.6 into the following form:

Definition 4.7. Let A, $B \in \boldsymbol{F}_L(F)$ be regular. B is said to be syntactically inferred from A, if there exists a finite sequence of the form $A_0, \ldots, A_n \in \boldsymbol{F}_L(F)$, where $B \le A_n$, and for each $k \in \{0, \ldots, n\text{-}1\}$, A_{k+1} can be obtained from A_k as the following steps:

(1) There exists a Q-Horn clause $(\forall x_1)\cdots(\forall x_m)(p \to q)$ in F such that

(i) if $q_0 \in F$, where the free variables are $x_1, \ldots, x_r$ and $q_0 \ne q$, then

$$A[\,(\forall x_1)\cdots(\forall x_m)(q_0)\,] = A_{k+1}[\,(\forall x_1)\cdots(\forall x_r)(q_0)\,];$$

or (ii) $A_{k+1}[(\forall x_1)\cdots(\forall x_m)(q)]$

$$= [A_k[(\forall x_1)\cdots(\forall x_m)(p)] \otimes A[(\forall x_1)\cdots(\forall x_m)(p \to q)]] \vee A[(\forall x_1)\cdots(\forall x_m)(q)].$$

(2) There exists a Q-Horn clause $\varphi = (\forall x_1) \cdots (\forall x_k)\ (q)$ and a term $t_{i_1}, \cdots, t_{i_r}$ ($i_1, \cdots, i_r \in \{1, \ldots, k\}$) without free variables, such that φ is transformed into φ^* by replacing the variables x_{i_j} $(j = 1, \cdots, r)$ in q as t_{i_j}, where the remaining variables are bounded by the qualifiers, and $A_{k+1}(\varphi^*) = A[(\forall x_1)\cdots(\forall x_k)(q)] \vee A(\varphi^*)$.

The mapping $Ded(A): F \to L$ is given as follows:

$$Ded(A)(\varphi) = \vee\{B(\varphi);\ B \text{ is syntactically inferred from } A\},$$

where φ is the Q-Horn clause in the form of (2) in Definition 4.1.

For an universal qualifier, if $q \in F$, x is an individual variable, then $\gamma(\forall xq)_D = \wedge_{d \in D} \gamma(q)_{D(x/d)}$, where $D(x/d)$ is an interpretation from the interpretation D by replaced x as d.

Lemma 4.1. Let p, $q \in F$, x does not appear in p as a free variable. Then

$$\forall x(p \to q) = p \to \forall xq.$$

Corollary 4.1. Let p, $q \in F$, both x and y does not appear in p as a free variable. Then $\forall x \forall y(p \to q) = p \to \forall x \forall yq$.

Corollary 4.2. Let p, $q \in F$, $x_1, \ldots, x_k$ do not appear in p as the free variables. Then $(\forall x_1)\ldots(\forall x_k)(p \to q) = p \to (\forall x_1)\ldots(\forall x_k)(q)$.

Lemma 4.2. Let p, $q \in F$, x does not appear in p as a free variable. Then

$$\gamma(\forall xq)_D \ge \gamma(\forall xp)_D \otimes \gamma(\forall x(p \to q))_D$$

holds for any interpretation D.

Corollary 4.3. Let $p, q \in F$, $x_1, \ldots, x_k$ do not appear in p as the free variables. Then $\chi((\forall x_1)\ldots(\forall x_k)(q))_D \geq \chi((\forall x_1)\ldots(\forall x_k)(p))_D \otimes \chi((\forall x_1)\ldots(\forall x_k)(p \to q))_D$
holds for any interpretation D.

Theorem 4.2. (Soundness) Let $A \in \boldsymbol{F}_L(F)$ be regular. Then

$$Ded(A)(\varphi) \leq Con(A)(\varphi)$$

holds for any lattice-valued Q-Horn clause φ in the form of (2) in Definition 4.1.

Theorem 4.3. (**Completeness**) Let $A \in \boldsymbol{F}_L(F)$ be regular. Then

$$Ded(A)(\varphi) \geq Con(A)(\varphi)$$

holds for any lattice-valued Q-Horn clause φ in the form of (2) in Definition 4.1.

It follows from Theorems 4.2 and 4.3 that if $A \in \boldsymbol{F}_L(F)$ is regular, then $Con(A)=Ded(A)$. That means the established linguistic-valued Q-Horn clause logic is sound and complete, which will provide a support and theoretical foundation for further establishing lattice-ordered linguistic-valued Prolog language.

4 Conclusions

Based on the key idea of the symbolic approach acts by direct reasoning on linguistic truth values, we characterized the set of linguistic values by a lattice-valued algebraic structure (lattice implication algebra) and investigated the corresponding logic systems with linguistic truth values, and furthermore, investigated the automated reasoning scheme based on linguistic truth-valued logic system, while the resolution method as well as its theorems of soundness and completeness were given on proving the satisfiability of logical formulae with respect to a certain linguistic truth-value level, and a linguistic-valued quasi-Horn clause logic were investigated with its soundness and completeness theorem being given.

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