Extended Linguistic Hierarchies for dealing with Multi-granular Contexts in Decision Making

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In decision making problems dealing with linguistic information and multiple sources of information it may happen that the sources have different degree of knowledge about the problem then they provide their information in different linguistic term sets defining a multigranular linguistic context. Different approaches have dealt with this type of information that present different limitations. In this contribution we extend the structure of Linguistic Hierarchies in order to improve and make more flexible the management of multigranular linguistic information in Decision Making problems.

Keywords: Linguistic variables, multigranularity linguistic contexts, linguistic hierarchies, decision-making

1. Introduction

Decision Making (DM) problems can present quantitative or qualitative aspects. Those problems that present quantitative aspects can be assessed by means of precise numerical values, on the other hand, where the aspects are qualitative, the use of the fuzzy linguistic approach[8] has obtained successful results in problems of different areas dealing with qualitative aspects.[1,2] An important concept dealing with linguistic information is the *granularity of uncertainty* that indicates the cardinality of the linguistic term set used to assess the linguistic variables. So in DM with qualitative information may happen that the experts, involved in the problem, have different degree of knowledge, then it seems suitable that each expert could express their preferences in different linguistic term sets based on their own knowledge.

In the literature different approaches[3,5,6] have been developed to deal with Multi-Granular Linguistic Information ($MGLI$). Such approaches conduct the multigranular linguistic information in an unique linguistic term set in order to accomplish computing with words processes. The approaches presented in [3,5] produce loss of information and hence a lack of precision in the final results. To improve these approaches, another approach based on linguistic hierarchies (LH) was presented in [7] that guarantees precision in the CW (computing with words) processes. Although the use of LH improves the precision of the CW processes but it presents some limitations to deal with any linguistic term set. In this contribution, we propose an Extension of the Linguistic Hierarchies (ELH) to deal with $MGLI$ without loss of information and with contexts where any linguistic term set can be used.

In order to do that, the contribution is structured as follows. Section 2 reviews the fuzzy linguistic approach, the 2-tuple linguistic representation model and the Linguistic Hierarchies, Section 3 presents an Extension of Linguistic Hierarchies and finally in Section 4 we shall point out some concluding remarks.

2. Linguistic Background

Here we review briefly some concepts to understand our proposal to improve the management of multigranular linguistic information in DM problems.

2.1. *2-tuple Linguistic Representation Model*

The 2-tuple fuzzy linguistic representation model represents the linguistic information by means of a 2-tuple, (s, α), where s is a linguistic label and α is a numerical value that represents the value of the symbolic translation.

Definition 1.[4] *Let β be the result of an aggregation of the indices of a set of labels assessed in a linguistic term set S, i.e., the result of a symbolic aggregation operation. $\beta \in [0, g]$, being $g + 1$ the cardinality of S. Let $i = round(\beta)$ and $\alpha = \beta - i$ be two values, such that, $i \in [0, g]$ and $\alpha \in [-.5, .5)$ then α is called a Symbolic Translation.*

Based on the symbolic translation concept, a linguistic representation model which represents the linguistic information by means of 2-tuples (s_i, α), $s_i \in S$ and $\alpha \in [-.5, .5)$ was developed in [4]. This model defines a set of transformation functions between numerical values and 2-tuples (Δ) and viceversa Δ^{-1} and has associated a computational model (see [4] for further detail). This model is the computational base of the LH.

2.2. *Linguistic Hierarchies*

The hierarchical linguistic structure was introduced in [7] in order to improve the precision in the processes of CW in linguistic multi-granular contexts. Our aim in this contribution, is to extend the Linguistic Hierarchies to facilitate the management of any linguistic term set in such structure.

A *linguistic hierarchy* is a set of levels, where each level is a linguistic term set with different granularity from those of the remaining levels of the hierarchy. Each level belonging to a linguistic hierarchy is denoted as $\mathbf{l}(\mathbf{t}, \mathbf{n}(\mathbf{t}))$, t indicates the level of the hierarchy and *n(t)* indicates the granularity of the linguistic term set of the level t.

We assume levels containing linguistic terms whose membership functions are triangular-shaped, symmetrical and uniformly distributed in $[0, 1]$. In addition, the linguistic term sets have an odd number of elements.

The levels belonging to a linguistic hierarchy are ordered according to their granularity. We define a linguistic hierarchy, LH, as the union of all levels t: $LH = \bigcup_t l(t, n(t))$ We are going to review the methodology to build linguistic hierarchies under a set of rules and conditions and its computational model presented in [7].

2.2.1. *Building Linguistic Hierarchies*

To build a linguistic hierarchy, we must take into account that its hierarchical order is given by the increase of the granularity of the linguistic term sets in each level.

We start from a linguistic term set, S, over the universe of the discourse U in the level t:$S = \{s_0, ..., s_{n(t)-1}\}$ being $s_k, (k = 0, ..., n(t)-1)$ a linguistic term of S, in LH, S is noted as, $S^{n(t)}$, where the granularity of uncertainty of $S^{n(t)}$ is $n(t)$: $S^{n(t)} = \{s_0^{n(t)}, ..., s_{n(t)-1}^{n(t)}\}$. A LH should satisfy the following rules, that we call, *linguistic hierarchy basic rules*:

(1) To preserve all *former modal points* of the membership functions of each linguistic term from one level to the following one.
(2) To make *smooth transitions between successive levels*. The aim is to build a new linguistic term set, $S^{n(t+1)}$. A new linguistic term will be added between each pair of terms belonging to the term set of the previous level t. To carry out this insertion, we shall reduce the support of the linguistic labels in order to keep place for the new one located in the middle of them.

Generally, we can say that the linguistic term set of level $t+1$ is obtained from its predecessor as: $l(t, n(t)) \rightarrow l(t+1, 2 \cdot n(t) - 1)$. As we can see with this way of building LH generates some limitations in the term sets that can belong to the hierarchies (see Table 3.1)

2.2.2. *Transformation Functions between Levels of a Linguistic Hierarchy*

In [7], a computational model was introduced to deal with LH in a precise way. Such model unifies the MGLI into an any linguistic term set of the LH by means of a transformation function between labels from the level t and a label in level $t' = t + \alpha$, with $\alpha \in Z$.

$$TF^t_{t'} : S^t \times [-.5, .5) \longrightarrow S^{t'} \times [-.5, .5)$$

$$TF^t_{t'}(s_i^{n(t)}, \alpha^{n(t)}) = \Delta(\frac{\Delta^{-1}(s_i^{n(t)}, \alpha^{n(t)}) \cdot (n(t') - 1)}{n(t) - 1}) \tag{1}$$

The transformation function, $TF^t_{t'}$, between linguistic terms in different levels of the linguistic hierarchy, LH, is bijective (see [7]).

3. Building Extended Linguistic Hierarchies

We have observed in DM problems that the experts require the use of multigranular context with terms sets of 5 and 7 or 7 and 9 labels. In this section, we are going to develop a new methodology to build linguistic hierarchies and its computational model in order to manage this type of context.

3.1. *Extended Building Linguistic Hierarchies*

Our proposal of extended linguistic hierarchies (ELH) will keep its hierarchical based on the increasing value of the granularity of the linguistic term sets in each level.

To build an ELH first, it will be fixed the term sets, $t, t+1, ...$, we need to deal with them in our problem, such that $n(t) < n(t+1)$.... This process does not follow the building rules of LH. However to build an ELH once all necessary term sets have been fixed, our process will add a new term set with the largest granularity t_{LG}, such that $n(t_{LG}) > n(t)$, $\forall t < t_{LG}$, $t \in ELH$. This term set, $S^{n(t_{LG})}$ must keep all the modal points of the other term sets, $S^{n(t)}$. To do so, its granularity is computed as: $n(t_{LG}) = (\prod_t n(t) - 1) + 1$

	LH	LH
	$l(t, n(t))$	$l(t, n(t))$
$t = 1$	$l(1, 3)$	$l(1, 7)$
$t = 2$	$l(2, 5)$	$l(2, 13)$
$t = 3$	$l(3, 9)$	

Table 3.1. Linguistic Hierarchies

	ELH	ELH
	$l(t, n(t))$	$l(t, n(t))$
$t = 1$	$l(1, 5)$	$l(1, 7)$
$t = 2$	$l(2, 7)$	$l(2, 9)$
t_{GL}	$l(3, 25)$	$l(3, 49)$

Table 3.2. Extended LH

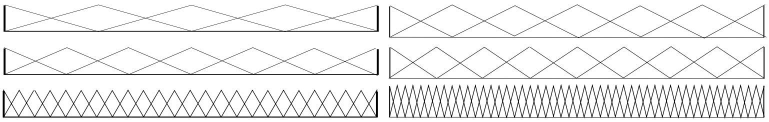

Fig. 1. *ELH* of 5, 7 and 25 labels and *ELH* of 7, 9 and 49 labels

Table 3.2 shows examples of ELH with term sets with 5 and 7, and 7 and 9 terms (graphically Figure 1). We can observe that the last level (t_{LG}) contains all the former modal points of the membership functions of each linguistic term, such as we imposed in the building process.

3.2. *Computational Model*

Now, we need to develop a computational model to accomplish CW without loss of information in ELH. We will use the linguistic 2-tuple computational model and the transformation functions, TF, designed for linguistic hierarchies. But in ELH the information cannot be unified in any level because the level chosen must keep modal points. So for ELH the transformation function between levels is based on eq. (1), but in this case one of the levels (t or t') must be t_{LG}, this way guarantees the transformations between any level and the level t_{LG} (and vice versa) of a ELH are carried out without loss of information.

Then the computational processes are carried out as follows:

- First, information should be unified in t_{LG}: the linguistic terms, $s_i^{n(t)}$, are transformed into the level LG of the ELH.

$$(s_j^{n(t)}, \alpha) \Rightarrow TF_{t_{LG}}^{t}(s_j^{n(t)}, \alpha) = (s_k^{n(t_{LG})}, \alpha'),$$

  The 2-tuple computational model is used to make the processes of CW over the linguistic 2-tuples expressed in the term set, $S^{n(t_{LG})}$. The results are expressed by linguistic 2-tuples in the level LG.

- Once the results have been obtained in the term set, $S^{n(t_{LG}}$, by means of linguistic 2-tuples, can be transformed in the initial linguistic term set, $S^{n(t)}$, by means of the transformations:

$$TF_t^{t_{LG}}(s_f^{n(t_{LG})}, \alpha_f) = (s_k^{n(t)}, \alpha),$$

4. Concluding Remarks

We have introduced an Extension of the Linguistic Hierarchies[7] in order to improve and make flexible the management of multigranular context.The ELH provides a computational model to accomplish computing processes without loss of information.

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