Extended Belief Rule Base Inference Methodology

Jun Liu[1], Luis Martinez[2], Ying-Ming Wang[3]

1.School of Computing and Mathematics, Univ. of Ulster at Jordanstown Northern Ireland, UK
2.School of Computing, Dept. of Computer Science, University of Jaén, 23071 Jáen, Spain
3.School of Public Administration, Fuzhou University Fuzhou, China
j.liu@ulster.ac.uk;martin@ujaen.es;msymwang@hotmail.com

Abstract

A belief Rule-base Inference Methodology using the Evidential Reasoning approach (RIMER) has been developed recently, which is an extension of traditional rule based systems and is capable of representing more complicated causal relationships using different types of information with uncertainties. A rule-base in RIMER is designed with belief degrees embedded in all possible consequents of a rule, where it is assumed that all the consequents are independent of each other in order to accommodate the assumption imposed on the evidential reasoning (ER) algorithm being used. To overcome this limitation, in the paper, we extend the RIMER approach to the case of fuzzy consequents, that is, each consequent can be defined as a fuzzy linguistic term because of vagueness and inexactness. In such cases, the intersection of adjacent two fuzzy sets is no longer an empty set, which results in the above ER algorithm inapplicable during the inference process, instead, the inference of the belief rule-based system is implemented using an extended fuzzy ER algorithm. This work extends the applicality and feasibility of the RIMER approach.

1 Introduction

A new methodology has been proposed recently [4] for modeling a hybrid rule-base using a belief structure and for inference in the belief rule-based system using the evidential reasoning (ER) approach [6, 8, 9]. The methodology is referred to as a belief Rule-base Inference Methodology using the Evidential Reasoning approach – *RIMER*. In the *RIMER* approach, a rule-base designed on the basis of the belief structure, called *belief rule-base*, is used to capture nonlinear causal relationships as well as uncertainty. The inference of a rule-based system is implemented using the ER approach. *RIMER* has been applied to the safety analysis of offshore systems [1, 2] and is applicable to a wide range of areas such as risk and safety analysis, quality assessment, and fault diagnosis. The optimization model of RIMER has also been introduced in [5].

In an established belief rule-base, input for each antecedent is transformed into a distribution on the referential values of this antecedent. This distribution describes the degree of each antecedent being activated. Moreover, the antecedents of an *IF-THEN* rule form an overall attribute, called *a packet antecedent attribute*. The activation weight of a rule can be generated by aggregating the degrees to which all antecedents in the rule are activated. In this context, an *IF-THEN* rule can be considered as an evaluation problem of a packet antecedent attribute being assessed to an output term in the consequent of the rule with certain degrees of belief.

In the *RIMER* approach does not take account of vagueness or fuzzy uncertainty of the consequent assessment grade in the IF-THEN rule, where it is assumed that each consequent assessment grade as independent crisp terms, which limit the applicality of the approach. It would be more natural to assume that the consequent assessment grades can be dependent, which may overlap in their meanings. For example, the assessment grades "*low*" and "*very low*" are difficult to be expressed as clearly distinctive crisp sets, but quite natural to be defined as two dependent fuzzy sets. In other words, the intersection of the two fuzzy sets may not be empty.

In addition, the rules in the belief rule base may come in different ways, such as extracted from experts or by examining historical data, or self-learned from training data. Especially considering the rule-based learnt from data, the input and output are both numerical case. Many fuzzy systems that automatically

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generate fuzzy rules from numerical data have been proposed, e.g., some fuzzy learning algorithms to construct the membership functions of the input variables and the output variables of fuzzy rules and to induce the fuzzy rules from the numerical training data set. In such case, the output variables in the IF-THEN rules are dependent instead of being independent.

In the paper, we extend the *RIMER* approach to the case of fuzzy consequents, that is, each consequent can be defined as a fuzzy linguistic term because of vagueness and inexactness. In such cases, the intersection of adjacent two fuzzy sets is no longer an empty set, so the inference of the belief rule-based system is implemented using an extended fuzzy ER algorithm. This work extends the applicality and feasibility of the RIMER approach.

The rest of this paper is organized as follows. RIMER approach is briefly reviewed in Section 2. The extended RIMER approach to deal with fuzzy consequents is proposed in Section 3. Conclusions are drawn in Section 4.

2 RIMER

The *RIMER* approach is summarized in this section and more details can be found in the references [4]. To take into account belief degrees, attribute weights and rule weights in a rule, suppose a belief rule-base is given by R={R_1, R_2,…, R_L} with the k^{th} rule represented as follows:

R_k: *IF U* is A^k *THEN D* with belief degrees β^k, with a rule weight θ_k and attribute weights δ_{k1}, δ_{k2},…, δ_{kT_k} (1)

where U represents the antecedent attribute vector (U_1,…, U_{T_k}), A^k the packet antecedents { A_1^k ,…, $A_{T_k}^k$ }, and A_i^k (i=1,…, T_k) the referential value of the i^{th} antecedent attribute in the k^{th} rule; T_k is the number of antecedent attributes used in the k^{th} rule. Suppose T is the total number of antecedent attributes used in the rule base, D the consequent vector (D_1,…, D_N), and β^k the vector of the belief degrees (β_{1k} ,…, β_{Nk}) for $k\in\{1,\ldots, L\}$, and β_{ik} ($i\in\{1,\ldots, N\}$ the belief degree to which D_i is believed to be the consequent if in the k^{th} packet rule the input satisfies the packet antecedents A^k. θ_k is the relative weight of the k^{th} rule and δ_{k1},…, δ_{kTk} are the relative weights of the T_k antecedent attributes used in the k^{th} rule. L is the number of all the packet rules in the rule-base. If $\sum_{i=1}^{N}\beta_{ik}=1$, the k^{th} packet rule is said to be complete; otherwise, it is incomplete. Rule (1) is referred to as *a belief rule*. A belief rule-base established using belief rules can be summarized using a belief rule expression matrix shown in Table 1:

Table 1: Belief rule expression matrix for a rule-base

Belief Output	Input					
	A^1 (w_1)	A^2 (w_2)	…	A^k (w_k)	…	A^L (w_L)
D_1	β_{11}	β_{12}	…	β_{1k}	…	β_{1L}
⋮	⋮	⋮	…	⋮	…	⋮
D_i	β_{i1}	β_{i2}	…	β_{ik}	…	β_{iL}
⋮	⋮	⋮	…	⋮	…	⋮
D_N	β_{N1}	β_{N2}	…	β_{Nk}	…	β_{NL}

In the matrix, w_k is the activation weight of A^k, which measures the degree to which the k^{th} rule is weighted and activated. w_k is calculated by:

$$w_k=\frac{\theta_k*\prod_{i=1}^{T_k}\left(\alpha_i^k\right)^{\bar{\delta}_i}}{\sum_{i=1}^{L}[\theta_i*\prod_{l=1}^{T_k}\left(\alpha_l^i\right)^{\bar{\delta}_i}]},\ \text{with}\ \bar{\delta}_i=\frac{\delta_i}{\max_{i=1,\ldots,T_k}\{\delta_i\}} \quad (2)$$

where it is assumed that $\theta_k\in[0,\ 1]$ (k=1,…, L) and $\delta_i\in[0,\ 1]$ (i=1,…, T_k). α_i^k (i=1,…, T_k), called the *individual matching degree,* is the degree of belief to which the input for U_i belongs to A_i^k of the i^{th} individual antecedent in the k^{th} rule, and $\alpha_i^k\in[0,\ 1]$. α_i^k could be generated using various ways depending on the nature of an antecedent attribute and the available data [4]. In this paper we assume that α_i^k (i=1,…, T_k) are already given.

Based on the above belief rule expression matrix, we can use the ER approach to combine rules and generate final conclusions.

Having represented each rule using Equation (1), the ER approach can be directly applied as follows. First, transform the degrees of belief β_{jk} for all j=1,…, N, k=1,…, L into basic probability masses using the following recursive evidential reasoning (ER) algorithm [8, 9]:

$$m_{j,k}=w_k\beta_{j,k}\ ,\ j=1,\Lambda\ ,N\ ;$$

$$m_{D,k}=1-\sum_{j=1}^{N}m_{j,k}=1-w_k\sum_{j=1}^{N}\beta_{j,k}\ ,$$

$$\bar{m}_{D,k}=1-w_k\ ,\ \text{and}\ \tilde{m}_{D,k}=w_k(1-\sum_{j=1}^{N}\beta_{j,k})$$

For all $k=1,\Lambda\ ,L$, with $m_{D,k}=\bar{m}_{D,k}+\tilde{m}_{D,k}$ for all $k=1,\Lambda\ ,L$ and $\sum_j^L w_j=1$. The probability mass assign ed to the consequent set D, which is unassigned to any individual consequent, is split into two parts, one caused by the relative importance of the k^{th} packet antecedent A^k (or $\bar{m}_{D,k}$) and the other by the incompleteness of the k^{th} packet antecedent A^k (or $\tilde{m}_{D,k}$).

Then, aggregate all the packet antecedents of the L rules to generate the combined degree of belief in each possible consequent D_j in D. Suppose $m_{j,I(k)}$ is the combined degree of belief in D_j by aggregating the first k packet antecedents ($A^1,\ldots, A^k$) and $m_{D,I(k)}$ is the remaining degree of belief unassigned to any consequent. Let $m_{j,I(1)} = m_{j,1}$ and $m_{D,I(1)} = m_{D,1}$. Then the overall combined degree of belief β_j in D_j is generated as follows:

$$\{D_j\}: m_{j,I(k+1)} = K_{I(k+1)}[m_{j,I(k)}m_{j,k+1} + m_{j,I(k)}m_{D,k+1} + m_{D,I(k)}m_{j,k+1}]$$

$$m_{D,I(k)} = \bar{m}_{D,I(k)} + \tilde{m}_{D,I(k)}, \; k = 1,\Lambda ,L$$

$$\{D\}: \tilde{m}_{D,I(k+1)} = K_{I(k+1)}[\tilde{m}_{D,I(k)}\tilde{m}_{D,k+1} + \tilde{m}_{D,I(k)}\bar{m}_{D,k+1} + \bar{m}_{D,I(k)}\tilde{m}_{D,k+1}]$$

$$\{D\}: \; \bar{m}_{D,I(k+1)} = K_{I(k+1)}[\bar{m}_{D,I(k)}\bar{m}_{D,k+1}]$$

$$K_{I(k+1)} = \left[1 - \sum_{j=1}^{N}\sum_{\substack{t=1\\ t\neq j}}^{N} m_{j,I(k)}m_{t,k+1}\right]^{-1}, \; k = 1,\Lambda ,L-1$$

$$\{D_j\}: \; \beta_j = \frac{m_{j,I(L)}}{1-\bar{m}_{D,I(L)}}, \; j = 1,\Lambda ,N$$

$$\{D\}: \; \beta_D = \frac{\tilde{m}_{D,I(L)}}{1-\bar{m}_{D,I(L)}}$$

β_D represents the remaining belief degrees unassigned to any D_j. It has been proved that $\sum_{j=1}^{N}\beta_j + \beta_D = 1$ [8, 9]. The final conclusion generated by aggregating the L rules, which are activated by the actual input vector $A^*=\{A^{*k}, k=1,\ldots, L\}$ can be represented as follows

$$S(A^*)=\{(D_j, \; \beta_j), j=1,\ldots, N\} \qquad (3)$$

Here we suppose that all the L rules are independent of each other, which means that the packet antecedent $A^1,\ldots, A^L$ are independent of each other. Then, aggregate all the packet antecedents of the L rules to generate the combined degree of belief in each possible consequent D_j in D. An overall analytical ER algorithm was also given in [3].

Take for example the following belief rule in safety analysis:

R_k: *IF* the ***failure rate*** is <u>*frequent*</u> and the ***consequence severity*** is <u>*critical*</u> and the ***failure consequence probability*** is <u>*unlikely*</u> *THEN* the ***safety estimate*** is {(*Good*, 0), (*Average*, 0), (*Fair*, 0.7), (*Poor*, 0.3)}

where {(*Good*, 0), (*Average*, 0), (*Fair*, 0.7), (*Poor*, 0.3)} is a belief distribution representation for safety consequent, representing that we are 70% sure that safety level is *Fair*, and 30% sure that safety level is *Poor*. In this belief rule, s***afety estimate*** is the only output fuzzy variable used to produce safety evaluation for a particular cause to technical failure. This variable is described linguistically, which is described and determined by the above three parameters. In safety assessment, it is common to express a safety level by degrees to which it belongs to such linguistic variables as "*Poor*", "*Fair*", "*Average*", and "*Good*" that are referred to as safety expressions, so the safety estimate can be regarded as the safety classification in which "*Poor*", "*Fair*", "*Average*", and "*Good*" are qualitatively regarded as independent.

3 Extended RIMER

In the RIMER approach reviewed above, the consequent assessment grades are assumed to be crisp and independent of each other. In many situations, however, an assessment grade may represent a vague or fuzzy concept or standard and there may be no clear cut between the meanings of two adjacent grades. In this section, we will drop the above assumption and allow the grades to be fuzzy and dependent. To simplify the discussion and without loss of generality, fuzzy sets will be used to characterise such assessment grades and it is assumed that only two adjacent fuzzy grades have the overlap of meanings. This represents the most common features of fuzzy uncertainty.

Suppose a general set of fuzzy consequent grades $\{D_j\}(j=1,\ldots, N)$ are dependent on each other, which may be either triangular or trapezoidal fuzzy sets or their combinations. Assuming that only two adjacent fuzzy consequent grades may intersect, we denote by $D_{j,j+1}$ the fuzzy intersection subset of the two adjacent fuzzy consequent grades D_j and D_{j+1} (see Fig. 1).

Since fuzzy consequent grades and belief degrees are used, then rule defined in (1) contains both fuzzy sets (grades) and belief degrees. The former can model fuzziness or vagueness and the latter incompleteness or ignorance.

In the derivation of Equations (3), it was assumed that the consequent evaluation grades are independent of each other. Due to the dependency of the adjacent fuzzy consequent grades on each other as shown in Fig. 1, the ER algorithm used in RIMER can no longer be

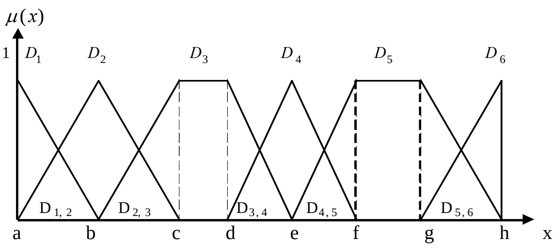

Fig.1. The mutual relationships between fuzzy consequent grades

employed without modification to aggregate rules assessed using such fuzzy grades. However, the evidence theory provides scope to deal such fuzzy assessments. Based on the ideas similar to those used to develop the non-fuzzy evidential reasoning algorithm [8, 9], a fuzzy evidential reasoning algorithm has been proposed in [7]. The new challenge is that the intersection of two adjacent evaluation grades D_j and D_{j+1} is $D_{j,j+1}$, which is not empty as shown in Fig. 1. Another difference is that the normalisation has to be conducted after all pieces of evidence have been combined in order to preserve the property that the generated belief and plausibility functions still represent the lower and upper bounds of the combined degrees of belief.

Following the assumptions on the fuzzy assessment grades made in the previous subsection, based on the belief decision matrix as shown in Equation (1), it is proven in [7] that the following analytical (non-recursive) fuzzy ER algorithm can be used to aggregate the L rules generate the combined degree of belief in each possible consequent D_j in D:

$$\{D_j\}:\ m(D_j)=k\left\{\prod_{k=1}^{L}(m_{j,k}+m_{D,k})-\prod_{k=1}^{L}m_{D,k}\right\}; \qquad (4)$$

$$\{\overline{D}_{j,j+1}\}:$$

$$m(\overline{D}_{j,j+1})=k\mu_{D_{j,j+1}}^{\max}\left\{\prod_{k=1}^{L}(m_{j,k}+m_{j+1,k}+m_{D,k})\right.$$
$$\left.-\prod_{k=1}^{L}(m_{j,k}+m_{D,k})-\prod_{k=1}^{L}(m_{j+1,k}+m_{D,k})+\prod_{k=1}^{L}m_{D,k}\right\}, \quad (5)$$

$$\{D\}:\ \tilde{m}(D)=k\left\{\prod_{k=1}^{L}m_{D,k}-\prod_{k=1}^{L}\overline{m}_{D,k}\right\} \qquad (6)$$

$$\{D\}:\ \overline{m}(D)=k\left[\prod_{k=1}^{L}\overline{m}_{D,k}\right] \qquad (7)$$

$$k=\left\{\sum_{j=1}^{N-1}(1-\mu_{H_{j,j+1}}^{\max})\left(\prod_{k=1}^{L}(m_{j,k}+m_{D,k})-\prod_{k=1}^{L}m_{D,k}\right)\right.$$
$$+\sum_{j=1}^{N-1}\mu_{H_{j,j+1}}^{\max}\left(\prod_{k=1}^{L}(m_{j,k}+m_{j+1,k}+m_{D,k})-\prod_{k=1}^{L}(m_{j+1,k}+m_{D,k})\right)$$
$$\left.+\prod_{k=1}^{L}(m_{N,k}+m_{D,k})\right\}^{-1}$$

$$\{D_j\}:\qquad \beta_n=\frac{m(D_j)}{1-\overline{m}(D)},\ j=1,\ldots,N \qquad (8)$$

$$\{\overline{D}_{j,j+1}\}:\ \beta_{j,j+1}=\frac{m(\overline{D}_{j,j+1})}{1-\overline{m}(D)},\ j=1,\ldots,N-1 \qquad (9)$$

$$\{D\}:\qquad \beta_D=\frac{\tilde{m}(D)}{1-\overline{m}(D)} \qquad (10)$$

where $\overline{D}_{j,j+1}$ is a normalized fuzzy subset for the fuzzy intersection subset $D_{j,j+1}$ whose maximum degree of membership is represented by $\mu_{D_{j,j+1}}^{\max}$ and is usually less than one.

$D_{j,j+1}$ is normalised to $\overline{D}_{j,j+1}$ as shown in Equations (5) so that $D_{j,j+1}$ can be measured as a formal fuzzy set with the maximum membership degree being one, therefore assessed in the same scale as the other defined fuzzy evaluation grades such as D_j (see Fig. 2). The normalisation of $D_{j,j+1}$ seems logical because the probability mass $m(\overline{D}_{j,j+1})$ assigned to the fuzzy intersection subset is directly related to the height of $\overline{D}_{j,j+1}$. In other words, how D_j and D_{j+1} are interrelated is thus taken into account in the calculation of the belief assigned to their intersection. Without the normalisation, $m(\overline{D}_{j,j+1})$ would remain constant as long as D_j and D_{j+1} intersect, however small or large the intersection might be.

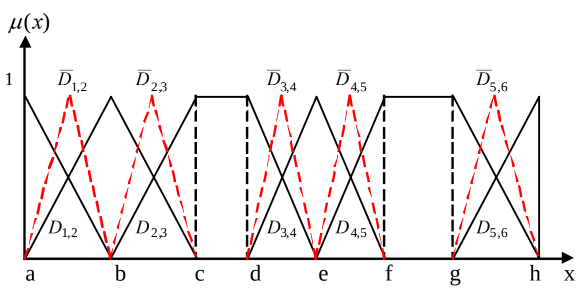

Fig. 2. Normalized fuzzy intersection subsets

Since $\overline{D}_{j,j+1}$ (or $D_{j,j+1}$) is not an originally defined fuzzy evaluation grade, however, its degree of belief (or $\beta_{j,j+1}$) should eventually be reassigned to

D_j and D_{j+1}. The question is how to find the relations between the two different sets of fuzzy assessment grades D_j (and D_{j+1}) and $\overline{D}_{j,j+1}$, so that $\overline{D}_{j,j+1}$ can be equivalently represented by D_j (and D_{j+1}) in some sense. The detailed assignment approach has been discussed and proposed in [7], and is summarized below:

Fig. 3 shows the typical relative position relations between one basic fuzzy assessment grade $\overline{D}_{j,j+1}$ and two general assessment grades D_j and D_{j+1}. From Fig. 6 one can see that $\overline{D}_{j,j+1}$ lies completely between D_j and D_{j+1}, and has no intersection with any other general fuzzy assessment grades. Therefore, it is sufficient to use only D_j and D_{j+1} to represent $\overline{D}_{j,j+1}$. Suppose $\overline{D}_{j,j+1}$ intersects D_j with an area of $(S_j + S_{j,j+1})$ and D_{j+1} with an area of $(S_{j,j+1} + S_{j+1})$, where $S_{j,j+1}$ is the common area of $\overline{D}_{j,j+1}$ intersecting both D_j and D_{j+1}. The minimum distance between the peaks of $\overline{D}_{j,j+1}$ and D_j is denoted by d_j and that between the peaks of $\overline{D}_{j,j+1}$ and D_{j+1} by d_{j+1}.

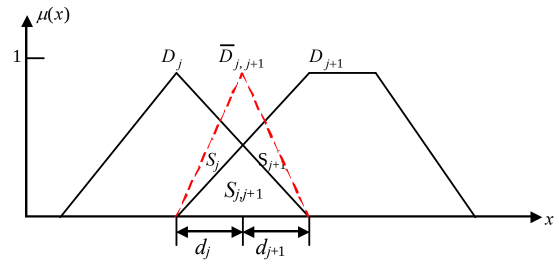

Fig.6. The reassignment of the belief degree of fuzzy intersection subset

It seems logical that the belief degree of $\overline{D}_{j,j+1}$ belonging to an assessment grade, say D_j, is related to S_j, S_{j+1}, $S_{j,j+1}$, d_j and d_{j+1}. Intuitively, a large S_j and a small S_{j+1} together with a small d_j and a large d_{j+1} should imply a high degree of belief to which $\overline{D}_{j,j+1}$ belongs to D_j.

We can solve the problem of reassigning the belief degree of the fuzzy intersection subset $\overline{D}_{j,j+1}$. Since $\overline{D}_{j,j+1}$ intersects D_j and D_{j+1} only (see Fig. 6), the degree of belief assigned to $\overline{D}_{j,j+1}$ should only be reassigned to the two fuzzy grades D_j and D_{j+1}. From Fig. 6, the allocation of $S_{j,j+1}$ should be related to the distances d_j and d_{j+1} as well as the areas S_j and S_{j+1}. In order to model the allocation, the following two allocation factors AF_j and AF_{j+1} are introduced (see Fig. 6):

$$AF_j = \frac{1}{2}\left[(1 - \frac{d_j}{d_j + d_{j+1}}) + \frac{S_j}{S_j + S_{j+1}}\right] \qquad (11)$$

$$AF_{j+1} = \frac{1}{2}\left[(1 - \frac{d_{j+1}}{d_j + d_{j+1}}) + \frac{S_{j+1}}{S_n + S_{j+1}}\right] \qquad (12)$$

It is obvious that $AF_j + AF_{j+1} = 1$. Also, if d_j and S_{j+1} are both zero, then $AF_{jn} = 1$ and $AF_{j+1} = 0$; if d_{j+1} and S_j are both zero, then $AF_j = 0$ and $AF_{j+1} = 1$. The allocation factors are used to assign the belief degrees to which $\overline{D}_{j,j+1}$ is allocated to D_j and D_{j+1} as follows:

$$Bel(\overline{D}_{j,j+1} \subset D_j) = \frac{S_j + AF_j \cdot S_{j,j+1}}{S_j + S_{j,j+1} + S_{j+1}} \qquad (13)$$

$$Bel(\overline{D}_{j,j+1} \subset D_{j+1}) = \frac{S_{j+1} + AF_{j+1} \cdot S_{j,j+1}}{S_j + S_{j,j+1} + S_{j+1}} \qquad (14)$$

Thus, the belief degree $\beta_{j,j+1}$ can be divided into two parts:

$\beta_{j,j+1} Bel(\overline{D}_{j,j+1} \subset D_j)$, and

$\beta_{j,j+1} Bel(\overline{D}_{j,j+1} \subset D_{j+1})$.

The former should be added to β_j and the latter to β_{j+1}. Therefore, the final belief degree that supports the fuzzy assessment grade D_j should include three parts:

$$\beta_j + \beta_{j,j+1} Bel(\overline{D}_{j,j+1} \subset D_{j+1}) + \beta_{j,j+1} Bel(\overline{D}_{j,j+1} \subset D_j)$$

for $j = 2, \mathrm{K}, N-1$. The belief degree for D_1 is given by

$\beta_1 + \beta_{1,2} Bel(\overline{D}_{1,2} \subset D_1)$ and

$\beta_N + \beta_{N-1,N} Bel(\overline{D}_{N-1,N} \subset D_N)$ for D_N.

The belief degree which supports the whole set $D = \{D_1, \mathrm{K}, D_N\}$ is still β_D. For convenience, we denote the above final belief degrees by $\beta_{1F}, \beta_{2F}, \ldots, \beta_{NF}$ and β_D . Therefore, the final conclusion generated by aggregating the L rules, which are activated by the actual input vector $A^*=\{A^{*k}$, $k=1,\ldots, L\}$ can be represented as follows as

$$S(A^*) = \{(D_j, \beta_{jF}), j = 1, \ldots, N\}.$$

Different from the ER algorithm in RIMER that is of a recursive nature, the new fuzzy ER algorithm provides an analytical means for combining all attributes in one go without iteration, thus providing scope and flexibility for sensitivity analysis and optimisation.

6 Conclusion

In the RIMER approach in [4] does not take account of vagueness or fuzzy uncertainty of the consequent assessment grade in the IF-THEN rule, where it is assumed that each consequent assessment grade as independent crisp terms, which limit the applicality of the approach. To overcome this limitation, in the paper, we extended the RIMER approach to the case of fuzzy consequents, that is, each consequent can be defined as a fuzzy linguistic term because of vagueness and inexactness. The inference of the belief rule-based system is implemented using an extended fuzzy ER algorithm. This work extended the applicality and feasibility of the RIMER approach. The corresponding optimization models will be further investigated.

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