sion makers it is a suitable approach for evaluation processes.

The aim of this contribution is to present a sensory evaluation model that provides and manages a unbalanced linguistic evaluation framework such that the experts can express their opinions in it. To do so, we use a methodology to deal with Unbalanced Linguistic Information [10] based on the Linguistic Hierarchical and the 2-tuple representation model [11, 12] together an evaluation model based on the decision analysis [9].

This contribution is structured as follows, section 2 reviews some linguistic concepts, section 3 reviews in short a methodology to deal with unbalanced linguistic information model. Section 4 presents our proposal for the sensory evaluation model. Finally, Section 5 points out some conclusions.

2. Linguistic background

In this section, we review some linguistic concepts necessary to understand our proposal.

2.1. 2-tuple linguistic representation model

In [12] was presented a linguistic representation model based on linguistic 2-tuples that carries out processes of computing with words (CW) in a precise way when the linguistic term sets are symmetrical and uniformly distributed. This model is based on the concept of symbolic translation.

The 2-tuple fuzzy linguistic representation model represents the linguistic information by means of a 2-tuple, (s, α), where s is a linguistic label and α is a numerical value that represents the value of the symbolic translation.

Definition 1.[12] *Let β be the result of an aggregation of the indices of a set of labels assessed in a linguistic term set S, i.e., the result of a symbolic aggregation operation. $\beta \in [0, g]$, being $g + 1$ the cardinality of S. Let $i = round(\beta)$ and $\alpha = \beta - i$ be two values, such that, $i \in [0, g]$ and $\alpha \in [-.5, .5)$ then α is called a Symbolic Translation.*

This linguistic representation model defines a set of functions to make transformations between linguistic 2-tuples and numerical values:

Definition 2.[12] *Let $S = \{s_0, ..., s_g\}$ be a linguistic term set and $\beta \in [0, g]$ a value supporting the result of a symbolic aggregation operation, then the 2-tuple that expresses the equivalent information to β is obtained with the following function:*

$$\Delta : [0, g] \longrightarrow S \times [-0.5, 0.5)$$

$$\Delta(\beta) = \begin{cases} s_i & i = round(\beta) \\ \alpha = \beta - i & \alpha \in [-.5, .5) \end{cases}$$

where round is the usual rounding *operation, s_i has the closest index label to "β" and "α" is the value of the symbolic translation.*

Proposition 1.[12] *Let $S = \{s_0, ..., s_g\}$ be a linguistic term set and (s_i, α) be a 2-tuple. There is always a function Δ^{-1}, such that, from a 2-tuple it returns its equivalent numerical value $\beta \in [0, g] \subset \mathcal{R}$.*

$$\Delta^{-1} : S \times [-.5, .5) \longrightarrow [0, g]$$

$$\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta.$$

Remark 1: From definitions 1 and 2 and proposition 1, it is obvious that the conversion of a linguistic term into a linguistic 2-tuple consist of adding a value 0 as symbolic translation:

$$s_i \in S \Longrightarrow (s_i, 0)$$

The 2-tuple representation model has developed a computational model presented in [12]

2.2. Hierarchical linguistic context

The hierarchical linguistic structure was used in [11] to improve the precision of the processes of CW in linguistic multi-granular contexts. It will be used in this contribution to manage the unbalanced linguistic term sets.

A *linguistic hierarchy* is a set of levels, where each level is a linguistic term set with different granularity from the remaining of levels of the hierarchy. Each level belonging to a linguistic hierarchy is denoted as $\mathbf{l}(\mathbf{t}, \mathbf{n}(\mathbf{t}))$, being:

1. *t*, indicates the level of the hierarchy,
2. *n(t)*, the granularity of the linguistic term set of the level *t*.

We assume levels containing linguistic terms whose membership functions are triangular-shaped, symmetrical and uniformly distributed in $[0, 1]$. In addition, the linguistic term sets have an odd number of elements.

The levels belonging to a linguistic hierarchy are ordered according to their granularity, i.e., for any two consecutive levels t and $t+1$, $n(t+1) > n(t)$. This provides a linguistic refinement of the previous level.

From the above concepts, we define a linguistic hierarchy, LH, as the union of all levels t:

$$LH = \bigcup_t l(t, n(t))$$

Given a LH, $S^{n(t)}$ denotes the linguistic term set of LH corresponding to the level t of LH with a granularity of uncertainty of $n(t)$:

$$S^{n(t)} = \{s_0^{n(t)}, ..., s_{n(t)-1}^{n(t)}\}$$

Generally, we can say that the linguistic term set of level $t+1$, $S^{n(t+1)}$, is obtained from its predecessor, $S^{n(t)}$, as:

$$l(t, n(t)) \rightarrow l(t+1, 2 \cdot n(t) - 1)$$

A graphical example of a linguistic hierarchy is shown in Fig. 2:

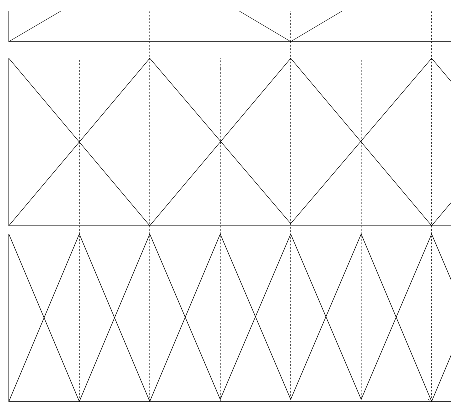

Fig. 2: Linguistic hierarchy of 3,5 and 9 labels.

In [11] was defined a transformation function between labels from different levels to carry out processes of CW in multi-granular linguistic information contexts without loss of information, it has been defined as follows:

$$TF_{t'}^{t} : l(t, n(t)) \longrightarrow l(t', n(t'))$$

$$TF_{t'}^{t}(s_i^{n(t)}, \alpha^{n(t)}) = \Delta(\frac{\Delta^{-1}(s_i^{n(t)}, \alpha^{n(t)}) \cdot (n(t') - 1)}{n(t) - 1})$$

Proposition 2.[11] *The transformation function between linguistic terms in different levels of the linguistic hierarchy is bijective:*

$$TF_t^{t'}(TF_{t'}^{t}(s_i^{n(t)}, \alpha^{n(t)})) = (s_i^{n(t)}, \alpha^{n(t)})$$

This result guarantees the transformations between levels of a linguistic hierarchy are carried out without loss of information.

3. Unbalanced linguistic information

Most of problems modeling information with linguistic assessments use linguistic variables assessed in linguistic term sets whose terms are uniform and symmetrically distributed [13]. However, there exist problems in which it is more suitable to assess their assessments by means of linguistic term sets that are not uniform and symmetrically distributed. We call this type of term sets as, *unbalanced linguistic term sets*. In some cases, the unbalanced linguistic information appears either due to the nature of the linguistic variables that participate in the problem, or in problems dealing with scales in which it is necessary to assess preferences with a greater granularity on a side of the scale than on the other one, as for example it happens in the sensory scale, Fig. 1.

In [10] was developed a methodology to obtain a semantic representation algorithm for unbalanced linguistic term sets that provides semantics to the linguistic terms belong to a unbalanced linguistic term set. This methodology acts in two different aims.

First, it defines an algorithm to build the semantics for an unbalanced linguistic term sets using Linguistic Hierarchies, a further and detailed description can be found in [10]. The algorithm returns a *Hierarchical semantic representation,* $LH(S)$ for an unbalanced linguistic term set $\mathcal{S} = \{s_i,\ i = 0, ..., g\}$ and obtains its representation in the Linguistic Hierarchy, LH.

The semantic obtained $LH(S) = \{s_{I(i)}^{G(i)},\ i = 0, ..., g\}$,it is such that $\forall s_i \in S\ \exists l(t, n(t)) \in LH$ that contains a label $s_k^{n(t)} \in S^{n(t)}$, in such a way that $I(i) = k$ and $G(i) = n(t)$, being I and G functions that assign to each label $s_i \in S$ the index of the label that represents it in the linguistic hierarchy and the granularity of label set of linguistic hierarchy in which it is represented, respectively.

Second, the methodology defines a computational model for unbalanced linguistic term sets

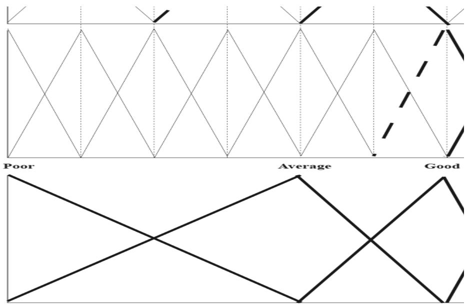

Fig. 3: Semantic representation of the sensory evaluation in LH.

based on the 2-tuple computational model. To accomplish the processes of CW without loss of information dealing with LH and 2-tuples. The algorithm proposed in [10] builds a structure as the Table 3 with information that supports the computations with unbalanced labels (see Table 3). This table reports which label of the LH represents a label $s_i \in S$ and additionally uses a boolean function noted as $Brid$ to indicate when a label is represented by means of two different labels in the LH.

S	$LH(S)$	$Brid(S)$
$s_0 = F$	$s_{I(0)}^{G(0)} = s_0^3$	False
$s_1 = D$	$s_{I(1)}^{G(1)} = s_1^3$	True
$s_2 = C$	$s_{I(2)}^{G(2)} = s_3^5$	True
$s_3 = B$	$s_{I(3)}^{G(3)} = s_7^9$	False
$s_4 = A$	$s_{I(4)}^{G(4)} = s_8^9$	False

Table 1: $LH(S)$ and $Brid(S)$.

To accomplish the CW processes were introduced, two unbalanced linguistic transformation functions that converts a unbalanced linguistic term $s_i \in S$ into the linguistic term in the LH $s_k^{n(t)} \in LH = \bigcup_t l(t, n(t))$ and vice versa such a way the 2-tuple computational model can be used.

1. LH: Transformation function that associates with each unbalanced linguistic 2-tuple (s_i, α), $s_i \in S$ its respective linguistic 2-tuple in LH $(s_k^{n(t)}, \alpha), s_k^{n(t)} \in LH$.

   $LH : (S \times [0.5, -0.5)) \rightarrow (LH \times [0.5, -0.5))$ such that $\forall (s_i, \alpha_i) \in (S \times [0.5, -0.5)) \Longrightarrow LH(s_i, \alpha_i) = (s_{I(i)}^{G(i)}, \alpha_i)$.

2. LH^{-1}: Transformation function that associates with each linguistic 2-tuple expressed in LH its respective unbalanced linguistic 2-tuple in S.

   $LH^{-1} : (LH \times [0.5, -0.5)) \rightarrow (S \times [0.5, -0.5))$, $\forall (s_k^{n(t)}, \alpha_k) \in (LH \times [0.5, -0.5)) \mid s_k^{n(t)} \in S^{n(t)}$, being t a level of LH, then is defined by cases in [10]

4. Unbalanced linguistic information sensory evaluation model

The aim of this contribution is to propose a Sensory Evaluation model based on the linguistic decision analysis [9] to deal with unbalanced linguistic information whose mathematical formalism will be the linguistic 2-tuple model in order to obtain an evaluation framework where the experts can express their preferences in unbalanced linguistic term sets with different discrimination levels on both sides of the scale. The decision analysis scheme that will use our proposal for the sensory evaluation model consists of the following phases:

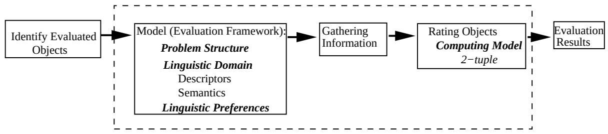

Fig. 4: Sensory Evaluation Scheme based on linguistic 2-tuple decision analysis.

- *Identify Evaluated Objects*
- *Evaluation Framework*: this phase defines the evaluation context in which the experts will express their preferences about the evaluated objects. Here it will be chosen the linguistic descriptors and their semantics.
- *Gathering Information*: the experts express their sensorial knowledge about the objects.
- *Rating Objects*: in this proposal the unbalanced linguistic computing model is used to rate the different objects. There must be chosen the aggregation operators that will be used

in the rating process.

- *Evaluation Results*: The rates obtained in the before phase will serve to analyze the different evaluated objects.

The following subsections present in further detail the main phases of the linguistic sensory evaluation model proposed.

4.1. Evaluation framework

This phase defines the evaluation framework, such that, the problem structure is defined and the linguistic descriptors and semantics that will be used by the experts to provide their information about the sensory features of the evaluated objects are chosen. In some sensory evaluation problems could be that experts need a greater level of distinction in one side of the evaluation scale than in the other one, we propose a unbalanced linguistic evaluation framework where the experts can express their opinions in it. In this paper the linguistic decision analysis is based in a Multi-Experts Decision Making (MEDM) context [14]. Therefore the evaluation framework could be as:

- $E = \{e_1, \ldots, e_n\}$ panel of experts.
- $O = \{o_1, ..., o_m\}$ set of evaluated objects.
- $F = \{f_1, ..., f_h\}$, set of evaluated sensorial features for each object.
- $S = \{s_0, ..., s_g\}$, unbalanced linguistic term set

Applying the representation algorithm of unbalanced linguistic information to represent the unbalanced labels of $S = \{s_0 : P(Poor), s_1 : A(Average), s_2 : G(Good), s_3 : VG(VeryGood), s_4 : E(Excellent)\}$. shown in Fig.1 whose semantics are obtained by the algorithm presented in [10] are those ones shown in Fig.3.

Furthermore, the algorithm provides information to control the representation of S in the CW processes (see Table 3).

4.2. Gathering information

The experts provide their sensory subjective preferences using an unbalanced linguistic term set,S, fixed in the evaluation framework by means of utility vectors that contain a linguistic assessment for each evaluated feature.

- e_i provides his/her preferences in S by means of a utility vector:

  $$U_i = \{u^i_{11}, ..., u^i_{1h}, u^i_{21}, ..., u^i_{2h}, ..., u^i_{m1}, ..., u^i_{mh}\}$$

  where $u^i_{jk} \in S$ is the assessment provided to the feature f_k of the object o_j by the expert e_i.

Due to the fact, that the evaluation model will use the unbalanced linguistic computing model the linguistic preferences provided by the experts will be transformed into linguistic 2-tuples according to the *Remark 1.*

4.3. Rating objects

Given that the semantics of the unbalanced term set belong to different levels of LH we cannot operate directly with them. So we will unify the semantics of these labels in a level of the LH, called *basic representation level* (t_{HGLS}) which will support the computation processes of unbalanced linguistic assessments [11]. We choose as t_{HGLS} the level of LH used in the representation algorithm which has associated the highest granularity label set (HGLS). We transform into t_{HGLS} the preferences of the experts for every feature of the objects expressed in S by means of the set of transformation functions LH between levels of LH presents in the subsection 2.2.

1. *Computing collective evaluations for each feature*:
   Once the unbalanced linguistic assessments are represented in t_{HGLS}, the rating process will compute a collective value for each feature, using an aggregation operator, AG_1, on the assessments provided by the experts represented in t_{HGLS}

   $$u_{jk} = AG_1(u^1_{jk}, ..., u^n_{jk})$$

2. *Computing a collective evaluation for each object*: the final aim of the rating process is to obtain a global evaluation, of each evaluated object according to all the experts and features that take part in the evaluation process. To do so, this process will aggregate the collective features values for each object using an aggregation operator, AG_2

   $$u_j = AG_2(u_{j1}, ..., u_{jh})$$

The aggregation operators,AG_1 and AG_2, will depend on each evaluation problem taking into account if all experts or features are equally important or there are experts or features more important than the others.

The aggregation result is expressed in $S^{n(t_{HGLS})}$. If we want to express the aggregation result expressed in S. This is achieved by applying the transformation function LH^{-1} to the results obtained by AG_2.

5. Conclusions

The sensory evaluation is an evaluation process in which the information provided by the experts involves uncertainty because it is acquired via human senses. Therefore this information usually is vague and uncertain, could be that experts need a greater level of distinction in one side of the evaluation scale than in the other one.In this contribution we have presented a sensory evaluation model that offers an unbalanced linguistic evaluation framework to the experts in order to offer a greater flexibility to express their knowledge in the evaluation process.

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References

[1] L.A. Zadeh. The concept of a linguistic variable and its applications to approximate reasoning. *Information Sciences, Part I, II, III*, 8,8,9:199–249,301–357,43–80, 1975.

[2] L. Martínez. Sensory evaluation based on linguistic decision analysis. *International Journal of Aproximated Reasoning*, 44 Num 2:148–164, 2007.

[3] G.B. Dijksterhuis. *Multivariate Data Analysis in Sensory and Consumer Science, Food and Nutrition*. Press Inc. Trumbull, Connecticut, USA, 1997.

[4] D. Ruan and X. Zeng (Eds.). *Sensory Evaluation: Methodologies and Applications*. Springer, 2004.

[5] H. Stone and J.L. Sidel. *Sensory Evaluation Practice*. Academic Press Inc., San Diego, CA, 1993.

[6] C.T. Chen. Applying linguistic decision-making method to deal with service quality evaluation problems. *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, 9(Suppl.):103–114, 2001.

[7] C.H. Cheng and Y. Lin. Evaluating the best main battle tank using fuzzy decision theory with linguistic criteria evaluation. *European Journal of Operational Research*, 142:174–186, 2002.

[8] L. Martínez, J. Liu, J.B. Yang, and F. Herrera. A multi-granular hierarchical linguistic model for design evaluation based on safety and cost analysis. *International Journal of Intelligent Systems.*, 20(12):1161–1194, 2005.

[9] R.T. Clemen. *Making Hard Decisions. An Introduction to Decision Analisys*. Duxbury Press, 1995.

[10] F. Herrera, E. Herrera-Viedama, and L. Martínez. A fuzzy linguistic methodology to deal with unbalanced linguistic term sets. *IEEE Transactions on Fuzzy Systems*, page In press, 2007.

[11] F. Herrera and L. Martínez. A model based on linguistic 2-tuples for dealing with multigranularity hierarchical linguistic contexts in multiexpert decision-making. *IEEE Transactions on Systems, Man and Cybernetics. Part B: Cybernetics*, 31(2):227–234, 2001.

[12] F. Herrera and L. Martínez. A 2-tuple fuzzy linguistic representation model for computing with words. *IEEE Transactions on Fuzzy Systems*, 8(6):746–752, 2000.

[13] F. Herrera and L. Martínez. The 2-tuple linguistic computational model. Advantages of its linguistic description, accuracy and consistency. *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, 9(Suppl.):33–49, 2001.

[14] F. Herrera, E. Herrera-Viedma, and L. Martínez. A hierarchical ordinal model for managing unbalanced linguistic term sets based on the linguistic 2-tuple model. In *Proceedings of the Eurofuse Workshop on Preference Modelling and Applications*, pages 201–206, Granada (Spain), April 2001.